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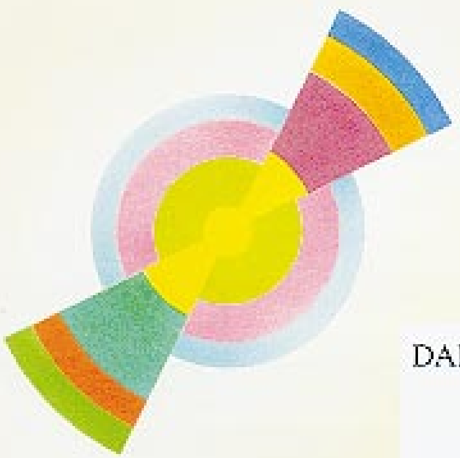

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THE STRUCTURE OF THE PROTON PHENOMENOLOGY [1]

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INTRODUCTION

Deep inelastic scattering of an electron on a proton is a very interesting process to probe with high precision the structure of the proton. In deep inelastic scattering a highly virtual photon of mass Q^2 interacts with pointlike constituents (partons) of the proton. In the Breit frame the photon carries no energy and the proton has a momentum P proportionnal to $\sqrt{Q^2}$. For high Q^2, P is large and the proton looks like a highly contracted pancake. The time τ of interaction is proportional to $\frac{1}{\sqrt{Q^2}}$; for small τ we can safely suppose that the proton scatters incoherently on each parton. The cross section for deep inelastic scattering depends on the parton distribution seen when we look inside the proton with a resolution time τ.

The cross section for longitudinaly and transverse polarised photon (resp. σ_L and σ_T) can be written using two independent structure fonctions, $F_L(x,Q^2)$ and $F_2(x,Q^2)$ where x is the fraction of impulsion carried by the struck parton. In the quark-parton model

$$F_2(x,Q^2) = \sum_i {e_i}^2 x(q_i(x,Q^2) + \bar{q}_i(x,Q^2)) \tag{1}$$

where $q_i(x,Q^2)$ is the probability of finding a quark (of flavor i) localised within the transverse region $R_T \sim \frac{1}{Q}$ carrying the fraction x of the parton momentum.

HERA experiments, H1 and ZEUS, are particulary well suited for deep inelastic analysis and F_2 measurements. HERA provides an energy in the center of mass frame of $\sqrt{s} \sim 300 GeV$ by colliding an electron of energy $27.5 GeV$ and a proton of $820 GeV$. If we note p and q the quadrivectors of the proton and the photon the kinematic variables are defined as $Q^2 = -q^2$ and $x = \frac{Q^2}{2pq}$.

With HERA, we have accessed to a new kinematical range at small x where precise tests of the theory of stong interactions (perturbative QCD) can be done. The recently published 1994 results from HERA experiments [1] on the proton structure function F_2 have reached a high level of precision. It covers an extended kinematical range. In particular, it reaches very low values of x ($x \sim 10^{-5}$) and Q^2 ($Q^2 \sim 1.5\, GeV^2$). These data confirm with high statistics the strong rise of F_2 when x becomes very small, first noticed in 1992 experiments. In this new regime there are two main theoretical predictions to describe the evolution of F_2 with respect to x or Q^2, the DGLAP [2]and

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[2]DGLAP stands for Dokshitzer, Gribov, Lipatov, Altarelli, Parisi

BFKL [3] evolution equations.

The purpose of this contribution is to present briefly the main aspects of these predictions and then comment experimental ideas which can be used to discriminate between these two alternatives. We will show that two observables are good candidates for this analysis, $\lambda = \frac{\partial \ln F_2}{\partial \ln 1/x}$ and $R = \frac{F_L}{F_2 - F_L} = \frac{\sigma_L}{\sigma_T}$. ($F_L$ is the longitudinal structure function and λ enables to quantify the stong rise of F_2 at small x).

