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Extraction of proton wavefunction from Compton scattering

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Calculations are presented for real Compton scattering on the nucleon in a PQCD formalism by starting from different quark distribution amplitudes of the nucleon. In a second step we present the formalism to extract this distribution amplitude in a model independent way from hard exclusive Compton scattering data.

1 Introduction : hard exclusive reactions

Exclusive reactions at high momentum transfer can be used as a new tool in order to extract the quark distribution amplitudes of the nucleon. Besides the study of hadron form factors, the next simplest process to probe the structure of the nucleon is Compton scattering off protons at large momentum transfer.

Within the formalism of Brodsky-Lepage [1], we present a calculation [2] of the amplitudes for Compton scattering on a proton at large momentum transfer as the convolution of the distribution amplitudes of the nucleon and the hard scattering operator. In the calculations, care was taken to integrate the propagator singularities, by performing the calculations using two independent integration methods. The different numerical implementation of these singularities is very probably at the origin of the two different results obtained by two groups [3 4] who performed these calculations before.

In order to extract the distribution amplitudes in a model independent way from proton Compton scattering data at a future high energy accelerator like ELFE [5], a perturbative QCD (PQCD) formalism is proposed in which the distribution amplitude of the proton is expanded in terms of a given set of basis functions. Contrary to Refs.[34], which used a given model distribution amplitude based on the knowledge of the few lowest moments from QCD sum rules, in the present approach the unknown coefficients are considered as parameters to be extracted from experiment. This will allow to extract the nucleon distribution amplitude directly from experiments in the kinematical range of a proposed 15-30 GeV electron accelerator [5].

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[a]Talk presented at the Workshop on Virtual Compton Scattering VCS 96, Clermont-Ferrand, June 1996.

2 PQCD formalism for Compton scattering

The observables for Compton scattering are evaluated from the Compton scattering amplitude which depends on the helicity of the participating particles : $h, h' = \pm\frac{1}{2}$ for incoming and outgoing protons (with momenta p and p$'$) and $\lambda, \lambda' = \pm 1$ for incoming and outgoing photons (with momenta k and k$'$). In leading order PQCD, only the amplitudes where hadron helicity is conserved are nonzero. Hadron helicity conservation is an important consequence of PQCD at high momentum transfer [6]. On the quark level, it results from neglecting of the small (current) quark masses and from the vector coupling of the gauge particles. On the hadron level, helicity conservation results in addition to the previous arguments from the dominance of valence Fock states with zero angular momentum projection. From the eight resulting nonzero helicity amplitudes only 3 are independent due to parity invariance and time reversal invariance.

The leading order PQCD calculation of the Compton scattering amplitude is performed in the framework of the factorization hypothesis of Brodsky-Lepage [1]. This hypothesis can be visualized as in Fig.1. The amplitude for a hard exclusive reaction is factorized as a product of : (i) the distribution amplitude $\phi(x_1, x_2, x_3)$ for the proton assuming only valence quarks with momentum fractions x_1, x_2, x_3 $(x_1+x_2+x_3 = 1)$, (ii) a hard scattering amplitude T_H calculable in PQCD, (iii) the distribution amplitude $\phi(y_1, y_2, y_3)$ that the three outgoing quarks reform into a proton so as to contribute to the exclusive channel.

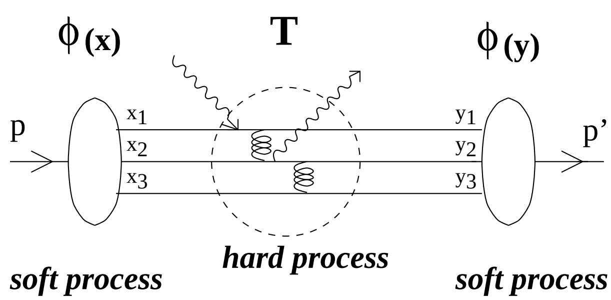

Figure 1: *Factorisation of the hard scattering Compton amplitude.*

The relation between the distribution amplitude ϕ and the proton state (in the infinite momentum frame) which is made up of three collinear moving valence quarks is given by

$$
\begin{aligned}
|p, h = +\frac{1}{2}\rangle = \quad & f_N \int dx_1 \int dx_2 \int dx_3 \, \delta(1 - x_1 - x_2 - x_3) \\
& \times \frac{1}{8\sqrt{6}} \, \{\phi_1(x_1, x_2, x_3) \, |u_\uparrow(x_1) \, u_\downarrow(x_2) \, d_\uparrow(x_3)\rangle \\
& \qquad + \phi_2(x_1, x_2, x_3) \, |u_\uparrow(x_1) \, d_\downarrow(x_2) \, u_\uparrow(x_3)\rangle \\
& \qquad + \phi_3(x_1, x_2, x_3) \, |d_\uparrow(x_1) \, u_\downarrow(x_2) \, u_\uparrow(x_3)\rangle\}. \quad (1)
\end{aligned}
$$

