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Deep-Inelastic Onium Scattering

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Recently, a dipole model approach to the QCD computation of onium-onium scattering from the perturbative resummation of leading logarithms has been proposed[1,2,3]. In this framework, each onium state describes a small (i.e. massive) colour-singlet quark- antiquary pair whose infinite momentum wavefuntion is itself composed of colour dipoles at the moment of the interaction. These onia interact locally in impact-parameter space via 2-gluon exchange between the component dipoles. The overall total cross-section has been shown to reproduce the well-known[4] QCD Pomeron exchange proton structure function at $x < 10^{-2}$, with only 3 free parameters for 100 experimental points, and then we confront our predictions with the most recent set of data at lower Q^2.

1.

$$\frac{d^2\sigma}{d^2b} =$$

(2)

is the 2-gluon exchange amplitude between dipoles of size respectively X_1 and X_2, and

$$\eta\left(X_{0i}; b_i, X_i; Y/2\right) \simeq \frac{X_{0i}}{X_i} \frac{1}{\pi b_i^2} \int_{-\infty}^{+\infty} \frac{d\nu_i}{\pi} \times$$
$$\times \quad 2i\nu_i \left\{\frac{b_i^2}{X_{0i} X_i}\right\}^{2i\nu_i} \exp\left\{\frac{N_c \alpha_s}{\pi} \chi\left(\nu_i\right) Y/2\right\}, \quad (3)$$

is the multiplicity distribution of dipoles for given initial size X_{0i}, impact parameter b_i, size X_i and rapidity $Y/2$.

As usual,

$$\chi(\nu_i) = 2\Psi(1) - \Psi\left(\frac{1}{2} + 2i\nu_i\right) - \Psi\left(\frac{1}{2} - 2i\nu_i\right) \; ; \; \Psi(\gamma) \equiv \frac{d \log \Gamma}{d\gamma}, \quad (4)$$

denotes the Mellin-transformed BFKL kernel. Note that formula (3) can be obtained from the wave functional at infinite-momentum [2] within the approximations $b_i/X_i, b_i/X_{0i} << 1$, $\nu_i \approx 0$.

Inserting expressions (2) and (3) inside formula (l), let us proceed further with a determination of the cross-section keeping the two scales X_{01} and X_{02}

small, but assuming a rather large ratio $\frac{X_{01}}{X_{02}}$. We first perform the independent integrations on b_1, b_2 and ℓ to get:

$$
\begin{aligned}
\frac{d^2\sigma}{d^2b} \simeq\ & 4\ \frac{\alpha^2}{b^2} X_{01} X_{02} \int\int \frac{dX_1}{X_1^2}\frac{dX_2}{X_2^2}\ \frac{X_<^2}{4} \left[1 + \ln\left(\frac{X_>}{X_<}\right)\right] \\
\times\ & \int\int \frac{d\nu_1}{\pi}\frac{d\nu_2}{\pi}\,(2i\nu_1 + 2i\nu_2)\left(\frac{b^2}{X_{01}X_1}\right)^{2i\nu_1}\left(\frac{b^2}{X_{02}X_2}\right)^{2i\nu_2} \\
\times\ & \exp\left(\frac{N\alpha Y}{2\pi}\left(\chi(\nu_1) + \chi(\nu_2)\right)\right),
\end{aligned}
\tag{5}
$$

where $X_<$ (resp. $X_>$) is the smaller (resp. larger) of X_1 and X_2. Note that both values are required to verify $X < b$. After further integration over X_1, X_2 (exact) and over ν_1, ν_2 (saddle-point) one gets:

$$
\begin{aligned}
\frac{d^2\sigma}{d^2b} &= 4\ \alpha^2\ \frac{X_{01}X_{02}}{\pi b^2}\ a\ \exp\left((\alpha_p - 1)\,Y - a\ \left(\ln^2\frac{b}{X_{01}} + \ell n^2 \frac{b}{X_{02}}\right)\right) \\
&= 4\ \alpha^2\ \frac{X_{01}X_{02}}{\pi b^2}\ a\ \exp\left((\alpha_p - 1)\,Y - \frac{a}{2}\ln^2\frac{b^2}{X_{01}X_{02}}\right) \\
&\times\ \exp\left(-\frac{a}{2}\ln^2\frac{X_{01}}{X_{02}}\right)
\end{aligned}
\tag{6}
$$

where, using the conventional notations, α_p is the "intercept" of the BFKL singularity and a the "diffusion" coefficient at rapidity Y[4]. One has

$$
\begin{aligned}
\alpha_p - 1 &= \frac{\alpha N_c}{\pi}\chi(0) \equiv \frac{\alpha N_c}{\pi} 4\ \ln 2 \\
a \equiv a(Y) &= \left[-\frac{\alpha N_c}{4\pi}\chi''(0) Y\right]^{-1} = \left[\frac{\alpha N_c}{\pi} 7\zeta(3) Y\right]^{-1}.
\end{aligned}
\tag{7}
$$

