Neutral weak currents in pion electroproduction on the nucleon.

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Abstract

Parity violating asymmetry in inclusive π^0 or π^+ meson production by longitudinally polarized electrons on unpolarized protons, is calculated as a function of the momentum transfer square k^2 and the total energy W of the πN-system. We consider the Δ-contribution in the s-channel, the standard Born contributions and the vector mesons (ρ and ω) exchanges in the $t-$channel. The parity-odd term is the sum of two contributions. The main term (found to be linear in k^2) comes from the isovector component of the electromagnetic currents. It is model independent and can be calculated exactly in terms of fundamental constants. The second term is isoscalar in nature. Near threshold and in the Δ-region, it is found to be much smaller (in absolute value) than the isovector one.

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I. INTRODUCTION

Parity violation (PV) was discovered in 1956 in nuclear beta-decay by C.S. Wu [1], following a suggestion of T.D. Lee and C. N. Yang [2]. In 1960, Ya. Zeldovich [3] pointed out that PV should lead to parity-odd (P-odd) terms also in electron-hadron interactions. These are now considered as a manifestation of the electroweak interaction, whose properties are dictated by the Standard Model (SM). Several P-odd observables have been studied since, in two types of PV experiments, namely in atomic physics [4,5] (at very low energy and momentum transfer) and in electron scattering (at relatively high energies and non-zero momentum transfers).

At first, these experiments were aiming at testing the SM and measuring the Weinberg angle. A pioneering experiment was performed at SLAC on a deuterium target [6], followed 10 years later by experiments at Mainz on ^{9}Be [7] and Bates on ^{12}C [8]. Their determination of the Weinberg angle were confirmed later on, within their stated accuracy of 10%, by high energy experiments. Since $sin^2\theta_W$ is now known to three decimal places [$sin^2\theta_W$ = 0.23124(24)] [9], the emphasis of e-p scattering today, is to make use of the SM to learn about the internal structure of the nucleon.

Until recently, it has been assumed that the nucleon was only made of u and d valence or sea quarks, but there are indications that the nucleon carries also hidden strangeness:

- the Σ-term (deduced from the pion-nucleon scattering length) is very different from the theoretical value calculated within the chiral perturbation theory (which is a realization of the SM at low energy), indicating that 35% of the nucleon mass might be carried out by strange quarks. See [10] and refs. herein.

- experiments of polarized Deep-Inelastic-Scattering (DIS) of leptons show that up to 10-20% of the nucleon spin could be carried by strange quarks [11–14],

- elastic scattering of neutrinos and anti-neutrinos by protons can only be explained by taking into account strange quarks in the nucleon [15,16],

- a natural explanation of the strong violation of the OZI-rule in $p\overline{p}$ annihilation [17,18] and of ϕ- production [19] or η-meson production [20] in nucleon-nucleon interactions takes into account a nucleon (antinucleon) strange sea.

These experiments are sensitive to various aspects of nucleon structure: for example, the Σ-term and NN or $N\overline{N}$ experiments are sensitive to the *scalar* part of the hadronic current, polarized DIS is sensitive to the *vector-axial* current and elastic scattering of neutrinos/anti-neutrinos to the *axial* current. In this respect, PV in electron-nucleon scattering seems the most attractive way of measuring the strange *vector* current, thanks to a clean theoretical interpretation through the SM.

The SAMPLE collaboration at MIT-Bates, has measured PV asymmetries at $-k^2$= 0.1 GeV^2 and large angle [21], which allowed them to obtain the first experimental determination of the weak magnetic form-factor of the proton. From this measurement and the knowledge of the proton and neutron electromagnetic form-factors, one could extract a strange magnetic form-factor consistent with zero within the stated uncertainties.

A recent measurement with a deuteron target [22], which is much less sensitive to G_M^s and is thus essentially determining the axial proton current, shows that the isovector axial form-factor $G_A^e(T=1)$ has not the sign predicted by theory. This could be the result of a large anapole contribution [23]. When combined with the earlier SAMPLE result [21], one obtains a very small strange magnetic moment for the proton $\mu_s = 0.01 \pm 0.29(stat) \pm 0.31(syst) \pm 0.07(th)$.

Another experiment, done by the HAPPEX collaboration at Jefferson Lab, [24,25] has done a measurement at $-k^2$= 0.48 GeV^2 and small scattering angle $\theta_e = 35^0$ where the sensitivity to the weak electric form factor G_E^Z is increased. Here the measured asymmetry $A = (-14.2 \pm 2.2) \cdot 10^{-6}$ is consistent with the SM prediction in the absence of $< s\bar{s} >$ components in the nucleon sea. From this asymmetry, one can deduce the following contribution to the strange form-factor:

$$G^s_{HAPPEX} = G^s_E + 0.39 G^s_M = 0.025 \pm 0.020 \pm 0.14,$$

again compatible with zero within the error bars. In this kinematics, the axial form-factor has a negligible effect.

Although disappointing at first sight, these results have stimulated a strong interest and many predictions have been published, whether within quark models [26,27], QCD sum rules [28] or Chiral Perturbation Theories [29,30]. These calculations predict that while G_M^s is essentially constant as a function of k^2, G_E^s may vary rapidly. They also indicate that there might be some cancellation between G_E^s and G_M^s which are predicted of different signs. Therefore new e-p experiments are being set up in order to check these predictions: at $-k^2 = 0.225$ GeV^2 at MAMI-Mainz [31], at $-k^2 = 0.1$ GeV^2 and forward angles by the HAPPEX collaboration to do a Rosenbluth separation of G_E^s and G_M^s in combination with the SAMPLE results, and finally a full separation of G_E^s and G_M^s in the momentum transfer range $|k^2| = 0.12$-1.0 GeV^2 is foreseen by the G^0 collaboration at Jefferson Lab [32].

The reactions $e + p \to e + p + \pi^0$ and $e + p \to e + n + \pi^+$ are of practical interest for experimentalists as they may contaminate the elastic peak. It is therefore important to determine their own asymmetries since, if they are much larger than or, even, of different sign from the elastic one, they might be a source of errors or large uncertainties. This applies also to the estimation of possible background in SLAC E-158 experiment [33], which aims to measure the left-right asymmetry in Möller scattering, $e^- + e^- \to e^- + e^-$. The knowledge of P-odd asymmetries for pion electroproduction is also important for the estimation of parity violating asymmetry in inclusive pion electroproduction for proton, in the region of the Δ-resonance [34].

In 3-body reactions, besides the weak PV asymmetries, there are also strong (parity-conserving) interactions, due to the so-called 5^{th} response function [35], which are generally much larger (of the order of $10^{-2} - 10^{-3}$ instead of $10^{-5} - 10^{-6}$) than PV asymmetries but which cancel in inclusive reactions or when detectors have an azimuthal asymmetry.

Pion production has been studied previously [23,36–42] in quasi 2-body models with stable isobars, i.e. $e^- + N \to e^- + \Delta$. A more complete calculation including background (Born) terms with pseudovector πN coupling with the Δ treated as a Rarita-Schwinger field with phenomenological πN electromagnetic transition currents can be found in [43].

In the present study, we calculate PV asymmetries in inclusive $N(e,e')N\pi$ electroproduction, starting from threshold up to the Δ-region in an approach differing from [43] in many aspects:

- the main improvement consists in including ω- and ρ-exchange in the t-channel for $\gamma^*(Z^*) + N \to \pi + N$ (where $\gamma^*(Z^*)$ is a virtual photon (boson)),
- we use a different parametrization for the Δ contribution, which is free from off-mass shell effects and slightly different values of mass and width,
- crossing symmetry is treated differently than in Ref. [43],
- we use a pseudoscalar πNN interaction in order to identify possible off-mass-shell effects,
- finally we use a specific parametrization for the asymmetry, which separate, in a model independent way, the main (isovector) contribution (which depends only on the Fermi constant G_F, the fine structure constant α and $sin^2\theta_W$) and the smaller isoscalar part.

II. P-ODD BEAM ASYMMETRY FOR $e^- + N \to e^- + N + \pi$

We shall consider here the processes $e^- + N \to e^- + N + \pi$, where N is a nucleon (p or n) and π is a pion (π^0 or π^+). We take into account two standard mechanisms, $\gamma-$ and $Z-$ boson exchanges (Fig. 1), predicted by the SM. The matrix element can be written in the following form:

$$\mathcal{M} = \mathcal{M}_\gamma + \mathcal{M}_Z,$$

$$\mathcal{M}_\gamma = -\frac{e^2}{k^2}\ell_\mu \mathcal{J}_\mu^{(em)},$$

$$\mathcal{M}_Z = \frac{G_F}{2\sqrt{2}}\left(g_v^{(e)}\ell_\mu + g_a^{(e)}\ell_{\mu,5}\right)\left(\mathcal{J}_\mu^{(nc)} + \mathcal{J}_{\mu,5}^{(nc)}\right), \tag{1}$$

where G_F is the Fermi constant of the weak interaction, $\mathcal{J}_\mu^{(em)}$ is the electromagnetic current for $\gamma^*+N \to N+\pi$, $\mathcal{J}_\mu^{(nc)}$ and $\mathcal{J}_{\mu,5}^{(nc)}$ are the vector and vector-axial parts of the neutral weak current for $Z^* + N \to N + \pi$. The four-vectors ℓ_μ and $\ell_{\mu,5}$ are the vector and vector-axial parts of the neutral weak current of a point-like electron:

$$\ell_\mu = \overline{u}(k_2)\gamma_\mu u(k_1), \quad \ell_{\mu,5} = \overline{u}(k_2)\gamma_5\gamma_\mu u(k_1) \tag{2}$$

where k_1 (k_2) is the four-momentum of the initial (final) electron. The notation for the particle four momenta is explained in Fig. 1. In the Standard Model the constants $g_a^{(e)}$ and $g_v^{(e)}$ are determined by the following expressions: $g_a^{(e)} = 1, \quad g_v^{(e)} = 1 - 4\sin^2\theta_W$.

