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# Reaction mechanisms in two-pion photoproduction on the proton: I- Meson exchange picture.

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## Abstract

The recent **availability** of more precise results for the  $\gamma p \rightarrow p\pi^+\pi^-$  cross section, **as** well as the first ever **measurements** of the  $\gamma p \rightarrow p\pi^0\pi^0$ , and  $\gamma p \rightarrow n\pi^+\pi^0$  channels **calls** for a revival and **an** extension of the effective Lagrangians model of Lücke and Söding. The updated model we present here not only accounts for the contribution of the  $\Delta(1230)$  Born terms, corrected for absorptive effects, but also explicitly considers the formation of the dominant baryonic resonances in the **s-channel** **as** well as in the Born terms. The model makes use of no free parameters. While it reproduces well the  $p\pi^+\pi^-$  and  $p\pi^0\pi^0$  cross sections, its failure to account for half of the cross-section of the  $n\pi^+\pi^0$  channel, recently determined at Mainz, is perplexing and call for further improvements. Finally, we witness, in the  $\pi^0\pi^0$  channel, the first direct evidence of the excitation of the  $P_{11}$  resonance by real photons. The extension of the model in the CEBAF energy range is also discussed.

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# 1 Introduction

Due to experimental factors, the study of double pion photoproduction reaction mechanisms on nucleons has historically been limited to the study of the  $p\pi^+\pi^-$  channel. The two principal experiments relating to this subject, each performed on bubble chambers using bremsstrahlung beams [1, 2], relied on the detection of at least two charged particles to positively identify this reaction. Those involving more than one neutral particles in the final state, such as the other two possible channels,  $p\pi^0\pi^0$  and  $n\pi^+\pi^0$ , were then deemed to be unmeasurable. Furthermore, the poor statistics characteristic of such experiments only allowed for the determination of the general trends of these processes.

It is only recently, with the advent of the high-precision tagged photon beam located at the 855 MeV CW electron accelerator MAMI-B, in Mainz, combined with the  $4\pi$  detector Daphne, that more complete measurements have become possible. By not only providing better statistics, but also by the ability to detect neutral particles, a new set of constraints is available to add to our understanding of the mechanisms involved [3]. CEBAF, additional, will soon provide the opportunity to extend these studies to higher energies.

On the theoretical front, Lücke and Söding attempted to reproduce the available  $p\pi^+\pi^-$  data within the framework of an effective Lagrangians model [4]. The graphs which they calculated are depicted in fig.1-I, Ha, and IIIa. It was then clearly shown that the dominant contribution was essentially due to the presence of an intermediate  $\Delta^{++}\pi^-$  state. The model also accounted for unitary absorptive corrections, which are due to the coupling to other open channels. It was also shown that the formation in the s-channel (Ha) of the  $D_{13}(1520)$  resonance strongly interferes with the contact term (Ia), as both lead to an S-wave  $\Delta^{++}\pi^-$  final state. More recently Gomez-Tejedor and Oset [5] added many more nucleon Born diagrams, whose total contribution, however, is small. Their not accounting for unitary absorptive corrections, unfortunately, prevented them from achieving a good energy dependence, especially above 700 MeV.

We have extended the pioneering work of Lücke and Söding and considered every diagram depicted on figure 1. As such, we have explicitly included the contribution of every baryonic resonances known to decay appreciably into the double pion channel [6] and which is strongly coupled to the  $\gamma N$  channel [7]. This is meant to include decays into the usual  $\Delta\pi$  channel, as well as decays into nucleon-meson systems. Since we are primarily interested here in the Mainz energy range, we consider only the  $P_{11}(1440)$ ,  $D_{13}(1520)$  and  $D_{33}(1700)$  resonances. Some, such as the  $S_{11}(1535)$ , decay almost exclusively in the  $\pi N$  and  $\eta N$  channels and others, as the  $P_{33}(1600)$  or  $S_{31}(1620)$ , have vanishing radiative decay widths and need not be added into our analysis. We have not considered the contribution of the  $F_{15}(1700)$ , which mainly decays in the  $N\rho$  channel ; it will be considered elsewhere. Two new features of our

Resonance	Mass in MeV/c <sup>2</sup>	Central Mass Widths in MeV					
		Full	Partial				
		r	$\Gamma_{\pi\Delta}$	$\Gamma_{\gamma\pi}$	$\Gamma_{\rho N}$	$\Gamma_{\sigma N}$	$\Gamma_{\pi N}$
D <sub>13</sub>	1520.	120.	24.	.3	24.	0.	72.
D <sub>33</sub>	1700.	300.	150.	.9	24.	0.	81.
P <sub>11</sub>	1440.	350.	88.	.3	0.	43.	220.

Table 1: Parameters of the resonances used in the model.

model are the Born terms (fig.1-IV) associated to each baryonic resonance and the  $\rho$  contact term which contributes only in the  $n\pi^0\pi^+$  channel.

It is the philosophy of this article to fix the numerical values of each of the parameters associated with the resonances, listed in table 1, to those found in the literature [6, 7, 8]. We stress, therefore, that we rely on no free parameters to fit the model to the available data.

## 2 The Model

### 2.1 Notations and cross-sections

We adhere to the convention defined by Bjorken and Drell and use their normalization factors, as well as set  $\hbar = c = 1$ . We introduce now the notation which will be found throughout this article. Every kinematical quantity is calculated in the laboratory frame unless otherwise specified. Stated quantities are evaluated in the rest frame of the  $\pi N$  resonant state, be it a  $\Delta$ , a D<sub>13</sub>, a D<sub>33</sub>, or a P<sub>11</sub>.  $\vec{k}$ , and  $\vec{\epsilon}$  are respectively the momentum and the polarization of the photon.  $(\mu_{1,2}^0, \vec{\mu}_{1,2})$  are the 4-vector momenta of the two pions according to the convention shown in the diagrams of figure 1.  $E_{i,f}$ , and  $m_{i,f}$  are the energy and masses of the target and outgoing nucleon, and  $\vec{p}_f$  stands for the momentum of the outgoing nucleon.  $m_\pi$  is the mass of the pion.  $\vec{S}$  is the spin 3/2 to spin 1/2 transition operator.  $Q$  is the total pion-nucleon (A) energy in the barycentric system, defined as  $Q^2 = (p_\Delta^0)^2 - |\vec{p}_\Delta|^2$ .  $\chi_i$  and  $\chi_f$  are the magnetic quantum numbers of the nucleon in the initial and the final state.  $M_X$ , and  $\Gamma_X$  stand for the mass and the full width of the X-resonance.  $W$  is the total available energy of the system. Finally,  $\sqrt{s}$  is the invariant mass of the  $\pi\pi$  system, while  $m_\sigma$ , and  $m_\rho$  are the on-shell masses of the  $\sigma$  and  $\rho$  mesons. The parameters describing these mesons are listed in table 2

The complete expression describing the cross-section is:

$$d\sigma = (2\pi)^4 \delta^4(P_i - P_f) \left( \frac{1}{2k} \right) \left( \frac{m_i}{E_i} \right) \left( \frac{1}{2\mu_1^0} \right) \left( \frac{1}{2\mu_2^0} \right) \left( \frac{m_f}{E_f} \right)$$

Meson	Mass in MeV/c <sup>2</sup>	Full width in MeV
$P$	770.	151.
$\sigma$	800.	800.

Table 2: Parameters of the mesons used in the model

Graph		$p\pi^+\pi^-$		$n\pi^0\pi^+$		$p\pi^0\pi^0$
		$(p\pi^+)\pi^-$	$(p\pi^-)\pi^+$	$(n\pi^0)\pi^+$	$(n\pi^+)\pi^0$	$(p\pi^0)\pi^0$
Born $\Delta$	$C_\pi$	-1	$\frac{1}{3}$	$\frac{\sqrt{2}}{3}$	0.	0.
$D_{13} \rightarrow \pi\Delta$	$C_D$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{\sqrt{2}}{3}$
$P_{11} \rightarrow \pi\Delta$	$C_D$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{\sqrt{2}}{3}$
$D_{13} \rightarrow \rho N$	$C_\rho$	$-\frac{1}{\sqrt{6}}$	0.	$\frac{1}{\sqrt{3}}$	0.	0.
$P_{11} \rightarrow \sigma N$	$C_\sigma$	$\frac{1}{\sqrt{3}}$	0.	0.	0.	$-\frac{1}{\sqrt{3}}$
$D_{33} \rightarrow \pi\Delta$	$C_D$	$-\sqrt{\frac{2}{5}}$	$\sqrt{\frac{8}{45}}$	$\frac{4}{3\sqrt{5}}$	$-\frac{1}{3\sqrt{5}}$	$\sqrt{\frac{2}{45}}$
Born $P_{11}$	$C_B$	0.	2	$-\sqrt{2}$	0.	0.
Born $D_{13}$	$C_B$	0.	2	$-\sqrt{2}$	0.	0.
$\rho$ contact	$C'_\rho$	0.	0.	1.	0.	0.
$\rho$ diffractive	$C''_\rho$	1.	0.	0.	0.	0.

Table 3: Isospin coefficients

$$\times \frac{1}{(2\pi)^9} d\vec{\mu}_1 d\vec{\mu}_2 d\vec{p} \frac{1}{2} \frac{1}{2} \sum_{\text{spins}} |\mathcal{T}_{FI}(\vec{\mu}_1, \vec{\mu}_2) + \mathcal{T}_{FI}(\vec{\mu}_2, \vec{\mu}_1)|^2 \quad (1)$$

where the two  $\frac{1}{2}$  factors are the averages taken over the spin of the nucleon and the photon polarization. Integrating over the momenta of each undetected particle lead to the various differential and integrated cross-sections.  $\mathcal{T}_{FI}$  is the coherent sum of the matrix elements describing the various reaction channels, which we will now write out explicitly.

## 2.2 The $\pi\Delta$ Born terms

The diagrams corresponding to the  $\pi\Delta$  Born terms are depicted on fig.1-I. The contact diagram leads solely to a S-wave  $\Delta\pi$  final state. A rapid rise in cross-section as a function of energy is generally considered to be a direct manifestation of this term. The pion exchange term, on the other hand, leads to S, P and higher orbital angular momenta in the final state, and rises more smoothly. The last two terms, though strictly required to insure gauge invariance, are numerically unimportant in our energy range [4]. In the following discussion we will only concentrate on the contact and the pion exchange terms and ignore any possible contribution from the

other two diagrams.

The matrix element, subject to the above conditions, then takes the form [9]:

$$T(\vec{\mu}_1, \vec{\mu}_2) = -eC_\pi G_3^2 \frac{2Q}{Q^2 - M_\Delta^2 - iM_\Delta \Gamma_\Delta} \sqrt{\frac{E_i + m_i}{2m_i}} \sqrt{\frac{E_f + m_f}{2m_f}} \times \left\langle \chi_f \left| \vec{S} \cdot \vec{\mu}_2 \vec{S}^\dagger \cdot \left[ \frac{(\vec{\mu}_1^* - \vec{k}^*)(2\vec{\mu}_1 - \vec{k}) \cdot \vec{\epsilon}}{(\mu_1 - k)^2 - m_\pi^2} + \vec{\epsilon} \right] \right| \chi_i \right\rangle \quad (2)$$

where  $C_\pi$  reflects the isospin structure of the vertex. Its values, depending on the reaction, are listed in table 3.  $G_3$  is the  $\pi N \Delta$  coupling constant and  $\Gamma_\Delta$  is the energy dependent  $\Delta$  width, as defined in ref [10]. Note that the first term in the bracket corresponds to the pion exchange Born term while the second corresponds to the contact Born term.

### 2.3 The $D_{13} + \pi \Delta$ channel

The  $D_{13}$  resonance is excited both by Electric and Magnetic multiples. Their radiative widths are respectively  $\Gamma_\gamma^E = 0.315$  MeV and  $\Gamma_\gamma^M \simeq 0.210$  MeV [7]. They are related to the corresponding coupling constants by:

$$\Gamma_\gamma^E = \frac{1}{3\pi} e^2 G_\gamma^E \frac{m_i}{M_{D_{13}}} k$$

and

$$\Gamma_\gamma^M = \frac{3}{4\pi} e^2 G_\gamma^M \frac{m_i}{M_{D_{13}}(m_i + \sqrt{m_i^2 + k^2} + 3)^2}$$

Ultimately, the  $D_{13}$  decays appreciably into  $\pi \Delta$  S and D states. The corresponding partial widths are respectively [6]  $\Gamma_{\pi\Delta}^S \simeq 7 \pm 4$  MeV and  $\Gamma_{\pi\Delta}^D \simeq 18 \pm 5$  MeV, and are related to the corresponding coupling constants by:

$$\Gamma_{\pi\Delta}^S = \frac{1}{2\pi} \frac{M_\Delta}{M_{D_{13}}} G_\pi^{S^2} \mu_1$$

and

$$\Gamma_{\pi\Delta}^D = \frac{2}{3\pi} \frac{m_\pi^2 + \mu_1^2}{M_{D_{13}} M_\Delta} G_\pi^{D^2} \mu_1^5$$

The matrix elements, depicted in fig.1.IIa, describing the electric excitation of the  $D_{13}$  resonance related to its decay have the same structure and may be combined in the same compact form:

$$T = C_D e G_\gamma^E G_3 G_3^* \frac{2Q}{(Q^2 - M_\Delta^2 + iM_\Delta \Gamma_\Delta)} \frac{2W}{(W^2 - M_{D_{13}}^2 + iM_{D_{13}} \Gamma_{D_{13}})} \times$$

$$\sqrt{\frac{E_i + m_i}{2m_i}} \sqrt{\frac{E_f + m_f}{2m_f}} \left\langle \chi_f \left| \vec{S} \cdot \vec{\mu}_2 \vec{S}^\dagger \cdot \vec{\epsilon} \right| \chi_i \right\rangle \quad (3)$$

provided that

$$G_3^* = G_\pi^S + \frac{2}{3} G_\pi^D \mu_1^2 \frac{\sqrt{m_\pi^2 + \mu_1^2}}{M_\Delta}$$

The isospin coefficient  $C_D$  is listed in table 3.

It is worth pointing out that Eq. 3 clearly exhibits the same structure as the contact part of Eq. 2. This is the origin of the well known maximum interference between the  $D_{13}$  s-channel formation term and the Born contact term.

The matrix element corresponding to the magnetic excitation of the  $D_{13}$  resonance is obtained in the same way, but is not explicitly written here.

