$\gamma^*\gamma^*$ total cross-section in the dipole picture of BFKL dynamics

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The total $\gamma^*\gamma^*$ cross-section is derived in the Leading Order QCD dipole picture [1] of BFKL dynamics [2], and compared with the one from 2-gluon exchange. The Double Leading Logarithm approximation of the DGLAP cross-section is found to be small in the phase space studied. Cross sections are calculated for realistic data samples at the e^+e^- collider LEP and a future high energy linear collider. Next to Leading order corrections to the BFKL evolution have been determined phenomenologically, and are found to give very large corrections to the BFKL cross-section, leading to a reduced sensitivity for observing BFKL effects.

1. Differential cross-sections

The total $\gamma^*\gamma^*$ cross-section is derived in the Leading Order QCD dipole picture of BFKL dynamics. This could be a good test of the BFKL equation which an be performed at e$^+$-e$^-$ colliders (LEP or linear collider LC). The advantage of this process is that it is a purely perturbative process.

In this study, we compare the 2-gluon and the BFKL cross-sections. Defining y_1 (resp. y_2), and Q_1^2 (resp. Q_2^2) to be the rapidities and the squared transfered energies for both virtual photons, one gets

$$d\sigma_{e^+e^-}(Q_1^2, Q_2^2; y_1, y_2) = \frac{4}{9}\left(\frac{\alpha_{e.m}^2}{16}\right)^2 \alpha_s^2\, \pi^2 \sqrt{\pi}\, \frac{dQ_1^2}{Q_1^2}\frac{dQ_2^2}{Q_2^2}\frac{dy_1}{y_1}\frac{dy_2}{y_2}\frac{1}{Q_1\,Q_2} \frac{e^{\frac{4\alpha_s N_c}{\pi} Y \ln 2}}{\sqrt{\frac{14\alpha_s N_c}{\pi} Y\zeta(3)}} \times e^{-\frac{\ln^2 \frac{Q_1^2}{Q_2^2}}{\frac{56\alpha_s N_c}{\pi} Y\zeta(3)}} \left[2l_1 + 9t_1\right]\left[2l_2 + 9t_2\right] , \quad (1)$$

for the BFKL-LO cross-section, where $t_1 = \frac{1+(1-y_1)^2}{2}$, $l_1 = 1 - y_1$, and an analogous definition for t_2 and l_2, and $Y = \ln \frac{s y_1 y_2}{\sqrt{Q_1^2 Q_2^2}}$. The 2-gluon cross-section has been calculated exactly within the high energy approximation (NNNLO calculation) and reads

$$d\sigma_{e^+e^-}(Q_1^2, Q_2^2; y_1, y_2) = \frac{dQ_1^2}{Q_1^2}\frac{dQ_2^2}{Q_2^2} \frac{dy_1}{y_1}\frac{dy_2}{y_2}\frac{64(\alpha_{e.m}^2\alpha_s)^2}{243\pi^3}\frac{1}{Q_1^2} \Big[t_1 t_2 \ln^3\frac{Q_1^2}{Q_2^2} + (7t_1t_2 + 3t_1l_2 + 3t_2l_1)\ln^2\frac{Q_1^2}{Q_2^2} + \left((\frac{119}{2} - 2\pi^2)t_1t_2 + 5(t_1l_2 + t_2l_1) + 6l_1l_2\right)\ln\frac{Q_1^2}{Q_2^2} + \left(\frac{1063}{9} - \frac{14}{3}\pi^2\right)t_1t_2 + (46 - 2\pi^2)(t_1l_2 + t_2l_1) - 4l_1l_2\Big] . \quad (2)$$

Figure 1 shows the differential cross-sections in the BFKL, DGLAP Double Leading Logarithm (DLL) and 2-gluon approximation, as a function of $\ln Q_1^2/Q_2^2$ and for three values of Y. The cross-sections on the left hand side are calculated using the unintegrated exact formulae, for respectively the BFKL, DGLAP (in the double Leading Log approximation) and 2-gluon exchange cross-sections. Also the phenomenological HO-BFKL cross-sections, as detailed in section 2, are given.