1 THE DGLAP MODEL

Let us first notice that the full x and Q^2 dependence of the structure functions of the proton is not given by the theory, there are only predictions for evolution equations of these structure functions (according to which Feynman diagrams are summed up). The DGLAP model (leading order) is a sum of $(\alpha_S \ln Q^2)^n$ terms and this model describes an evolution in Q^2 [2]. (We will see later that the BFKL model which sums up $(\alpha_S \ln 1/x)^n$ terms corresponds to an evolution in $1/x$). We define the singlet and the non singlet structure functions $F_S(x, Q^2)$ and $F_{NS}(x, Q^2)$

$$F_2(x, Q^2) = F_S(x, Q^2) + F_{NS}(x, Q^2) \tag{2}$$

with F_S completely symmetric in quark flavors and F_{NS} completely antisymmetric. For example $F_2 \sim F_{NS}$ at high x ($x > 0.2$) because the behaviour of F_2 at high x is dominated by the valence quarks and then by its non singlet part. On the contrary the low x regime is essentially driven by the singlet structure function (sea quarks) and the gluon distribution F_G, then $F_2 \sim F_S$ and the DGLAP evolution equations (in Q^2) exhibit couplings between F_S and F_G. These couplings are direct consequences of the elementary processes of perturbative QCD : $q \to q + g$, $\bar{q} \to \bar{q} + g$, $g \to q + \bar{q}$, $g \to g + g$, where the last process is typical of non abelian gauge theories. We will not give a complete derivation of the DGLAP equations but the idea is to sum up the $(\alpha_S \ln Q^2)^n$ contributions of diagrams like the one of Fig. 1. These calculations are performed with a strong ordering in ${k_T}^2$ and within the kinematic range $\alpha_S(Q^2) \ln 1/x << 1$ for $\alpha_S(Q^2) << 1$. DGLAP evolution equations can also be seen as a consequence of the renormalisation group equation and the operator product expansion.

At this stage we essentially deal with the predictions on $\lambda = \frac{\partial \ln F_2}{\partial \ln 1/x}$ at low x, we analyse in a next section the other observable $R = \frac{F_L}{F_2 - F_L}$. Thus, let us now determine the predictions on $\lambda(Q^2)$ in the DGLAP scheme. First, we consider the inverse Mellin transforms of the singlet structure function F_S, namely

$$F_S(x, Q^2, \mu^2) = \frac{1}{2i\pi} \int_{\omega_0 - i\infty}^{\omega_0 + i\infty} x^{-\omega} F_S(\omega, Q^2, \mu^2) \mathrm{d}\omega \tag{3}$$

where the integration line $Re\omega = \omega_0$ is at the right of all singularities of $F_S(\omega, Q^2, \mu^2)$ and μ^2 defines the scale of the initial condition of the Q^2 evolution. In the DGLAP model $F_S(\omega, Q^2, \mu^2)$ and $F_G(\omega, Q^2, \mu^2)$ (the gluon distribution in Mellin-moment space) verify

$$\begin{pmatrix} F_S(\omega, Q^2, \mu^2) \\ F_G(\omega, Q^2, \mu^2) \end{pmatrix} = K(\omega, Q^2, \mu^2) \begin{pmatrix} F_S(\omega, \mu^2) \\ F_G(\omega, \mu^2) \end{pmatrix} \tag{4}$$

where $K(\omega, Q^2, \mu^2)$ is the Mellin transform of the DGLAP matrix kernel at the scale Q^2. The rightmost singularity of this kernel lies at $\omega_0 = 0$. Also $F_S(\omega, \mu^2)$ is the Mellin

[3]BFKL stands for Balitskii, Fadin, Kuraev, Lipatov

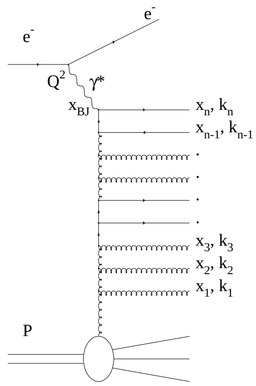

Figure 1: *Diagrammatic representation of quark and gluon rungs contributing to deep inelastic scattering.*

transform of the singlet input which is either regular or singular at the right of $\omega_0 = 0$, $F_G(\omega, \mu^2)$ is the Mellin transform of the gluon input. We then get two classes of models satisfying a DGLAP evolution.

(i) Either the rightmost ω-plane singularity is fixed by the DGLAP kernel singularity at $\omega = 0$, we obtain the Glück, Reya, Vogt (GRV) type parametrization [3]. In this case, at a given Q^2 near the $\omega = 0$ singularity we can show that

$$F_S(\omega, Q^2, \mu^2) \simeq \omega f(\omega) \exp\left(\frac{4N_c}{\omega} - a\right)\xi \tag{5}$$

where $f(\omega)$ is an input-dependent function regular at $\omega = 0$ by assumption and a is a constant ($a \sim 109/10$), $f(\omega)$ can be expanded as

$$f(\omega) = f(0)[1 + \Sigma_i \omega^i b_i] \tag{6}$$

Then we can determine $\frac{\partial \ln F_S}{\partial \ln \frac{1}{x}}$ using the following property of Bessel functions

$$\frac{1}{2i\pi} \int_{\omega_0 - i\infty}^{\omega_0 + i\infty} \omega^n \mathrm{e}^{\;\omega \ln 1/x} \mathrm{e}^{\frac{4N_c}{\omega}\xi} \mathrm{d}\omega = \left(\frac{4N_c\xi}{\ln 1/x}\right)^{\frac{n+1}{2}} I_n[2(4N_c \xi ln 1/x)^{\frac{1}{2}}] \tag{7}$$