The P-odd asymmetry in the scattering of longitudinally polarized electrons can be written as:

$$A = \frac{N_+ - N_-}{N_+ + N_-} = -\frac{G_F|k^2|}{2\sqrt{2}\pi\alpha}\frac{W^-}{W^{(em)}}, \tag{3}$$

with two different contributions to W^-:

$$W^- = g_a^{(e)}\widetilde{W_1} + g_v^{(e)}\widetilde{W_2}, \tag{4}$$

where $W^{(em)}$ is proportional to $\overline{|\mathcal{M}_\gamma|^2}$:

$$W^{(em)} = \ell_{\mu\nu}W_{\mu\nu}^{(em)}, \quad W_{\mu\nu}^{(em)} = \overline{\mathcal{J}_\mu^{(em)}\mathcal{J}_\nu^{(em)*}}, \tag{5}$$

$$\ell_{\mu\nu} = 2\left(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} - g_{\mu\nu}k_1\cdot k_2\right), \tag{6}$$

and the overline in Eq. (5) stands for the sum over the final nucleon polarizations and the average over the polarizations of the initial nucleon in the process $\gamma^* + N \to N + \pi$. The quantities $\widetilde{W_1}$ and $\widetilde{W_2}$ in Eq. (4) characterize the interference of the electromagnetic hadronic current $\mathcal{J}_\mu^{(em)}$ with the vector and axial parts of the weak neutral current:

$$\widetilde{W_1} = \ell_{\mu\nu}W_{\mu\nu}^{(v)}, \quad W_{\mu\nu}^{(v)} = \frac{1}{2}\overline{\mathcal{J}_\mu^{(em)}\mathcal{J}_\nu^{(nc)*}}, \tag{7}$$

$$\widetilde{W_2} = \ell_{\mu\nu}^{(a)}W_{\mu\nu}^{(a)}, \quad W_{\mu\nu}^{(a)} = \frac{1}{2}\overline{\mathcal{J}_\mu^{(em)}\mathcal{J}_{\nu,5}^{(nc)*}}, \tag{8}$$

and

$$\ell_{\mu\nu}^{(a)} = 2i\epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}, \tag{9}$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the usual antisymmetric tensor.

Due to $g_v^{(e)} \ll g_a^{(e)}$, we can neglect the $\widetilde{W_2}$ contribution (the second P-odd contribution, which is induced by the axial part of the neutral weak current, is more model dependent and it will be the object of a detailed analysis in a subsequent paper).

In this approximation, the P-odd asymmetry is solely determined by the vector part of the hadronic neutral weak current:

$$A = -\frac{G_F|k^2|}{2\sqrt{2}\pi\alpha}\frac{\widetilde{W_1}}{W^{(em)}}, \tag{10}$$

In order to calculate the ratio $\widetilde{W_1}/W^{(em)}$, we shall use the isotopic structure of the vector neutral current, which holds in the SM when neglecting the contributions of the isoscalar quarks (s, c,...):

$$\mathcal{J}_\mu^{(nc)} = 2\mathcal{J}_\mu^{(1)} - 4\sin^2\theta_W \mathcal{J}_\mu^{(em)} =$$

$$2(1-2sin^2\theta_W)\mathcal{J}_\mu^{(em)} - 2\mathcal{J}_\mu^{(0)}, \tag{11}$$

where $\mathcal{J}_\mu^{(0)}$ and $\mathcal{J}_\mu^{(1)}$ are the isoscalar and isovector components of the electromagnetic hadronic current. Considering the specific isotopic structure of $\mathcal{J}_\mu^{(nc)}$, Eq. (11), the asymmetry A for any process $\vec{e}+N \to e+N+\pi$ can be written as:

$$A = -\frac{G_F|k^2|}{2\sqrt{2}\pi\alpha}\left[1-2\sin^2\theta_W + \Delta^{(s)}\right], \tag{12}$$

where the quantity $\Delta^{(s)}$ results from the interference of the isoscalar component $\mathcal{J}_\mu^{(0)}$ of the electromagnetic current with the full electromagnetic current in $\mathcal{J}_\mu^{(em)}$ i.e.:

$$\Delta^{(s)} = \frac{W^{(0)}}{W^{(em)}}, \quad W^{(0)} = -\ell_{\mu\nu}\overline{\mathcal{J}_\mu^{(em)}\mathcal{J}_\nu^{(0)*}}. \tag{13}$$

One can see from Eq. (12) that the isovector part of the electromagnetic current induces a definite contribution to the P-odd asymmetry A, which is model independent and can be predicted in terms of the fundamental constants G_F, α and $\sin^2\theta_W$ only. Note that this contribution depends on the single kinematical variable k^2. Therefore, for reactions such as $e^- + N \to e^- + \Delta$, $e^- + d \to e^- + d + \pi^0$, where the electromagnetic current is pure isovector (and therefore $\Delta^{(s)} = 0$), the asymmetry can be predicted exactly:

$$A = -\frac{G_F|k^2|}{2\sqrt{2}\pi\alpha}\left[1 - 2\sin^2\theta_W\right], \tag{14}$$

in agreement with ref. [37] and neglecting the small contributions from the axial hadronic current, which is not considered here (note that $\frac{G_F}{2\sqrt{2}\pi\alpha} = 1.8\cdot 10^{-4}/\mathrm{GeV}^2$). In particular, for the reaction $e^- + p \to e^- + \Delta^+$ this model-independent estimate of A together with the possibility of a precise measurement of the P-odd asymmetry, open new ways to look for new physics [36] and to study effects due to the axial current.

In the next section, we will show that the quantity $\Delta^{(s)}$, in the near-threshold region for $e^- + N \to e^- + N + \pi$, as well as in the region of the Δ excitation, can be considered as a small correction to the main isovector contribution. Therefore, the uncertainty in the estimate of $\Delta^{(s)}$ will affect very little the results.

From Eq. (12) it appears that the inclusive asymmetry A depends on the variables E_1 and W only through the correction $\Delta^{(s)}$: $\Delta^{(s)} = \Delta^{(s)}(k^2, W, E_1)$. Taking into account the longitudinal and transversal polarizations of the virtual γ and Z-boson, the following representation for the correction $\Delta^{(s)}$ can be written (in case of a single channel: $e + p \to e + p + \pi^0$ or $e + p \to e + n + \pi^+$):

$$\Delta^{(s)} = \frac{\sigma_T^{(s)} + \epsilon\dfrac{(-k^2)}{\widetilde{k_0^2}}\,\sigma_L^{(s)}}{\sigma_T + \epsilon\dfrac{(-k^2)}{\widetilde{k_0^2}}\,\sigma_L}, \tag{15}$$

$$\epsilon^{-1} = 1 - 2\frac{(-\vec{k}^2)}{k^2}\tan^2\frac{\theta_e}{2},\quad \widetilde{k_0} = \frac{W^2 + k^2 - m^2}{2W},$$

where $\sigma_T(k^2, W)$ and $\sigma_L(k^2, W)$ are the total cross sections of virtual photon absorption in $\gamma^* + N \to N + \pi$:

$$\sigma_L = \int \overline{\left|\mathcal{J}_z^{(em)}\right|^2} d\Omega_\pi,\quad \sigma_T = \int \overline{\left(\left|\mathcal{J}_x^{(em)}\right|^2 + \left|\mathcal{J}_y^{(em)}\right|^2\right)} d\Omega_\pi. \tag{16}$$

We use here a coordinate system in which the z-axis is along the three momentum of the virtual photon, and $\mathcal{J}_x^{(em)}$, $\mathcal{J}_y^{(em)}$ and $\mathcal{J}_z^{(em)}$ are the space components of the hadronic electromagnetic current.

The interference contributions $\sigma_L^{(s)}$ and $\sigma_T^{(s)}$ are defined as follows:

$$\sigma_L^{(s)}(k^2, W) = \int d\Omega_\pi \mathcal{R}e\ \overline{\mathcal{J}_z^{(em)}\mathcal{J}_z^{(0)*}},$$
$$\sigma_T^{(s)}(k^2, W) = \int d\Omega_\pi \mathcal{R}e\ \left[\overline{\mathcal{J}_x^{(em)}\mathcal{J}_x^{(0)*} + \mathcal{J}_y^{(em)}\mathcal{J}_y^{(0)*}}\right], \tag{17}$$

where $\vec{\mathcal{J}}^{(0)}\left(\mathcal{J}_x^{(0)}, \mathcal{J}_y^{(0)}, \mathcal{J}_z^{(0)}\right)$ are the space components of the isoscalar part of the hadronic electromagnetic current. The lines above the products of the components of the electromagnetic currents mean the sum over the polarizations of the final nucleons and the average over the polarizations of the initial nucleons.

The inclusive asymmetry for $p(\vec{e}, e')N\pi$ with the contribution of two channels $p + \pi^0$ and $n + \pi^+$ in the final state, is determined by the following expressions:

$$A = -\frac{G_F|k^2|}{2\sqrt{2}\pi\alpha}\left[1 - 2\sin^2\theta_W + \Delta_{incl}^{(s)}\right],$$
$$\Delta_{incl}^{(s)} = \frac{\Delta^{(s)}(\gamma^* p \to n\pi^+) + R\Delta^{(s)}(\gamma^* p \to p\pi^0)}{(1+R)}, \tag{18}$$

with

$$R = \frac{\sigma_T(\gamma^* p \to p\pi^0) + \epsilon\dfrac{(-k^2)}{\widetilde{k_0^2}}\sigma_L(\gamma^* p \to p\pi^0)}{\sigma_T(\gamma^* p \to n\pi^+) + \epsilon\dfrac{(-k^2)}{\widetilde{k_0^2}}\sigma_L(\gamma^* p \to n\pi^+)}.$$

Therefore, the P-odd inclusive asymmetry A for $p(\vec{e}, e')N\pi$, $N\pi = (p + \pi^0) + (n + \pi^+)$ is determined by a set of four total cross sections:

$$\sigma_T(k^2, W),\ \sigma_L(k^2, W),\ \sigma_T^{(s)}(k^2, W),\ \text{and}\ \sigma_L^{(s)}(k^2, W),$$

for each $\gamma^* + p \to n + \pi^+$ and $\gamma + p \to p + \pi^0$ processes (8 in total), as functions of two independent kinematical variables k^2 and W.

In the present calculation we shall use the following parametrization of the spin structure of the matrix element for $\gamma^* + N \to N + \pi$, in terms of six standard contributions:

$$\mathcal{M}(\gamma^* N \to N\pi) = \chi_2^\dagger \mathcal{F}\chi_1,$$

$$\mathcal{F} = i\vec{e} \cdot \hat{\vec{k}} \times \hat{\vec{q}} f_1 + \sigma \cdot \vec{e} f_2 + \vec{\sigma} \cdot \hat{\vec{k}}\, \vec{e} \cdot \hat{\vec{q}} f_3 + \vec{\sigma} \cdot \hat{\vec{q}}\, \vec{e} \cdot \hat{\vec{q}} f_4 \tag{19}$$

$$+\vec{e} \cdot \hat{\vec{k}} (\vec{\sigma} \cdot \hat{\vec{k}} f_5 + \vec{\sigma} \cdot \hat{\vec{q}} f_6),$$

where χ_1 and χ_2 are the two-component spinors of the initial and final nucleons, $\vec{e}$ is the three-vector of the virtual photon polarization, $\hat{\vec{k}}$ and $\hat{\vec{q}}$ are the unit vectors along the 3-momentum of the γ^* and π in the CMS of the $\gamma^* + N \to N + \pi$-reaction.