We note that the 2-gluon cross-section is almost always dominating the DGLAP one in the Double Leading Log approximation. The saddle point approximation turns out to be a very good approximation for the BFKL cross-section and is not displayed in the figure (saddle-point results are close to the exact calculation up to 5% in the high Y region, and up to 10% at lower Y. A similar conclusion was reached in [5]). We note that the difference between the BFKL and 2-gluon cross-sections increase with Y.

On the right side of Figure 1, curves for the exact LO and saddle-point DGLAP calculations are shown, as well as the full NNNLO (eq. 2) result and the LO (eq. 2, $\ln^3 Q_1^2/Q_2^2$ term only) result for the 2-gluon cross-section. Unlike for the BFKL calculation, for the DGLAP case the saddle-point approximation appears to be in worse agreement with the exact calculation, and overestimates the cross-section by one order of magnitude, which is due to the fact that we are far away from the asymptotic regime. The comparison between the DGLAP-DLL and the 2-gluon cross-section in the LO approximation shows that both cross-sections are similar when Q_1 and Q_2 are not too different (the dashed line describes the value $Q_1^2/Q_2^2 = 2$), so precisely in the kinematical domain where the BFKL cross-section is expected to dominate. However, when Q_1^2/Q_2^2 is further away from one, the LO 2-gluon cross-section is lower than the DGLAP one, especially at large Y. This suggests that the 2-gluon cross-section could be a good approximation of the DGLAP one if both are calculated at NNNLO and restricted to the region where Q_1^2/Q_2^2 is close to one. In this paper we will use the exact NNNLO 2-gluon cross-section in the following to evaluate the effect of the non-BFKL background, since the 2-gluon term appears to constitute the dominant part of the DGLAP cross-section in the region $0.5 < Q_1^2/Q_2^2 < 2$.

2. Integrated cross-sections

Results based on the calculations developed above will be given for LEP (190 GeV centre-of-mass energy) and a future Linear Collider (500 - 1000 GeV centre-of-mass energy). $\gamma^*\gamma^*$ interactions are selected at e^+e^- colliders by detecting the scattered electrons, which leave the beampipe, in forward calorimeters. Presently at LEP these detectors can measure electrons with an angle θ_{tag} down to approximately 30 mrad. For the LC it has been argued [5] that angles as low as 20 mrad should be reached. Presently angles down to 40 mrad are foreseen to be instrumented for a generic detector at the LC.

Let us first specify the region of validity for the parameters controlling the basic assumptions made in the previous chapter. The main constraints are required by the validity of the perturbative calculations. The "perturbative" constraints are imposed by considering only photon virtualities Q_1^2, Q_2^2 high enough so that the scale μ^2 in α_S is greater than 3 GeV^2. μ^2 is defined using the Brodsky Lepage Mackenzie (BLM) scheme [6], $\mu^2 = \exp(-\frac{5}{3})\sqrt{Q_1^2 Q_2^2}$ [6]. In this case α_S remains always small enough such that the perturbative calculation is valid. In order that gluon contributions dominates the QED one, Y is required to stay larger than $\ln(\kappa)$ with $\kappa = 100$. (see Ref. [6] for discussion). Furthermore, in order to suppress DGLAP evolution, while maintaining BFKL evolution will constrain $0.5 < Q_1^2/Q_2^2 < 2$ for all nominal calculations. The comparison between the DGLAP-DLL and the 2-gluon cross-section in the LO approximation shows that both cross-sections are similar when Q_1 and Q_2 are not too different ($0.5 < Q_1^2/Q_2^2 < 2$), so precisely in the kinematical domain where the BFKL cross-section is expected to dominate. However, when Q_1^2/Q_2^2 is further away from one, the LO 2-gluon cross-section is lower than the DGLAP one, especially at large Y. This suggests that the 2-gluon cross-section could be a good approximation of the DGLAP one if both are calculated at NNNLO and restricted to the region where Q_1^2/Q_2^2 is close to one. In this paper we will use the exact NNNLO 2-gluon cross-section in the following to evaluate the effect of the non-BFKL background, since the 2-gluon term appears to constitute the dominant part of the DGLAP cross-section in the region $0.5 < Q_1^2/Q_2^2 < 2$.