## 2.4 The $D_{13} \rightarrow N\rho$ channel

The  $D_{13}$  resonance has also been found [6] to decay in the  $N\rho$  channel. The matrix element describing this channel, as depicted on fig.1.IIb, is given by:

$$\begin{aligned} \mathcal{T} = & C_\rho g_{\rho\pi\pi} (eG_\gamma^E) g_\rho^* \sqrt{\frac{E_i + m_i}{2m_i}} \sqrt{\frac{E_f + m_f}{2m_f}} \times \\ & \frac{1}{M_{\pi\pi}^2 - m_\rho^2 + im_\rho \Gamma_\rho W^2 - M_{D_{13}}^2 + iM_{D_{13}} \Gamma_{D_{13}}} \left\langle \chi_f \left| \vec{S} \cdot (\vec{\mu}_1 - \vec{\mu}_2) \vec{S}^\dagger \cdot \vec{\epsilon} \right| \chi_i \right\rangle \end{aligned} \quad (4)$$

the isospin coefficient  $C_\rho$  is listed in table 3, and  $G_\gamma^E$  is the coupling constant at the  $\gamma p D_{13}$  vertex. The coupling constant  $g_{\rho\pi\pi}$  describing the decay of the  $\rho$  into two pions is related to its width  $\Gamma_\rho$  via

$$\Gamma_\rho = \frac{1}{12\pi} \frac{g_{\rho\pi\pi}^2}{m_\rho^2} k^3$$

The coupling constant  $g_\rho^*$  describing the decay of the  $D_{13}$  into the p-nucleon system is related to the corresponding width by

$$\Gamma_{\rho N} = \frac{(g_\rho^*)^2 (E_f + m)}{12\pi M_{D_{13}}} \rho(W)$$

$\rho(W)$  is the momentum of the p-meson weighted by its mass distribution, as per ref [6]. It takes the form:

$$\rho(W) = \int_{2m_\pi}^{W-m} \frac{q}{2\pi} \frac{\Gamma_\rho}{(Q - m_\rho)^2 + (\Gamma_\rho/2)^2} dQ$$

where  $Q$  stands here for the mass of the  $\pi\pi$  system, and  $q$  is its total momentum in the  $D_{13}$  rest frame.

## 2.5 The $D_{33}$ decay channels

The expression for the amplitudes of these channels is given by Eqs.3 and 4, with the relevant values of the mass and coupling constants listed in tables 1 and 3.

## 2.6 The $P_{11} \rightarrow \pi \Delta$ channel

The matrix element describing this channel, shown on fig.1-IIIa, with the intermediate P-wave  $\pi \Delta$  state, is written:

$$\mathcal{T} = \frac{ieG_\gamma^P G_\pi C_D 2W 2Q}{(W^2 - M_{P_{11}}^2 - iM_{P_{11}}\Gamma_{P_{11}})(Q^2 - M_\Delta^2 - iM_\Delta\Gamma_\Delta)} \times \sqrt{\frac{E_i + m_i}{2m_i}} \sqrt{\frac{E_f + m_f}{2m_f}} \langle \chi_f | \vec{S} \cdot \vec{\mu}_2^* \vec{S}^\dagger \cdot \vec{\mu}_1^* \vec{\sigma} \cdot \vec{k} \times \vec{\epsilon} | \chi_i \rangle \quad (5)$$

where  $C_D$  is the isospin constant as per eq. 3. We identify  $G_\pi$  as being related to the partial width  $\Gamma_\Delta$ , listed in table 1, by:

$$\Gamma_{\Delta\pi} = \frac{1}{\pi} G_\pi^2 \frac{M_\Delta}{M_{P_{11}}} (\mu_1)^3$$

and  $G_\gamma^P$  by:

$$\Gamma_{\gamma\pi} = \frac{e^2}{3\pi} (G_\gamma^P)^2 (k)^3 \frac{m}{M_{P_{11}}}$$

In order to reflect the relative phase of the experimental multiples [6], and partial waves [7],  $G_\gamma^P$  should be  $< 0$ . The total width  $\Gamma_{P_{11}}$  is taken with the appropriate energy dependence.

## 2.7 The $P_{11} \rightarrow N\sigma$ channel

The  $P_{11}$  resonance has also been found to decay into a  $N(\pi\pi)_{S=wave}^{I=0}$  mode [6]. We have accordingly parameterized this  $(\pi\pi)_{S=wave}^{I=0}$  state by the so-called  $\sigma$  particle, as shown on fig. 1-IIIb. The matrix element for this channel is written as:

$$\mathcal{T} = \frac{ieg_{\sigma\pi\pi} m_\sigma G_\gamma^P C_\sigma g_\sigma^* \sqrt{\frac{E_i + m_i}{2m_i}} \sqrt{\frac{E_f + m_f}{2m_f}} \times \frac{1}{2W} \frac{1}{M_{\pi\pi}^2 - m_\sigma^2 + im_\sigma\Gamma_\sigma W^2 - M_{P_{11}}^2 + iM_{P_{11}}\Gamma_{P_{11}}} \langle \chi_f | \vec{\sigma} \cdot \vec{k} \times \vec{\epsilon} | \chi_i \rangle \quad (6)$$

where  $G_\gamma^P$  describes the  $\gamma p P_{11}$  vertex.  $C_\sigma$  is the usual isospin coefficient. The coupling constant  $g_{\sigma\pi\pi}$  is related to the decay width of the  $\sigma$  into two pions

$$\Gamma_\sigma = \frac{g_{\sigma\pi\pi}^2}{4\sqrt{2}\pi} k$$

and  $g_\sigma^*$  is related to the decay width of the  $P_{11}$  into a  $\sigma$ -nucleon system

$$\Gamma_{N\sigma} = \frac{(g_\sigma^*)^2}{6\pi} \frac{m}{M_{P_{11}}} \rho(W)$$

## 2.8 The $\rho$ contact term

It is necessary to include  $p$  production Born mechanisms. To date, we have only included the contact term (fig.1-V). It contributes only in channels where a charged  $p$  can be produced. The matrix element is:

$$\mathcal{T} = eC'_\rho \frac{g_\rho(1+\kappa_\nu)}{2m} g_{\rho\pi\pi} \frac{\langle \chi_f | \vec{\sigma} \cdot \vec{\epsilon}_\gamma \times (\vec{\mu}_1 - \vec{\mu}_2) | \chi_i \rangle}{M_{\pi\pi}^2 - m_\rho^2 + im_\rho \Gamma_\rho} \sqrt{\frac{E_i + m_i}{2m_i}} \sqrt{\frac{E_f + m_f}{2m_f}}$$

where  $C'_\rho$  is the isospin coefficient, and, as found in ref [11],  $g_\rho^2/4\pi=0.40$ , and  $\kappa_\nu=6$ .

## 2.9 The diffractive $\rho$ production term

**In** our analysis, we have assumed that the  $p$  meson is diffractively produced via the exchange of the Pomeron, and subsequently decays into a  $\pi^+\pi^-$  pair. The corresponding amplitude is:

$$\mathcal{T} = C''_\rho g_{\rho\pi\pi} \left( \frac{q_\rho}{k} \right)_{c.m.} \frac{1}{M_{\pi\pi}^2 - m_\rho^2 + im_\rho \Gamma_\rho} \vec{J}_{\gamma p \rightarrow p\rho} \cdot (\vec{\mu}_1 - \vec{\mu}_2) \quad (7)$$

where  $\vec{J}_{\gamma p \rightarrow p\rho} \cdot \vec{\epsilon}_\rho$  is given by Eq. B.8 in Ref. [12]. Since this amplitude leads to a constant cross-section with varying photon energy, we've resorted to multiplying it by the  $\left( \frac{q_\rho}{k} \right)_{c.m.}$  factor so as to reproduce its proper threshold behavior.

### 2.10 The $\pi D_{13}$ Born term

As the model is extended to higher energies, we find it necessary to introduce terms of the  $\pi\Delta$  Born type, substituting the various resonances for the  $\Delta$  (fig.1-IV). Once again, we only calculate the contact, and pion exchange terms. We get:

$$\mathcal{T} = - \frac{2WeG_{D_{13}}^2 C_B}{W^2 - M_{D_{13}}^2 + iM_{D_{13}}\Gamma_{D_{13}}} \sqrt{\frac{E_i + m_i}{2m_i}} \sqrt{\frac{E_f + m_f}{2m_f}} \times$$

$$\begin{aligned}
& \langle \chi_f | \vec{\sigma} \cdot \left[ \frac{\vec{R}}{E_{D_{13}} + M_{D_{13}}} - \frac{\vec{p}_f}{E_f + m_f} \right] \vec{S} \cdot \vec{\mu}_2 \\
& \vec{S}^\dagger \cdot \left[ \vec{\epsilon} + \frac{\vec{R}(\vec{\epsilon} \cdot \vec{R})}{M_{D_{13}}(E_{D_{13}} + M_{D_{13}})} - \vec{q}^* \frac{(2\vec{\mu}_1 - \vec{k}) \cdot \vec{\epsilon}}{(q^*)^2 - m_\pi^2} \right] \vec{\sigma} \cdot \left[ \frac{\vec{p}_i}{E_i + m_i} - \frac{\vec{R}}{E_{D_{13}} + M_{D_{13}}} \right] | \chi_i \rangle
\end{aligned} \tag{8}$$

where starred quantities are now calculated in the  $D_{13}$  resonance rest frame.  $\vec{R}$  stands for the momentum of the emitted resonance.  $C_B$  is the isospin coupling constant. The coupling constant  $G_{D_{13}}$  is related to  $\pi N$  partial decay width through

$$\Gamma_{\pi N} = \frac{G_{D_{13}}^2}{12\pi} \frac{q^{*5}}{M_{D_{13}}(E_f + m_f)}$$

One recognizes, in the second to last bracket of Eq. 8, the contact term (proportional to  $\vec{S}^\dagger \cdot \vec{\epsilon}$ ) and the pion exchange term.

## 2.11 The $\pi P_{11}$ Born term

In a similar fashion we compute the matrix element for the  $\pi P_{11}$  Born term. The element takes the form:

$$\begin{aligned}
\mathcal{T} = & \frac{-2Weg^{*2}C_B}{W^2 - M_{P_{11}}^2 + iM_{P_{11}}\Gamma_{P_{11}}} \sqrt{\frac{E_i + m_i}{2m_i}} \sqrt{\frac{E_f + m_f}{2m_f}} \times \\
& \langle \chi_f | \vec{\sigma} \cdot \left[ \frac{\vec{R}}{E_{P_{11}} + M_{P_{11}}} - \frac{\vec{p}_f}{E_f + m_f} \right] \\
& \vec{\sigma} \cdot \left[ \frac{\vec{\epsilon}}{m_i + M_{P_{11}}} + \left( \frac{\vec{p}_i}{E_i + m_i} - \frac{\vec{R}}{E_{P_{11}} + M_{P_{11}}} \right) \frac{(2\vec{\mu}_1 - \vec{k}) \cdot \vec{\epsilon}}{(\mu_1 - k)^2 - m_\pi^2} \right] | \chi_i \rangle
\end{aligned} \tag{9}$$

where  $g^{*2}/4\pi = 10.34$  is related to the  $\pi N$  partial width through

$$\Gamma_{\pi N} = \frac{3g^{*2}}{4\pi} \frac{q^{*3}}{M_{P_{11}}(E_f + m_f)}$$

One recognizes, in the last bracket of Eq. 9, the contact term (proportional to  $\vec{\sigma} \cdot \vec{\epsilon}$ ) and the pion exchange term.

### 3 Unitarity

Adding a resonant contribution to a non resonant (real) background violates unitarity. While methods to restore unitarity are well known [13] when one resonance dominates and when only one channel is open, the game becomes more tricky when several channels are open. Near threshold, where a few partial waves contribute and/or a few channels are open, a coupled channel approach may prove to be a useful guide to analyze the data. This is almost impossible at high energies where more channels are open. There approximate methods should be used, the most popular of which is the Eikonal Absorption Model [14].

This model assumes that after being produced through the Born terms the pion and the A, or the emitted resonance, scatter without being deviated. The only effect is a lost of flux due to the coupling to other open channels, which suppresses the Born term contribution. Each partial wave should then be multiplied by an absorptive factor which is related to the imaginary part of the corresponding elastic scattering phase shift, and eventually to the reaction cross-section and the slope of the elastic scattering angular distribution at forward angle.

Since we are interested in understanding the main trend of the data in a wide energy range, this is the method which we have retained. We leave for a later publication a coupled channel treatment more appropriate to the threshold region.

Accordingly with the analysis of Lücke and Söding [4], we have multiplied the contact Born terms, each leading to a unique spin final state ( $J=\frac{3}{2}$  for the  $\pi\Delta$  channel for instance), by the asymptotic expression:

$$S_{ff}^{1/2} = \sqrt{1 - \xi_{\Delta} \exp[-(J - 1/2)^2 / 2A_{\Delta} R^{\star 2}]}$$

where  $\xi_{\Delta}=1$ ,  $A_{\Delta}=8 \text{ GeV}^2$ , and  $R^{\star}$  is the momentum of the recoiling resonance in the overall cm. frame. Initial state absorption effects have also been taken into account above the  $\rho$  production threshold, according to the vector dominance model, through the multiplicative factor :

$$S_{ii}^{1/2} = \sqrt{1 - \xi_{\rho} \exp[-(J - 1/2)^2 / 2A_{\rho} q_{\rho}^2]}$$

with  $\xi_{\rho}=1$ , and  $A_{\rho}=5.5 \text{ GeV}^2$ . The momentum of the  $\rho$ , in the overall c.m. frame, is  $q_{\rho}^2$ .

Since the pion exchange graphs (fig.1-IB, and IVB) project out on many partial waves, they ought to be evaluated singly before applying the absorptive corrections. To simplify the resulting calculations, thereby avoiding a multipole expansion, we found it more convenient to multiply the pion exchange terms by a dipole form factor with a 900 MeV cut-off mass.

## 4 Comparison with experiment

### 4.1 The high energy sector of the $p\pi^+\pi^-$ channel

As is clearly demonstrated on Fig. 2, the absorptive corrections significantly reduce the Born cross-section and restore the agreement with the measured values of the  $\Delta^{++}\pi^-$  cross section in the energy range  $2 < E_\gamma \leq 5$  GeV. The  $\pi\Delta$  contact term, which dominates at low energy, is almost completely suppressed above  $E_\gamma \simeq 2$  GeV, where the pion exchange Born term dominates. At high energy, the  $\rho$  diffractive production mechanism accounts for the strength in the  $\gamma p \rightarrow \rho p$  channels. Below  $E_\gamma = 2$  GeV, the excess of experimental cross-section indicates the need to include higher mass, and strongly decaying into the  $\rho$  channel, resonances than those taken into account in the present study.

We find also that, above  $E_\gamma = 1$  GeV, the  $\pi P_{11}$  and  $\pi D_{13}$  Born terms contribute significantly to the cross section of the full  $\pi^+\pi^-$  channel. They, however, do not exhibit the proper high energy behavior since our model clearly overestimates the data above  $E_\gamma = 2$  GeV. On the other hand, both the  $\Delta^{++}$  and  $\rho$  production channels are correctly reproduced. Therefore, following the recipe described in the appendix, we have replaced the usual pion exchange amplitudes by Regge Pole exchange amplitudes which are known to exhibit the correct high energy behavior. Fig. 3 shows the marked improvement in the description of the data at high energy after this modification has been applied. In the following discussion, we only make use of the results of the modified model. We leave to a later paper a more comprehensive study of the implications of Regge phenomenology.

Figs. 4 and 5 show respectively  $p\pi^+$  and  $\pi^+\pi^-$  mass spectra at  $E_\gamma = 1$  and 3 GeV. The model agrees fairly well with the normalized data from ref [1]. The  $\Delta^{++}$  and the  $\rho$  clearly appear. On fig.4, the excess of experimental cross-section in the  $p\pi^+$  spectrum, around 2 GeV, strongly suggests some contributions from Born Terms associated with A resonance in this mass range.

