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DAPNIA/SPhN-96-24 08/1996

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D. Lacroix and R. Dayras

DAPNIA

Département d'Astrophysique, de Physique des Particules, de Physique Nucléaire et de l'Instrumentation Associée
CEA/Saclay F-91191 Gif-sur-Yvette Cédex

Couverture : Sylvie Colard et Marc Savalle

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Adresse : DAPNIA, Bâtiment 141
CEA Saclay
F -9 I I 9 I Gif-sur-Yvette Cedex

DLNA: a simple One-dimensional dynamical model as a possible interpretation of fragment size distribution in nuclear multifragmentation

D. Lacroix*, R. Dayras.

CEA, DAPNIA/SPhN CE-Saclay, 91191 Gif-Sur-Yvette Cedex, France.

(August 21, 1996)

Abstract

We discuss the possibility of interpreting multifragmentation data obtained from Heavy-Ion collisions at intermediate energies, by a new type of model: the DLNA (Dynamical Limited Nuclear Aggregation). This model is connected to a more general class of models presenting Self-Organization Criticality (SOC). We show that the fragment size distributions exhibit a power-law dependence comparable to those obtained in second-order phase transition or percolation models. We have also studied fluctuations in term of scaled-factorial moments and cumulants: no signal of intermittency is seen.

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*Service militaire actif, Scientifique du Contingent.

1.Introduction

In this paper, we discuss the possible origins of multifragmentation at intermediate energies in Heavy-Ion collisions. This phenomenon is an intermediate type of reactions that occurs when the energy deposited in the system is sufficient to break it into many pieces some of them being Intermediate Mass Fragments (IMF: $Z \geq 3$). When selecting this class of events and averaging over the events, the averaged probability $P(s)$ of having a fragment of mass s is very close to a power-law function:

$$P(s) \sim s^{-\tau}, \tag{1}$$

τ is the power-law exponent associated with the averaged distribution. More generally, it is possible to define a parameter ε that characterizes the distance from the optimal value of a characteristic parameter x. The general form of the number of fragments with mass s is

$$N(s,\varepsilon) \sim s^{-\tau} \cdot f(s\ \varepsilon^{\sigma}) \qquad\qquad \tau, \sigma > 0, \tag{2}$$

where $\varepsilon = x - x_c$ with the condition $f(0) = cst$. In our case, ε was tentatively associated with the excitation energy deposited into the system. This power-law is well-known to exist in second-order phase transition [1] or in percolation models [2] in the continuous limit at the critical value x_c of the parameter. Furthermore, an event by event statistical study of fluctuations in the fragment size distribution shows that break-up of excited nuclei is comparable to percolation on a network [4] even if the different signals are not clearly understood [5,6]. Similarities between critical phenomenon and Heavy-Ion collisions at intermediate energy were interpreted as a possible signal of phase transition in nuclear matter [3,4] although it is not obvious to extrapolate properties from finite size systems to infinite ones. Up to now, no real proof of a second-order phase transition emerging from nuclear data exists. It is thus interesting to look for other possible interpretations of the nuclear data properties. In particular, we want to discuss another class of models: the Self-Organized Critical models (SOC) [7]. The concept of self-organized criticality was first introduced to describes systems that evolves "spontaneously" to critical behaviour without the tuning of some parameter(s)

as it is the case in second order phase transition. In particular, systems can evolve spontaneously through SOC and give scaling properties as for instance power-law dependence of some characteristic entity sizes [8]. It is interesting also to notice that those systems are considered far from equilibrium and this enables us to look for a possible dynamical origin of the fragment size distribution. Following the work of Hwa in Heavy-Ion collisions at high energy [9], we propose a simple one-dimensional model of dynamical cluster formation: the DLNA (Dynamical Limited Nuclear Aggregation). In this models, we consider a set of clusters initially randomly distributed on a lattice with given kinetic energies. According to their relative velocity, clusters evolve on the lattice and can collide. Depending on their relative energy, the clusters may break into many pieces or fuse. A final mass distribution of fragments is obtained. This model, based on some simple rules, is comparable to the random walk of N interacting bodies on a lattice (Appendix A presents our method of considering bodies numerically).

In the next section, we present the general features of the model. We have tried to incorporate as much as possible, realisctic aspects in order to enable comparison with experimental data. In the last section, we present first results and discuss some possible improvement of the model.