We will not discuss here all the phenomenological results, and some detail can be found in [7], as well as the detailed calculations. We first study the effect of the tagged electron energy and angle. We first study the effect of increasing the LC detector acceptance for electrons scattered under small angles and the ratio of the 2-gluon and the BFKL-LO cross-sections increase by more a factor 3 if the tagging angle varies between 40 and 20 mrad. The effect of lowering the tagging energy is smaller. An important issue on the BFKL cross-section is the importance of the NLO corrections and we adopt a phenomenological approach to estimate the effects of higher orders. First, at Leading Order, the rapidity Y is not uniquely defined, and we can add an additive constant to Y. A second effect of NLO corrections is to lower the value of the so called Lipatov exponent in formula 1. In the fit of the proton structure function F_2 measured by the H1 collaboration [4], the value of the Lipatov exponent $\alpha_{I\!P}$ was fitted and found to be 1.282 [3,8], which gives an effective value of α_s of about 0.11. The same idea can be applied phenomenologically for the $\gamma^*\gamma^*$ cross-section. For this purpose, we modify the scale in α_S so that the effective value of α_S for $Q_1^2 = Q_2^2 = 25$ GeV^2 is about 0.11.

Finally, the results of the BFKL and 2-gluon cross-sections are given in Table 1 if we assume both effects. The ratio BFKL to 2-gluon cross-sections is reduced to 2.3 if both effects are taken into account together. In the same table, we also give these effects for LEP with the nominal selection and at the LC with a detector with increased angular acceptance. The column labelled 'LEP*' gives the results for the kinematic cuts used by the L3-collaboration who have recently presented preliminary results [9]. The cuts are $E_{tag} = 30$ GeV and $\theta_{tag} > 30$ mrad and $\mu^2 > 2$ GeV^2. For this selected region the difference between NLO-BFKL and 2-gluon cross-section is only a factor of 2.4. A cut on Q_1^2/Q_2^2, as done for the other calculations in this paper, would help to allow a more precise determination of the 2-gluon 'background'.

Another idea to establish the BFKL effects in data is to study the energy or Y dependence of the cross-sections, rather than the comparison with total cross-sections itself. To illustrate this point, we calculated the BFKL-HO and the 2-gluon cross-sections, as well as their ratio, for given cuts on rapidity Y (see table 2). We note that we can reach up to a factor 5 difference ($Y \geq 8.5$) keeping a cross-section measurable at LC. The cut $Y \geq$ 9. would give a cross-section hardly measurable at LC, even with the high luminosity possible at this collider. Cuts on Y will be hardly feasible at LEP because of the low cross-sections obtained already without any cuts on Y.

	BFKL$_{LO}$	BFKL$_{NLO}$	2-gluon	ratio
LEP	0.57	3.1E-2	1.35E-2	2.3
LEP*	3.9	0.18	6.8E-2	2.6
LC 40 mrad	6.2E-2	6.2E-3	2.64E-3	2.3
LC 20 mrad	3.3	0.11	3.97E-2	2.8

Table 1
Final cross-sections (pb), for selections described in the text.

Y cut	BFKL$_{NLO}$	2-gluon	ratio
no cut	1.1E-2	3.97E-2	2.8
$Y \geq 6.$	5.34E-2	1.63E-2	3.3
$Y \geq 7.$	2.54E-2	6.58E-3	3.9
$Y \geq 8.$	6.65E-3	1.43E-3	4.7
$Y \geq 8.5$	1.67E-3	3.25E-4	5.1
$Y \geq 9.$	5.36E-5	9.25E-6	5.8

Table 2
Final cross-sections (pb), for selections described in the text, after different cuts on Y

3. Conclusion

We have discussed here the difference between the 2-gluon and BFKL $\gamma^*\gamma^*$ cross-sections both at LEP and LC. The LO BFKL cross-section is much larger than the 2-gluon cross-section. Unfortunately, the higher order corrections of the BFKL equation (which we estimated phenomenologically) are large, and the 2-gluon and BFKL-HO cross-sections ratios are reduced to a factor two to four. The Y dependence of the cross-section remains a powerful tool to increase this ratio and is more sensitive to BFKL effects, even in the presence of large higher order corrections. Further more, the higher order corrections to the BFKL equations were treated here only phenomenologically, and we noticed that even a small change on the BFKL pomeron intercept implies large changes on the cross-sections. The uncertainty on the BFKL cross-section after higher order corrections is thus quite large. We thus think that the measurement performed at LEP or at

LC should be compared to the precise calculation of the 2-gluon cross-section after the kinematical cuts described in this paper, and the difference can be interpreted as BFKL effects. A fit of these cross-sections will then be a way to determine the BFKL pomeron intercept after higher order corrections.