Fig. 6 shows the double differential cross-section  $d\sigma/dM_{p\pi^+}dt$  at  $E_\gamma = 5$  GeV. The four-momentum transfer, between the incoming photon and the outgoing  $\pi^-$ , is  $t = (k - \mu_1)$ . In the  $t$  distribution, the direct  $A^{++}$  contribution (Fig.1.I) appears clearly above that of the background due to the other processes.

We next applied a cut on the mass distribution of fig. 6 in order to concentrate on the contribution from the  $A^{++}$ . This cut corresponds to  $1170 \leq M_{p\pi^+} \leq 1400$  MeV. Fig. 7 shows the results from this operation and allows us to compare the resulting distribution with the experimental data available at  $E_\gamma = 5$  GeV. We note that the  $A^{++}$  contribution decreases quickly with increasing  $t$ , as is expected, but that above  $-t \simeq 2$  GeV<sup>2</sup> the  $p\pi^+$  background dominates becoming comparable to the experimental data. It should be noted, however, that this experiment [15] was

performed with a continuous Bremsstrahlung beam and the  $\Delta^{++}$  really couldn't be reliably identified. We suggest that this experiment should be repeated using the intense monochromatic real photon beam soon to be made available at CEBAF.

## 4.2 The low energy sector

We now compare the predictions of our model with most of the available experimental data obtained at energies up to  $E_\gamma=1$  GeV, and in particular with the recent high quality data from Daphne. This is the domain where meson exchange models are expected to be more valid and where absorptive corrections should not dominate.

### 4.2.1 The $p\pi^+\pi^-$ Channel

In this reaction channel, every mechanism, except the direct  $\rho$  production (fig.1-V), is present. We find, as shown on fig.8, that more than half of the cross-section, as well as the overall shape, are explained by the presence of the  $\pi\Delta$  born term (fig.1-I). Both the  $\Delta^{++}\pi^-$  and the  $\Delta^0\pi^+$  intermediate states have been explicitly calculated. Around  $E_\gamma=600$  MeV, the remainder of the cross section is essentially due to the interference of the  $D_{13}(1520)$  resonance in the s-channel with the contact part of the  $\pi\Delta$  Born term. Around  $E_\gamma = 900$  MeV, the additional shoulder showing up in the cross section is due to interference of the  $D_{33}(1700)$  resonance with the Born term. Contributions from the other mechanisms shown in fig.1, though small, have also been explicitly included.

Our results, using the central value of parameters extracted from the analysis of other experiments [8], are in good accord with the Daphne data. A better fit to the cross-section could have been obtained by slight adjustment of some of the parameters, within their error bars, but we stress again that this is not the object of this paper.

### 4.2.2 The $p\pi^0\pi^0$ Channel

This channel is characterized by the absence of any of the Born terms. This is due to the fact that since  $\pi^0$  cannot couple to photons, graphs I-b and IV-b of fig.1 are forbidden. By gauge invariance arguments, this implies that the contact parts, graphs I-a, and IV-a, are suppressed. Finally, as the  $\rho$  may not decay into two  $\pi^0$ , graphs II-b, and V are also absent. This makes the  $p\pi^0\pi^0$  channel ideally suited for the study of the nucleon resonances, since only graphs II-a and 111 are allowed to remain.

The calculated cross-section is shown on fig.9, along with the contribution from

the appropriate diagrams, and the experimental data obtained with Daphne. We have found that major part of the cross-section is essentially due to the presence of the  $P_{11}(1440)$  resonance decaying into the  $\sigma$ . The parameters describing the various couplings to the  $\sigma$  are the same which have been obtained in the analysis of the  $\pi N \rightarrow \pi\pi N$  channel [6]. One should note also the suppression of the strength of the  $D_{33}(1700)$  due to the isospin coupling constant. Finally, Fig. 10 shows the mass spectra of the two emitted neutral pions as well as of the  $p\pi^0$  pair at photon energies of 1 and 3 GeV. The contributions of the A and  $\sigma$  emission channels are clearly separated, especially at the higher energy.

#### 4.2.3 The $n\pi^+\pi^0$ Channel

This last channel turns out to be the most difficult to interpret, as it contains subtle aspects of the previous two. Half of the Born terms are present, since only the charged pion can couple to the photon. Graphs Ia,b and IVa,b must, therefore, be calculated for the  $\Delta^0\pi^+$  channel only. The only graph which is totally absent is III-b, which, for this channel, violates charge conservation. The results of our calculations are shown on fig.11, along with the data obtained with Daphne, and the contribution from the more important mechanisms. Every known resonance which could contribute in the energy range under consideration has been included. There are still no free parameters involved.

Fig.12 shows a comparison of our model with data obtained on the reaction  $\gamma n + p\pi^-\pi^0$ , which is the isobaric mirror of reaction presently being discussed. While in the Daphne energy range the model greatly underestimates the experimental data, it is encouraging to note that for energies above 1 GeV it does reproduce the data quite well, in spite of not having included Fermi motion effect in the originally measured  $\gamma D + \pi^-\pi^0 pp$  reaction cross-section. One should note the sizable contribution of the  $p$  production contact term. Its shape reflects the manner in which we have implemented absorption in the  $p$  channel, namely no absorption below the  $p$  production threshold.

For completeness, the mass spectra of the emitted  $\pi^-\pi^0$  pair are shown on Fig.13 at photon energies of 1 and 3 GeV. The contribution of the  $p$  production channel appears clearly.

#### 4.2.4 Discussion and prospects

This discrepancy in the  $n\pi^+\pi^0$  channel between the experimental results and the model is disconcerting and calls for improvements. On the one hand, there is a high level of confidence in the experimental data [3]. On the other hand, the good agreement for the other two channels does not allow much freedom in the present

version of the model. Two improvements are worth considering.

We suspect that the manner in which we have dealt with unitarity, while correct at high energies, might not be appropriate in the Daphne energy range. Though it takes correctly into account the loss of flux of the emitted particles, due to the coupling to the other open channels, it does not deal correctly with the addition of a resonant contribution to the Born background, nor with possible increase of the cross-section due to **charge exchange rescattering mechanisms**. A **complete implementation of unitarity, in a full coupled channel approach**, seems to be the way to proceed but lies outside the scope of this paper.

The second improvement will be to include the contributions of higher mass resonances, both in the s-channel and the Born Terms, and to play with their parameters in the limits of the range allowed by the analysis of other experiments. However, before playing such a game, the prerequisite is to master the non resonant background and to know how to add resonant contributions in a unitary way.

These issues, as well as the effects of other speculative mechanisms, will be addressed at a later time.

## 5 Conclusion

By improving the model of Lücke and Söding, we have reproduced the  $p\pi^+\pi^-$  data to an acceptable level. At low energy, we do agree with the older premise that most of the cross-section is due to the presence of the  $\Delta\pi$  born term, and especially its contact part. Effects of the  $D_{13}(1520)$  are also evident by interference with the contact part of the Born term. We have shown, in addition, the contribution of the  $D_{33}(1700)$ , which also interferes with the Born term. Finally, the contributions of the Born terms associated with the  $\pi P_{11}$  and  $\pi D_{13}$  channels become more important as the energy increases.

Our most important result lies in our analysis of the  $p\pi^0\pi^0$  data. The good agreement between our model and the experiment is of its own accord extremely interesting. It shows clearly, for the first time, a direct evidence for the production of the  $P_{11}(1440)$  resonance by real photons. This is an important result in view of understanding the nature of the  $P_{11}(1440)$  which, in the naive quark constituent model, is thought to be the radial excitation of the nucleon. Our results tend to be in conflict with this simplest representation as real transverse photons may not excite such a mode. This indicates a more complicated structure. We have also shown that this resonance manifests itself through its strong decay into two  $\pi^0$ , in a relative S-state, as parameterized by the  $\sigma$  meson.

At low energies, the discrepancy between the model and a recent accurate mea-

surement of the  $\gamma p \rightarrow n \pi^+ \pi^0$  reaction cross-section calls for a more involved treatment of unitarity.

When extended to higher energies, this meson exchange model provides us with a fair accounting of the trends of the experimental data. However, a description based on the exchange of Regge trajectories is more appropriate. A full implementation of Regge phenomenology will be the matter of a next paper.

Nevertheless, the present version of the model provides us already with a good starting point to simulate experiments planned at existing facilities such as CEBAF.

## Appendix

When extrapolated at high energies, meson exchange Born amplitudes are known to lead to the wrong energy ( $W$ ) as well as transfer momentum ( $-t$ ) dependence of the corresponding cross-sections, which often violate unitarity bounds. Regge Pole Exchange phenomenology offers us with a powerful way to extrapolate far from meson poles, while satisfying unitary constraints at high energies (see for instance Ref [16]).

Accordingly with Ref. [17], we have replaced the pion propagator, in Eq. 8 and Eq. 9, by the corresponding Regge amplitude :

$$\frac{1}{t - m_\pi^2} \rightarrow \frac{1}{\Gamma(\alpha + 1)} \left( \frac{s}{s_0} \right)^\alpha \frac{\pi \alpha'}{2} \frac{1 + e^{-i\pi\alpha}}{\sin \pi\alpha} \quad (\text{A.1})$$

where  $s = W^2$  is the total c.m. energy squared,  $s_0 = 1 \text{ GeV}$ , and  $\Gamma(\alpha)$  is the Gamma function.

The Regge trajectory is defined as :

$$\alpha \equiv \alpha(t) = \alpha' [t - m_\pi^2] \quad (\text{A.2})$$

with  $\alpha' = 0.7 \text{ GeV}^{-1}$ .

It is easy to verify that this Regge amplitude does coincide with the pole amplitude  $1/(t - m_\pi^2)$ , near the pole as  $t \rightarrow m_\pi^2$ .

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## Figure Captions.

Fig.1 : The graphs considered in the present model.

Fig.2 : Plots of the  $\gamma p \rightarrow \Delta^{++}\pi^-$ ,  $\gamma p \rightarrow \rho p$  (top) and  $\gamma p \rightarrow p\pi^+\pi^-$  (bottom) cross-sections from threshold up to 5 GeV. In the top part, the results from the complete model are shown by the solid line. The Born term (fig.1-I) is the hashed curve. We show also, by the dot-dashed curve, the total  $\Delta^{++}\pi^-$  cross section when the absorptive corrections are turned off. The dotted curve corresponds to the contribution of diffractive  $\rho$  production. In the bottom plot, the results from the complete model are displayed with and without the diffractive  $\rho$  production by the dot-dashed and dashed curve respectively. The complete  $\Delta^{++}\pi^-$  channel is once again described by the solid curve. Finally, the  $D_{13}$  and the  $P_{11}$  Born terms (fig.1-IV) are shown respectively as the dotted and dot-dashed curve. The experimental data are from Ref. [1] :  $A^{++}$  (o),  $\rho$  (o) and full  $\pi^+\pi^-$  (A) channels.

Fig.3 : The full curve shows the effects of the Reggeization of the pion exchange  $P_{11}$  and  $D_{13}$  Born Term on the total cross-section of the  $\pi^+\pi^-$  channel. The dash-dotted curve shows the results of the meson exchange model (as in Fig. 2), while the dashed curve shows the contribution of the  $A^{++}$  emission channel.

Fig.4 : The  $p\pi^+$  mass spectrum in the reaction  $\gamma p \rightarrow p\pi^+\pi^-$  at  $E_\gamma = 1$  and 3 GeV. The experimental data come from Ref. [1].

Fig.5 : The  $\pi^-\pi^+$  mass spectrum in the reaction  $\gamma p \rightarrow p\pi^+\pi^-$  at  $E_\gamma = 1$  and 3 GeV. The experimental data come from Ref. [1].

Fig.6 : The momentum dependence of the  $p\pi^+$  mass spectrum in the reaction  $\gamma p \rightarrow p\pi^+\pi^-$  at  $E_\gamma = 5$  GeV.

Fig.7 : The full histogram shows the momentum dependence of  $\pi^-$  emitted in the reaction  $\gamma p \rightarrow \Delta^{++}\pi^-$  at  $E_\gamma = 5$  GeV. The dashed histogram represents the contribution of the background below the  $A^{++}$  peak in the mass range  $1170 < M_{p\pi^+} \leq 1400$  MeV. The experimental data come from Ref. [15]. The rise of experiment at large  $t$  indicates a possible contribution of graph Ic which is not included in this calculation.

Fig.8 : The cross-section of the  $\gamma p \rightarrow p\pi^+\pi^-$  channel below  $E_\gamma = 1.1$  GeV. The complete model is shown on the top part of the figure by the solid line. The contribution from the Born term (fig.1-I), the  $D_{13}$  and the  $P_{11}$  Born terms (fig.1-IV) are, respectively, the dotted line, the long dashed and the long dot-dashed curves. The dashed curve, the dot-dashed, and the closely spaced dotted curves display the results obtained with the Born+ $D_{13}$ , the Born+ $D_{33}$  and the Born+ $D_{13}$ + $D_{33}$  alone, showing clearly the interference between these terms. The other, lesser, contributions are shown on the bottom part of the figure. The  $P_{11}$  graphs described by fig.1-IIa, and IIb are the solid, and dot-dashed lines. The  $D_{13}$  graphs fig.1-IIa, and

b, are the dotted, and dashed lines. The  $D_{33}$ , depicted in fig.1-IIa, is the hashed line. The experimental data are from Refs.[1]( $\Delta$ ), [2]( $\circ$ ), and [3] ( $\bullet$ ).

Fig.9 : The cross-section of the  $\gamma p \rightarrow p\pi^0\pi^0$  channel below  $E_\gamma = 1.1$  GeV. The total cross section is plotted as the solid curve. The  $D_{13}$ , and the  $D_{33}$  contributions (fig.1-IIa) are, respectively, the short and long-dotted curves. The  $P_{11}$  graphs from fig.1-IIIa, and IIIb are the dot-dashed, and dashed curves. The data is from Ref. [3].