The DLNA model

After the initial collision of the two clusters, one or more excited nuclei are created. The purpose of DLAN is to modelize a possible dynamical way of break-up of this system. Considering that our system is azimuthally symmetrical, we only take the radial position of clusters. For simplicity, we consider that clusters are located on an oriented one-dimensional lattice, they move, collide, break according to some simple rules. In the present section, we present these rules, part of them are included in ref. [9]. Our model is developed with a particular care of physical picture so that it could easily be compared with experimental data.

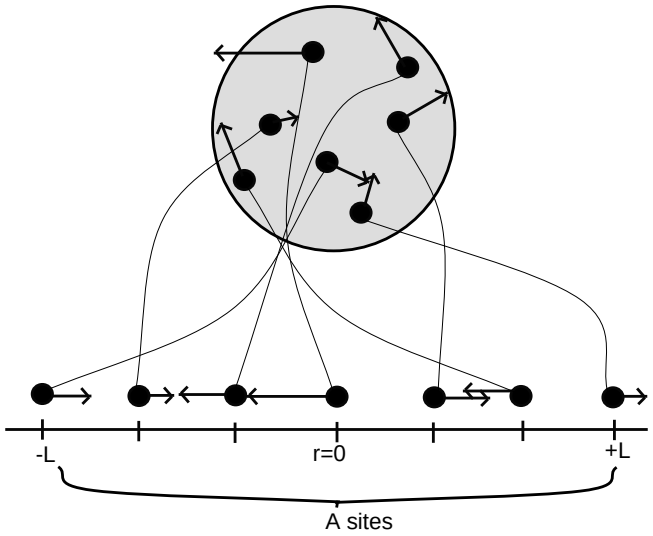

FIG. 1. *Illustration of the initialization.* Positions are randomly distributed, linear momenta are also randomly distributed according to the method explained in appendix B.

The region of the interaction is mapped to a one-dimensional interval between $-L$ and $+L$ at the initial time. This region is divided into A sites, where A is the size of the heated system (see fig. 1).

Initialization of positions and linear momenta- At the initial time, positions of nucleons within the nucleus are randomly distributed in the interval $r = -L$ and $r = +L$ in order to have only one nucleon per site. At the beginning, each nucleon is considered as a cluster of size 1 (no distinction is made between protons and neutrons). The momenta are randomly distributed with only the total energy conservation constraint. (In appendix A, we present our method of considering clusters numerically.)

$$\sum_{i=1}^{A} \frac{{p_i}^2}{2m_i} = E_{tot} \tag{3}$$

In this formula, E_{tot} is the initial energy deposited in the system. The velocity could be positive of negative, in order to have the total momentum equal to zero on average (at the first step $r = 0$ is the center of mass of the system. The excitation energy (E_i^* of a single particle is equal to zero). Here, excitation energy is the difference of energy from the ground

state ($E^* = 0$ in the ground state, including internal energy) so that the total excitation energie is identical to E_{tot}. We were particularly careful, during the initialization, in the way of distributing E_{tot} among the nucleons. This method, detailed in appendix B, provides a simple way of covering uniformally the momentum phase space with the energy conservation constraint. After initialization, we have A clusters of size 1 with some momentum and no excitation energy ($energy(k) = 0$ see appendix B).

Reactions: fusion or break-up- After having the initialization phase, the clusters can collide. Considering that the interaction is a short-range interaction, we consider that clusters collide if they overlap on the lattice. Then, we have retained essentially two possible collision effects as illustrated in fig. 2.