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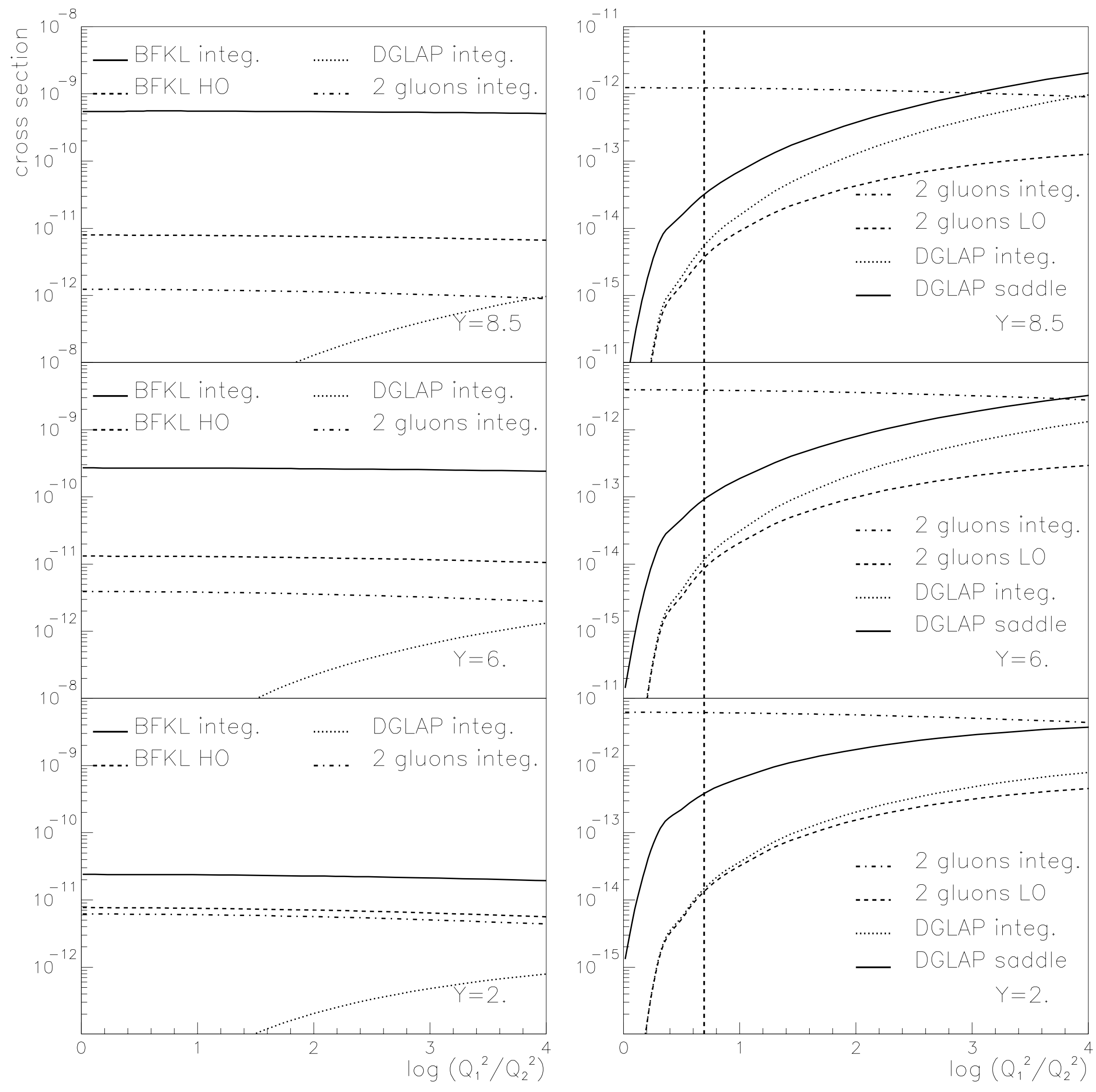

Figure 1. Differential cross-sections for different values of Y (see text).