As the proton state has to be symmetric in flavor-spin variables, there are two relations between the three amplitudes ϕ_1, ϕ_2 and ϕ_3 :

$$
\begin{aligned}
\phi_2(x_1, x_2, x_3) \; &= \; -\left[\phi_1(x_1, x_2, x_3) + \phi_1(x_3, x_2, x_1)\right] , \\
\phi_3(x_1, x_2, x_3) \; &= \; \phi_1(x_3, x_2, x_1) \, . \qquad (2)
\end{aligned}
$$

The proton state as given by Eq.1 is anti-symmetric in color by taking the three-quark states as

$$
|u_\uparrow(x_1) \, u_\downarrow(x_2) \, d_\uparrow(x_3)\rangle \; \equiv \; \frac{1}{\sqrt{6}} \, \epsilon_{ijk} \, |u^i_\uparrow(x_1) \, u^j_\downarrow(x_2) \, d^k_\uparrow(x_3)\rangle \, , \qquad (3)
$$

where i, j, k are the color indices. The mass scale in the proton state of Eq.1 is given by f_N (wavefunction at the origin). This is a nonperturbative parameter for which estimates exist within the framework of QCD sum rules [7] [8] [9] : $f_N \approx 0.52 \, . \, 10^{-2} \, GeV^2$. This value will be used in the calculations of this paper. Furthermore, the independent distribution amplitude ϕ_1 in Eq.1 will be denoted by ϕ_N in the following. The distribution amplitude can be expanded in terms of orthonormal Appel polynomials $A_n(x_1, x_2, x_3)$ [7] as

$$
\phi_N(x_1, x_2, x_3, Q^2) = 120 x_1 x_2 x_3 \sum_n a_n(Q^2) A_n(x_1, x_2, x_3) \, , \qquad (4)
$$

where the momentum transfer (Q^2) dependence is shown explicitely. Within the framework of QCD sum rules, theoretical calculations of the first six expansion coefficients a_n at $Q^2 \approx 1 - 2GeV^2$ exist in the literature [7 8 9]. This has motivated the construction of several model distribution amplitudes. In this paper we will perform calculations with the distribution amplitudes CZ [7], COZ [8] and KS [9], which are given by (the slow evolution in Q^2 is neglected)

$$
\begin{aligned}
\phi_N^{CZ}(x_1, x_2, x_3) = \quad & \phi_{as} \, \{ \, 1.69 \, - \, 9.26 \, x_1 \, - 10.94 \, x_3 \\
& \quad + 22.70 \, x_1^2 \, + \, 13.45 \, x_3^2 \, + \, 9.26 \, x_1 x_3 \, \} \, , \qquad (5) \\
\phi_N^{COZ}(x_1, x_2, x_3) = \quad & \phi_{as} \, \{ \, 5.880 \, - \, 25.956 \, x_1 \, - 20.076 \, x_3 \, \} \, ,
\end{aligned}
$$

$$+36.792\, x_1^2 \,+\, 19.152\, x_3^2 \,+\, 25.956\, x_1 x_3 \; \} \; , \quad (6)$$

$$\phi_N^{KS}(x_1,x_2,x_3) = \quad \phi_{as} \; \{ \; 8.40 \,-\, 26.88\, x_1 \,-\, 35.28\, x_3$$
$$+35.28\, x_1^2 \,+\, 37.80\, x_3^2 \,+\, 30.24\, x_1 x_3 \; \} \; , \quad (7)$$

where the asymptotic distribution amplitude ϕ_{as} is given by $\phi_{as}(x_1,x_2,x_3) = 120\, x_1\, x_2\, x_3$. The distribution amplitudes CZ, COZ and KS have a caracteristic shape and predict that in a proton, the u-quark with helicity along the proton helicity carries about 2/3 of its longitudinal momentum.