III. MODEL FOR $e^- + N \to e^- + N + \pi$

We use here the standard approach for the calculation of the electromagnetic current for the $\gamma^* + N \to N + \pi$ processes, which describes satisfactorily well the existing photo- and electro-production data, in the region of W starting from threshold, $W = m + m_\pi$, up to $W \simeq 1.3$ GeV (the Δ excitation region). This approach takes into account the following three contributions:

- Born terms in the s, t and u channels,
- vector meson (ω and ρ) exchanges in the t-channel,
- Δ-isobar excitation in the s channel.

Using the isotopic structure of the 'strong' vertices on the diagrams (Fig. 2), the scalar amplitudes for each $\gamma^* + N \to N + \pi$ process can be written as:

$$f_i = \sqrt{(E_1 + m)(E_2 + m)} \left[a_s f_{i,s} + a_u f_{i,u} + a_t f_{i,t} \right.$$

$$\left. + a_\rho f_{i,\rho} + a_\omega f_{i,\omega} + a_\Delta f_{i,\Delta} \right], \tag{20}$$

where $f_{i,s} ... f_{i,\Delta}$ characterize the contributions of the different Feynmann diagrams to the scalar amplitudes f_i, $i = 1-6$, E_1 (E_2)is the energy of the initial (final) nucleon. The isotopic numerical coefficients $a_s ... a_\Delta$ for the two processes $\gamma^* + p \to p + \pi^0$ and $\gamma^* + p \to n + \pi^+$ are shown in Table 1.

One can see now that, in the framework of the considered approach, the main contributions to $\mathcal{J}_\mu^{(em)}$ have an isovector nature:

- Δ-excitation in π^+ and π^0 production,
- π^+-exchange for π^+ production,
- ω-exchange for π^0 production,
- contact term for π^+ production (in the case of a pseudovector πNN-interaction),
- $s+u$ Born contributions.

Therefore, the isoscalar electromagnetic current can only contain the following contributions:

- $\rho-$exchange for π^0 and π^+-production,
- the isoscalar part of the $s+u$-diagrams.

However these isoscalar contributions are small in comparison with the corresponding isovector ones. Indeed, the ρ-exchange term is smaller than the ω -exchange term, due to the following reasons: $g_{\rho\pi\gamma} \simeq \frac{1}{3} g_{\omega\pi\gamma}$: suppression at electromagnetic vertices; and $g_{\rho NN} \simeq \frac{1}{6} g_{\omega NN}$: suppression at the strong vertex.

In the same way, the isoscalar Born contribution due to the nucleon magnetic moment, for example, is smaller than the isovector contribution: $\frac{|\mu_p+\mu_n|}{|\mu_p-\mu_n|} \approx 10^{-2}$.

This clearly shows that $\Delta^{(s)}$ can be considered a small correction to the model-independent prediction of Eq. (14). Let us briefly discuss now the properties of the suggested model, for the $\gamma^* + p \to N + \pi$ processes.

A. Born contribution

Using a pseudoscalar πNN-interaction, we can write the relativistic invariant expression for the matrix element of the $\gamma^\star + p \to n + \pi^+$ reaction in the following form:

$$\mathcal{M}_B = eg(\mathcal{M}_s + \mathcal{M}_u + \mathcal{M}_t),$$

$$\mathcal{M}_s = \overline{u}(p_2)\gamma_5 \frac{\hat{p_2}+\hat{q}+m}{s-m^2}\left(F_{1p}\hat{e} + F_{2p}\frac{\sigma_{\mu\nu}e_\mu k_\nu}{2m}\right)u(p_1),$$

$$\mathcal{M}_u = \overline{u}(p_2)\left(F_{1n}\hat{e} + F_{2n}\frac{\sigma_{\mu\nu}e_\mu k_\nu}{2m}\right)\frac{\hat{p_2}+\hat{q}-m}{u-m^2}\gamma_5 u(p_1),$$

$$\mathcal{M}_t = \frac{(2e\cdot q - e\cdot k)}{t-m_\pi^2}\overline{u}(p_2)\gamma_5 u(p_1),$$

where s, t, and u are the standard Mandelstam variables: $s = (p_2+q)^2$, $t = (p_1 - p_2)^2$, $u = (p_2 - k)^2$, k is the four-momentum of γ^*, e_μ is the four-vector of the virtual photon polarization, g is the πNN coupling constant (for a pseudoscalar interaction), $F_{1p}(k^2)$ and $F_{2p}(k^2)$ ($F_{1n}(k^2)$ and $F_{2n}(k^2)$) are the Dirac and Pauli electromagnetic form factors of the proton (neutron). The electromagnetic form factors of the nucleon are usually parametrized in form of a k^2-dependence of the electric (G_{EN}) and magnetic (G_{MN}) form factors:

$$F_{1N}(k^2) = \frac{G_{EN}(k^2) - \tau G_{MN}(k^2)}{1-\tau},$$

$$F_{2N}(k^2) = \frac{-G_{EN}(k^2) + G_{MN}(k^2)}{1-\tau}, \quad \tau = \frac{k^2}{4m^2}.$$

A simple dipole dependence of G_{Ep}, G_{Mp} and G_{Mn}:

$$G_{Ep}(k^2) = G_{Mp}(k^2)/\mu_p = G_{Mn}(k^2)/\mu_n =$$

$$\left[1 - \frac{k^2}{0.71\ \mathrm{GeV}^2}\right]^{-2} = G_D,$$

with $\mu_p = 2.79, \mu_n = -1.91$, has been considered as a good parametrization of the existing experimental data, while $G_{En}(k^2)$=0, in a wide region of space-like momentum transfer,. However a recent direct measurement [44] of the ratio G_{Ep}/G_{Mp} shows some deviation of G_{Ep} from a dipole behavior, in the region $0 \leq -k^2 \leq 3.5$ GeV^2. This high precision experiment is based on the measurement of the polarization of the final protons in $\vec{e}+p \to e+\vec{p}$, in the elastic scattering of longitudinally polarized electrons [45].

This effect should be taken into account in future calculations, as well as the fact that G_{En} deviates from zero, at least in the region $k^2 \leq 1$ GeV^2. The last direct measurements of G_{En}, in $\vec{e}+\vec{d} \to e+X$ [46] confirm previous parametrizations [47,48]. A recent derivation of G_{En} up to $-k^2 = 3.5$ GeV^2 has been done in [49]. In the Vector Dominance Model (VDM) approach, the pion electromagnetic form factor $F_\pi(k^2)$ is described by: $F_\pi(k^2) = \left(1 - \frac{k^2}{{m_\rho}^2}\right)^{-1}$, where m_ρ is the ρ-meson mass. Effects of possible variations of these form factors have been extensiveley analyzed in the framework of the present model in [50], up to large momentum transfer square: $-k^2 \leq 2$ GeV^2. In the present case these variations can be considered a second order correction for the small quantity $\Delta^{(s)}$, Eq. (18), and will be quantitatively discussed at the end of Section IV.

Note that the electromagnetic current for the reaction $\gamma^* + p \to p + \pi^0$, corresponding to the sum of the Born diagrams in the s and u-channels, is conserved for any form factors F_{1p} and F_{2p} in the whole kinematical region. This is not the case for the reaction $\gamma^* + p \to n + \pi^+$. A possible way to avoid this difficulty is to renormalize the matrix element $\mathcal{M}_B(\gamma^* p \to n\pi^+)$ in the following way:

$$\mathcal{M}_B \to \mathcal{M}'_B = \mathcal{M}_B + eg\frac{e \cdot k}{k^2}\overline{u}(p_2)\gamma_5 u(p_1)\left(-F_{1p} + F_{1n} + F_\pi\right), \tag{21}$$

The electromagnetic current, corresponding to the new Born matrix element $\mathcal{M}'_B$ is conserved for any form factor. Such a procedure changes only σ_L, without any effect on the transversal cross-sections $\sigma_T(k^2, W)$ and $\sigma_T^{(s)}(k^2, W)$. Moreover, this additional term which restores the gauge invariance, has evident isovector nature, and it does not contribute to $\mathcal{J}_\mu^{(0)}$. This implies that for the calculation of the main term of the asymmetry A, Eq. (18), which is isovector, we do not have problems with the gauge invariance, for pion electroproduction. This is also an advantage of the present method, based on the separation of isovector and isoscalar components of hadronic currents. The scalar amplitudes f_i, corresponding to different diagrams of the Born mechanism, are given in the Appendix.

