Collimation Depths for the TESLA Interaction Region

O. Napoly
CEA/Saclay, DAPNIA-SEA

11th October 1999

Abstract

The formalism for the rectangular beam collimation required by the clearance of the synchrotron radiation emitted in low beta quadrupoles is developed and applied to the case of the TESLA final focus system.

1 Collimation depths for uncoupled collimation and optics

We assume that collimation is provided by a pair of horizontal collimators with full gap ($g_x^{(i)}, i = 1,2$) and a pair of vertical collimators ($g_y^{(i)}, i = 1,2$) in such a way that both transverse planes and both phases are covered. The collimator gaps can be normalized with respect to the RMS beam sizes at the collimator

$$g_{x,y}^{(i)} = 2(N_{x,y}^{(i)} \cdot \sigma_{x,y}^{(i)})$$

The aim of the paper to determine the required values of the *collimation depths* $N_{x,y}^{(i)}$ such that the synchrotron radiation emitted by the collimated beam in the final doublet (or low-beta quadrupoles) is cleared by a given aperture in front of the doublet. This aperture can be the bore of the doublet itself, or the one of the opposite doublet, inboard or outboard of the IP, or some mask aperture.

Having considered an $x - y$ uncoupled geometry of the collimators, in contrast with circular collimators for instance, and assuming an uncoupled beam line optics in the beam delivery system (BDS), the tranverse phase space distribution of the collimated beam at a given position s along the beam line, is $x - y$ decoupled :

$$\rho_\perp(X, Y; s) = \rho_H(X; s) \cdot \rho_V(Y; s)$$

with $X = (x, x')$ and $Y = (y, y')$. Also important is the collimated transverse phase space *volume* which, for an uncoupled distribution, can be written as the cartesian product of the horizontal and vertical volumes :

$$V_\perp(s) = V_H(s) \times V_V(s)$$

To evaluate the impact of synchrotron radiation in the final doublet quadrupoles, we need first to characterize the phase space volume $V_\perp(s)$ for all positions s along the doublet quadrupoles where photons are emitted, and then propagate it, as if filled up by the photons, along a drift space through the relevant aperture. We will do this by transporting, using electron optics, the phase space volume to the interaction point (IP), used as a reference point, and then backward to doublet.

2 Phase space volume of the collimated beam for linear optics

The derivation of the phase space volume $V_\perp(s)$ is a decoupled problem and we therefore work it out in one plane, say the horizontal one. The horizontal volume $V_H(s)$ at any position s can simply be mapped from the acceptance of the two horizontal collimators :

$$V_H(s) = \left\{ X \mid x^2(s^{(1)}) < (g_x^{(1)}/2)^2 \text{ and } x^2(s^{(2)}) < (g_x^{(2)}/2)^2 \right\}$$

with

$$x(s^{(i)}) = \left(M^{(i)}(s)\right)^{-1}_{11}(X)$$

where $M^{(i)}(s)$ is the 2×2 map from the collimator i to the position s. At the linear order, the map $M^{(i)}(s)$ is given by the transfer matrix $R^{(i)}(s)$ from the horizontal collimator i to the position s, and one can write :

$$x(s^{(i)}) = \left(R^{(i)}(s)\right)^{-1}_{11}(X) = R^{(i)}_{22}(s)x - R^{(i)}_{12}(s)x'$$

As a consequence, the phase volume $V_H(s)$ is the intersection of the stripes contained between the two parallel and symmetrical lines defined by

$$R^{(i)}_{22}(s)x - R^{(i)}_{12}(s)x' = \pm g_x^{(i)}/2$$

for each collimator i. In the case of two collimators, $V_H(s)$ is in general a trapezoid. With n collimators, it is a $2n$-polygon with parallel opposite sides. These polygones are completely defined by the position of their corners which can be transported along the beam lines. In the doublet, the non-linear aberrations are expected to be small and we will use the linear optics to determine the volume $V_H(s)$. Higher order or tracking studies must be done to validate this approximation.

