

Scalar meson production in nucleon-nucleon collisions near threshold

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Abstract

We establish the model-independent spin structure of the matrix elements for the near-threshold scalar meson production in pp - and np -collisions, when the final particles are emitted in S-state. Polarization phenomena are derived in a general form. The properties of the t -channel dynamics, which is based on different meson exchanges, are studied in terms of the s -channel parametrization of the matrix element. The prediction of a 'realistic' model, based on $\pi + \sigma$ -exchanges are also presented.

I. INTRODUCTION

The strong and electromagnetic decays of the scalar mesons S ($S = \sigma, f_0,$ and a_0) are recently object of large theoretical and experimental interest. The structure of these mesons is not yet fully understood and the coupling constants of their decays not determined (see [1–8] and refs. herein). As an example, the electromagnetic constants $g_{\rho\sigma\gamma}$ and $g_{\omega\sigma\gamma}$ are

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very important for the solution of different problems in hadron electrodynamics. In the near threshold region, the constant $g_{\rho\sigma\gamma}$ drives the σ -contribution to the differential cross section of the process $\gamma + p \rightarrow p + p + \rho^0$ [9–11]. The coupling constant $g_{\omega\sigma\gamma}$ is important for the estimation of the effects of the meson exchange currents for the deuteron electromagnetic form factors [12–14], particularly in the region of large momentum transfer. Both these constants play a major role in the interpretation [15] of the HERMES effect [16], concerning the anomalous behavior of the electroproduction cross section on nuclei at low Q^2 - where the cross section is enhanced for longitudinally polarized virtual photons and depleted for transversally polarized photons. Moreover, the constants $g_{\rho\sigma\gamma}$ and $g_{\omega\sigma\gamma}$ enter in the interpretation of different radiative decays of vector mesons, like $\rho^0(\omega) \rightarrow \pi^0\pi^0\gamma$ and $\rho^0(\omega) \rightarrow \pi^+\pi^-\gamma$ [17].

Two-photon decays of scalar mesons, $S \rightarrow 2\gamma$, which are important for the estimation of the corresponding t -channel contributions to the amplitudes of real and virtual Compton scattering on nucleons [18–21], could be estimated on the basis of the $g_{V\sigma\gamma}$ coupling constants, in the VDM approach.

In the framework of the effective Lagrangian approach, the calculation of the cross section for scalar meson photo- and electro-production on nucleons requires the knowledge of the $g_{V\sigma\gamma}$ and g_{NNS} -coupling constants. The strong coupling constants g_{NNS} [22,23] enter in various calculations in hadron dynamics, not only with respect to the NN-potential, but also for different observables for processes like $\pi + N \rightarrow S + N$ [24,25], $N + N \rightarrow S + d$ [25–28], and $N + N \rightarrow S + N + N$ [24].

The feasibility of the experimental study of scalar meson production depends essentially on the nature and on the rate of the decay. If, for example, the decay $f_0 \rightarrow K\bar{K}$ or $a_0 \rightarrow K\bar{K}$ dominate, these mesons could be observed in the $K\bar{K}$ effective mass distribution, close to the kaon mass, as a resonant contribution. Such effects have been observed in πN -collisions [4,29–34]. The interpretation of the data needs an adequate theoretical approach for the process $\pi + N \rightarrow N + S^0$. It was shown that the reaction $p + p \rightarrow p + p + K^+K^-$ in the kinematical conditions of the DISTO [35] or at COSY the presence of the f_0 signal is hindered

by a large background. Therefore, in principle, the process $n + p \rightarrow n + p + K^+ K^-$ could be more favorable, because the cross section is one order higher, whereas the background is comparable with respect to pp -collisions.

Note that the experimental study of $K^+ K^-$ -production in the process $p + p \rightarrow p + p + K^+ + K^-$, at an energy excess $Q=17$ MeV over threshold [36], gives the following value: $\sigma(pp \rightarrow pp f_0) = (1.84 \pm 0.29_{-0.35}^{+0.25})$ nb, including the statistical and systematic errors. The extension of this study is foreseen [37–39].

In this paper we derive the most general and model independent properties for the processes of scalar meson production in NN -collisions, in the threshold region, where the theoretical analysis is essentially simplified. The spin structure of the corresponding matrix elements contains a set of sixteen independent amplitudes in the general case, eight amplitudes for coplanar kinematics and only three independent amplitudes in the threshold region.

This paper is organized as follows. In Section II, using the selection rules with respect to Pauli principle, P-parity and total angular momentum, we establish the spin structure of the matrix elements for the processes $p + p \rightarrow p + p + S^0$ and $n + p \rightarrow n + p + S^0$ and analyze the polarization phenomena for these processes in a model independent way. In Section III we transform (using the two-component Fierz transformation) different t -channel contributions, which are described by a definite set of Feynman diagrams, to the universal s -channel parametrization and find the expression for the corresponding partial threshold amplitudes. In Section IV we discuss the predictions of a realistic model based on $\sigma + \pi$ -exchanges. A short discussion of final state interactions (FSI) is done in Section V. In the Conclusions we summarize the results obtained.