Defining $\bar{\omega} = (\frac{4N_c\xi}{\ln 1/x})^{\frac{1}{2}}$ and $v = 2(4N_c\xi \ln 1/x)^{\frac{1}{2}}$, it is straightforward to see that

$$\lambda \simeq \frac{\partial \ln F_S}{\partial \ln \frac{1}{x}} \simeq \bar{\omega} \frac{I_3(v)}{I_2(v)} \tag{8}$$

and the terms $b_i, i > 1$ are negligible (which means that λ is independent of the input). Moreover $\frac{\partial^2}{(\partial \ln Q^2)^2} \left(\frac{\partial \ln F_S}{\partial \ln \frac{1}{x}} \right) < 0$. This gives us an information on the concavity of the function $\lambda(Q^2)$ in the DGLAP model with the rightmost singularity imposed by the DGLAP kernel.

(ii) Or we have the López, Barreiro, Ynduráin (LBY) type parametrization where the rightmost singularity lies at the right of $\omega = 0$ due to a singular input $F_s(\omega, Q^2, \mu^2)$.

Moreover its location is essentially not modified by the perturbative evolution [4]. (Let us observe that the BFKL dynamics also leads to a rightmost singularity at ω greater than $\omega = 0$. We will come back to this later.) Starting from the LBY formulation of the singlet structure function

$$F_S(x, Q^2) = [B_s(Q^2)x^{-\lambda_s} + C_s(Q^2)](1-x)^{\nu(Q^2)} \tag{9}$$

where B_s, C_s and ν are Q^2-dependent functions [4]. $\lambda_s > 0$ defines the location of the rightmost $\omega-$plane singularity and is Q^2-independent, but for charm and bottom thresholds. At small x

$$\lambda \simeq \frac{\partial \ln F_S}{\partial \ln \frac{1}{x}} \simeq \lambda_s, \tag{10}$$

up to the correction due to the phenomenological factor $C_s(Q^2)$.

2 THE BFKL MODEL

As already mentioned in this approach one sums up contributions of the type $(\alpha_S \ln 1/x)^n$ at low x [5]. There are two main features in this model : (1) BFKL evolution is an evolution from high longitudinal momentum partons to low longitudinal ones and (2) the evolution occurs at a fixed tranverse momentum (fixed Q^2). Then one can view the x evolution of the BFKL equation as creating the small x part of the wavefunction of the proton or simply as the "dressing" of a high momentum quark or gluon in the proton with low x gluons. In this model an interesting prediction (leading order) can be derived for the gluon distribution $F_G(x, Q^2)$

$$F_G(x, Q^2) \quad \sim h(Q^2)\; x^{-\alpha_P} \tag{11}$$

$$\alpha_P \quad = \frac{4\bar{\alpha} N_C \ln 2}{\pi} \leq 0.5 \tag{12}$$

showing a strong rise of F_G at low x and where α_P is the BFKL Pomeron intercept which is a constant, since the strong coupling constant $\bar{\alpha}$ is held fixed in this scheme. In this contribution we implement the BFKL dynamics using the recently developped QCD dipole model which is equivalent to BFKL for inclusive processes [6], [7]. We can then provide a prediction for $\lambda(Q^2)$. First we have for the singlet structure function

$$F_S = C a^{1/2} \frac{Q}{Q_0} e^{-\frac{a}{2} \ln^2 \frac{Q}{Q_0}}\; x^{-\alpha_P} \tag{13}$$

where $a = \left(\frac{\bar{\alpha} N_c}{\pi} 7\zeta(3) \ln \frac{1}{x}\right)^{-1}$. C and Q_0 are non-perturbative parameters to be determined by fitting with the data. We get

$$\lambda \simeq \frac{\partial \ln F_s}{\partial \ln \frac{1}{x}} = \alpha_p - 0.5 \frac{1}{\ln \frac{1}{x}} + \frac{1}{14 \frac{\bar{\alpha} N_c}{\pi} \zeta(3) \ln^2 \frac{1}{x}} \ln^2 \frac{Q}{Q_0} \tag{14}$$

Here we have, contrary to the GRV scheme $\frac{\partial^2}{(\partial \ln Q^2)^2} \left(\frac{\partial \ln F_S}{\partial \ln \frac{1}{x}} \right) > 0$.