Coming back to Fig.1, the factorized expression of the Compton helicity amplitudes can be written down as

$$M_{\lambda\lambda'}^{hh'} \quad = < p', h' | T_H(k',\lambda';k,\lambda) | p, h >$$
$$= \int dx_i dy_j \; \phi_N^*(y_j) \; T_H(h',\lambda',y_j;h,\lambda,x_i;s,t) \; \phi_N(x_i) \; , \qquad (8)$$

where s and t are the Mandelstam invariants. The evaluation of Eq.8 requires a four-fold convolution integral since there are two constraint equations ($x_1 + x_2 + x_3 = 1$ and $y_1 + y_2 + y_3 = 1$). In writing down Eq.8 we have assumed a sufficiently large momentum transfer so as to neglect the transverse momentum dependence in the wavefunction.

For the computation of the hard scattering amplitude T_H in Eq.8, the leading order PQCD contribution corresponds to the exchange of the minimum number of gluons (in the present case two) between the three quarks. The number of diagrams grows rapidly with the number of elementary particles involved in the reaction (42 diagrams for elastic nucleon form factor calculations, 336 diagrams in the case of Compton scattering). The calculation of the expressions of the diagrams for Compton scattering was double checked by us. Despite the large number of diagrams, the calculation of T_H is a parameter free calculation once the scale $\Lambda_{QCD} \approx 200 MeV$ in $\alpha_s(Q^2)$ is given. In a first stage, we simplified the calculations by approximating the x, y dependence in the gluon virtuality Q^2 in $\alpha_s(Q^2)$ by their average values for a given distribution amplitude. In a next stage, the full x, y dependence of $\alpha_s(Q^2)$ will be taken into account as in Ref.[10] for the calculation of the elastic form factors. In the latter case however, care has to be taken of the end point region ($x_i \approx 0, x_i \approx 1$) where the asymptotic formula for $\alpha_s(Q^2)$ is no longer valid. Let us also mention that the hard scattering amplitude has the s-dependence ($M \sim s^{-2}$) which leads to the QCD scaling laws [11]. For Compton scattering, PQCD predicts a behaviour for the cross section at fixed angle as $\frac{d\sigma}{dt} \sim s^{-6}$.

Once the hard scattering amplitude T_H is evaluated, the four-fold convolution integral of Eq.8 has to be performed to obtain the Compton helicity amplitudes. The numerical integration requires some care because the quark and/or gluon propagators can go on-shell which leads to (integrable) singularities. The different numerical implementations of these singularities are probably at the origin of the different results obtained in two previous calculations [3, 4]. In Refs.[3, 12], the propagator singularities were integrated by taking a finite value for the imaginary part $+i\epsilon$ of the propagator. Then the behaviour of the result was studied by decreasing the value of ϵ. To obtain convergence with a practical number of samples in the Monte Carlo integration performed in Refs.[3, 12], the smallest feasible value for ϵ was $\epsilon \approx 0.005$. In Ref.[4] the propagator singularities were integrated by decomposing the propagators into a principal value (off-shell) part and an on-shell part. To compare these methods, we implemented both of them and found for the $+i\epsilon$ method differences of the order of 10% for every diagram as compared with the result of our final method. It is not surprising that when summing hundreds of diagrams an error of 10% on every diagram can easily be amplified. To have confidence in the evaluation of the convolution of Eq.8, we compared the principle value integration method with a third independent method. This third method starts from the observation that the diagrams can be classified into four categories depending upon the number of propagators which can develop singularities : in the present case this number is 0, 1, 2 or 3. Besides the trivial case of zero singularities which can be integrated immediately, the diagrams with one or two propagator singularities can be integrated by performing a contour integration in the complex plane for one of the four integrations. For the most difficult case of three propagator singularities, we found it possible to evaluate it by performing two contour integrations in the complex plane. In doing so, one achieves quite a fast convergence because the integrations along the real axis are replaced by integrations along semi-circles in the complex plane which are far from the propagator poles. We checked this method by also implementing the principal value integration method and found the same result up to 0.1% for each type of singularity. The principal value method was found to converge much slower and is more complicated to implement especially for the case with three singularities due to the fact that the three principal value integrals are coupled.

3 Results with a model distribution amplitude

Having exposed the PQCD calculational framework for Compton scattering, we now come to the calculations which are performed with several model distribution amplitudes. Although the energy at which existing experiments were performed ($E_\gamma \approx 5$ GeV) is probably too low to motivate a PQCD calculation, we nevertheless show the comparison with these existing data for illustrative purpose.