The integrated cross-section $\sigma = \int d^2b\, \frac{d^2\sigma}{d^2b}$ reads:

$$
\sigma = 2\pi X_{01} X_2\ \alpha^2\ e^{(\alpha_p - 1)Y}\left[\frac{2a}{\pi}\right]^{1/2}\ \exp\left(-\frac{a}{2}\ln^2\frac{X_{01}}{X_{02}}\right) \tag{S}
$$

Our results summarized in formulae (6) and (S) deserve some comments. The essential feature is the scale-ratio dependent factor

$$\exp\left(-\frac{a}{2}\ln^2\left(\frac{X_{01}}{X_{02}}\right)\right) \equiv \exp\left(-\frac{\ln^2\left(\frac{X_{01}}{X_{02}}\right)}{2\,K\,(\alpha_p-1)\,Y}\right), \tag{9}$$

where we have used eq. (7) to define the constant K such that:

$$a^{-1} \equiv K\,(\alpha_p - 1)\,Y\,;\;\; K \equiv \frac{\chi''(0)}{4\chi(0)} = \frac{7\zeta(3)}{4\ln 2}\quad (\sim 3)\,.$$

When the scale ratio is equal to 1, one recovers for σ the known result[1].

Expression (9) gives a non-trivial logarithmic dependence on the scale independent variables typical of the reaction namely in $\frac{X_{01}}{X_{02}}$ and Y. The factor (9) does play a role if the typical sizes of the dipoles, while being both small in order to preserve the perturbative treatment of the process, are hierarchically different $\frac{X_{01}}{X_{02}} >> 1$. The interpretation of (9) as due to a genuine scaling-violation factor present in the dipole derivation of the BFKL singularity for deep-inelastic onium scattering will be made clear in the forthcoming discussion. We shall also make more precise the range of validity of the scale-dependent factor.

2. In order to describe deep-inelastic scattering on an onium state, and in particular to fulfill the requirement of k_T factorization[6], let us introduce the unintegrated structure function $F' = \frac{dF(x,Q^2)}{d\ln Q^2}$ and its formulation [1] in terms of the dipole distribution function inside the onium state. One writes[1]:

$$xF'(x) = 2\frac{\alpha N_c}{\pi}\int_{1/2-i\infty}^{1/2+i\infty}\frac{d\gamma}{2i\pi}\left(\frac{Q}{Q_0}\right)^{2\gamma} v_\gamma e^{\frac{\alpha N}{\pi}\chi(\gamma)\ln 1/x}, \tag{10}$$

where Q_0 correspond to some average over the dipole size distribution of the onium target (at least when γ stays in the vicinity of the critical value $\gamma_c = 1/2$). v_γ is defined by

$$v_\gamma = \int_1^\infty v(u)\, u^{-2\gamma-1} du, \tag{11}$$

[1] Note that the expression (6) for $\frac{d^2\sigma}{d^2b}$ is formally different from the analogous formula (10) of Ref. [2], even when $\frac{X_{01}}{X_{02}} = 1$. However the dominant contribution in (10), ref. [2] is valid for $a^{-1} \propto \ln\left(\frac{b^2}{X_{01}X_{02}}\right)$, which restores the equivalence between the two expressions.

where the function v(u) describes the factorized vertex as a non-perturbative input in general. We only know that v(u) $\rightarrow$ 1 when u becomes large enough.

Inserting (11) into formula (10), one finds the following expression:

$$F' = 2\frac{\alpha N_c}{\pi} \int_1^{\infty} v(u) \frac{du}{u} \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2i\pi} \left(\frac{Q}{uQ_0}\right)^{2\gamma} \exp\left(\frac{\alpha N}{\pi} \chi(\gamma) \ln 1/x\right). \quad (12)$$

Now, let us suppose that we are interested to determine the Mellin integral in a region where the ratio Q/Q_0 is large but not too large in order to keep the -y-integration near γ_c (these conditions will be made quantitative further on). Then the convolution (12) will be dominated by large values of $u \approx Q/Q_0$, and thus one may consider that $v(u) \approx 1$. The solution of (12) becomes straightforward, and after further integration in In Q', one gets the final answer:

$$xF(x) = \frac{\alpha N_c}{\pi} \int_{1/2-i\infty}^{1/2+i\infty} \frac{d\gamma}{2i\pi\gamma^2} \left(\frac{Q}{Q_0}\right)^{2\gamma} e^{\frac{\alpha N}{\pi}\chi(\gamma) \ln 1/x} \quad (13)$$

and, using a saddle-point method by expansion around γ_c, one obtains

$$xF(x) = \frac{\alpha N_c}{\pi} e^{(\alpha_p - 1) \ln 1/x} \left[\frac{a}{\pi}\right]^{1/2} \frac{Q}{Q_0} e^{-\frac{a}{2} \ln^2 \frac{Q}{Q_0}} \quad (14)$$

with the variables a, α_p defined in the same way as in (7), identifying Y $\equiv$ In l/z. We thus recover a formula similar to (S), but now appropriately defined in terms of the scale ratio Q/Q_0 where Q is the photon virtuality and Q_0, a typical scale related to the average dipole size in the target.