As an intermediate step, we first derive the *linear* collimated phase space volume V_H^* at the IP. Denoting by $R^{*(i)}$ the transfer matrix from the collimator i to the IP, the collimated phase space volume at the IP is given by :

$$V_H^*(s) = \left\{ X \mid \left(R^{*(i)}_{22}x - R^{*(i)}_{12}x'\right)^2 < (g_x^{(i)}/2)^2,\ i = 1, 2 \right\}$$

It is the interior region of the trapezoid defined by the two pairs of parallel and symmetrical lines:

$$\begin{aligned} R^{*(1)}_{22}x - R^{*(1)}_{12}x' &= \pm g_x^{(1)}/2 \\ R^{*(2)}_{22}x - R^{*(2)}_{12}x' &= \pm g_x^{(2)}/2 \end{aligned}$$

Although it is independent of the choice of the matched beam conditions in the BDS, this system can be conveniently expressed in terms of the nominal Twiss parameters such that $\alpha_x^* = 0$ at the IP. Denoting by $\beta_x^{(i)}$the beta-function at the collimator i and by $\Delta\psi_x^{(i)}$ the phase advance from the collimator i to the IP, the system can be written as

$$\begin{aligned} x\sqrt{\frac{\beta_x^{(1)}}{\beta_x^*}}\cos\Delta\psi_x^{(1)} - x'\sqrt{\beta_x^{(1)}\beta_x^*}\sin\Delta\psi_x^{(1)} &= \pm N_x^{(1)}\sqrt{\beta_x^{(1)}\epsilon_x} \\ x\sqrt{\frac{\beta_x^{(2)}}{\beta_x^*}}\cos\Delta\psi_x^{(2)} - x'\sqrt{\beta_x^{(2)}\beta_x^*}\sin\Delta\psi_x^{(2)} &= \pm N_x^{(2)}\sqrt{\beta_x^{(2)}\epsilon_x} \end{aligned}$$

which can easily simplified into

$$\begin{aligned} x\cos\Delta\psi_x^{(1)} - x'\beta_x^*\sin\Delta\psi_x^{(1)} &= \pm N_x^{(1)}\sqrt{\beta_x^*\epsilon_x} \\ x\cos\Delta\psi_x^{(2)} - x'\beta_x^*\sin\Delta\psi_x^{(2)} &= \pm N_x^{(2)}\sqrt{\beta_x^*\epsilon_x} \end{aligned}$$

In general this system of equations defines a *finite domain*. Only if the phase advances from collimators 1 and 2 to the IP are exactly identical (single phase collimation) is the determinant of the above system zero

$$\Delta = \beta_x^*\sin\left(\Delta\psi_x^{(1)} - \Delta\psi_x^{(2)}\right)$$

and the domain an *infinite* stripe between the two narrower parallel lines. Otherwise, the most important points are the corners of the trapezoid : since the trapezoid shape is invariant under charged (electrons) or neutral (photons) beam linear transport, the corners define the maximum extent of the beam and of the synchrotron radiation fan. The four corners are of course symmetric with respect to the origin. The two independent ones are given by

$$X_c^{*(\pm)} = \begin{pmatrix} \frac{N_x^{(2)} \sin \Delta\psi_x^{(1)} \pm N_x^{(1)} \sin \Delta\psi_x^{(2)}}{\sin\left(\Delta\psi_x^{(1)} - \Delta\psi_x^{(2)}\right)} \sqrt{\beta_x^* \epsilon_x} \\ \frac{N_x^{(2)} \cos \Delta\psi_x^{(1)} \pm N_x^{(1)} \cos \Delta\psi_x^{(2)}}{\sin\left(\Delta\psi_x^{(1)} - \Delta\psi_x^{(2)}\right)} \sqrt{\gamma_x^* \epsilon_x} \end{pmatrix}$$

Of particular interest is the case when collimator 1 is at the doublet phase, i.e. $\pi/2$ from the IP, and collimator 2 at the IP phase. In this case, collimator 1 intercepts the most dangerous particles with a large offset in the doublet, while collimator 2 limits their angles in the doublet. The coordinates of the corners simplify to

$$X_c^{*(\pm)} = \begin{pmatrix} +N_x^{(2)} \sqrt{\beta_x^* \epsilon_x} \\ \pm N_x^{(1)} \sqrt{\gamma_x^* \epsilon_x} \end{pmatrix}$$

This implies that the trapezoid bounding V_H^* is a rectangle, as expected from the choice of the phase advances.