B. Vector meson exchange

The matrix element $\mathcal{M}_V$, corresponding to vector meson exchange in the $t-$channel can be written in the following form:

$$\mathcal{M}_V = \frac{e g_{V\pi\gamma^*}(k^2)}{t-m_V^2}\epsilon_{\mu\nu\alpha\beta}e_\mu k_\nu \mathcal{J}_\alpha^{(V)} q_\beta, \tag{22}$$

$$\mathcal{J}_\alpha^{(V)} = \overline{u}(p_2)\left[\gamma_\alpha F_1^V(t) - \frac{F_2^V(t)}{2m}\sigma_{\alpha\beta}(p_1-p_2)_\beta\right]u(p_1),$$

where $g_{V\pi\gamma^*}(k^2)$ is the electromagnetic form factor for the $V\pi\gamma^*$-vertex, m_V is the vector meson mass, $F_1^V(t)$ and $F_2^V(t)$ are the "strong" form factors for the V^*NN vertex (with a virtual V-meson). In principle the "static" values of these form factors (i.e. for $t=0$), are related to the ωNN and ρNN coupling constants: $F_1^V(0) = g_{VNN}$, $F_2^V(0)/F_1^V(0) = \kappa_V$. An estimate for the ωNN coupling constants, based on the Bonn potential [51], gives:

$$\frac{g_{\omega NN}^2}{4\pi} = 20, \quad \kappa_\omega = 0$$

The ρNN coupling constants can be estimated from pion photoproduction data [52]: $\frac{g_{\rho NN}^2}{4\pi} = 0.55, \quad \kappa_\rho = 3.7.$

The VDM allows to write the following parametrization for the k^2 dependence of the electromagnetic form factor for the $\gamma^* + V \to \pi$ vertex (*hard* form factor):

$$g_{V\pi\gamma^*}(k^2) = \frac{g_{V\pi\gamma}(0)}{1-k^2/m_V^2}. \tag{23}$$

The $g_{V\pi\gamma}(0)$ coupling constant can be fixed by the width of the radiative decay $V\to\pi\gamma$, through the following formula:

$$\Gamma(V\to\pi\gamma) = \frac{\alpha}{24}|g_{V\pi\gamma}(0)|^2\left(1-\frac{m_\pi^2}{m_V^2}\right)^3.$$

The numerical estimate, is based on the following values [53]: $Br(\omega\to\pi^0\gamma) = \Gamma(\omega\to\pi\gamma)/\Gamma_\omega = (8.5\pm0.5)10^{-2}$, $\Gamma_\omega = (8.41\pm0.09)$ MeV, $Br(\rho^0\to\pi^0\gamma) = (6.8\pm1.7)\cdot10^{-4}$, $Br(\rho^\pm\to\pi^\pm\gamma) = (4.5\pm0.5)\cdot10^{-4}$, $\Gamma_\rho = (150.7\pm1.1)$ MeV. The following relation holds for the hadronic electromagnetic current: $Br(\rho^\pm\to\pi^\pm\gamma) = Br(\rho^0\to\pi^0\gamma)$. Therefore any violation of this relation is an indication of the presence of an isotensor component of the electromagnetic current, which is absent, however, at the quark level. So, a precise experiment with the simultaneous determination of the two coupling constants for $\rho^0\to\pi^0\gamma$ and

$\rho^{\pm} \to \pi^{\pm}\gamma$ would be very important. It would not only constitute a test of the isotopic properties of the hadronic electromagnetic current, but also have application in the calculation of the meson exchange current contributions (MEC) to the deuteron electromagnetic form factors.

Note that the relative sign of the $V-$exchange and Born contributions to the $\gamma^* + p \to N + \pi$ processes, is generally not known. So, we shall consider here both relative signs.

Finally we stress that the electromagnetic current, corresponding to vector meson exchange in the processes $\gamma^* + p \to N + \pi$ is automatically conserved, independently of the parametrization of the strong form factors $F_1(t)$ and $F_2(t)$ and the electromagnetic form factor $g_{V\pi\gamma^*}(k^2)$.

C. Δ-excitation

This contribution can be analyzed in a relativistic framework [43], considering a virtual Δ as a Rarita-Schwinger field with spin 3/2, but in this approach it is difficult to treat off-shell effects. First of all, this means that $\Delta-$exchange may contain contributions from a state with spin 1/2 as well as antibaryonic terms with negative P-parity and s=1/2 and 3/2. Therefore the description of the $\Delta-$isobar, with $\mathcal{J}^P = 3/2^+$, especially in the $s-$channel is not straightforward. To avoid these complications, we choose here a direct parametrization of the Δ contribution. Note that the CMS for $\gamma^* + p \to \Delta^+ \to N + \pi$ is the optimal frame, because the three-momentum of the Δ is zero, so that the Δ can be described by a two-component spinor, with a vector index, $\vec{\chi}$, which satisfies the following auxiliary condition: $\vec{\sigma} \cdot \vec{\chi} = 0$, typical for a pure spin 3/2 state. Using this condition, it is possible to find the following expression for the Δ-density matrix: $\rho_{ab} = \overline{\chi_a \chi_b^\dagger} = \frac{2}{3}(\delta_{ab} - \frac{i}{2}\epsilon_{abc}\sigma_c)$, with the normalization condition: $Tr\rho_{\alpha\alpha} = 2s_\Delta + 1 = 4$.

In this formalism the $\Delta N\pi$-vertex can be parametrized as follows: $\mathcal{M}_{\Delta N\pi} = g_{\Delta N\pi}\chi^\dagger \vec{\chi}\cdot\hat{\vec{q}}$, where χ is the 2-component spinor of the nucleon in the decay $\Delta \to N + \pi$, $\hat{\vec{q}}$ is the unit vector along the pion three momentum, in the Δ rest frame, and the constant $g_{\Delta N\pi}$

characterizes the width of the strong decay $\Delta \to N + \pi$.

Taking into account the conservation of the total angular momentum and of the P-parity in the electromagnetic decay $\Delta \to N + \gamma$ with production of M1 photons, the following expression can be written for the matrix element:

$$\mathcal{M}_{\Delta N\gamma} = eg_{\Delta N\gamma}\chi^{\dagger}\vec{\chi} \cdot \vec{e} \times \hat{\vec{k}}, \tag{24}$$

where $g_{\Delta N\gamma}$ is the constant of the magnetic dipole radiation (or the magnetic moment for the transition $\Delta \to N + \gamma$), $\vec{e}$ and $\hat{\vec{k}}$ are the photon polarization three-vector and unit momentum vector along the three-momentum of γ, respectively.

In the general case, the transition $\gamma^* + N \to \Delta$ must be described by three different form factors, corresponding to the absorption of $M1$, $E2_t$ (transversal) and $E2_\ell$ (longitudinal) virtual photons. But the existing experimental data about pion photo and electro-production on nucleons (in the Δ resonance region) indicate that the $M1$ term is dominant even for large k^2 [54], therefore in our analysis we will consider only this form factor.

We shall use the following formula for the k^2-dependence of the transition electromagnetic form factors:

$$G(k^2) = \frac{G(0)\ G_D(k^2)}{\left(1 - \frac{k^2}{m_x^2}\right)}. \tag{25}$$

The factor $(1 - \frac{k^2}{m_x^2})^{-1}$, with $m_x^2 = 6$ GeV^2, is included in order to take into account a steeper decreasing of $G(k^2)$ in comparison with the dipole behavior [54].

The normalization constant $G(0)$ can be found according to the following procedure. Let us calculate first the differential cross section for π^0-photoproduction:

$$\frac{d\sigma}{d\Omega}(\gamma p \to p\pi^0) = \frac{\alpha}{32\pi}\frac{q_\Delta^3}{k_\Delta}\frac{(E_{1\Delta} + m)(E_{2\Delta} + m)}{M_\Delta^4 \Gamma_\Delta^2} G^2(0)(5 - 3\cos^2\theta_\pi), \tag{26}$$

at $s = M_\Delta^2$, where the Δ-excitation in the $s-$channel is the main mechanism. So, our parametrization of the Δ-contribution describes correctly the angular dependence $(5 - 3\cos^2\theta_\pi)$, typical for the magnetic excitation of a $\frac{3}{2}^+$ state in $\gamma + p \to \Delta^+ \to N + \pi$.

Therefore, the total cross section can be written as:

$$\sigma_t(\gamma p \to p\pi^0) = \frac{\alpha}{2}\frac{q_\Delta^3}{k_\Delta}\frac{(E_{1\Delta}+m)(E_{2\Delta}+m)}{M_\Delta^4\Gamma_\Delta^2}G^2(0),$$

where:

$$E_{1\Delta} = \frac{M_\Delta^2+m^2}{2M_\Delta},\ E_{2\Delta} = \frac{M_\Delta^2+m^2-m_\pi^2}{2M_\Delta},$$
$$k_\Delta = \frac{M_\Delta^2-m^2}{2M_\Delta},\ q_\Delta = \sqrt{E_{2\Delta}^2-m^2}.$$

We can approximate with a good accuracy $\sigma_t(\gamma p \to p\pi^0)$ by a single $\Delta-$resonance contribution. For a numerical estimate of $G(0)$ we use $\sigma_t \simeq 250 \cdot 10^{-30}$ cm^2.

Note again that this procedure cannot determine the sign of $G(0)$. However for the $\gamma + p \to n + \pi^+$ reaction, there is a strong interference between the pion diagram and the $\Delta-$contribution. The comparison of the calculations using different signs with the experimental θ_π-dependence in the resonance region allows to fix the corresponding relative sign. Two remarks should be done about this procedure:

- there is no ambiguity concerning off-mass shell effects for the Δ-contribution, at least in the $s-$channel,
- this special contribution is gauge invariant.

We neglect in our consideration the Δ-exchange in the $u-$channel. The main reason to include this contribution is to have the crossing symmetry of the model. This is in principle an important property of the photoproduction amplitude, in particular in connection with the dispersion relations approach. However in the framework of phenomenological approaches, this symmetry is typically strongly violated. For example, the $s-$channel $\Delta-$ contribution induces an amplitude which is mostly complex (with a Breit-Wigner behavior), whereas the u-channel contribution results in a real amplitude. The inclusion of different form factors for the $s-$ and $u-$channel violates the crossing symmetry, which is particularly important for the Born contributions. This appears clearly for the reaction $\gamma + p \to p + \pi^0$, because here the crossing symmetry is correlated with the gauge invariance of the electromagnetic interaction, and the violation of the crossing symmetry has for direct consequence the violation of the current conservation. On the contrary, for the Δ-contribution, this important correlation is absent.

In any case, the *delta*-exchange in u-channel does not contribute to the isoscalar part of the electromagnetic current, and therefore does not affect our basic result, Eq. (18).

IV. NUMERICAL PREDICTIONS AND DISCUSSION

Having determined all the parameters of the model, it is possible in principle to calculate all observables for the processes $e^- + N \to e^- + N + \pi$ (on proton and neutron targets) in the kinematical region from threshold to the Δ-resonance region ($W \leq 1300$ MeV), for any pion production angle, θ_π, and k^2.

In order to test the present model, we used existing experimental data on the angular dependence of the differential cross sections for both the $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ reactions. This comparison allowed to fix empirically the relative sign of the different contributions: Born, Δ-excitation (in the $s-$channel) and vector meson exchange (in the $t-$channel). The relative signs of all three diagrams for the Born approximation in the case of the process $\gamma + p \to n + \pi^+$ are fixed by gauge invariance, but it is necessary to find the relative signs between the Born amplitudes, on one side, and the Δ-isobar and vector meson exchange contributions, on another side. The $\gamma + p \to n + \pi^+$ reaction is more sensitive to the signs of the Δ-contribution and ρ^+-exchange. Then the data about the differential cross sections for $\gamma + p \to p + \pi^0$ allow a further check and give a constrain for the $\omega-$exchange amplitude.