Interestingly enough, the conditions to obtain formula (14) from the saddle-point method are such that they fix the conditions of its applicability. Noting that the saddle point value is $\gamma^* = 1/2$ – a in Q/Q_0, the consistent approximation leading to (14) is given by:

$$a \ln Q/Q_0 \simeq \frac{\ln Q/Q_0}{\ln 1/z} \ll 1, \quad a \ln^2 Q/Q_0 \simeq \frac{\ln^2 Q/Q_0}{\ln 1/x} = O(1). \quad (15)$$

One thus realizes that the proposed parametrization should be valid for the region of moderate Q/Q_0 when compared to the range in l/z. This makes it an interesting parametrization for the H 13 RF\ range, provided one may extend the validity of (14) from an hypothetical onium initial state to the proton. This is the subject of the next section. Note that at higher Q^2 values, one expects the usual Double-Log-Approximation (DLLA) to become valid, which amounts[7] to consider in formula (13) the pole in l/y in the kernel $\chi(\gamma)$, (see (4)). In that case, however, the input function $v(\gamma)$ is not a-priori known.

Let us focus the discussion on the signification of the scale-dependent factor (9) with respect to the usual derivations of the BFKL contribution. Indeed, an inverse Mellin-transform similar to Eqns. (10-13) appears in the classical derivations of the BFKL singularity [4, 8], as well as in the dipole-model formulation[1, 2]. In some cases, like the production of a forward jet in deep-inelastic scattering[9], a similar scale factor, which depends on the ratio of the photon virtuality to the jet transverse momentun, has been taken into account. However, in that case, the physical goal was to emphasize the typical scale-independent BFKL contribution by choosing this scale-ratio as possible of 0(1) in order to damp the possible DGLAP evolution.

In our case, on contrary, the fact that both the dipole-model result of Eqn. (8) and the inverse-mellin transform leading to Eqn. (14) give similar results leads us to the conclusion that the scale-dependence we obtain is a quite general feature of the BFKL singularity and should be taken into account as a genuine scaling violation prediction of the whole theoretical scheme. It is also an incentive to extend our results from the original deep-inelastic onium reaction to the more practical case of proton inelastic scattering (with the assumption of neglecting non-perturbative effects).

3. In order to test the accuracy of the parametrisation obtained above, a fit using the published data of the H1 and Zeus experiments [5] was achieved. The parametrisation used for the fit is the following (see Eqn. (14)):

$$F_2 = C a^{1/2} \exp(1/Ka) \exp\left(\ln\frac{Q}{Q_0}\left(1 - \frac{a}{2}\ln\frac{Q}{Q_0}\right)\right) \quad (16)$$

where:

$$a^{-1} = K(\alpha_P - 1) \ln\frac{1}{x} \quad (17)$$

The parameters used in the fit are α_P, Q_0, and C. The data used for the fit were the published 93 data from the H 1 and Zeus experiments, with $x \leq 0.014$ and $Q^2 \leq 250.\ GeV^2$ [5], which corresponds to 100 measured points. This choice is motivated by the theoretical requirements, in particular the validity range defined in (15).

The results of the fit are given in figure 1. It can be noticed that the high x and Q^2 points were not present in the fit, (as expected, the behaviour in that kinematical region is not reproduced by the theoretical curve). The values of the parameters are the following : α_P = 1.243, Q_0 = 0.513, C = 0.090, for a χ^2 equal to 93.4. If we compare with the values expected by the theory, they are in fairly good agreement:, since one would expects the following range of values: $\alpha_P = 1 + \frac{\alpha N_C}{\pi} 4 ln2 \approx$ 1.3, and $C = \frac{\alpha N_C}{\pi} \frac{2}{\pi} \sqrt{\frac{2}{\pi}} \approx$ 0.1, while the value of Q_0 corresponds to an average radius of 1 GeV^{-1} for the dipole size in the proton which seems reasonable. These values are obtained for N_C = 3 and $\alpha = 0.12$.

If one would have performed separate fits using only the 60 H1 points (with $x \leq$ 0.013 and 4.5 $\leq Q^2 \leq$ 120), or only the 40 Zeus points (with $x \leq$ 0.014 and $8.5 \leq Q^2 \leq 250$), we get respectively : α_P = 1.214(1.351), Q_0 = 0.516(0.522), K = 0.114(0.043), and a χ^2 = 43.2(27.2). All in all, the fit of the data is remarkably good and the values of the parameters are rather close to those expected from the theoretical framework. As foreseen, the high Q^2 predictions are not so good as the parametrisation is not supposed to be valid in this domain, where the DGLAP equation is supposed to be more accurate.

To check the validity of this parametrisation, the values of the measured F_2 obtained by the H1 collaboration with the 1994 data ([5]) were compared with the parametrisation. It must be noticed that the parameters used in the comparison are kept the same as for the previous fit without new adjustment, and simply compared with the new data at lower Q^2. The comparison is shown in the figures 2, and the agreement between the measured points at low $Q^2 (Q^2 \geq 2 GeV^2)$ is perfect.