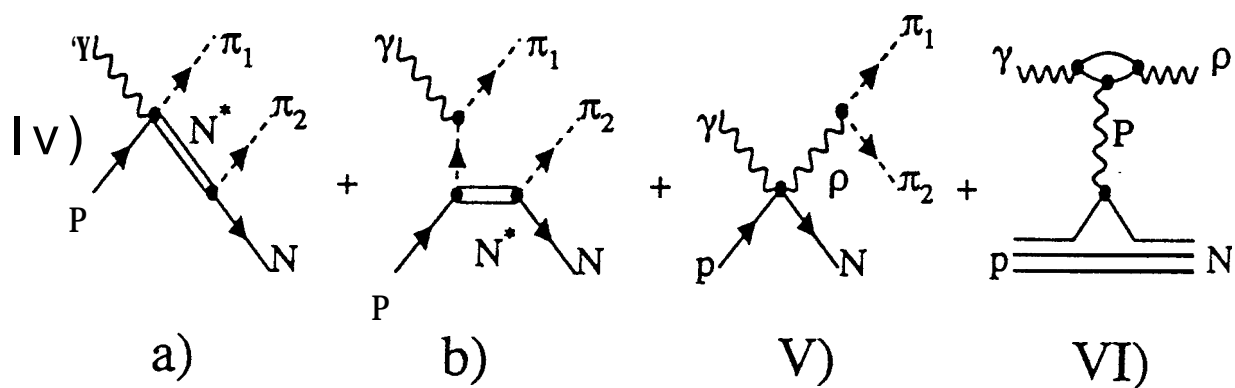
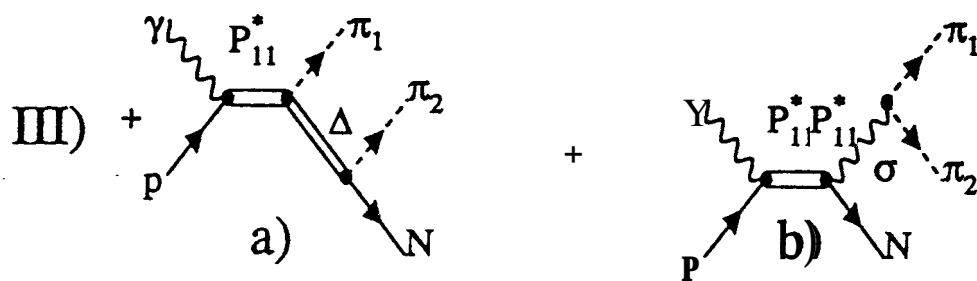
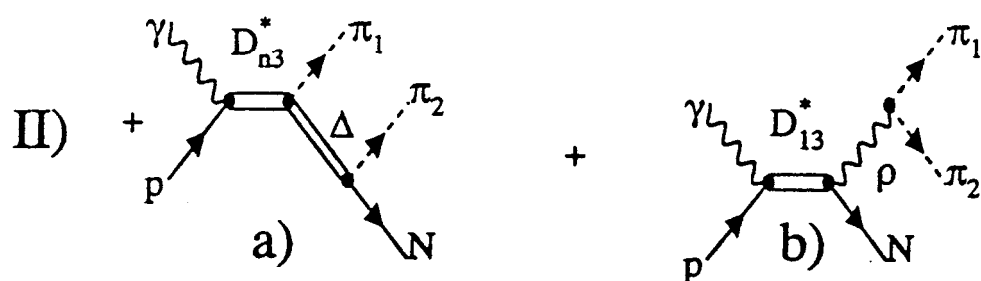
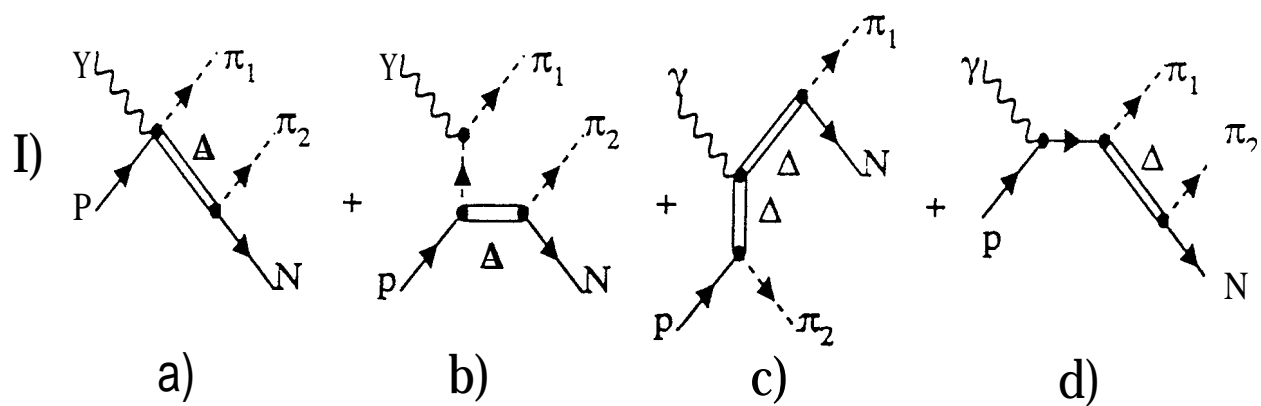
Fig.10 : The  $\pi^0\pi^0$  mass spectrum in the reaction  $\gamma p \rightarrow p\pi^0\pi^0$  at  $E_\gamma = 700$  MeV (a) and  $E_\gamma = 3$  GeV (b). The contribution of the  $\sigma$  production channel is the dashed histogram.

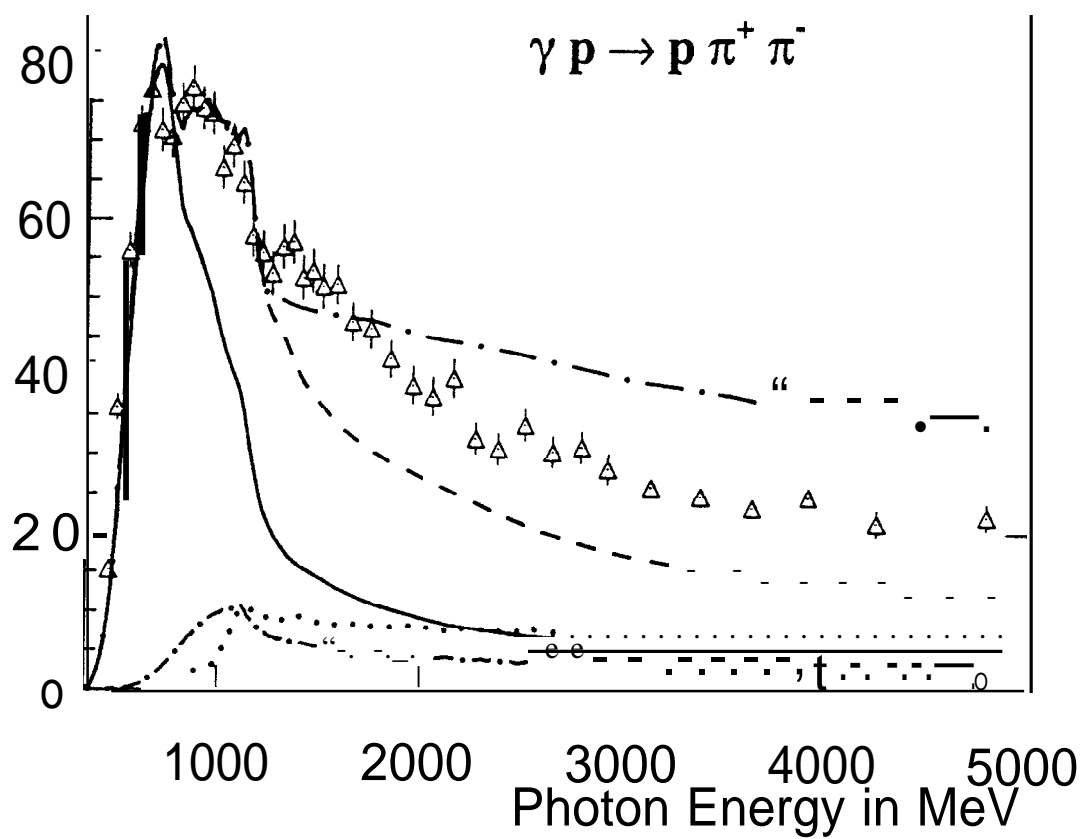
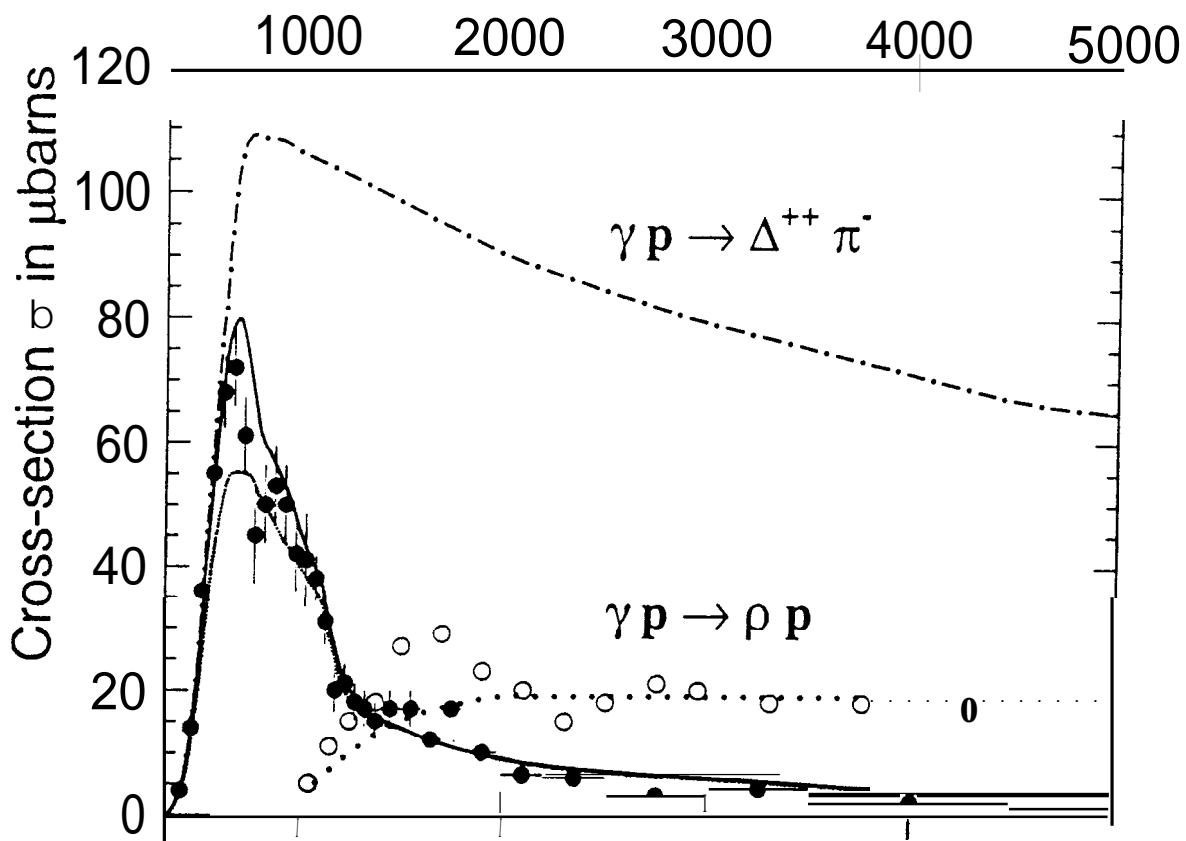
Fig.11 : The cross-section of the  $\gamma p \rightarrow n\pi^0\pi^+$  channel below  $E_\gamma = 1.1$  GeV. On the top part, the results describing the complete model are shown by the solid line. The contribution from the Born term (fig.1-I) is the dotted line. The  $D_{13}$ , and the  $P_{11}$  Born terms (fig.1-IV) are the dashed and the dot-dashed curves.  $\rho$  production (fig.1-V) is shown by the hashed line. On the bottom part, the other contributions are plotted. The  $P_{11}$  from fig. 1-IIIa is the solid curve. The  $D_{13}$  and  $D_{33}$  graphs from fig.1-IIa are the dotted, and hashed lines. The  $D_{13}$  from fig.1-IIb is shown by the dashed line. The data is from Ref. [3].

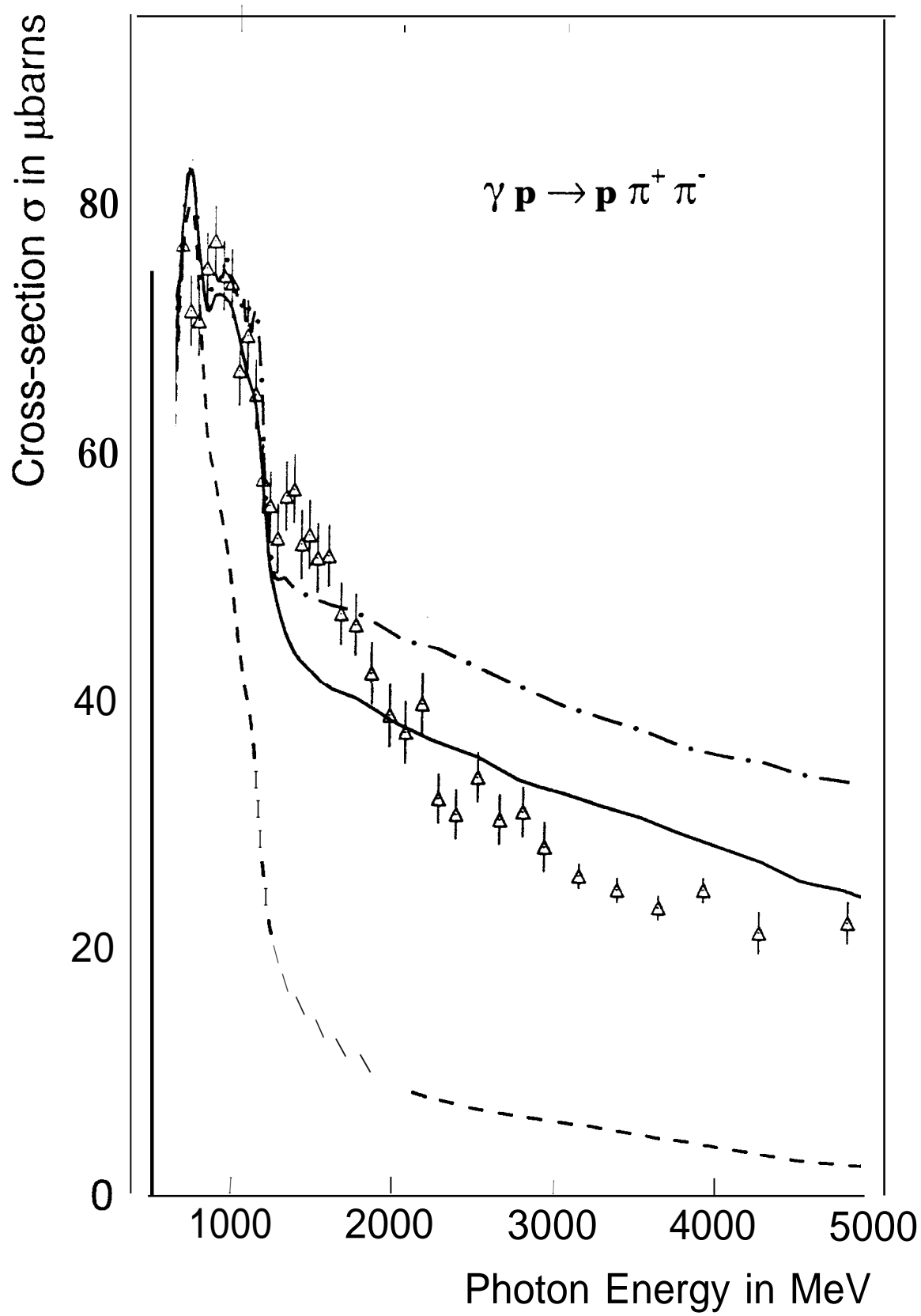
Fig.12 : The cross section of the reaction  $\gamma n \rightarrow p\pi^0\pi^-$  at high energy. The full histogram represents the result of the complete model. The dashed histogram represents the contribution of the A Born terms. The dotted and the dash-dotted histograms represent respectively the contribution of the  $D_{13}$  (including its  $\rho$  decay) and  $P_{11}$  resonances s-channel production. The cross-hatched area represents the contribution of the  $D_{33}$  resonance s-channel production (without including its  $\rho$  decay). Finally, the hatched area represents the contribution of the  $\rho$  contact term. The experimental data come from Ref. [1] (A) and Ref. [2] (.).