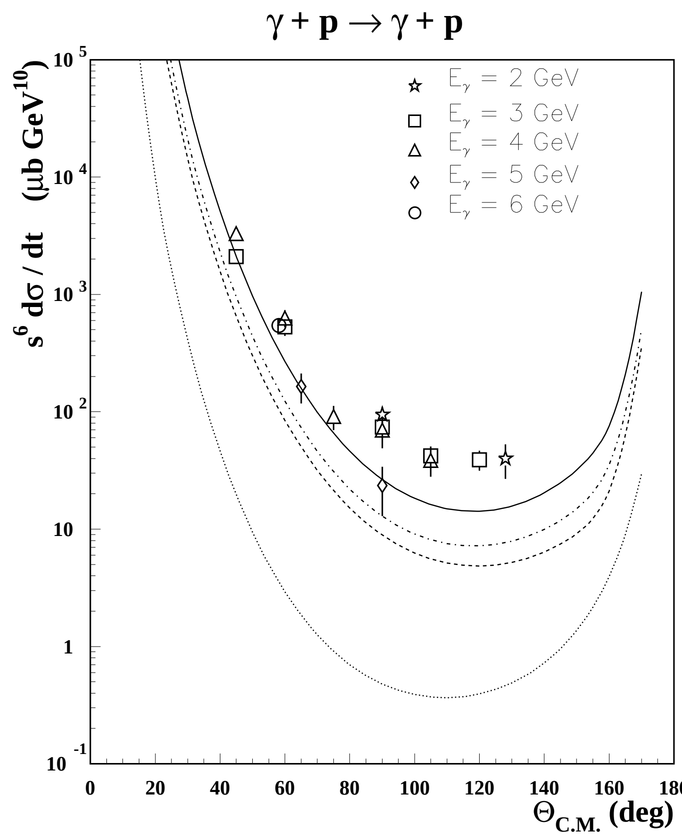

Figure 2: *Unpolarised Compton cross section on proton for different nucleon distribution amplitudes : KS (full line), COZ (dashed-dotted line), CZ (dashed line) and dotted line is the result using the asymptotic distribution amplitude. Data are from Refs.* [13] [14] [15].

In Fig.2, our result for the unpolarized real Compton differential cross section (multiplied by the scaling factor s^6) is shown as function of the photon cm angle. We first remark that the result with the asymptotic distribution amplitude is more than one decade below the results obtained with the QCD sum rules motivated amplitudes KS, COZ, CZ. The results with KS, COZ and CZ show a similar caracteristic angular dependence which is asymmetric around 90^o. Note that in the forward and backward directions, which are dominated by diffractive mechanisms, a PQCD calculation cannot be applied. Comparing the results obtained with KS, COZ and CZ, one notices that although these distribution amplitudes have nearly the same lowest moments, they lead to differences of a factor of two in the Compton scattering cross section. Consequently, this observable is sensitive enough to distinguish between various distribution amplitudes.

In Fig.3, we show the polarized Compton cross sections for the two helicity states of the photon and for a target proton with positive helicity. We remark that for all three distribution amplitudes there is a marked difference both in magnitude and angular dependence between the cross sections for the two photon helicities. Consequently, the resulting photon asymmetries change sign for different values of Θ_{CM}, which suggest that this might be a useful observable to distinguish between nucleon distribution amplitudes.

4 Model independent way to extract distribution amplitude from experiment

We now consider the possibility to extract the nucleon distribution amplitude in a model independent way from the Compton cross sections at high momentum transfer.

Extracting the distribution amplitude may proceed in two steps. In a first step one performs the convolution of the hard scattering amplitude with the basis functions (e.g. Appel polynomials) in the expansion (Eq.4) of the nucleon distribution amplitude :

$$T_H^{ij} = \int d[x] \int d[y] \; A_i(y) \; T_H(x,y) \; A_j(x) \; , \tag{9}$$

This first step is of course model independent. In a second step, starting from a general expansion of the distribution amplitude in terms of these basis functions :

$$\begin{aligned} \phi_N(x_1,x_2,x_3) = \quad & \phi_{as} \; \{ \; a_1 \; + \; a_2\, x_1 \; + a_3\, x_3 \\ & + a_4\, x_1^2 \; + \; a_5\, x_1 x_3 \; + \; a_6\, x_3^2 \; \} \\ & + a_7\, x_1^3 \; + \; a_8\, x_1^2 x_3 \; + \; a_9\, x_1 x_3^2 \; + \; a_1 0\, x_3^3 \; \} \\ & + \ldots \; \} \; , \end{aligned} \tag{10}$$

one extracts the expansion coefficients from a fit to the Compton scattering data. This is possible because the Compton scattering amplitude can be written as $M \;\sim\; \sum_i \sum_j \, a_i \, T_H^{ij} \, a_j$ where T_H^{ij} are the model independent matrix elements calculated in the first step.