Note that the sign of the $\rho-$exchange contribution relative to the Born contribution (in both the $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ reactions) has to be the same as the relative sign of meson exchange currents (due to the $\rho\pi\gamma^*$ meson-exchange mechanism in the calculation of the electromagnetic form factors of the deuteron) with respect to the amplitude in the impulse approximation for elastic $ed-$scattering. This represents an important link between very different physical problems.

In order to obtain a good description of the experimental data for $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ we introduced small corrections to the different contributions. For the reaction $\gamma + p \to p + \pi^0$ a form factor was added to the Born contribution. The $u-$channel nucleon contribution for $\gamma + p \to n + \pi^+$ can be neglected without violating gauge invariance, because its magnetic content satisfies alone the current conservation condition. As a matter of fact this contribution has a diverging behavior at large angles, which is typically corrected by introducing an *ad-hoc* form factor. We choose to replace this contribution with a somewhat simplified phenomenological (S-wave like) contribution: $a\left(1 - \frac{t}{1.2}\right)\frac{1.2 \text{ GeV}}{W}$, where a is a parameter which is adjusted in order to reproduce at best the π^0 photoproduction data.

We did not attempt to reproduce with a good accuracy the threshold behavior of the $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ amplitudes. A precise description of this behavior, in particular for the process $\gamma + p \to p + \pi^0$, can be obtained, for example, in the framework of the Chiral Perturbative Theory approach [55]. For inclusive calculations, a qualitative description of the data in the threshold region is sufficient.

The quality of our model is shown in Fig. 3, where we present the comparison of our predictions with the experimental data on the differential cross sections for the $\gamma + p \to p + \pi^0$ and $\gamma + p \to n + \pi^+$ reactions, in the kinematical region where our model can be considered a reasonable approach. Indeed the unpolarized differential cross sections are well described. We did not apply the model to polarization observables. In particular different $T-$odd observables, such as, for example, the target asymmetry or the polarization of the final nucleons, are very sensitive to the relative phases of the different contributions. A good description requires a very precise treatment of the unitarity condition as well as of T-invariance of the hadron electromagnetic interaction, which are not so important for the differential or total cross section. A further comparison with the existing electroproduction data is not conclusive, due to their limited accuracy [56].

Therefore, after having determined the relative signs of the different contributions, our model can be generalized to pion electroproduction.

Our aim is the calculation of the inclusive P-odd asymmetry A, in $p(\vec{e},e')X$, for the sum of two possible channels, $X = p + \pi^0$ and $X = n + \pi^+$. One can see, from Eq. (18), that such asymmetry is determined by the following ratios of inclusive cross sections:

$$R_L^{(s)} = \frac{\sigma_L^{(s)}(k^2,W)}{\sigma_T(k^2,W)}, \quad R_T^{(s)} = \frac{\sigma_T^{(s)}(k^2,W)}{\sigma_T(k^2,W)},$$

$$R_{LT} = \frac{\sigma_L(k^2,W)}{\sigma_T(k^2,W)},$$

for both channels, $\gamma^* + p \to p + \pi^0$ and $\gamma^* + p \to n + \pi^+$, and

$$R_{pn} = \frac{\sigma_T(\gamma^* p \to p\pi^0)}{\sigma_T(\gamma^* p \to n\pi^+)},$$

which characterizes the relative role of the two channels. The 2-dimensional plots of these ratios as functions of k^2 and W are shown in Fig. 4 and 5, for the reactions $\gamma^* + p \to p + \pi^0$ and $\gamma^* + p \to n + \pi^+$, respectively.

For π^0-electroproduction, both $R_L^{(s)}$ and $R_T^{(s)}$ are small corrections to A. In the considered kinematical region, they are positive and tend to decrease in the region of the Δ resonance, due to the dominance of the isovector resonance contribution. The behavior of all these ratios in the threshold region can be improved, as we discussed above.

In the case of the $\gamma^* + p \to n + \pi^+$ reaction, the corresponding corrections are also small, especially $R_L^{(s)}$. Note that $R_T^{(s)}$ is negative in the whole region of k^2 and W.

Combining these results it is possible to calculate the resulting asymmetry A for the sum of both channels, again in a 2-dimensional representation (Fig. 6). The dependence on the detailed electron kinematics for $p(\vec{e},e')X$ (energies of the initial and final electron and electron scattering angle) is contained in the single parameter ϵ, for which we used three different values: $\epsilon = 0,\ 1/2$ and 1. In order to extract the strong k^2 dependence of A, the "reduced" asymmetry $A_0 = -A/|k^2|$ is shown.

In this picture one can see that the behavior of A versus k^2 and W, in the region $1.08 \le W \le 1.26$ GeV and in a wide region of momentum transfer k^2, is smooth everywhere and negative (note the $-1/|k^2|$ factor in the formula). Such a behavior results from the isovector nature of the electroproduction processes which we have considered.

The role of the different contributions is illustrated in Figs. 7, 8 and 9. In Fig. 7 (Fig. 8) the ratio of the cross sections $R_L^{(s)}$ and $R_T^{(s)}$ is reported as a function of W, for two fixed values of $|k^2|$, (a) $|k^2| = 0.4$ GeV^2 and (b) 1.0 GeV^2, for the reaction $\gamma^* + p \to p + \pi^0$ ($\gamma^* + p \to n + \pi^+$). The Δ contribution (dashed-dotted line) vanishes, while the Born terms (dotted line) give the largest contribution at forward angles. The contribution given by the vector meson (ρ and ω) exchange diagrams is not so essential here.

The different contributions to the total asymmetry A are shown in Fig. 9. This figure is a projection of Fig. 6 showing the resulting reduced asymmetries $A_0 = -A/|k^2|$ as a function of W at a fixed value of the virtual photon polarization $\epsilon = 0.5$ and for two values of the momentum transfer (a) $|k^2| = 0.4$ GeV^2; (b) $|k^2| = 1.0$ GeV^2. The Δ-contribution only is constant as a function of W - due to its isovector dominance, the vector-meson exchange gives a rather small contribution at low W (below 1.2 GeV) and it is negligible above. The full calculation gives values of A varying smoothly from $-7 \cdot 10^{-5}$ at W=1.1 GeV (close to the elastic region) to $-8 \cdot 10^{-5}$ at W=1.25 GeV, in the region of the Δ at $|k^2| = 1$ GeV^2.

Now comparing elastic scattering and inclusive π- production (Fig. 9), we see that they are both negative and of the same order of magnitude. Moreover A is smaller in the region W=1.1 GeV (close to elastic scattering) and larger in the Δ region. Therefore we can conclude that a small admixture of π-production events in the region of the elastic peak, is not going to produce a large uncertainty in the elastic PV asymmetry although a quantitative estimate has to be made in each specific case, either by using Fig. 6 or from the corresponding numerical values available from the authors.

We studied quantitative effects of different choices of form factors for the electromagnetic vertexes. For illustration, we report, in Table II, numerical values for two ratios, $R_L^{(s)}$ and $R_T^{(s)}$, for the reaction $e + p \to e + p + \pi^0$, for $-k^2$= 0.5 GeV^2, in the resonance region. The effect of changing the electric neutron form factor, from G_{En} =0 to the value given by the parametrization [48], is less then 10%. The dependence on $g_{V\pi\gamma^*}$ is larger, when comparing the previous results based on Eq. (23), to the values obtained with a *soft* form factor:

$$g_{V\pi\gamma^*}(k^2) = \frac{g_{V\pi\gamma}(0)}{(1-k^2/m_V^2)^2}. \tag{27}$$

In any case the isoscalar correction is always very small, at least two order of magnitude smaller than the main (isovector) result for the asymmetry A.

V. CONCLUSIONS

We have calculated the k^2 and W-dependences of the P-odd asymmetry for inclusive scattering of longitudinally polarized electrons by unpolarized protons with π^0 or π^+ meson production. Using the known isotopic properties of the electromagnetic current for the processes $\gamma^* + p \to p + \pi^0$ and $\gamma^* + p \to n + \pi^+$ and the vector part of the weak neutral current for the processes $Z^* + p \to p + \pi^0$ and $Z^* + p \to n + \pi^+$, we have derived an original expression for the inclusive asymmetry $\mathcal{A}$. Without approximations, it is possible to determine the main (isovector) contribution to $\mathcal{A}$, which depends only on the variable k^2. The exact calculation of $\mathcal{A}$ is then reduced to the analysis of specific (small) isoscalar contributions to the electromagnetic currents.

We have calculated the amplitudes for $\gamma^* + p \to N + \pi$, taking into account three standard contributions: Born + vector-meson exchange+ Δ-excitation. All the necessary parameters: the coupling constants and different electromagnetic form factors are taken from other sources. Small adjustements of the parameters were done in order to obtain a good description of the experimental data on the differential cross sections for $\gamma+p \to p+\pi^0$ and $\gamma + p \to n + \pi^+$. The model gives the vector part of the weak neutral current which is the main contribution to P-odd effects in $e + N \to e + N + \pi$ processes.

The reduced asymmetry $\mathcal{A}_0$ varies very little as a function of the two basic kinematical variables, k^2 and W. In our approach this appears naturally from the fact that the isoscalar content of the electromagnetic current for $\gamma + N \to N + \pi$ is very small in the considered kinematical region.

The possibility to calculate this contribution as a small correction to the main contribution, opens a way to use P-odd observables in elastic and inelastic electron-proton scattering for the study of the relatively small axial contributions.

VI. APPENDIX

A. Born contribution: s-channel

The scalar amplitudes for $\gamma^* + p \to p + \pi^0$ are defined as

$$f_{1s} = f_{3s} = -\frac{g}{W-m}\frac{|\vec{k}||\vec{q}|}{(E_1+m)(E_2+m)}$$
$$\left[F_{1p}(k^2) + F_{2p}(k^2)\frac{W+m}{2m}\right],$$
$$f_{2s} = \frac{g}{W-m}\left[F_{1p}(k^2) - F_{2p}(k^2)\frac{W-m}{2m}\right],$$
$$+\frac{g}{W-m}\frac{\vec{k}\cdot\vec{q}}{(E_1+m)(E_2+m)}\left[F_{1p}(k^2) + F_{2p}(k^2)\frac{W+m}{2m}\right],$$
$$f_{4s} = 0,$$
$$f_{5s} = \frac{g}{(W+m)(E_1+m)}\left[-F_{1p}(k^2) + F_{2p}(k^2)\frac{E_1+m}{2m}\right],$$
$$f_{6s} = \frac{g}{(W-m)(E_2+m)}\frac{|\vec{k}|}{|\vec{q}|}$$
$$\left[-F_{1p}(k^2) + F_{2p}(k^2)\frac{E_1-m}{2m}\right],$$

with $|\vec{k}| = \sqrt{E_1^2 - m^2}$ and $|\vec{q}| = \sqrt{E_2^2 - m^2}$.