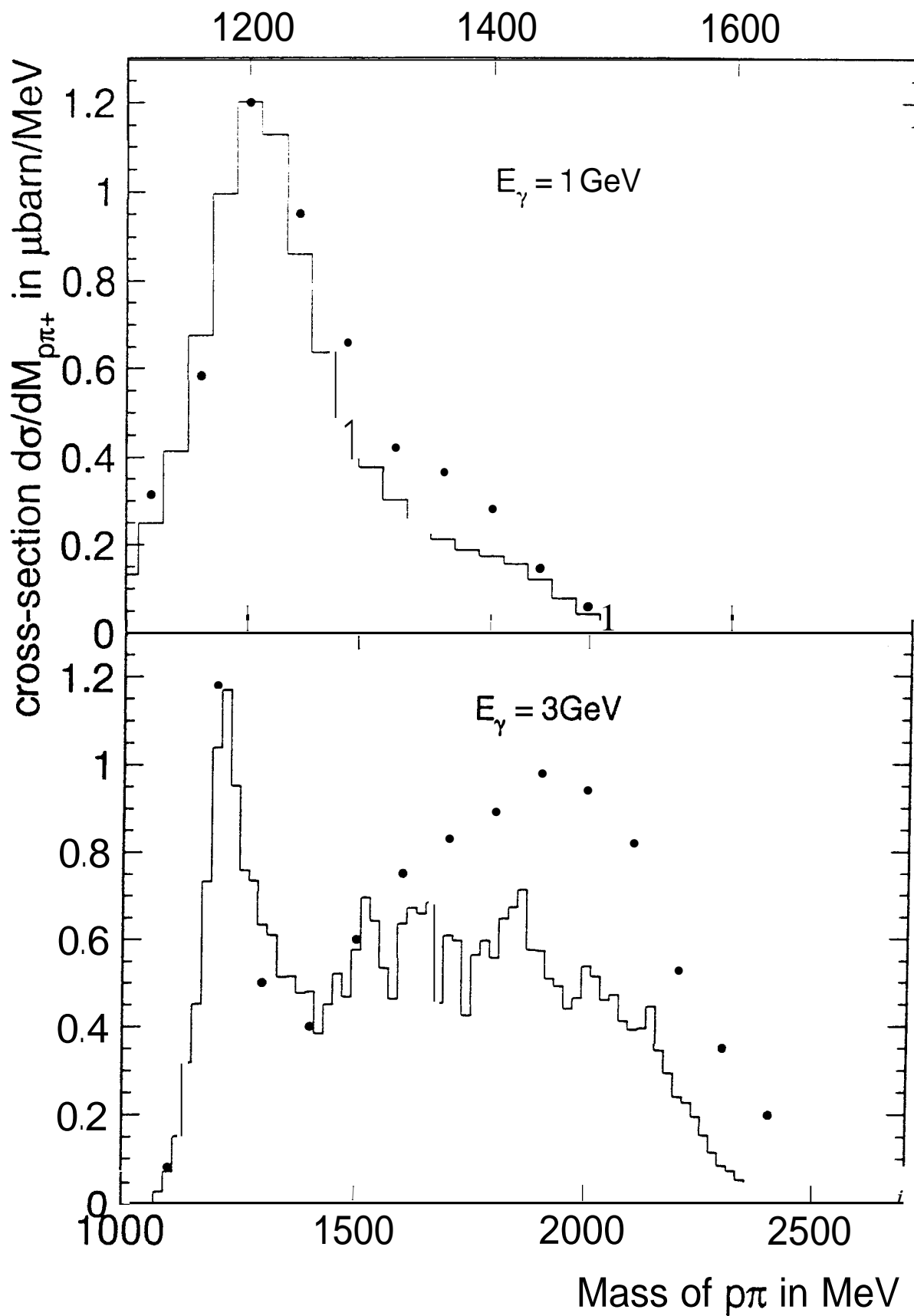
Fig.13 : The  $\pi^-\pi^0$  mass spectrum in the reaction  $\gamma n \rightarrow p\pi^0\pi^-$  at  $E_\gamma = 1$  and 3 GeV.