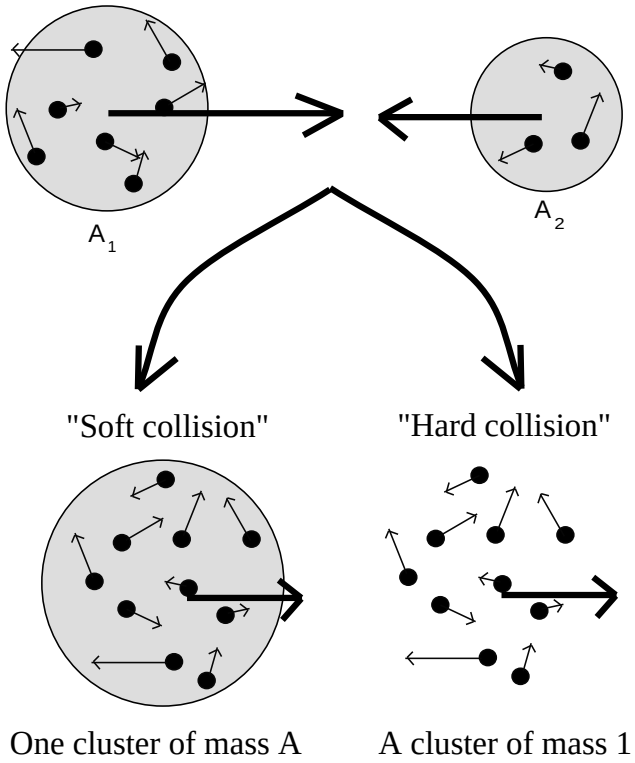

FIG. 2. *Reactions between clusters.* "Soft collisions" represent a fusion between the two clusters and "Hard collisions" represent a total breakup of the two clusters. ($A = A_1 + A_2$)

For classifying the type of reaction, we introduce a parameter p_E. Considering the energy associated with the collision ($energy(1+2)$) calculated in appendix B, we say that

the reaction is soft if $energy(1+2) < p_E \times (A_1 + A_2)$, where A_1 and A_2 are the masses of the initial clusters. In this case, the reaction is a fusion, and clusters form a new cluster of mass $A = A_1 + A_2$, whose excitation energy is equal to $energy(1+2)$ and whose momentum and position are those of the center of mass of the initial system. On the contrary, for hard collision $energy(1+2) > p_E \times (A_1 + A_2)$, the system break into $A_1 + A_2$ nucleons and the energies, momenta and positions are distributed according to the method of appendix B. For p_E, we have taken 8 MeV per nucleon, which is approximately the binding energy per nucleon in systems with mass $A > 12$. Note that, it is possible to improve the model with p_E depending of the colliding clusters sizes.

Expansion.- We have to simulate the random walk of N clusters on a lattice. We know that these clusters have velocities with different signs, so they can expand along two directions. At each step, we search which cluster has the biggest velocity $|\vec{v}_{max}|$. Then considering the i^{th} cluster, whose velocity is $\vec{v_i}$, we draw a random number x_i. If

$$0 \leq x_i \leq \left| \frac{\vec{v_i}}{\vec{v}_{max}} \right|$$

we do

$$position(i) = position(i) \pm 1$$

according to the sign of the velocity. If the relation above is not verified for x_i, $position(i)$ do not change. With this method, the more the program evolve, the faster it runs. (As we will see in the next section, the number of clusters decreases with time). The only inconvenient, is that we loose the notion of time because $|\vec{v}_{max}|$ changes from one step to the next.

Boundaries.- We introduce a freeze-out volume in our system. The clusters move on a lattice whose size is $2 \times L_{max}$, we have taken $L_{max} = 5 \times L$. If a cluster reaches $+L_{max}$ or $-L_{max}$ it is considered as being inactive, then we count it as a cluster in the event. One event is completed when all clusters are out of the lattice.We have numerically verified that the probability of having a reaction out of the lattice is negligeable. For each event, we have a repartition of clusters $(n_1, n_2, \cdots, n_A)$, where n_s is the number of clusters of size s. Note that, detected clusters are excited, it is then possible to imagine a de-excitation process after

the DLNA model. Figure 3 summarizes the architecture of the program.

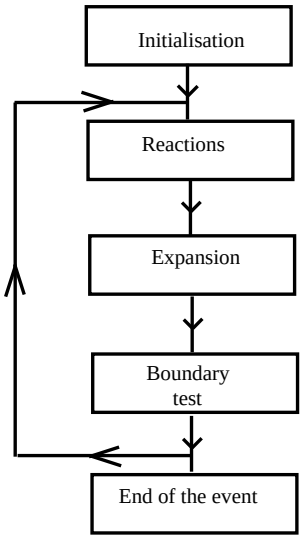

FIG. 3. *Event generator algorithm.*

After having explained the model, we present the first results obtained by statistical analysis.

Results and discussion

Size distribution

One of our first interest was to know if such a model could also exhibit an average size distribution with a power-law dependence. We have generated about 10000 events for three different energies initially deposited in a system of size $A = 50$: $E_{tot} = 8, 80, 1000\ MeV$ per nucleon. Figure 4 shows the fragment mass distribution. At low energy, an heavy residue is present. At high energy, only small fragments remain. At intermediate energy, we can clearly see a power-law dependence of the fragment size distribution for fragments with size $3 \leq s \leq 30$. The mass distribution can be fitted by a power-law function with an exponent $\tau \simeq 2.2$ which is very close to the value obtained in percolation models at the critical point [2]. The energies initially deposited in the system are not very realistic, a change of p_E can give a better approximation in order to recover experimental data.