B. Born contribution: u-channel

$$f_{1u} = \frac{g|\vec{k}||\vec{q}|}{u-m^2}\left\{F_{1p}(k^2)\frac{W+m}{(E_1+m)(E_2+m)} - \frac{F_{2p}(k^2)}{2m(E_1+m)}\right.$$
$$\left.\left[W + m + \left(m + \frac{m^2-k^2}{W}\right)\frac{E_\pi}{E_2+m} + \frac{2\vec{k}\cdot\vec{q}}{E_2+m}\right]\right\},$$
$$f_{2u} = \frac{g}{u-m^2}\left[F_{1p}(k^2)\left(W - m + \vec{k}\cdot\vec{q}\frac{W+m}{(E_1+m)(E_2+m)}\right)\right.$$

$$-\frac{F_{2p}(k^2)}{2m}\left((E_1-m)(W-m)+\tilde{k}_0\left(m+\frac{m^2-m_\pi^2}{W}\right)\right.$$
$$\left(-2\vec{k}\cdot\vec{q}+\left(m+\frac{m^2-k^2}{W}\right)\left(m+\frac{m^2-m_\pi^2}{W}\right)\right)$$
$$\left.\left.\frac{\vec{k}\cdot\vec{q}}{(E_1+m)(E_2+m)}\right)\right],$$

$$f_{3u}=\frac{g}{u-m^2}\frac{g|\vec{k}||\vec{q}|}{E_1+m}\left[-\frac{F_{1p}(k^2)}{E_2+m}\left(-m+\frac{m_\pi^2-m^2}{W}\right)-\frac{F_{2p}(k^2)}{2m}\right.$$
$$\left.\left(-W-m+\left(m+\frac{m^2-k^2}{W}\right)\frac{E_\pi}{E_2+m}+\frac{2\vec{k}\cdot\vec{q}}{E_2+m}\right)\right],$$

$$f_{4u}=\frac{g}{u-m^2}(E_2-m)\left[-2F_{1p}(k^2)+F_{2p}(k^2)\left(-1+\frac{W}{m}\right)\right],$$

$$f_{5u}=-\frac{g}{(u-m^2)(E_1+m)}\left[\left(-F_{1p}(k^2)+F_{2p}(k^2)\frac{E_1+m}{2m}\right)\right.$$
$$\left.\left(m+\frac{m_\pi^2-m^2}{W}\right)+F_{2p}(k^2)\frac{\vec{k}\cdot\vec{q}}{m}\right],$$

$$f_{6u}=\frac{g}{(u-m^2)(E_2+m)}\frac{|\vec{q}|}{|\vec{k}|}\left[-\left(F_{1p}(k^2)+F_{2p}(k^2)\frac{E_1-m}{2m}\right)\right.$$
$$\left.\left(m-\frac{m_\pi^2-m^2}{W}\right)+F_{2p}(k^2)\frac{\vec{k}\cdot\vec{q}}{m}\right],$$

where $u-m^2=k^2-2\tilde{k}_0E_2-2\vec{k}\cdot\vec{q}$, $\tilde{k}_0=\dfrac{W^2+k^2-m^2}{2W}$, and $E_\pi=W-E_2$.

C. Vector meson exchange: t-channel

$$f_{1V}=g_{V\pi\gamma^*}(k^2)g_{VNN}\frac{|\vec{k}||\vec{q}|}{m_V(t-m_V^2)}\left\{\left[1+\left(1+\frac{W}{m}\right)\kappa_V\right]\right.$$
$$\left(-1+\frac{\vec{k}\cdot\vec{q}}{(E_1+m)(E_2+m)}\right)$$
$$\left.+(1+\kappa_V)(W+m)\left(\frac{1}{E_1+m}+\frac{1}{E_2+m}\right)\right\},$$

$$f_{2V}=g_{V\pi\gamma^*}(k^2)\frac{g_{VNN}}{m_V(t-m_V^2)}\{(1+\kappa_v)[\tilde{k}_0(E_2-m)+$$

$$E_\pi(E_1 - m) - \vec{k}\cdot\vec{q}\left(\frac{\tilde{k}_0}{E_1+m} + \frac{E_\pi}{E_2+m}\right)\Bigg] +$$

$$\left[1 + \left(1 + \frac{W}{m}\right)\kappa_V\right]\frac{\vec{k}^2\vec{q}^2 - (\vec{k}\cdot\vec{q})^2}{(E_1+m)(E_2+m)}\Bigg\},$$

$$f_{3V} = g_{V\pi\gamma^*}(k^2)\frac{g_{VNN}|\vec{k}||\vec{q}|}{m_V(t-m_V^2)}$$

$$\Bigg\{(1+\kappa_V)\frac{E_\pi}{E_2+m} + \left[1 + \left(1 + \frac{W}{m}\right)\kappa_V\right]$$

$$\frac{(\vec{k}\cdot\vec{q})}{(E_1+m)(E_2+m)}\Bigg\},$$

$$f_{4V} = -g_{V\pi\gamma^*}(k^2)\frac{g_{VNN}}{m_V}$$

$$\frac{(E_1+m)(E_2+m)}{t-m_V^2}\left[1 + \left(1 + \frac{W}{m}\right)\kappa_V + \frac{(1+\kappa_V)}{E_1-m}\tilde{k}_0\right],$$

$$f_{5V} = g_{V\pi\gamma^*}(k^2)\frac{g_{VNN}}{m_V(t-m_V^2)}\frac{1+\kappa_V}{E_1+m}\left[t + \left(k^2 - m_\pi^2\right)\frac{m}{W}\right],$$

$$f_{6V} = -g_{V\pi\gamma^*}(k^2)\frac{g_{VNN}}{m_V(t-m_V^2)}\frac{1+\kappa_V}{E_1+m}\frac{|\vec{k}||\vec{q}|}{m_V(t-m_V^2)}$$

$$\left[t + \left(k^2 - m_\pi^2\right)\frac{m}{W}\right],$$

where

$$t - m_V^2 = m_\pi^2 - m_V^2 - 2\tilde{k}_0 E_\pi + 2\vec{k}\cdot\vec{q} + k^2.$$

D. One pion contribution: t-channel

$$f_{1t} = f_{2t} = 0,$$

$$f_{3t} = g\frac{2|\vec{k}||\vec{q}|}{t-m_\pi^2}\frac{F_\pi(k^2)}{E_1+m},$$

$$f_{4t} = -2g\frac{E_2-m}{t-m_\pi^2}F_\pi(k^2),$$

$$f_{5t} = -\frac{g}{t-m_\pi^2}F_\pi(k^2)\frac{2E_\pi - k_0}{E_1+m},$$

$$f_{6t} = -\frac{g}{t-m_\pi^2}F_\pi(k^2)\frac{|\vec{k}||\vec{q}|}{t-m_\pi^2}\frac{2E_\pi - k_0}{E_2+m}.$$

E. Calculation of the isoscalar amplitudes $f_i^{(s)}(\gamma^* p \to p\pi^0)$

The isoscalar amplitudes are:

$$f_i^{(s)}(\gamma^* p \to p\pi^0) = -f_{i,s}^{(s)} - f_{i,u}^{(s)} - f_{i,\rho}$$

where the contributions $f_{i,s}^{(s)}$ and $f_{i,u}^{(s)}$ are determined by the corresponding formulas, with the following substitutions:

$$F_{1p} \to F_{1s} = \frac{F_{1p} + F_{1n}}{2},$$
$$F_{2p} \to F_{2s} = \frac{F_{2p} + F_{2n}}{2}.$$

F. Gauge invariance of the suggested model

In the framework of the considered model, for the process of neutral pion electroproduction, $e + p \to e + p + \pi^0$, the corresponding hadronic electromagnetic current is conserved: $k \cdot \mathcal{J}_\mu^{(em)} = 0$ for any form factor in $\gamma^* NN$, $\gamma^*\pi\pi$, $\gamma^* V\pi$, and $\gamma^* N\Delta$-vertices.

In case of charged pion electroproduction, $e + p \to e + n + \pi^+$, a special contribution must be added to the matrix element:

$$\Delta\mathcal{M} = -\sqrt{2}g\frac{e \cdot k}{k^2}\gamma_5(F_{1p} - F_{1n} - F_\pi)$$

which results in additional contributions to the scalar amplitudes: $\Delta f_i(\gamma p \to n\pi^+)$:

$$\Delta f_1 = \Delta f_2 = \Delta f_2 = \Delta f_4 = 0$$

$$\Delta f_5 = \sqrt{2}g(E_1 - m)[F_{1p}(k^2) - F_{1n}(k^2) - F_\pi(k^2)]/k^2$$

$$\Delta f_6 = -\sqrt{2}g(E_2 - m)[F_{1p}(k^2) - F_{1n}(k^2) - F_\pi(k^2)]/k^2$$

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TABLES

TABLE I. Numerical coefficients for the different contributions to the Feynmann diagrams

Reaction	a_s	a_u	a_t	a_ρ	a_ω	a_Δ
$\gamma^* + p \to p + \pi^0$	-1	-1	0	+1	+1	$\sqrt{2}$
$\gamma^* + p \to n + \pi^+$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}$	0	+1

TABLE II. Numerical values for $R_L^{(s)}$ and $R_T^{(s)}$, for the reaction $e+p \rightarrow e+p+\pi^0$, for $-k^2$= 0.5 GeV^2, W=1.23 GeV and different form factors

$R_T^{(s)}$	$R_L^{(s)}$	G_{En}	$g_{V\pi\gamma^*}$
-0.00056	0.0059	0	$hard$ (Eq. 23)
-0.00050	0.0054	from [48]	$hard$ (Eq. 23)
-0.00104	0.0017	0	$soft$ (Eq. 27)

FIGURES

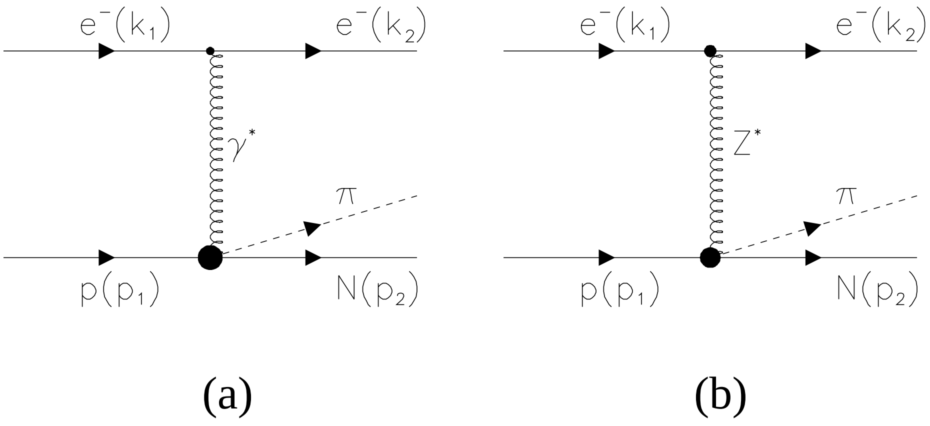

FIG. 1. Feynman diagrams for γ^*- and Z^*-boson exchanges in the processes $e^-+p \to e^-+N+\pi$.