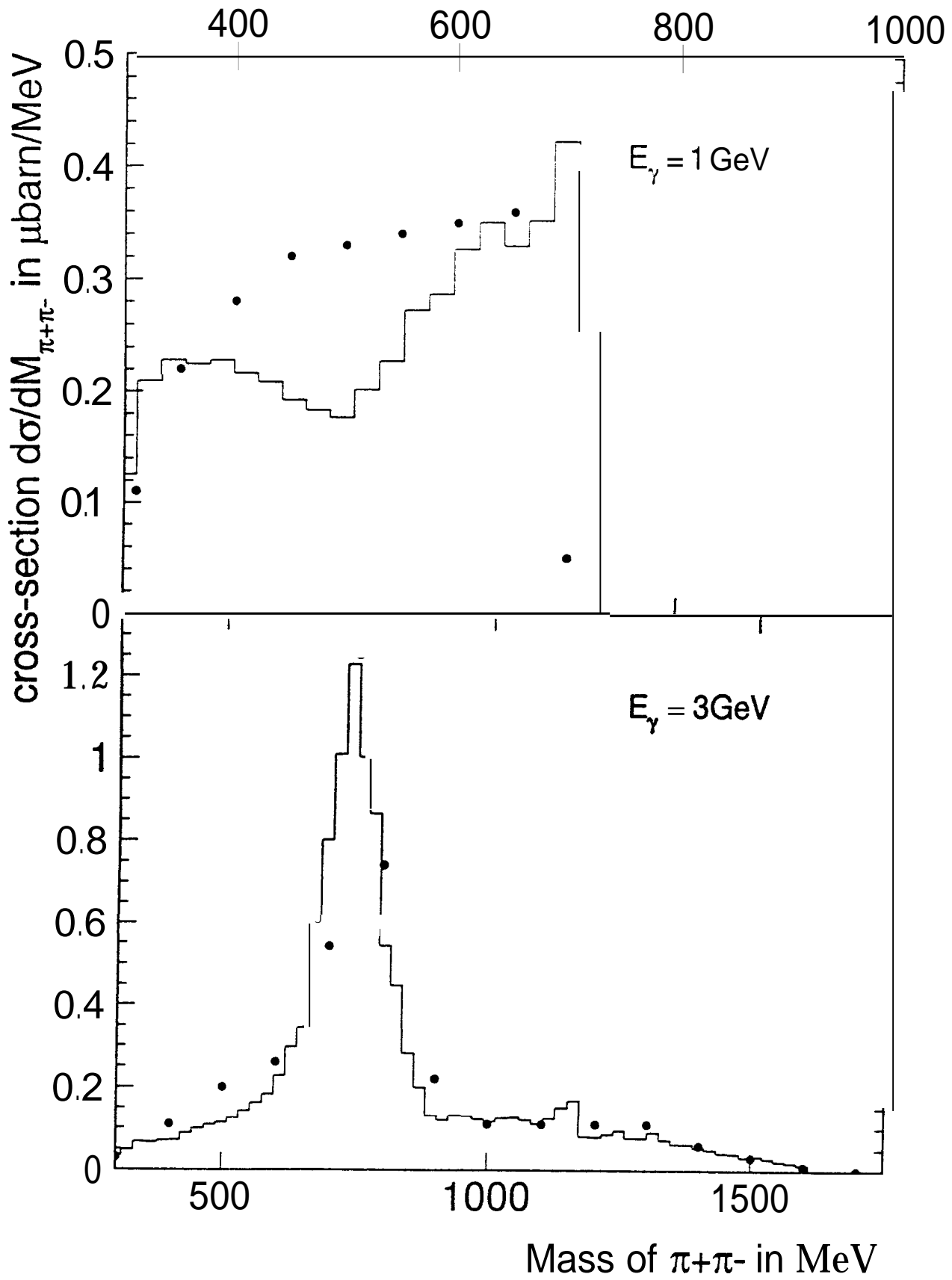
J.M. Laget, L.Y. Murphy FIG. 1



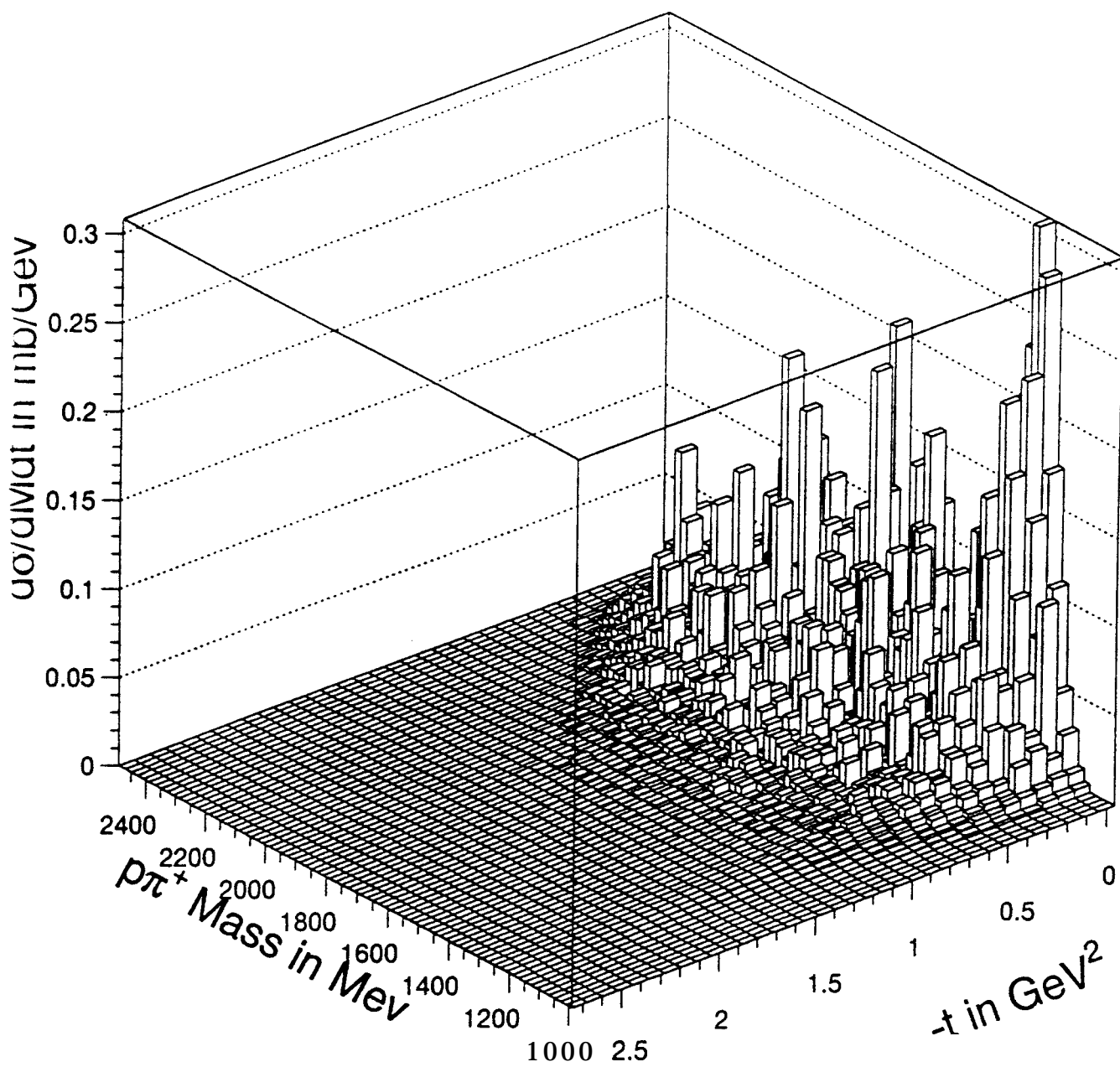


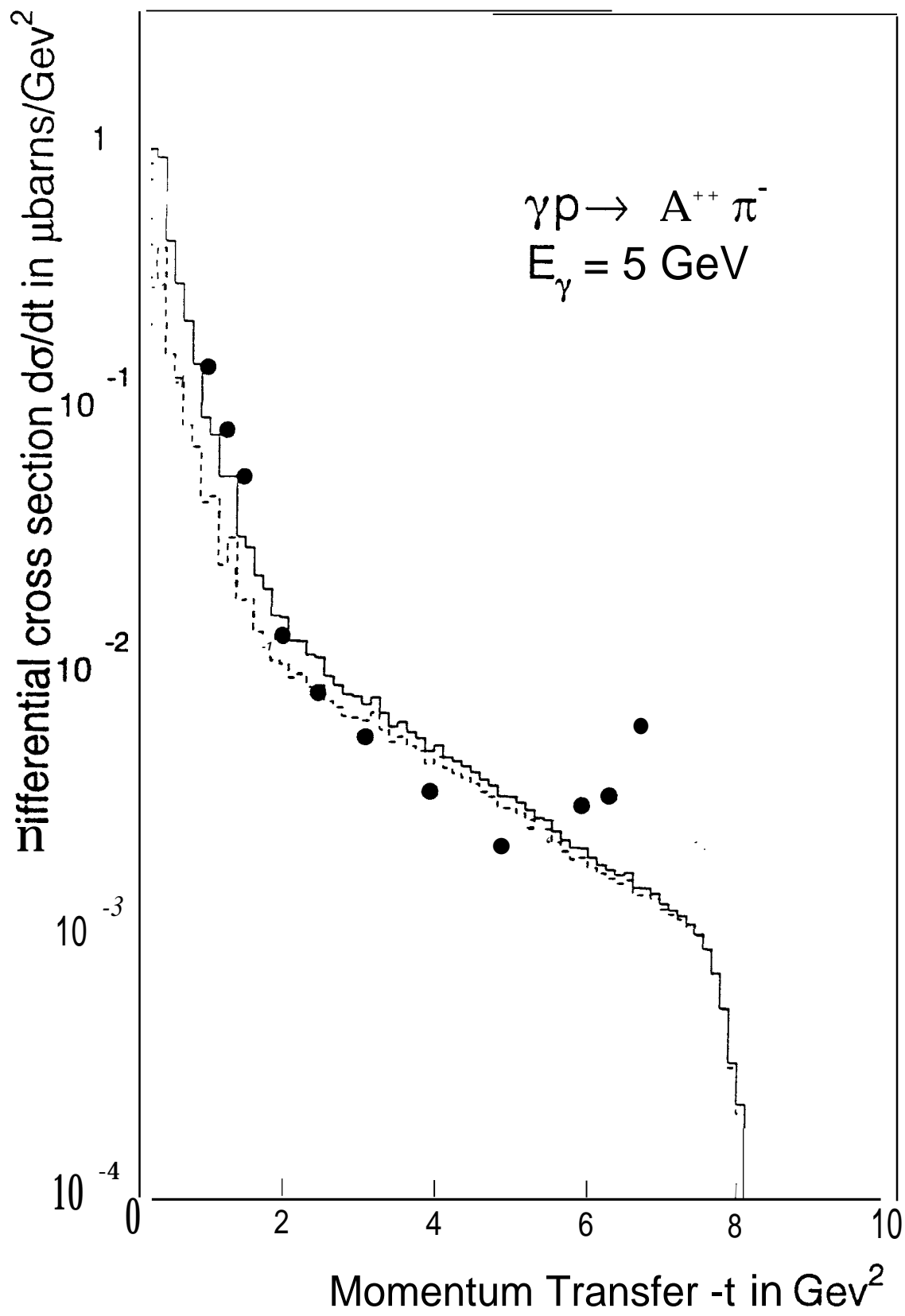


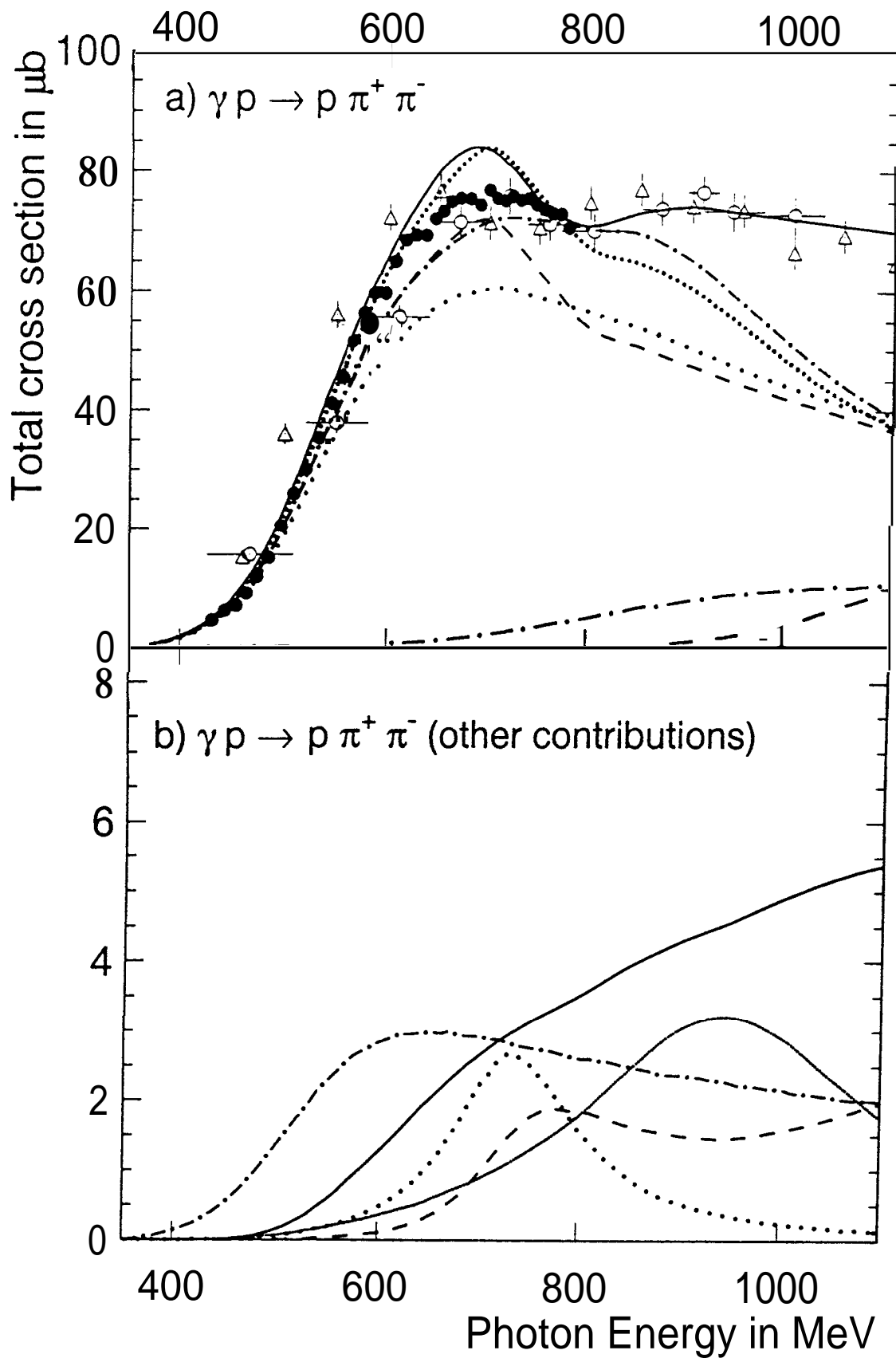


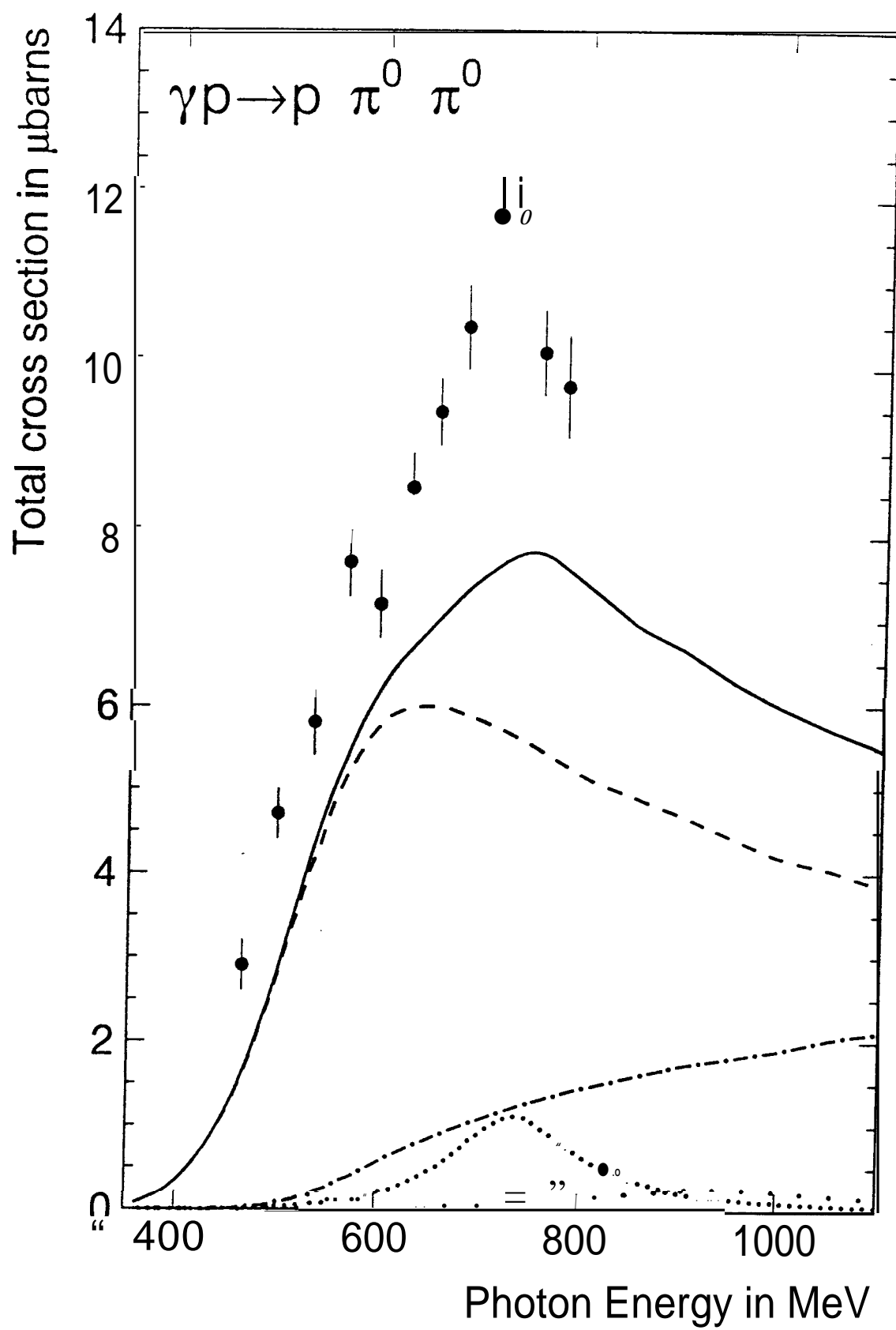


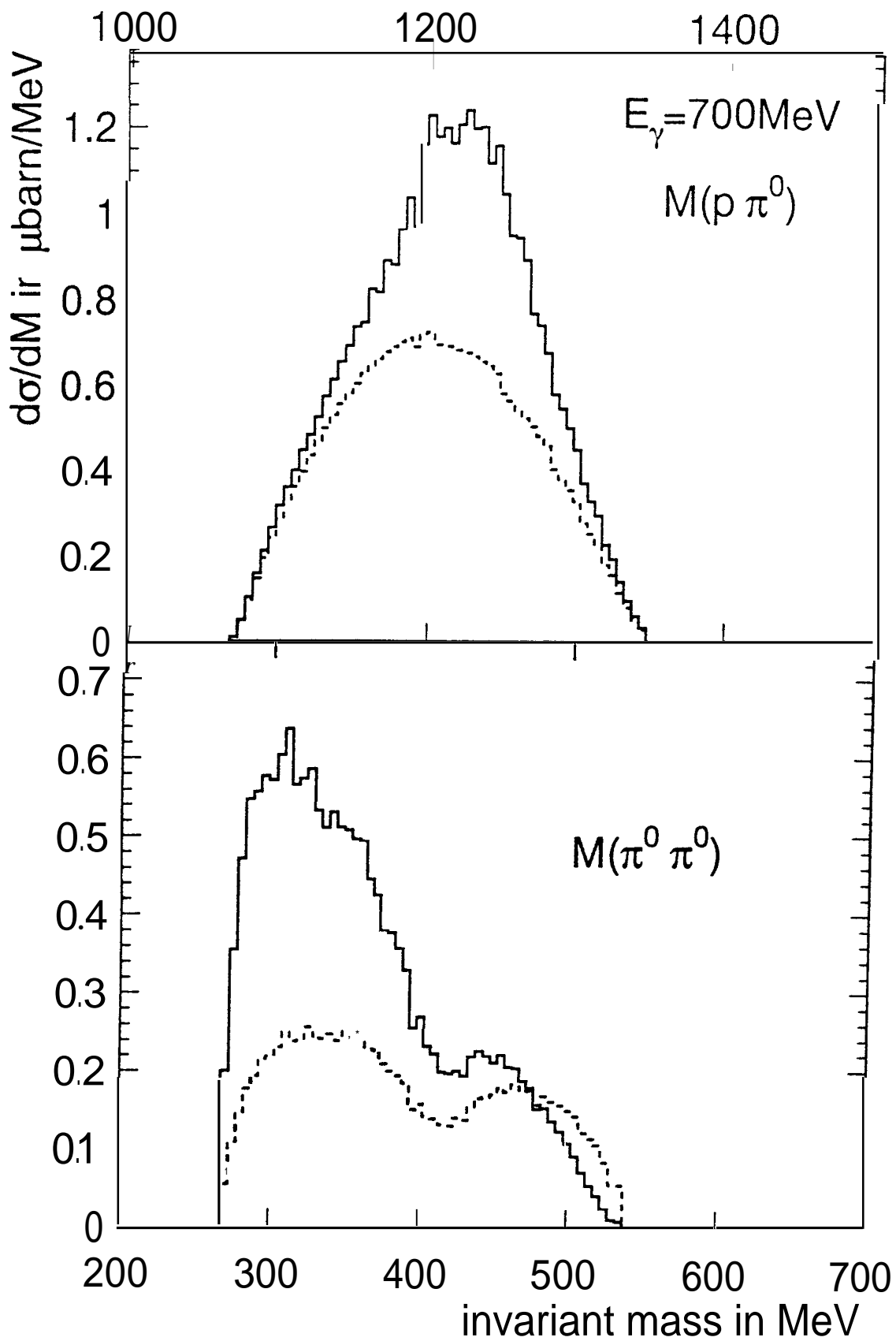
J.M.Laget, L.Y. Murphy FIG. 6

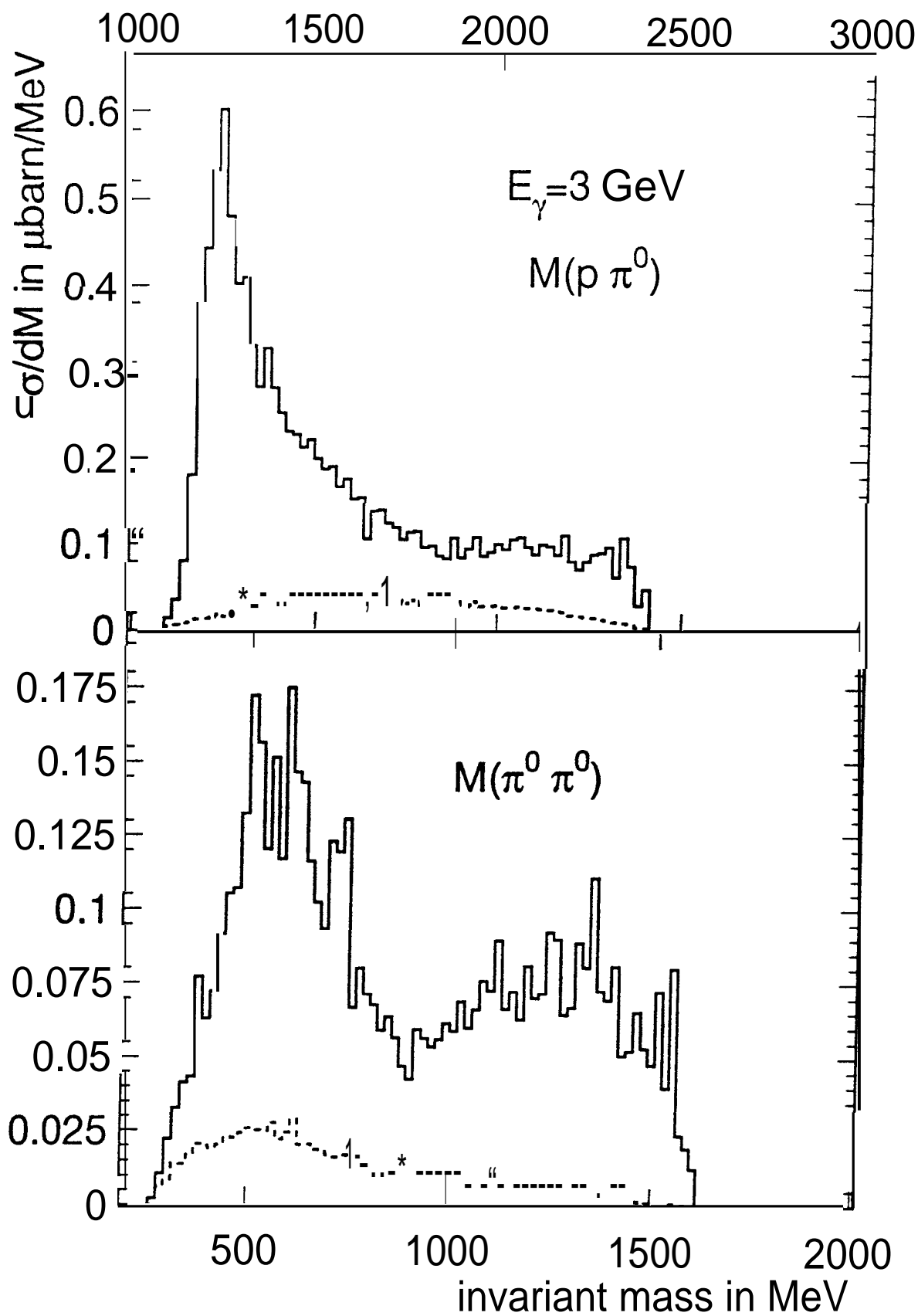


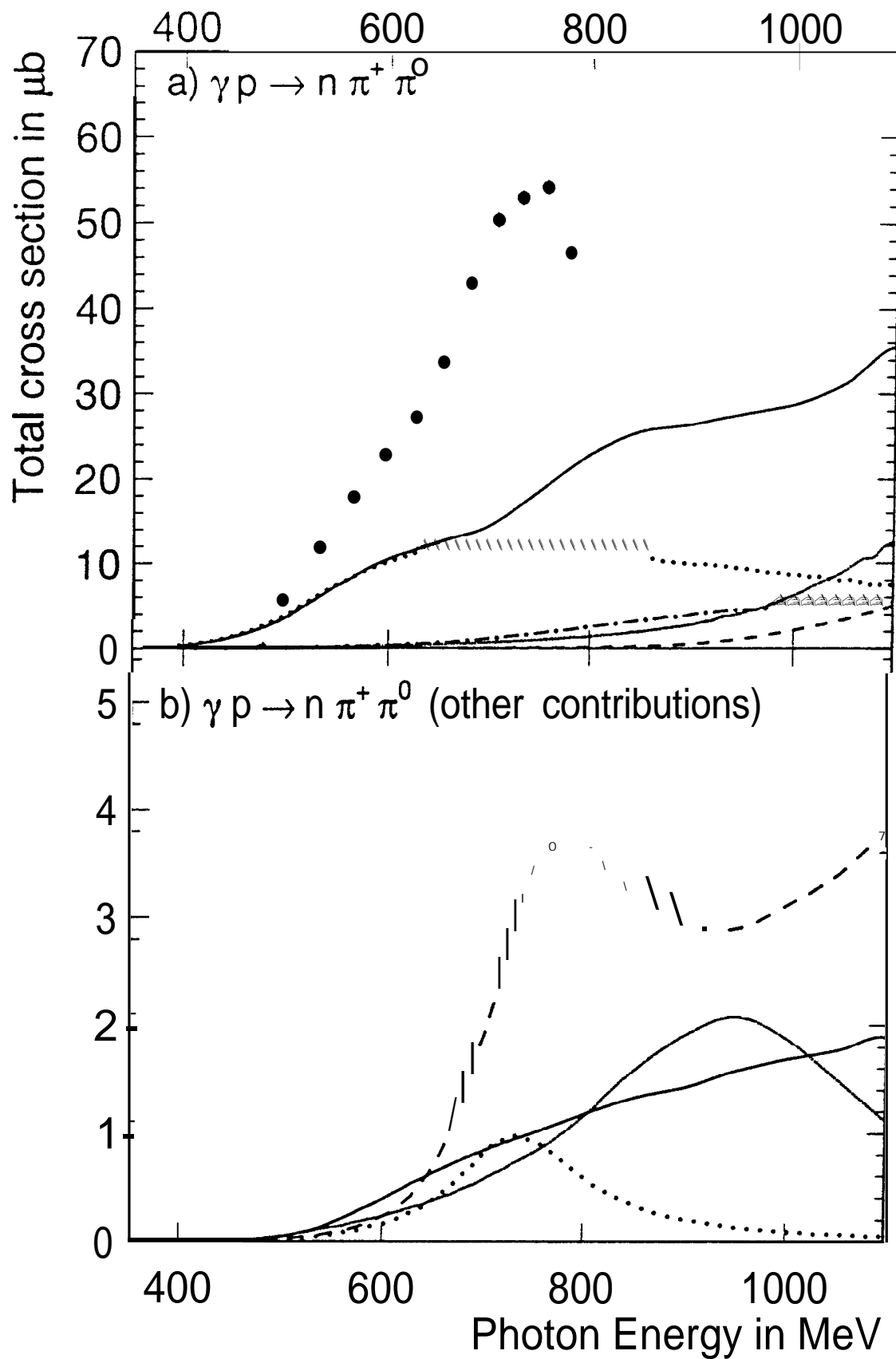


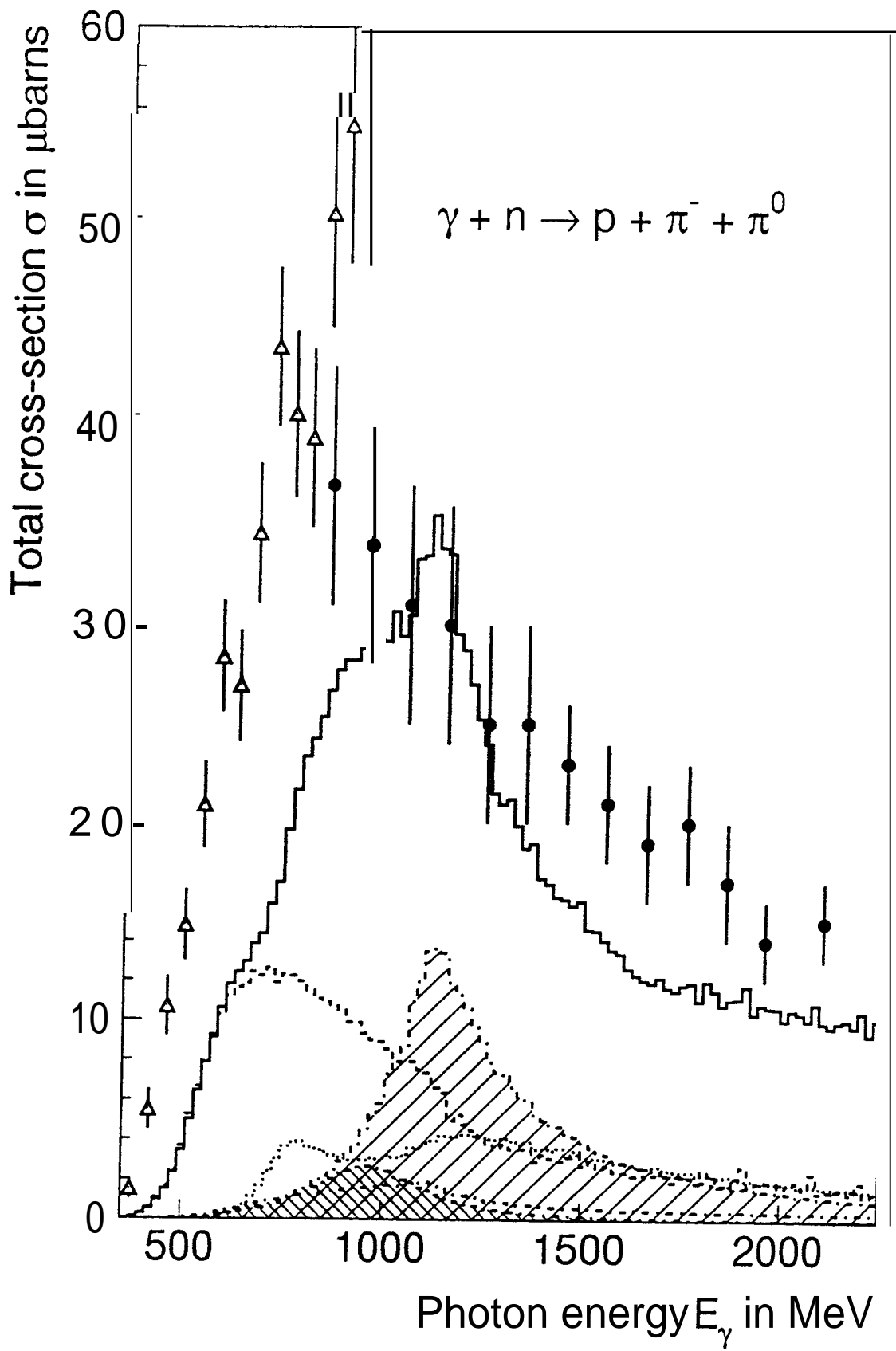


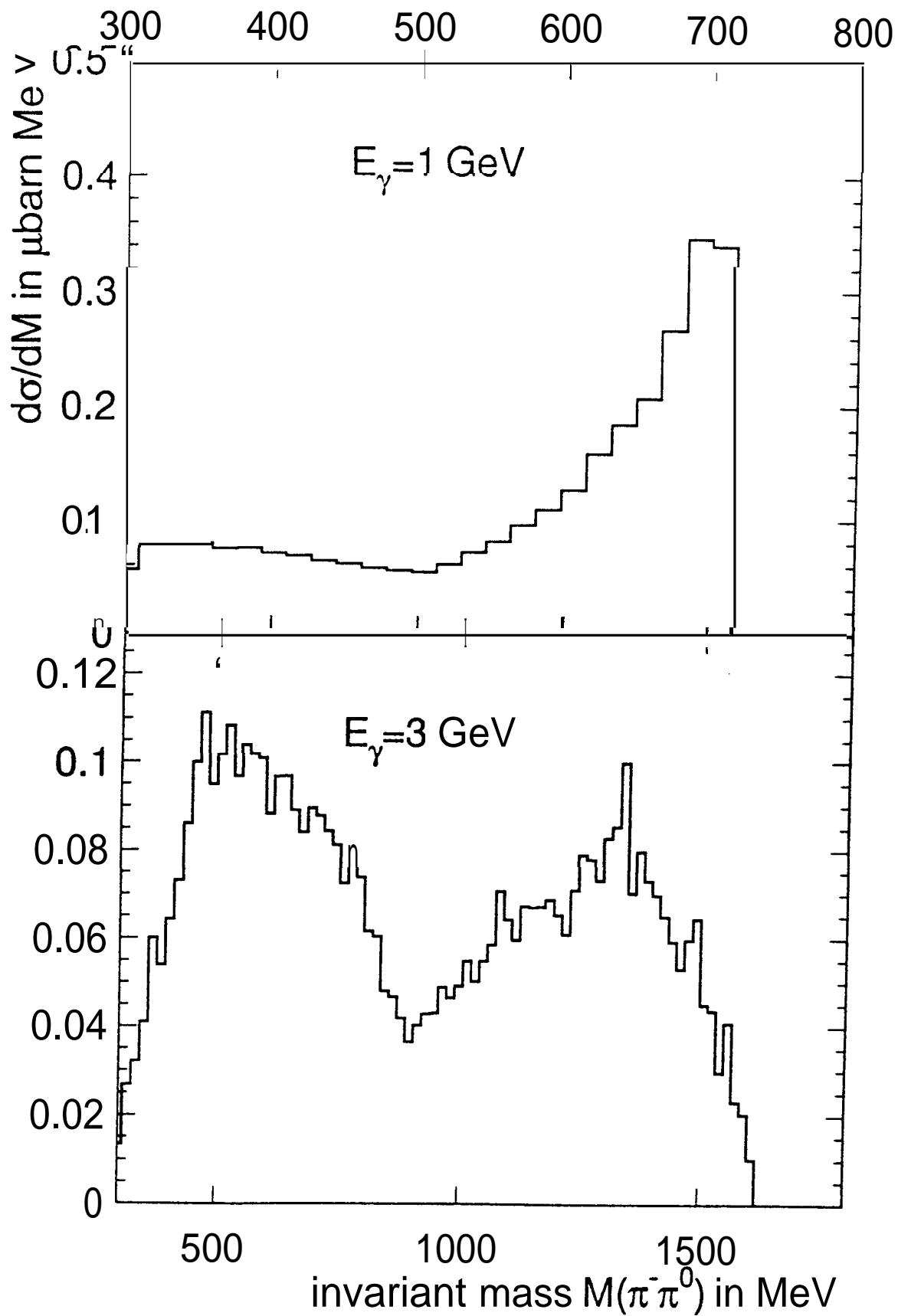


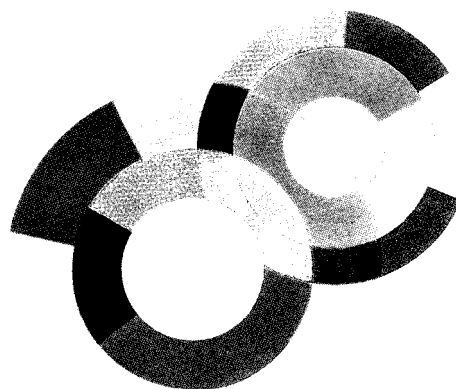












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at GeV incident energies

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Heating Nuclei with Light Ions at **GeV** incident energies.

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## INTRODUCTION

The subject of hot nuclei draws a number of engaging problems. Broadly speaking one can divide the interest into three; the heating dynamics <sup>1-3</sup>; properties of hot nuclear matter <sup>4</sup> and the decay processes <sup>5</sup>. All this is within the confines of a relatively small number of nucleons interacting through an interplay of short and long range forces <sup>5</sup>. In this contribution we report on a study of hot nuclei, where through an appropriate choice of incident channel and event selection, dynamical effects are attenuated and multifragmentation is limited. Herein, we aim at giving three preparatory results; (i) the  ${}^3\text{He}(1.8 \text{ GeV}) + {}^{nat}\text{Ag}$  can be described using an intranuclear cascade, INC, model; (ii) through a suitable selection of events we give a limit of the excitation energy that a nucleus can absorb without breaking into large pieces; (iii) we show that corresponding alpha decay is consistent with an, evaporative process.