3 Phase space volume of the synchrotron radiation

The corners of the phase space volume $V_H(s)$ at the location s in the doublet are then given by

$$X_c^{\pm}(s) = R^{-1}(s^*, s) \cdot X_c^{*(\pm)}$$

where s^* is the position of the IP and $R(s*, s)$ is the transfer matrix from s to the IP. At this position, synchrotron radiation fills up the volume with photons which are emitted at the same position and, neglecting the small $1/\gamma$ angular opening of the cone of emission, with the same angle in such a way that the photon phase volume coincides with $V_H(s)$.

The last step of the calculation consists in forward propagating the phase volume of the photons emitted at position s to the relevant aperture at the position L from the IP ($L > 0$ if downstream of the IP). This can be done by transporting the corners of $V_H(s)$ through a simple drift space of length $(s^* + L - s)$. The corners of the photon phase volume $V_H^\gamma(s, L)$ in this aperture are therefore given by

$$X_c^{\gamma\pm}(s, L) = D((s^* + L - s) \cdot R^{-1}(s^*, s) \cdot X_c^{*(\pm)}$$

where $D(l)$ is the transfer matrix of a drift with length l.

For a circular aperture of radius r, the transverse collimation depths are defined by the quadratic condition in $N_x^{(i)}$ and $N_y^{(i)}$

$$\sup_{(\pm)} \left(x_c^{\gamma\pm}(s, L)\right)^2 + \sup_{(\pm)} \left(y_c^{\gamma\pm}(s, L)\right)^2 = r^2$$

for *each* position s inside the upstream doublet. Synchrotron radiation clearance is achieved when the collimation depths $N_x^{(i)}$ and $N_y^{(i)}$ belong to the four dimensional domain defined as the intersection of the interior of all the 4D ellipsoids associated to each emission position s by the above equation. The best values of $N_x^{(i)}$ and $N_y^{(i)}$ are of course given by the 3D surface which bounds this domain. A large arbitrariness is still left in the choice of the collimation depths, and one lacks a criterion to define the *largest* set of these.

4 Symmetric collimation in a circular aperture

The problem reduces to a 2D analysis in the case of symmetric collimation where both phases are collimated at the same depths, $N_{x,y} \equiv N_{x,y}^{(1)} = N_{x,y}^{(2)}$. The coordinates $X_c^{\gamma\pm}$ and $Y_c^{\gamma\pm}$ of the phase volume corners are then simply proportional to N_x and N_y and the solution reduces to the following ellipse equation

$$\left(\frac{N_x}{N_x^{(0)}(s)}\right)^2 + \left(\frac{N_y}{N_y^{(0)}(s)}\right)^2 = 2$$

where the collimation depths $N_x^{(0)}(s)$ and $N_y^{(0)}(s)$ are defined by

$$\begin{aligned}\sup_{(\pm)} \left(x_c^{\gamma\pm}(s,L)\right)^2 &= r/\sqrt{2} \\ \sup_{(\pm)} \left(y_c^{\gamma\pm}(s,L)\right)^2 &= r/\sqrt{2}\end{aligned}$$

If $N_{x,y} = N_{x,y}^{(0)}(s)$, the synchrotron radiation fan emitted at position s is a square in the transverse xy-plane inscribed in the circular aperture. The values of $N_{x,y}^{(0)}(s)$ can be easily calculated and their minimum found numerically by discretising the final doublet into many slices.