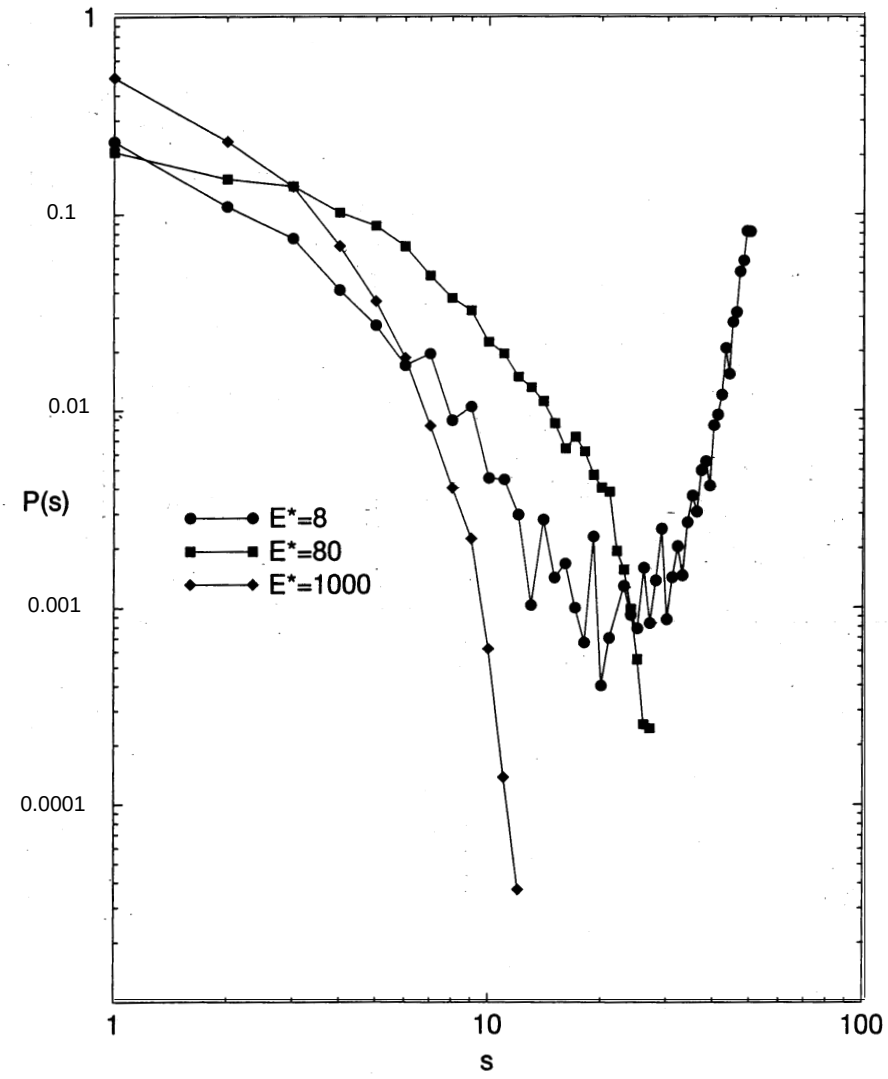

FIG. 4. *Size distribution of the DLNA model.* Log-log plot of the size distribution ($P(s)$ being the probability of having a cluster of size s) as a function of s, for three different initial conditions: $E_{tot} = 8, 80, 1000$ MeV per nucleon.

We conclude from this first study that power-law dependence of the averaged fragment size distribution can have many different origins. Second order phase transition is one of the most famous but it seems that models connected with SOC are also able to explain results of multifragmentation experiments in nuclear physics.

Intermittency in DLNA

We have also studied fluctuations around the average fragment size distribution with the scaled-factorial moments [4,10]. No intermittency is found in our model. We have also calculated scaled-factorial cumulants adapted to nuclear physics [6], unlike in percolation models, all cumulants are non-equal to zero. Normally cumulant of order p is equivalent to the p-body correlation function. This is not obvious in our case because of the difficulty of finding an unambiguous normalization for the factorial moments and cumulants. Nevertheless, having correlations between more than two clusters is not surprising taken into account the way of forming them with the energy conservation constraint described in appendix B.