Before indulging in the experimental set-up, it is practical to adopt a general outlook of the reaction processes involved in light ion induced reactions at GeV.A incident energies. Within a BUU<sup>6</sup> description, the projectile nucleons are subject to n-n collisions and are then, more often than not, ejected over a time scale compatible with  $\sim 15 \text{ fro/c}$ . This primary process leaves behind an abundant number of energetic,  $\pi, \Delta$  and nucleons. Over a time period covering say, 15 to 100 fro/c the residual system evolves through a sequence of dynamic evolutions with processes ( $\pi$  reabsorption, etc), in part, leading to thermalisation. As pointed out by Wang et al.<sup>3</sup>, the coupling to the mean field could have a determining effect on the final channel through the formation of spatial density variation. However, we consider that at 1.8 GeV the consequences of these effects are less marked than at 4.8 GeV<sup>3</sup>. Beyond 100 fro/c the mean field settles down to give a predominantly thermalised hot nucleus. Herein we consider an INC

description, where beyond  $\sim 30$  fm/c the resultant complex is assumed to have normal spherical, uniform density distributions with the energy being shared between thermal and rotational degrees of freedom. On average at 1.8 GeV, the angular momentum is  $20\hbar$  and the residual mass varies weakly with the calculated excitation energy<sup>7</sup>. As for the calculations presented, all impact parameters are considered with each event generated by the INC code<sup>1</sup> being fed into an evaporative routine, EUGENE<sup>8</sup>. Where necessary the events are filtered by the experimental acceptance (code FILTER). No effort was made to adjust the parameters of the INC and EUGENE.