Finally we can compare all the predictions with the data. As we can see on Fig. 2 the present accuracy of the data is not sufficient to distinguish between the different predictions but it appears a range of $Q^2 \in [1., 10.] GeV^2$ where new precise measurements could be very promising [8].

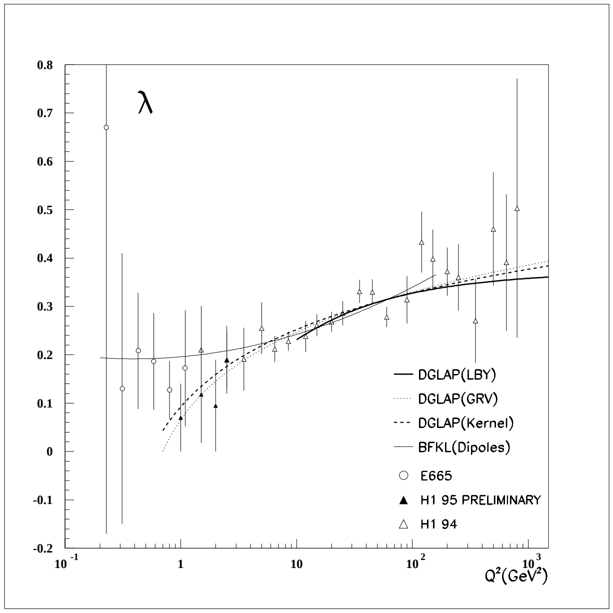

Figure 2: *Display of the data on λ compared with the prediction of the different parametrizations and evolution equations. DGLAP(Kernel) stands for* $\lambda \simeq \bar{\omega}\frac{I_3(v)}{I_2(v)}$ *and is verified to be in a good agreement with DGLAP(GRV).*

3 R MEASUREMENTS AND PREDICTIONS

$R = \frac{F_L}{F_2 - F_L}$ determination needs F_L measurement. There are several methods to get F_L, in the usual one we need at least two different beam energies. For example this can be achieved by decreasing the beam energy at the machine level or one can think of radiative events where a real photon is emitted from the incident electron which then losses one part of its initial energy. Other ideas are also in progress in the H1 collaboration and a precise measurement of F_L will become possible in a near future.

At this stage assuming $R \sim 0.3$ the total systematic error is expected to be of the order of 0.4. Statistical uncertainties depend on the luminosity of the machine and are expected to be of the order of 0.1 by running the experiment for a few weeks. Then we can compare with the theoretical predictions from DGLAP and BFKL (QCD dipole model) dynamics which are displayed on Fig. 3. We can not distinguish between them at this level of precision.

CONCLUSION

We have reminded the two types of QCD evolution equations describing the structure of the proton: the so called DGLAP and BFKL equations. These two descriptions cover two different physics images, thus it is an incentive question for experimentalists to find accurate observables (computable in the theory) which can afford to distinguish between them. We have shown that $\lambda = \frac{\partial \ln F_2}{\partial \ln 1/x}$ and $R = \frac{F_L}{F_2 - F_L}$ are good candidates for this analysis which is in progress in the H1 collaboration.

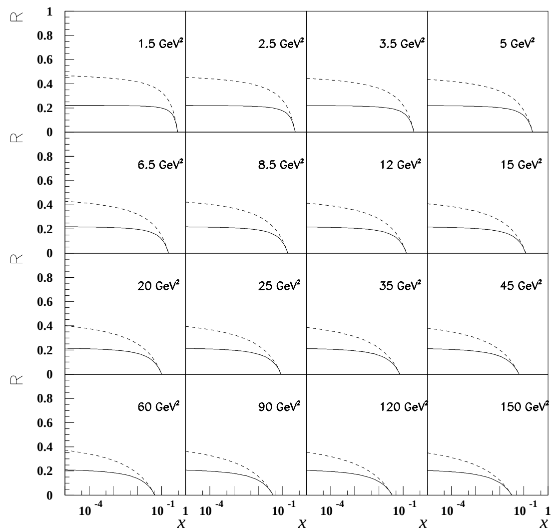

Figure 3: *Predictions on R, dashed line:DGLAP, continuous line:BFKL.*

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