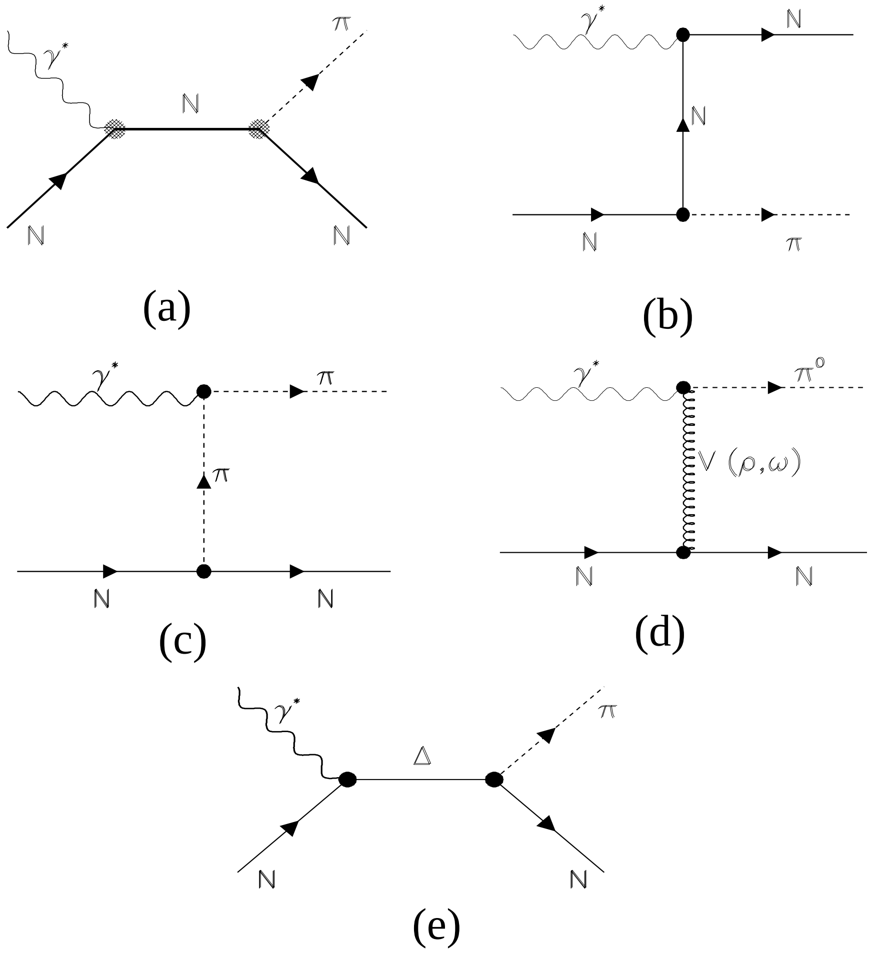

Fig. 2

FIG. 2. Feynman diagrams for $\gamma^* + p \to N + \pi$- processes.

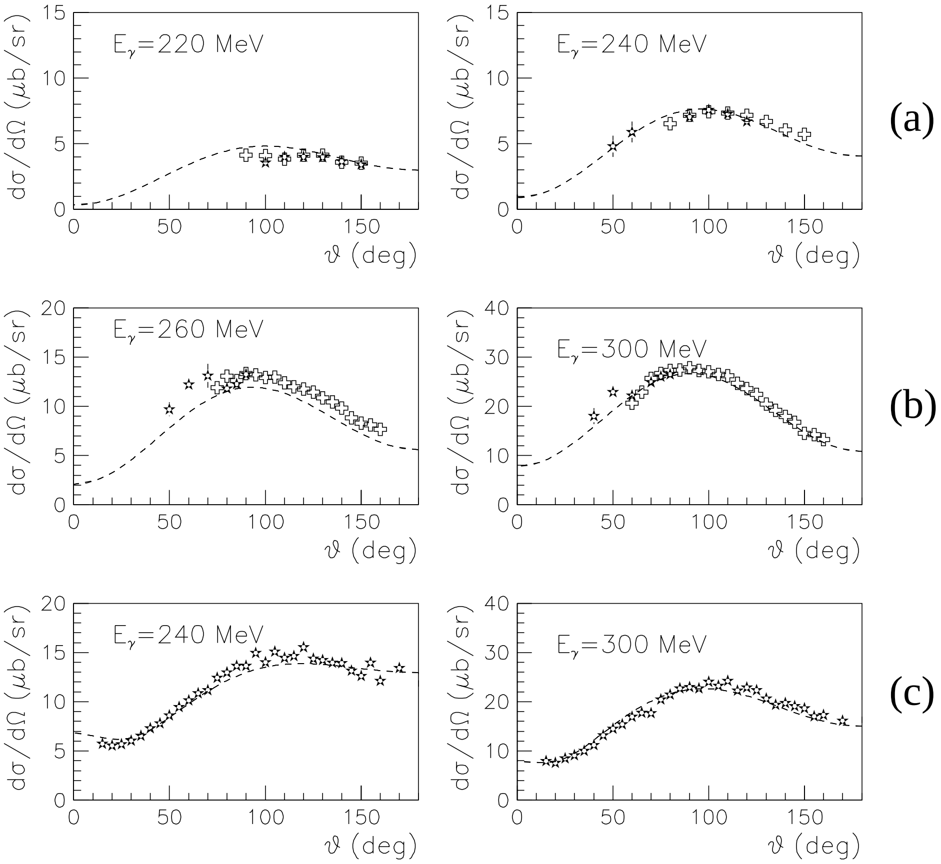

Fig. 3

FIG. 3. The angular dependence of the differential cross sections for the photoproduction processes: (a) and (b) $\gamma^* + p \to p + \pi^0$; open stars: data from ref. [59], open crosses: data from ref. [57], (c) $\gamma^* + p \to n + \pi^+$; data are from ref. [58]; the dashed line is the prediction of the present model.

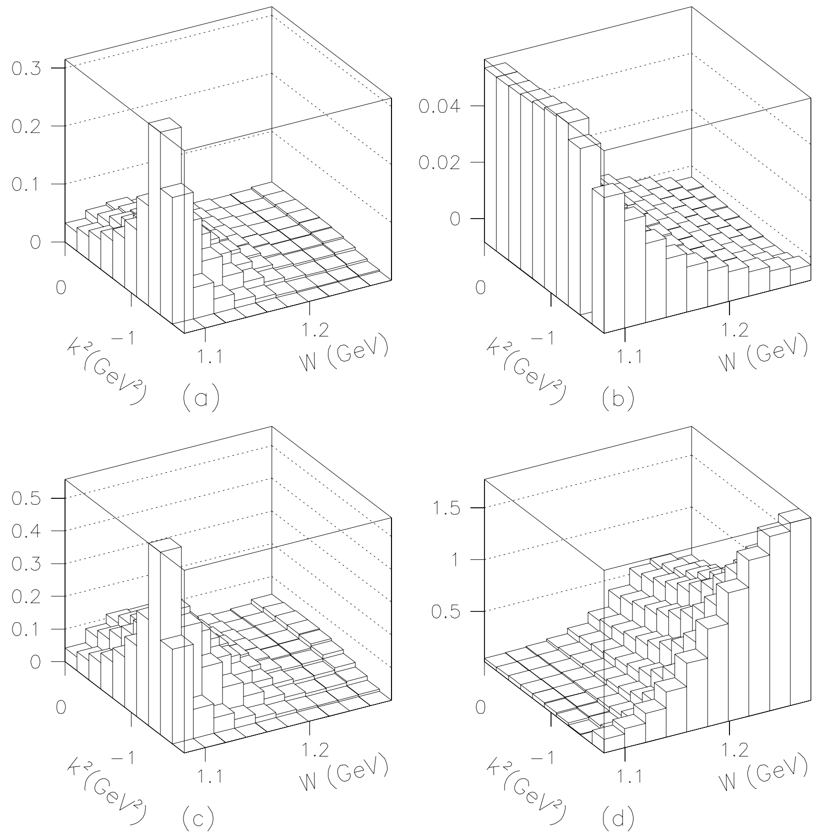

FIG. 4. The k^2 and W-dependences of the ratios of the total cross sections for the $e^- + p \rightarrow e^- + p + \pi^0$ reaction: (a) $R_L^{(s)}(k^2, W) = \sigma_L^{(s)}(k^2, W)/\sigma_T(k^2, W)$; (b) $R_T^{(s)}(k^2, W) = \sigma_T^{(s)}(k^2, W)/\sigma_T(k^2, W)$; (c) $R_{LT}(k^2, W) = \sigma_L(k^2, W)/\sigma_T(k^2, W)$; (d) $R_{np} = \sigma_T(p\pi^0)/\sigma_T(n\pi^+)$.