In conclusion, applying the dipole model to deep-inelastic onium scattering, we-have found the following results:

1) The dipole-dipole scattering cross-section between dipoles of unequal masses exhibit a non-trivial factor dependent of the ratio of the dipole sizes,

2) A similar factor appears in the structure function describing deep-inelastic scattering on an onium state. It depends on the ratio Q/Q_0, where Q is the virtuality of the photon and Q_0^{-1} is related to the average size of the dipole configurations of the onium. It plays the rôle of a genuine scaling-

violation contribution associated to the BFKL singularity

3) Extending the model to deep-inelastic proton scattering, we find a remarkably good description of the recent HERA data on the proton structure function. A 3-parameter fit gives a χ^2 value of less than 1 per point for the published H1 and ZEUS data, while the extrapolation of the resulting parametrization to the very recent low$-Q^2$ data is excellent. The 3 parameters found in the fit stick to the values expected from the theoretical framework

In view of the striking agreement between the theoretical dipole picture and the phenomenological description of the data at small x, we are led to think that the correct understanding of the proton constituent picture in this region requires the dipoles as the fundamental objects present during (and responsible of) the interaction.

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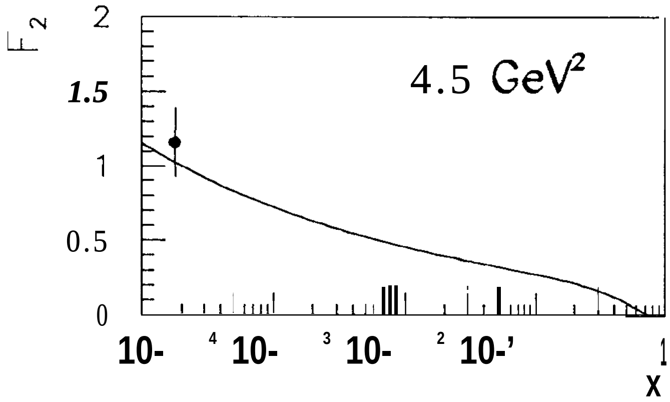
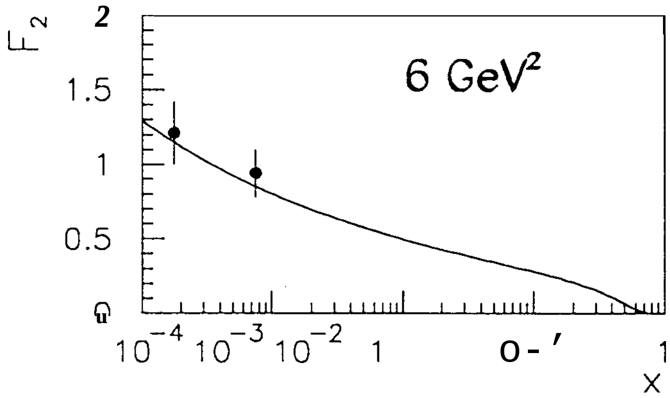


Figure 1 a



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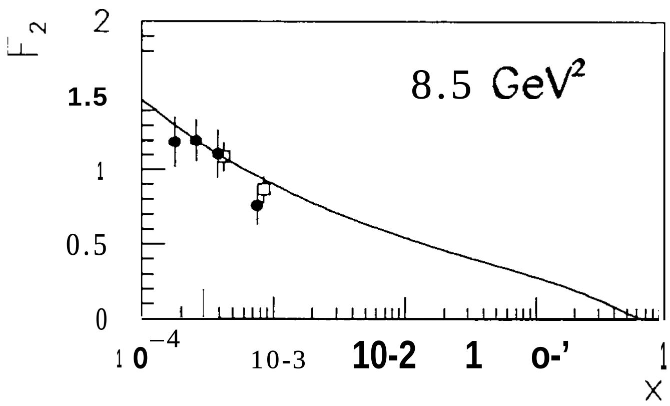

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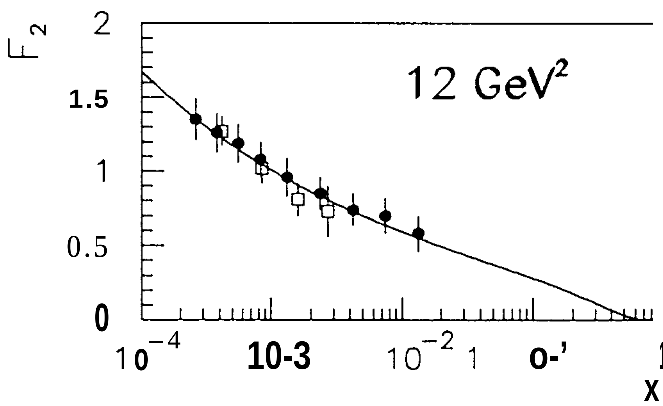