## Experimental Set-Up

The experiment was performed at the Laboratoire National Saturne using  $^3\text{He}$  beam at 1.8, 3.6 and 4.8 GeV. In this contribution we present the lower incident energy data set. The target was natural Ag of thickness  $1.08\text{ mg/cm}^2$ . Briefly, the experimental set-up consisted of essentially four parts. (i) To measure leading protons ARCOLE was used<sup>9</sup>. This consists of a forward plastic wall made up of 28 fast plastics and mounted so as to have a hole in the center for the beam. Light from each plastic was read out by two photomultipliers and gives the energy loss and impact position. This assembly covered an angular range of approximately  $2.5^\circ$  to  $12^\circ$  and was positioned to give a minimum flight path from the target of 4 meters. (ii) To detect heavy fragments, HF, a circular hodoscope, DELTA, which includes 30 high field Si detectors was used. The target-detector flight path was 60 cm and covered angles between  $5^\circ$  to  $10^\circ$ . (iii) Light charged particles ( $Z \leq 2$ ), LCP, and intermediate mass fragments ( $Z \leq 20$ ), IMF, were detected in an array called ISiS<sup>10</sup> which contains 162 triple detector telescopes in a tight geometry. Each telescope is composed of a gas-ionisation chamber, a fully depleted  $500\text{ }\mu\text{m}$  ion-implanted silicon detector and a 28 mm CsI(Tl) crystal. The geometrical acceptance is typically 70% and thresholds are better than 1 MeV. The charge, Z, resolution ranged up to 20. Mass resolution is obtained for those particles which punch through the Si crystal. (iv) An active collimator assembly was employed to veto the beam halo particles reaching ISiS.

Possibly the only distinct signal of the reaction primordial time that we captured with our set-up are the fast leading protons. In fig. 1 we give the overall proton multiplicity in ARCOLE with a minimum trigger of 2 particles in ISiS. The lower energy threshold in ARCOLE is approximately 50 MeV for protons. The dashed histogram gives the filtered proton multiplicity from the INC calculations and illustrates the fair description of the data.

## Analysis

Fig. 2 gives the superimposition of 26 mass vs velocity plots from DELTA for fragments under the minimum trigger condition. The mass was computed from the time between DELTA-ISiS and energy measurements. Correction due to time delay<sup>11</sup> and energy defect<sup>12</sup> were included. The latter was achieved through a coincident set-up with slowed down fission fragments in a separate measurement. Software velocity thresholds were set at  $0.25\text{ cm/ns}$ . We remark that in light ion induced reactions it is expected that the highest yield of HF should be for mass values close to that of the target<sup>13</sup>. The shift seen in the figure towards a mean mass of  $\sim 70$  amu is largely due

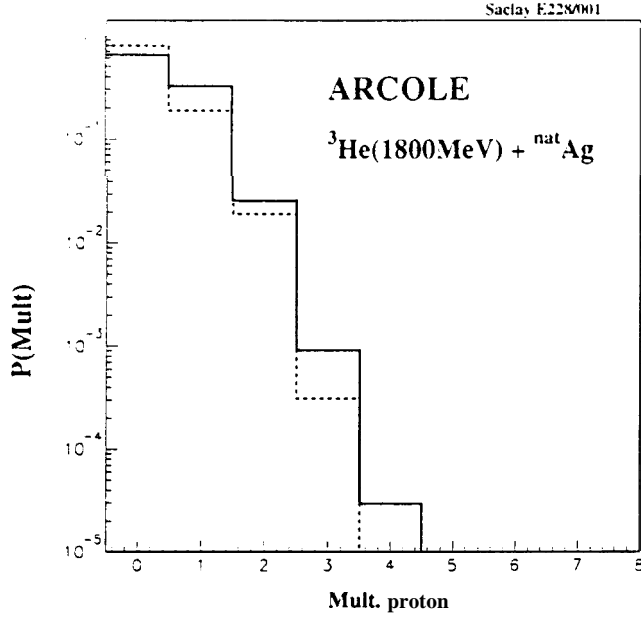


Figure 1. Probability for the multiplicity for high energy protons in the plastic wall ARCOLE with minimum trigger. The dashed histogram refers to the INC+EUGENE+ FILTER prediction.

to the target thickness energy threshold effects. It is important to note, however, that the fragments of interest here are the ones about  $(\text{mass}, \text{velocity}) = (45, 0.6)$ , which in fact are not strongly perturbed by the choice of target thickness.

Before detailing the analysis of how we assign excitation energies it is helpful to note and remark that; the data-model (INC+EUGENE+FILTER) comparison for events with minimum bias show a reasonable agreement for the global parameters. The total multiplicity,  $M_{\text{tot}}$ , is well described (fig. 3) for example, and this is also true for the total detected charge, IMF multiplicity,  $M_{\text{IMF}}$ , and so on. Less evident is the comparison of the linear momentum transfer. No inconsistency is found but the widths for the data and model are too wide to make a quantitative comparison. These results substantiate, in a circumstantial way, the model and allows us to examine our method to extract and correct the experimental excitation energies.

In this paragraph we describe the simple prescription we have employed to reconstruct what we call here, "thermal" excitation energy  $E_{th}$ , and mass,  $M_{th}$ . By thermal we refer to the energy and mass that is left over after the pre-equilibrium/cascade particles have escaped from the nuclear complex. Similarly, we consider  $Z_{th}$ . To compute these quantities experimentally we encounter three difficulties; (i) our set-up does not include a direct measure of neutron multiplicity and summed neutron kinetic energy. It is important to emphasise, that even with the present favourable choice of target, the calculated energy released through the neutrons can be as much as one half of  $E_{th}$ ; (ii) detector geometrical acceptance and in particular the mass resolution at low energy; (iii) establishing a criteria of rejecting particles within an event which are considered of non-thermal descent. These points are of no revelation. Different groups have developed different techniques to reduce source of error. Item (i) and (ii), for example, have

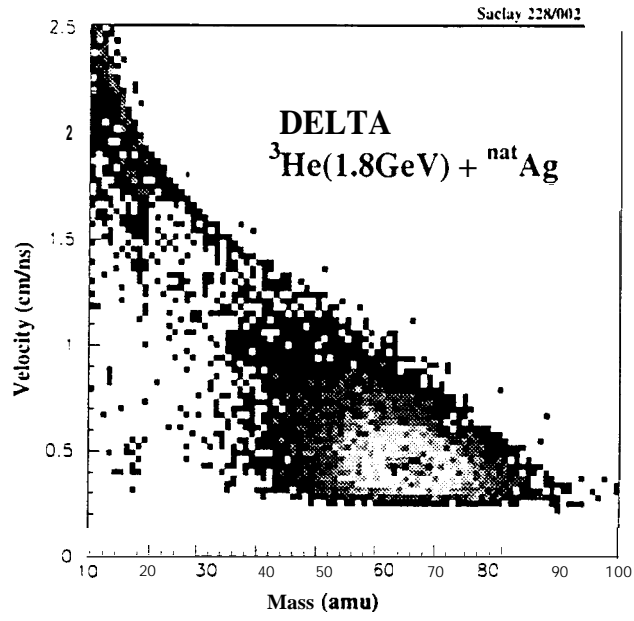


Figure 2. Residual mass versus velocity plot

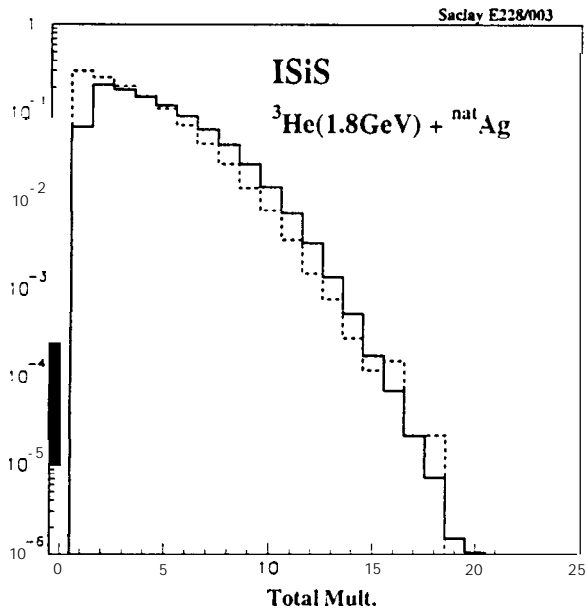


Figure 3. Probability for the total multiplicity data (solid histogram). The model predictions (dashed histogram) are normalised by a factor 1.2.

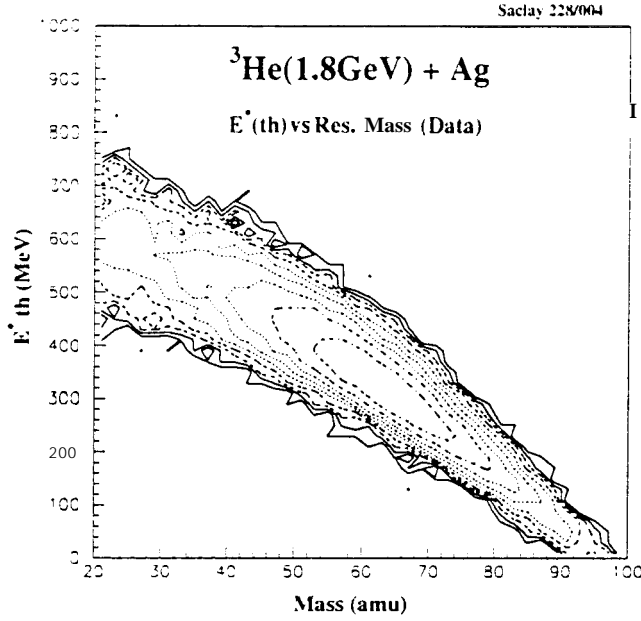


Figure 4. Experimental  $E_{th}$  as a function of residual mass.

been reasonably covered by the MBI + Ganil group at Cern <sup>14</sup>. The method employed here, and elsewhere <sup>15,16</sup>, to account for (iii) is somewhat brutal; within an event, only those LCPs with KEs smaller than a given value are treated. As for the IMFs they have practically no pre-equilibrium component. The total detected  $M_{tot}$  mass, which includes the HF is then obtained event-by-event. Where only the charge is detected, the mass is derived through a function giving values in the valley of stability. In the case of low energy  $Z=1$  these are given unit mass values. The total detected charge in ISIS and DELTA,  $Z_{th}$ , is obtained by transforming the HF mass into charge. The value of  $M_{th}$  is then obtained through a lookup table of ratios  $M_{th}/Z_{th}$  given by INC. The difference between  $M_{th}$  and  $M_{tot}$  gives the number of neutrons. Finally,  $E_{th}$  is obtained by summing over the KE of the selected particles and adding in the appropriate  $Q$ -values. For the neutrons we adopted a mean KE of 7.5 MeV/neutron, a value extracted from the INC+EUGENE code. It is important to stress that at this point we do not correct for geometrical acceptance. However with the above prescriptions and using the INC+EUGENE+FILTER as guide, we deduce the mean correction in extracted  $E_{th}$ . In the zone of interest, the correction in experimental  $E_{th}$  is of the order of 25%.

In fig. 4 we plot the residual mass as a function of  $E_{th}$ . This was obtained by demanding software triggers of a HF in DELTA for  $M_{IMF} = 0$ . Note that this condition on  $M_{IMF}$ , is equivalent to having  $\sim 40\%$  probability to have, in the event, an undetected IMF. Further, the HF velocity and accompanying charge distribution for events which do have IMFs in coincidence indicates that fission is not a competing process for the higher  $E_{th}$ . Projecting a  $E_{th}$  spectrum with a window on the residual mass of (40,45) yields a Gaussian like peak. Correction for the acceptance gives a mean excitation energy of 625 MeV. As is evident from the figure, shifting the mass window to lower values increases this  $E_{th}$ .

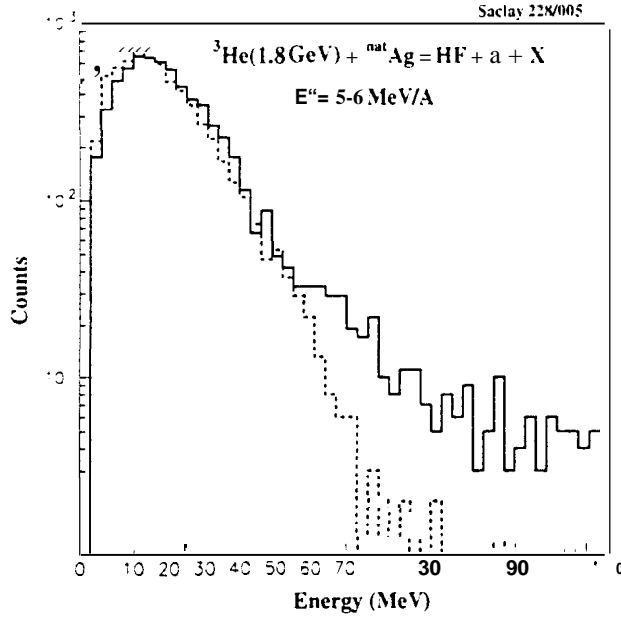


Figure 5. Energy spectrum for alphas in coincidence with HF for  $\epsilon^* = 5-6 \text{ MeV/A}$ .

The HF-coincident  $\alpha$  (note no mass resolution in ISIS for  $Z=2$  at low energy) particle spectra as a function of  $\epsilon^* = E_{th}/M_{th}$  and  $M_{th} \geq 35$  have been extracted and compared with the INC+EUGENE+FILTER calculations. In fig. 5 the spectra with a window of  $\epsilon^* = 5-6 \text{ MeV/A}$  are compared. Other comparisons, such as mass of the HF, HF- $M_{IMF}$  or HF- $M_{tot}$ , are of equivalent merit.

## Conclusion

In conclusion, we report on an experimental study of  ${}^3\text{He}(1.8 \text{ GeV}) + {}^{nat}\text{Ag}$  where we detect LCP + IMF with a large geometrical coverage in conjunction with heavy fragments and fast protons in the forward direction. The global parameters with this configuration are effectively reproduced with an INC + EUGENE description. An attempt is made to extract the highest excitation energy reached that decays in a non-multifragmentary fashion. Values of  $\epsilon^*/\text{binding energy}$  of 78% are obtained. The data-model comparison for the  $\alpha$ -spectra indicate that at relatively high excitation energy the system can still decay through an evaporative process.

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