5 Application to the TESLA Interaction Region.

The ellipses in the two dimensional plane (N_x, N_y) are plotted in Fig.1 for the case of the TESLA doublet with the high luminosity parameters[1]. The values of the *square* collimation depths $(N_x^{(0)}, N_y^{(0)})$ are also shown as stars. Three different apertures are considered[2]: the exit of the opposite doublet ($r = 24$ mm), the exit of the opposite mask ($r = 9$ mm) and the beam pipe at the vertex ($r = 11$ mm). From this plot, the proper choice of the necessary *square* collimation depths is easy to derive by considering the smallest of the allowed values. The resulting synchrotron radiation fan is shown in Fig.2.

Aknowlegdments

I am indebted to R. Brinkmann, P. Emma and N. Walker for many clarifying discussions.

References

[1] Reinhard Brinkmann. High luminosity with TESLA 500. Technical Report TESLA 97-13, DESY, 1997.

[2] O. Napoly, I. Reyzl, and N. Tesch. Interaction region layout, feedback and background issues for TESLA. In *International Workshop on Linear Colliders*, 1999. LCWS99, Sitges, Spain.

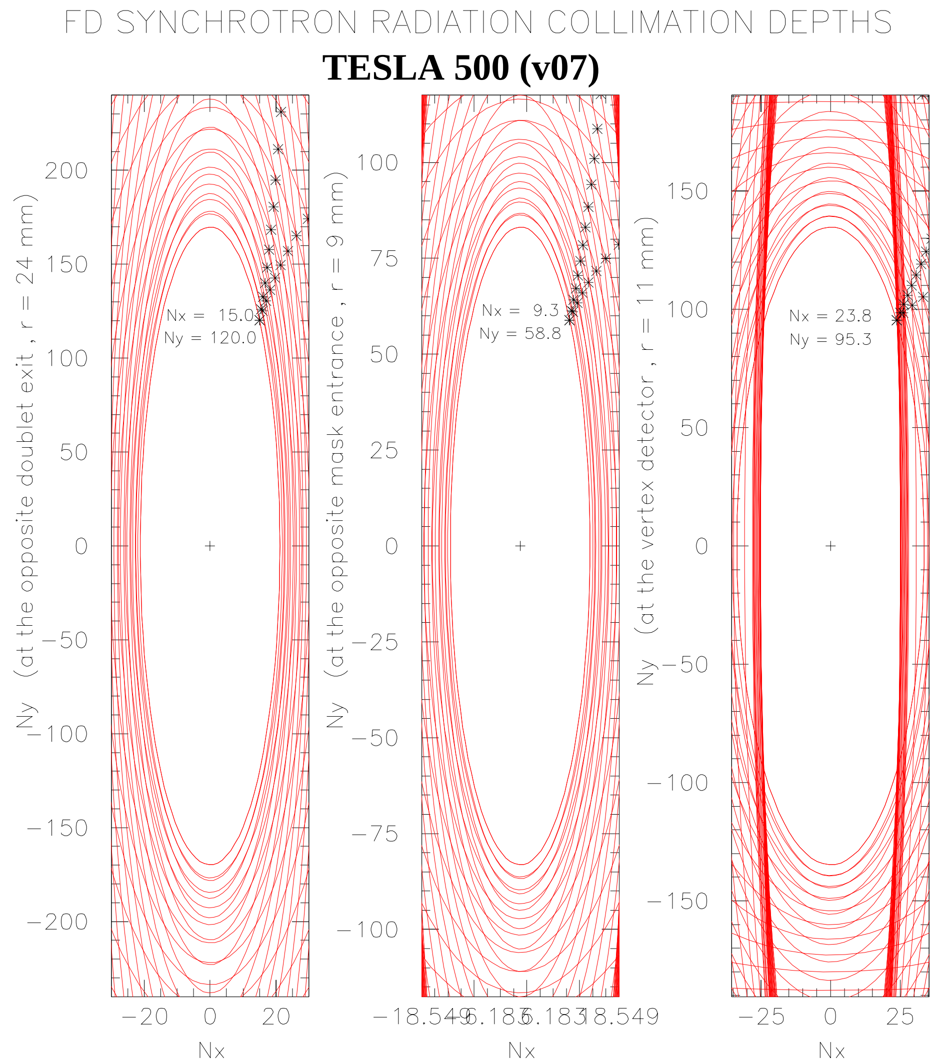

Figure 1: Allowed domain of collimation depths (N_x, N_y). Ellipses are drawn as a function of the position s along the final doublet.