To check this procedure, in a first stage we generated data starting from a model distribution amplitude. By putting error bars on these data, we studied the sensitivity to the experimental uncertainty of the extracted coefficients.

This fitting program can then be applied to experimental Compton scattering data in the scaling region. An experimental proposal to measure Real Compton Scattering for ELFE at DESY is underway[16]. This opens up prospects to study the nucleon distribution amplitude in a direct way.

References

1. S.J. Brodsky and G.P. Lepage, *Phys. Rev.* D **22**, 2157 (1980).
2. M. Vanderhaeghen, P.A.M. Guichon, J. Vande Wiele, in preparation.
3. G.R. Farrar and H. Zhang, *Phys. Rev.* D **41**, 3348 (1990); *Phys. Rev.* D **42**, 2413(E) (1990).
4. A.S. Kronfeld and B. Nizic, *Phys. Rev.* D **44**, 3445 (1991); *Phys. Rev.* D **46**, 2272(E) (1992).
5. J. Arvieux and B. Pire, *Prog.Part.Nucl.Phys.* ,, V (o)l.35, 299 (1995).
6. S.J. Brodsky and G.P. Lepage, *Phys. Rev.* D **24**, 2848 (1981).
7. V.L. Cernyak and A.R. Zhitnitsky, *Phys. Rep.* **112**, 173 (1984).
8. V.L. Chernyak, A.A. Ogloblin and I.R. Zhitnitskii ZPC **42**, 569 (1989).
9. I.D. King and C.T. Sachrajda, *Nucl. Phys.* B **279**, 785 (1987).
10. C.R. Ji, A.F. Sill, R.M. Lombard-Nelson, *Phys. Rev.* D **36**, 165 (1987).
11. S.J. Brodsky and G.R. Farrar, *Phys. Rev. Lett.* **31**, 1153 (1973).
12. G.R. Farrar, K. Huleihel and H. Zhang, *Nucl. Phys.* B **349**, 655 (1991);
13. M. Deutsch et al, PRD **8**, 3828 (1973).
14. J. Duda et al, ZPC **17**, 319 (1983).
15. M.A. Shupe et al, PRD **19**, 1921 (1979).
16. N. d'Hose and G.Tamas, Contribution to these proceedings.

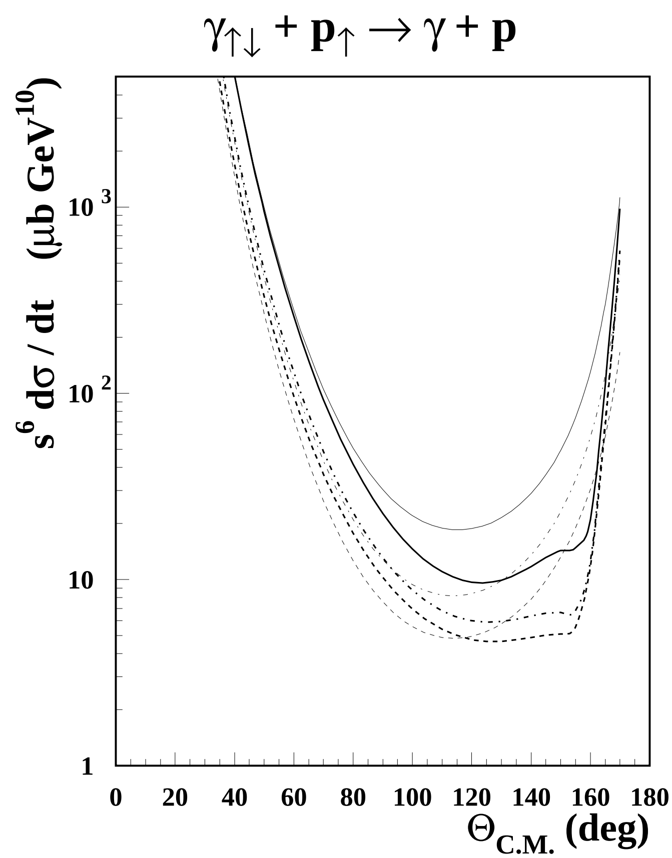

Figure 3: *Calculations for the polarised Compton cross section for two helicity states of the photon : $\lambda = +1$ (thick lines), $\lambda' = -1$ (thin lines). Results are shown with KS (full lines), COZ (dashed-dotted lines), CZ (dashed lines).*