Discussion

Although no intermittency is seen in the DLNA model, this model seems to be a good candidate for explaining nuclear multifragmentation data. In particular, we can imagine to create a two-dimensional model to take into account shape effects. We note also that only a part of the information is analyzed in this paper. We could thus have information about the energy transported by clusters or distribution of momenta after DLNA. This model would also be helpful for studying pre-equilibrium emission, if one imagine a temperature gradient at the initial stage of the reaction. In particular, it would be interesting to know if final created fragments have a memory of the gradient at the initial stage of the reaction.

The question of the origin of the power-law spectra in nuclear multifragmentation experiences is of great interest. Up to now, no unambiguous proof of second-order phase transition in nuclear matter exists. It is natural to search for alternative models in order to explain such type of behaviour. In particular, it would be very useful to find new statistical variables that could disentangle models from oneanother. Furthermore, studying variables that could distinguish between fluctuations coming from statistical effects and those coming from dynamical ones would be very interesting. Our model, because of its simplicity could be a good candidate for testing new variables.

Appendix A: Numerical traitment of clusters

For clarity, we explain the way of considering clusters in the DLNA model. The numerical traitment of N random walkers on a lattice needs a lot of numerical efforts. It is important to have an optimized program. We explain here the method of considering clusters. After initialization, an array, $Clust(k)$, of size 5 is associated to each nucleon.

$$\mathcal{C}lust(k)[5] \begin{bmatrix} C(k) \\ mass(k) \\ position(k) \\ momentum(k) \\ energy(k) \end{bmatrix}$$

$C(k)$ represents the cluster number to which the nucleon k belongs. At the initial time, all nucleons are considered as a single particle cluster, so all $C(k)_{(k=1,\cdots,A)}$ and $mass(k)$ are respectively equal to k and 1. $Position(k)$ is the site where the nucleon is and $energy(k)$ is the excitation energy associated with the cluster k (at the first step, the energy associated to each clusters is only "kinetic" energy, thus $energy(k) = 0$.

During the program execution, nucleons can form clusters with mass greater than 1. If we consider that n nucleons with number $(k_1, k_2 \cdots, k_n)$ form a cluster of size n, we associate to the new cluster, an array $Clust(k)$ belonging to one of the original nucleon with the number k such that:

$$k = \min_{i=1,n}(k_i)$$

all $C(k_i)$ are thus re-initialized to k. The signification of $C(k)$ is now clear:

-if $C(k) = k$, $Clust(k)$ contains information about a "real" cluster. In the above case, $mass(k)$ is equal to k. $Position(k)$ and $momentum(k)$ are those of the center of mass of the n nucleons. $Energy(k)$ are calculated with the momentum and the energy associated to the original nucleons according to the method explained in the appendix B. We see that $momentum(k)$ corresponds to the collective displacement of the cluster and $energy(k)$ is the energy associated with the motion of the nucleons inside the clusters.

-if $C(k) \neq k$, it means that the nucleon k belongs to the cluster $C(k)$, all the information is in $Clust(C(k))$.

When 2 clusters k_1 and k_2 fuse, information about the new cluster are in $Clust(k) = \min(k_1, k_2))$ and all numbers $C(k_i)$ associated to nucleons belonging to $\sup(k_1, k_2)$ are initialized to $\min(k_1, k_2)$. On the contrary, when a clusters k breaks into many nucleons $(k_i, i = 1, \cdots, n)$, all $C(k_i)$ are re-initialized to k_i, and nucleons are considered again as single particle clusters.

The interest of this type of programming is twofold. First, if we want to know information about clusters on the network, it is sufficient to test if $C(k) = k$. On the contrary, if we want to get more information on the property of nucleons forming the cluster k, we test if $C(k_i) = k$.

Appendix B: Momentum and Energy calculations

In this section, we present the way of calculating excitation energy of clusters created during reactions. The initialization at the first step is equivalent to the case of "hard collision" explained below. First, we consider the case of two cluster (k_1, k_2) colliding (see Fig. 2). Each cluster has a certain excitation energy $energy(k_1)$ and $energy(k_2)$ that reveals the internal motion of nucleons inside each cluster. If we consider the system formed by the two clusters just before the collision, we can associate to the total system an excitation energy defined as follow

$$energie(1+2) = E_{rel} + energy(k_1) + energy(k_2) \tag{4}$$

in this formula E_{rel} represent the relative energy of the clusters, it could be expressed by

$$E_{rel} = \sum_{i=1}^{2} \frac{1}{2} m_i (\vec{v}_i - \vec{v}_{cdm})^2 \tag{5}$$

this expression could be easily generalized for more than two clusters. $\vec{v}_i$ is the velocity in the laboratory and $\vec{v}_{cdm}$ is the center of mass velocity. E_{rel} represent a measurement of the violence of the collision. In our model two cases are possible (see fig. 2).