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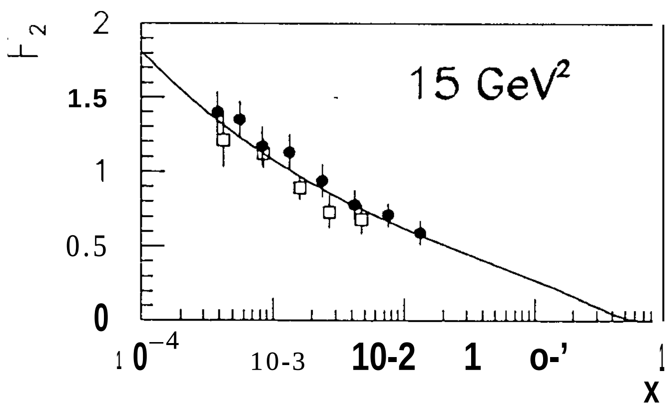

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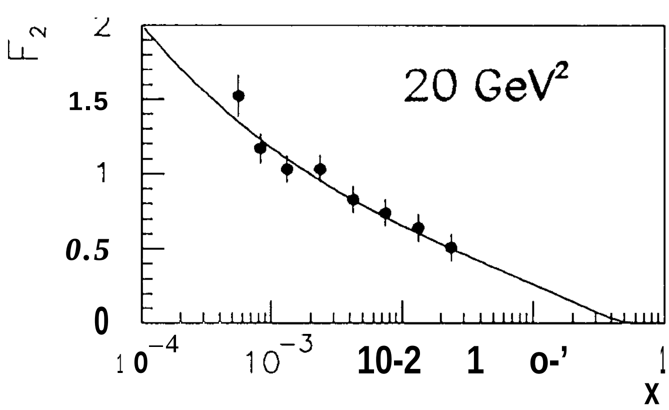

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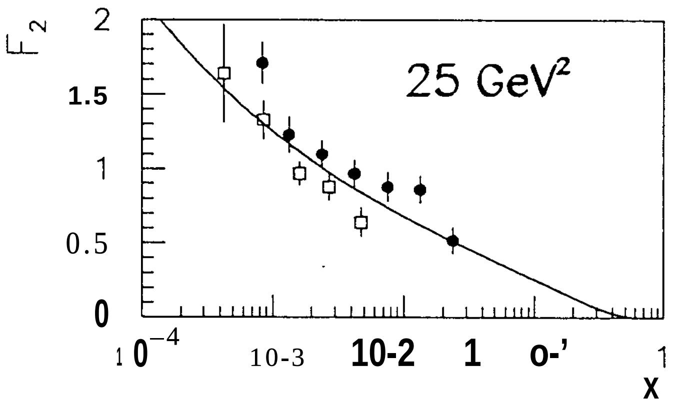

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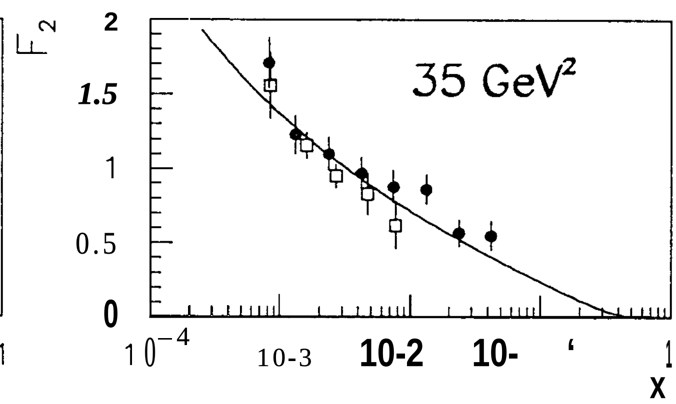

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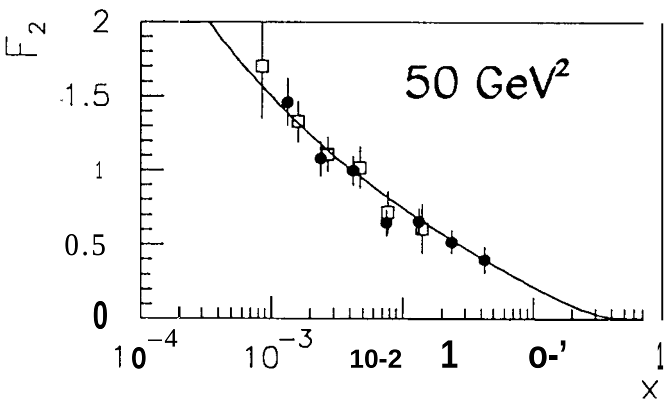

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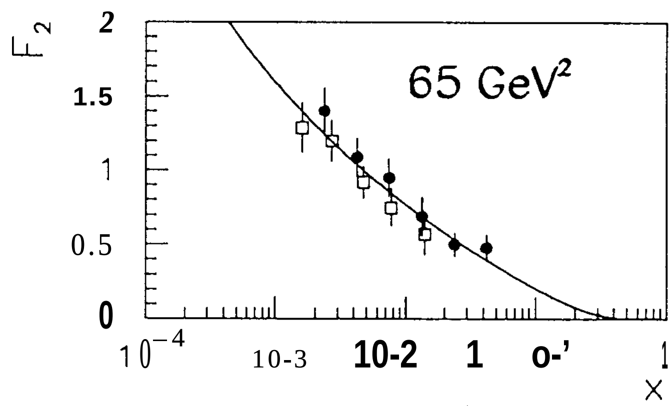