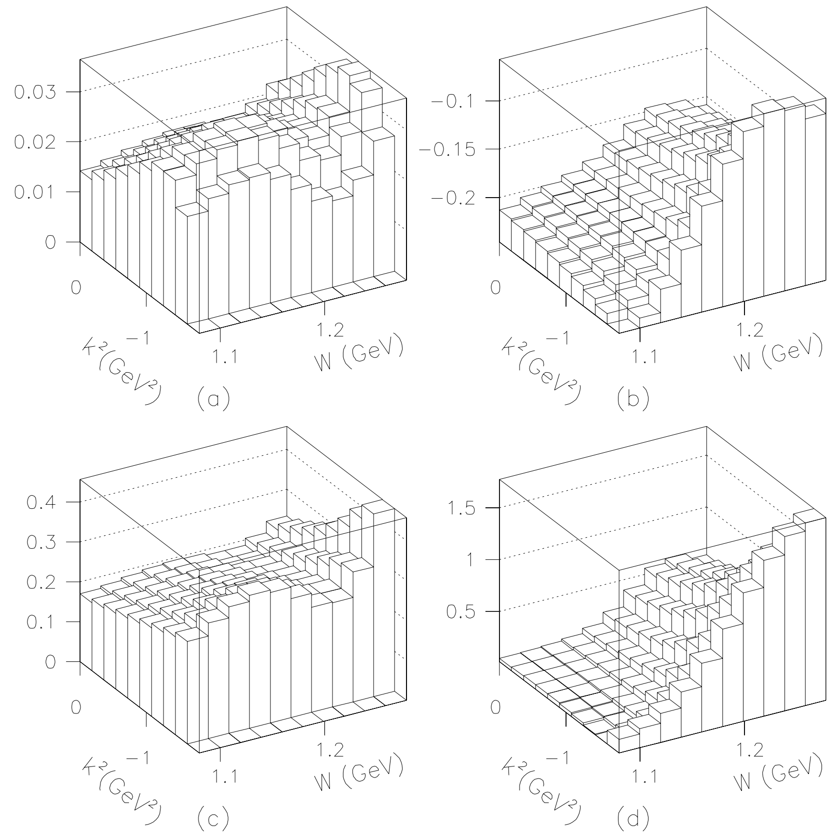

FIG. 5. The k^2 and W-dependences of the ratios of the total cross sections for the $e^- + p \rightarrow e^- + n + \pi^+$ reaction. Same conventions as in Fig. 4.

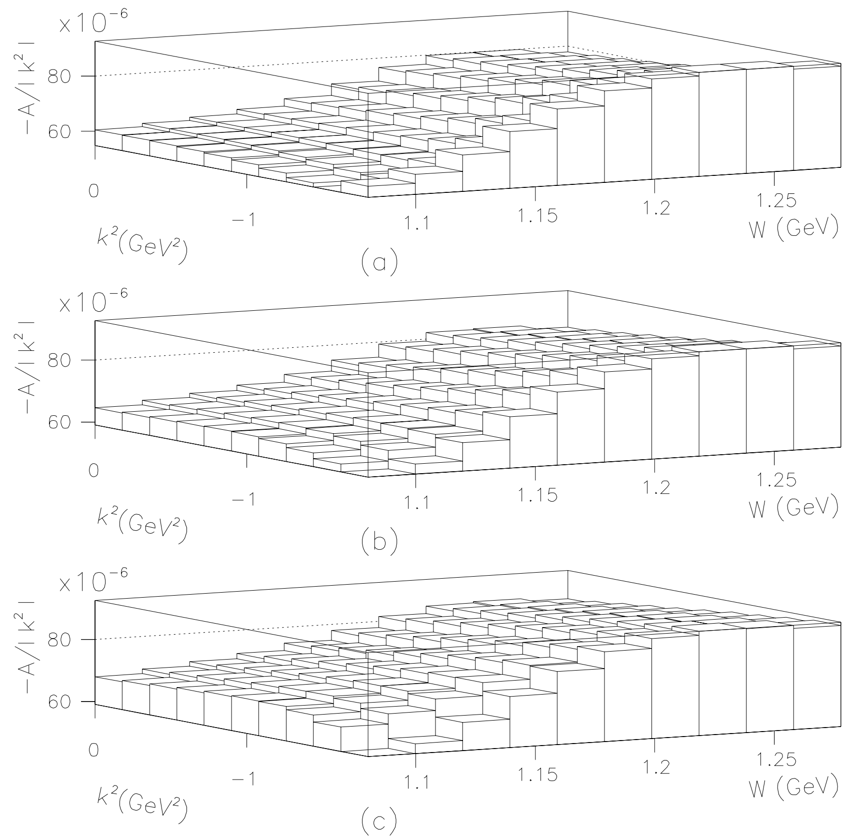

FIG. 6. The k^2 and W dependences of the reduced asymmetry $A_0 = -A/|k^2|$ (where A is the theoretical asymmetry to be compared to experimental data) for $p(\vec{e}, e')X$ at three different values of the virtual photon polarization ϵ: (a) $\epsilon = 0$; (b) $\epsilon = 0.5$; (c) $\epsilon = 1$.

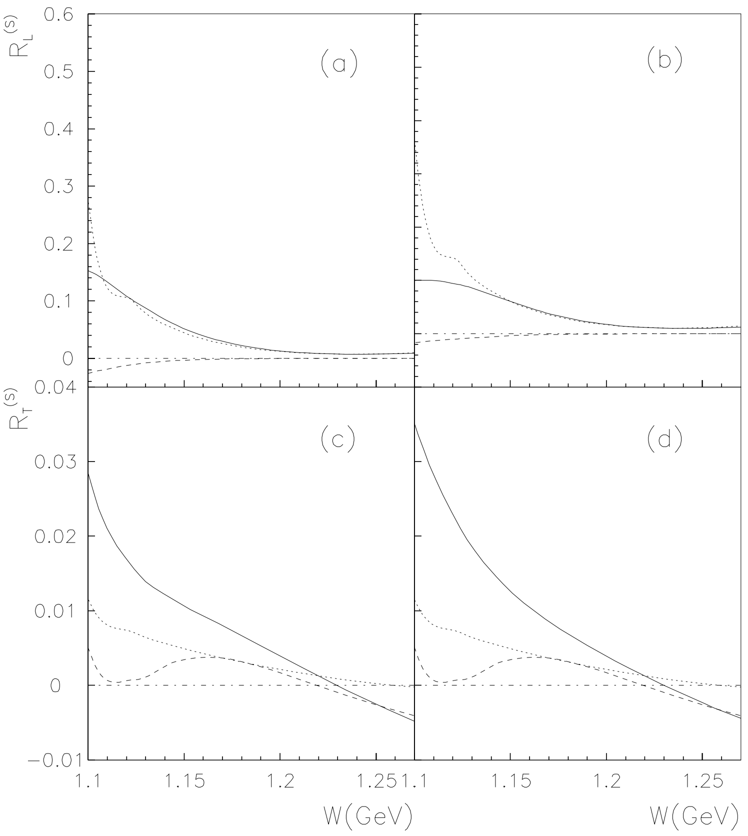

FIG. 7. The W-dependence of the ratios $R_L^{(s)}(k^2, W)$ and $R_T^{(s)}(k^2, W)$ for fixed values of k^2 a) and c) $-k^2 = 0.5$ GeV^2; b) and d) $-k^2 = 1.0$ GeV^2 for the $e^- + p \to e^- + p + \pi^0$ reaction. The curves represent the full calculation (full line), Δ-contribution only (dashed-dotted line), Δ + Born terms (dashed line), Δ + vector mesons (dotted line).

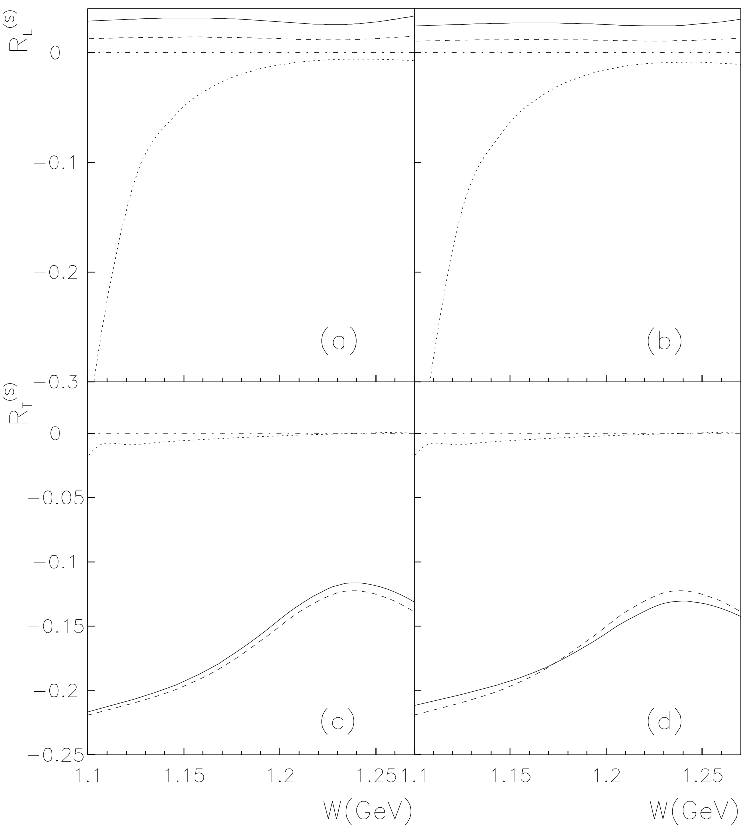

FIG. 8. The same as Fig. 7, for the $e^- + p \to e^- + n + \pi^+$ reaction

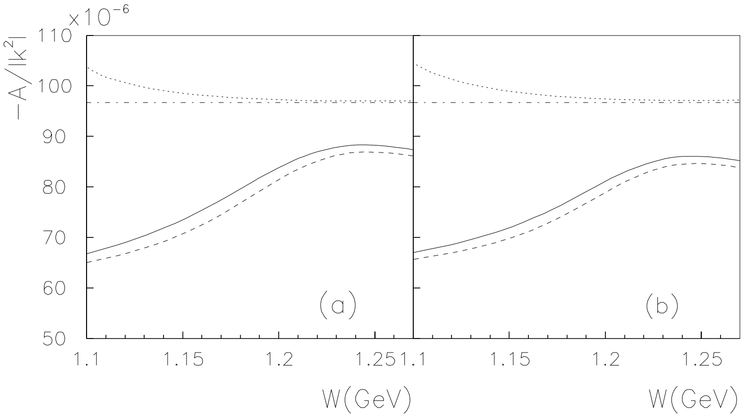

FIG. 9. The W-dependence of the reduced asymmetry A_0 for $\epsilon = 0.5$ and two values of k^2: (a) $-k^2 = 0.4$ GeV^2; (b) 1.0 GeV^2. Same conventions as in Fig. 7.