- In the case of *"soft" collisions*, the reaction is considered as a fusion. The mass of the

new cluster is the sum of the masses of the colliding clusters. The energy associated with the new cluster is simply equal to $energy(1+2)$. The position and the momentum are taken to be the position and the momentum of the center of mass of the total system just before the reaction.

- In the case of *"hard" collisions*, $energy(1+2)$ is big enough to break all bounds between nucleons belonging to both nuclei (see fig. 2). If the sizes of the clusters are respectively A_1 and A_2, we consider that we form $A_1 + A_2$ clusters of mass 1. The problem is then to distribute the $energy(1+2)$ among this nucleons. Note that, in the initialization case, $energy(1+2) = E_{tot}$ and $A = A_1 + A_2$. We present here the method we have used [11]. For simplicity, we consider only the case $A_1 = 1$ and $A_2 = 1$, for each nucleons, a random number x_1 and x_2 is drawn ($x_1, x_2, \cdots, x_{A_1+A_2}$ in the general case). A momentum is calculated in the center of mass of the system according to the formula

$$p_i = \sqrt{2.m_i \left(\frac{energy(1+2)}{A_1 + A_2} \right)} \tag{6}$$

If we consider the momentum space, where the i^{th} axis is associated to the momentum of p_i, our system could be seen as a point of coordinate $(p_1, p_2, \cdots, p_{A_1+A_2})$ (see. fig 5 for the case $A_1 + A_2 = 2$). The sign of p_i is chosen in order to have $\sum_{i=1}^{A_1+A_2} p_i \simeq 0$. At this stage, the energy conservation constraint is not satisfied. We calculate the coefficient α defined by

$$\alpha = \sqrt{\frac{energy(1,2)}{\sum_{i=1}^{A_1+A_2} \frac{{p_i}^2}{2.m_i}}} \tag{7}$$

If $\alpha \geq 1$ then we reject this step and choose new random numbers. If $\alpha \leq 1$ then we calculate new nuclear momenta given by $p'_i = \alpha.p_i$, with $i = 1, (A_1 + A_2)$. We then calculate $momentum(i)$ in the laboratory. This method enables to fill uniformally the momentum space with the total energy conservation constraint. Note that, in the case of a violent collision or in the initialization stage, the energies $energy(k_i)$ are re-initialized to 0 (see appendix A).

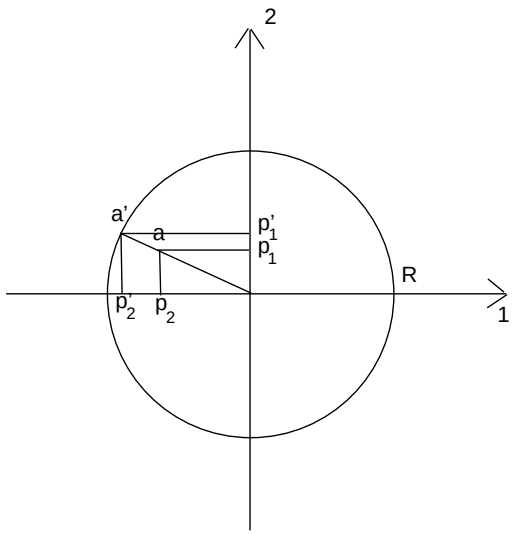

FIG. 5. *Illustration of the partitioning energy method.* We present the case $A_1 = 1$ and $A_2 = 1$. a is the point before multiplying by the factor α and $a^{'}$ the point after. The coordinates of $a^{'}$ correspond to the momenta of nucleons in the center of mass of the colliding clusters (the conservation of the total energy constraint is illustrated by the circle).

ACKNOWLEDGMENTS

One of us (D.L.) thanks the SPhN for its hospitality and physicists for their kindness. He wants particularly to thank P.Buchet, M. Colonna, J.L. Charvet, J. Cugnon, R. Legrain, L. Nalpas, R. Peschanski, E. Pollacco, C. Volant for having introduced him to nuclear physics and for everyday help and discussions.

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