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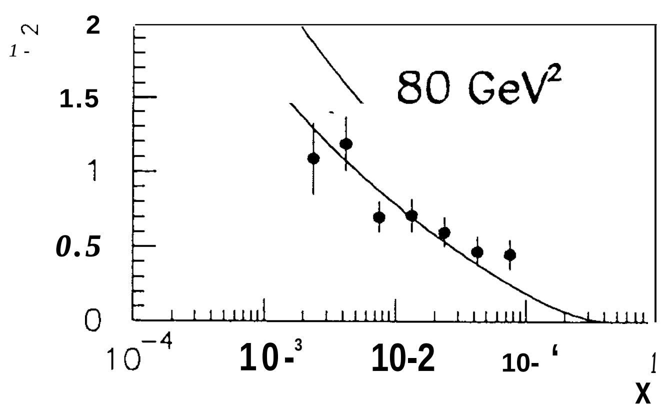

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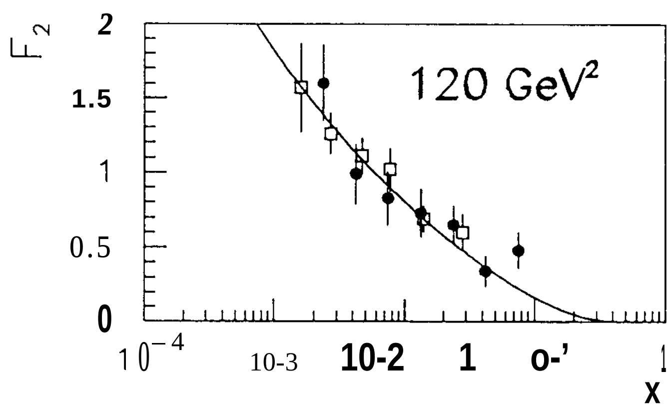

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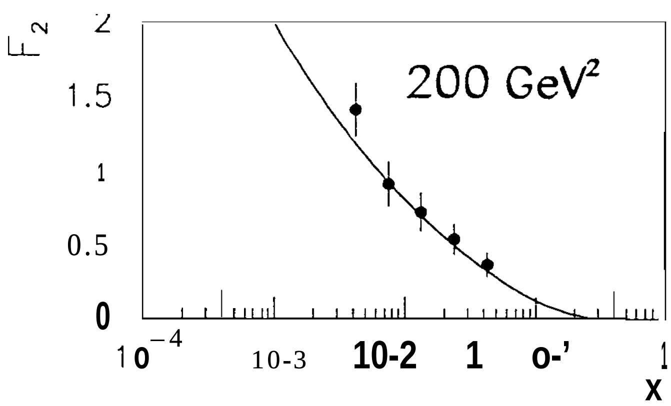

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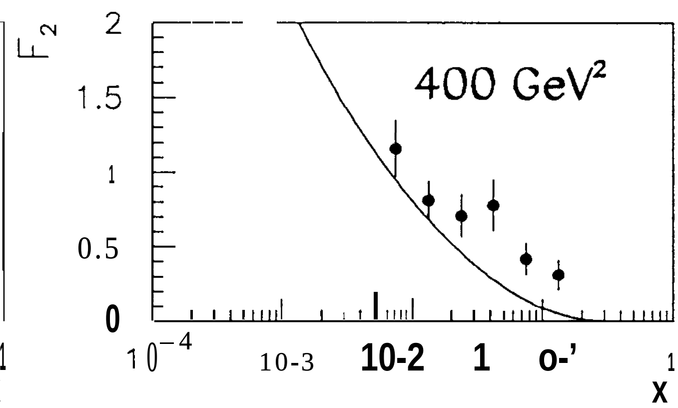

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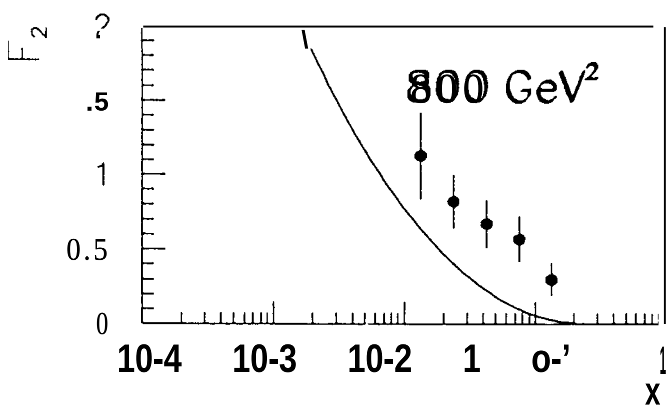
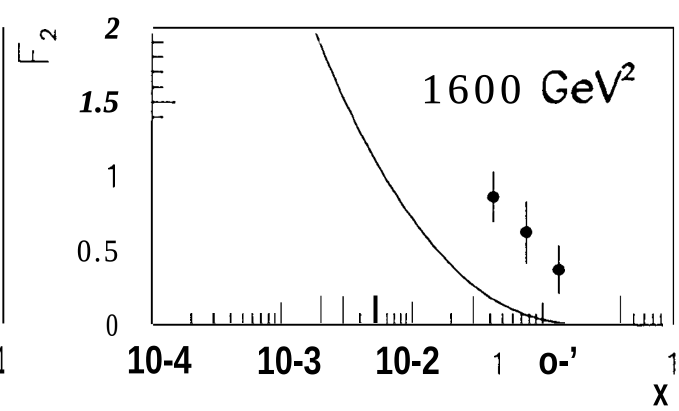