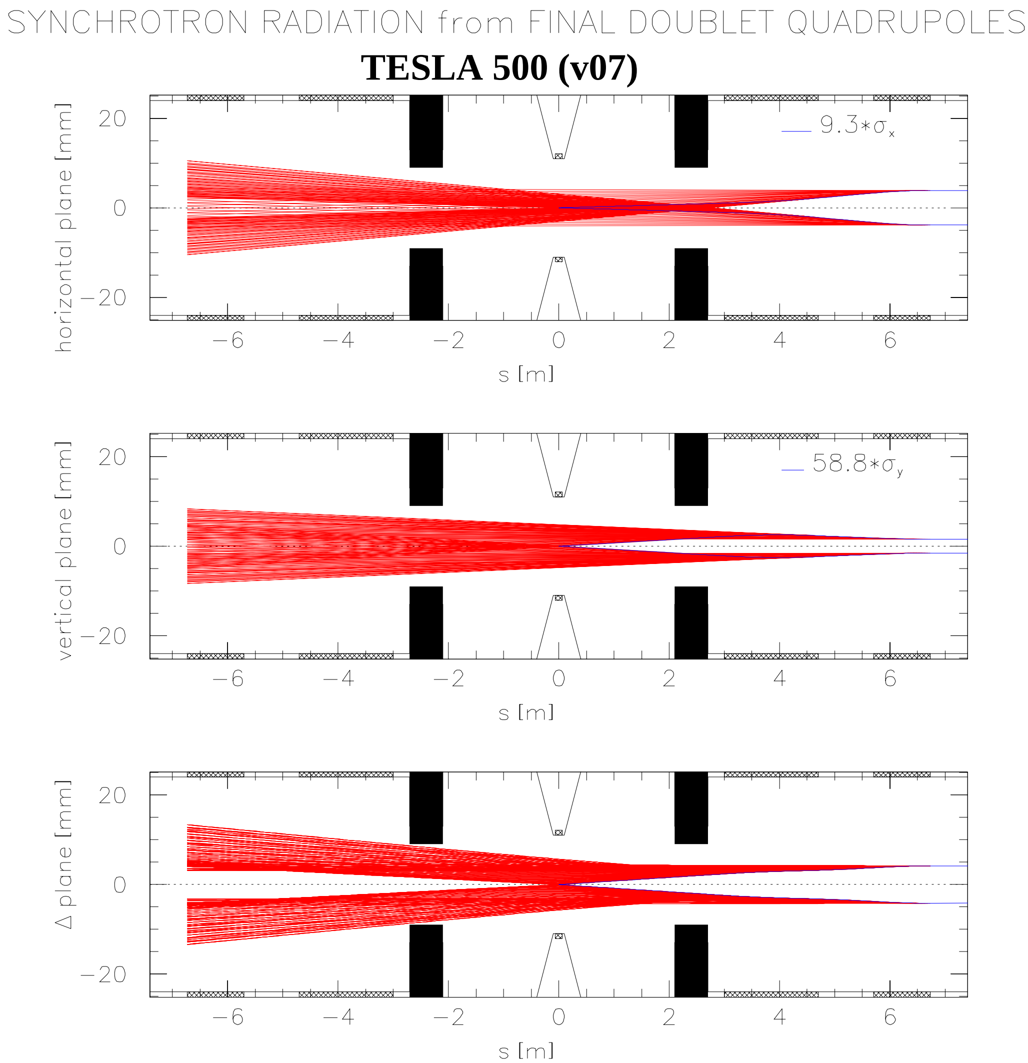

Figure 2: Synchrotron radiation fan through the TESLA IR. The bottom plot shows the diagonal extension of the corners of the photon phase space.