F gure **10**



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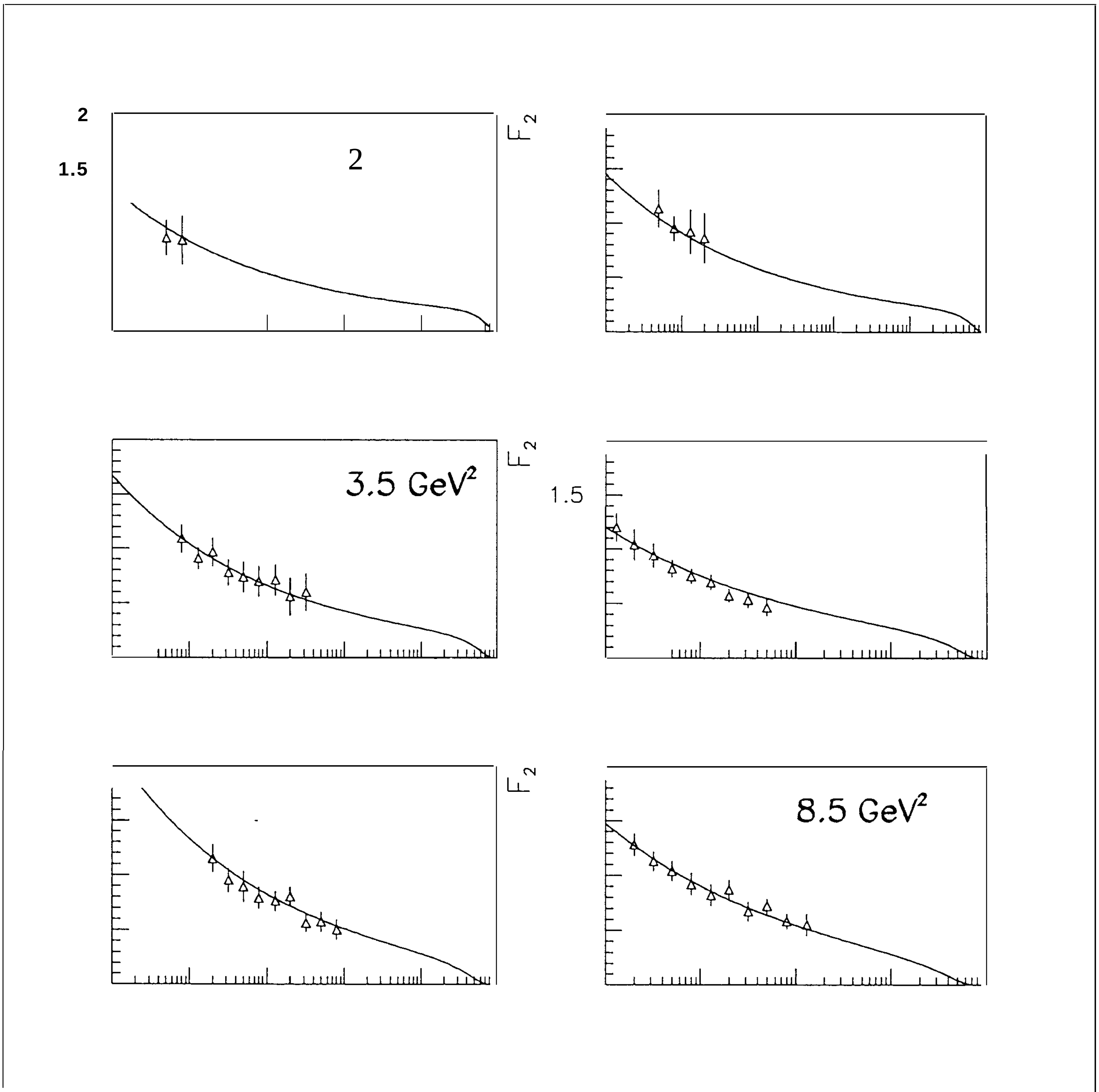

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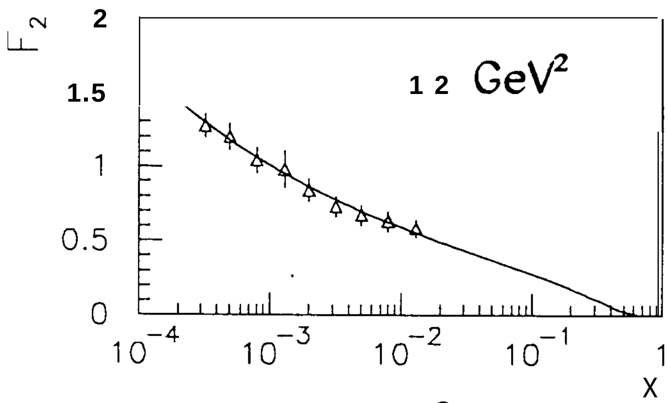

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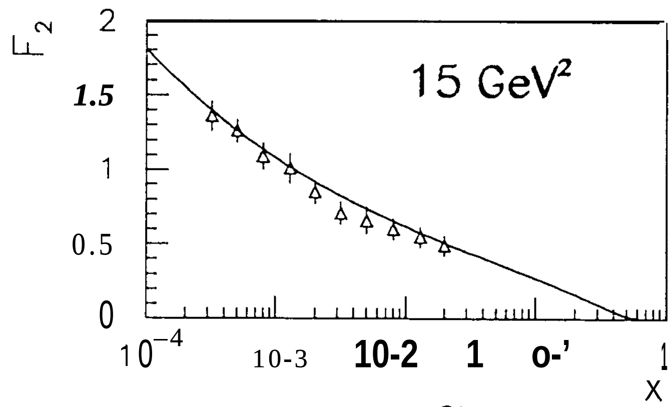

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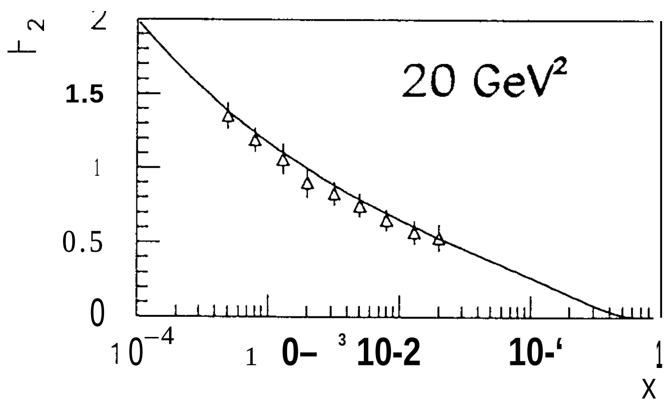
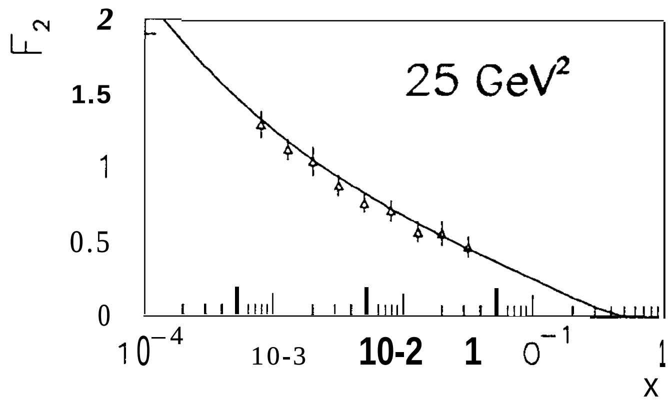


Figure 2i



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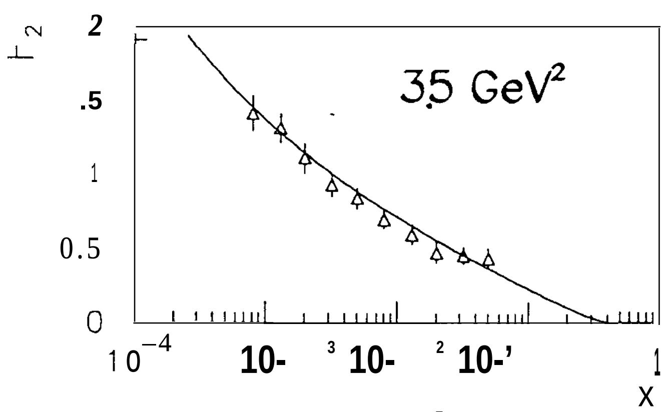

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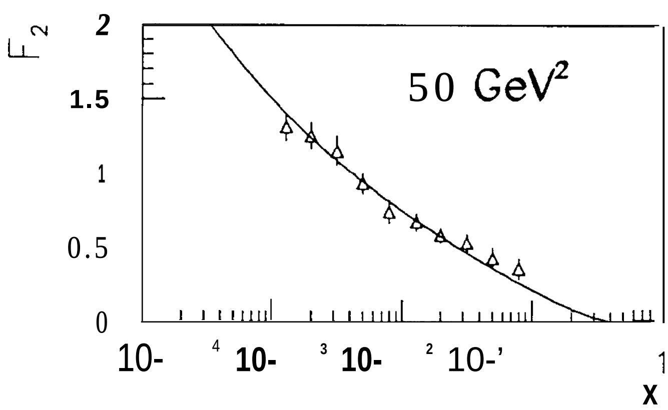

Figure 2l