II. SPIN STRUCTURE OF THE THRESHOLD MATRIX ELEMENT AND POLARIZATION OBSERVABLES

The spin structure of the matrix elements for scalar meson production in NN -collisions, $N + N \rightarrow N + N + S^0$, in the threshold region is determined by the selection rules with respect to P -parity, total angular momentum and by the Pauli principle. The threshold region, where all final particles in $N + N \rightarrow N + N + S^0$ are produced in relative S -state, can be rigorously described by a formalism based on the two-component nucleon spinor parametrization of the corresponding matrix element. This formalism can be built in model independent way [40], and has been previously successfully applied to vector [41], pseudoscalar [42], and strange [43] particle production in nucleon-nucleon collisions.

Let us explicitly derive the matrix element for the processes $p + p \rightarrow p + p + S^0$, where S^0 denotes a neutral scalar meson, $S^0 = \sigma, f_0$ or a_0^0 . Due to the Pauli principle, at the reaction threshold, only quantum numbers $j^P = 0^+$ are allowed, where j is the total angular momentum and P is the P -parity of the colliding protons. Only a single partial transition can take place, corresponding to:

$$S_i = 0, \ell = 0 \rightarrow j^P = 0^+ \rightarrow S_f = 0, \quad (1)$$

where S_i (S_f) is the total spin of the initial (final) protons and ℓ is the angular orbital momentum of the colliding protons.

The spin structure of the threshold matrix element for the transition (1), in the CMS of the considered reaction, can be parametrized in the following general form:

$$\mathcal{M}_{pp} = g(\chi_4^\dagger \sigma_y \tilde{\chi}_3^\dagger)(\tilde{\chi}_2 \sigma_y \chi_1), \quad (2)$$

where χ_1 and χ_2 (χ_3 and χ_4) are the two-component spinors of the initial (final) protons; g is the threshold partial amplitude, describing the singlet-singlet transition in the pp -system (with scalar meson production in the S -state), which is generally a complex function of three independent energies: W (the total invariant energy of the colliding particles), E_1 and

E_2 (the energies of the scattered protons). So, all the dynamics of the considered process is included in the amplitude g , but the exact form (2) of the spin structure of the matrix element results from a generalized quantum mechanical kinematics.

The Pauli matrix σ_y , in the parametrization (2), insures the correct transformation properties of the corresponding two-component spinor products, relative to rotation.

The presence of a single amplitude in (2) implies that the spin directions of all the protons are fixed, with definite relative angles and fixed modules. In other words, all polarization phenomena for $p + p \rightarrow p + p + S^0$ at threshold can be predicted without knowing the amplitude g , i.e. *in model independent way*. Moreover, these polarization effects are the same for any scalar meson: σ , f_0 or a_0^0 . However, the absolute value of the cross section, which depends on $|g^2|$, is different for different scalar mesons.

All T-odd polarization observables (i.e. one-spin and three-spin polarization correlations) are identically zero. The dependence of the cross section on the polarizations \vec{P}_1 and \vec{P}_2 of the colliding protons can be written as:

$$\frac{d\sigma}{d\omega}(\vec{P}_1, \vec{P}_2) = \left(\frac{d\sigma}{d\omega} \right)_0 (1 - \vec{P}_1 \cdot \vec{P}_2),$$

(typical for the singlet pp -interaction), where $d\omega$ is the phase space volume element for the three-particles final state and $(d\sigma/d\omega)_0$ is the differential cross section with all unpolarized protons in initial and final states.

All polarization transfer coefficients, characterizing the dependence of the polarization of any final proton from the polarization of the initial proton, vanish also.

The situation is very different in case of np - collisions, $n + p \rightarrow n + p + S^0$, in the threshold region, where there are three allowed transitions:

$$S_i = 0, \ell = 0 \rightarrow j^P = 0^+ \rightarrow S_f = 0,$$

$$S_i = 1, \ell = 0 \rightarrow j^P = 1^+ \rightarrow S_f = 1,$$

$$S_i = 1, \ell = 2 \rightarrow j^P = 1^+ \rightarrow S_f = 1.$$

The corresponding matrix element can be written in the following form:

$$\begin{aligned}
\mathcal{M}_{np} = & g_1(\chi_4^\dagger \sigma_y \tilde{\chi}_3^\dagger)(\tilde{\chi}_2 \sigma_y \chi_1) \\
& + g_2 \left[\chi_4^\dagger (\sigma_a - \hat{k}_a \vec{\sigma} \cdot \hat{k}) \sigma_y \tilde{\chi}_3^\dagger \right] \left[(\tilde{\chi}_2 \sigma_y (\sigma_a - \hat{k}_a \vec{\sigma} \cdot \hat{k}) \chi_1) \right] \\
& + g_3 (\chi_4^\dagger \vec{\sigma} \cdot \hat{k} \sigma_y \tilde{\chi}_3^\dagger)(\tilde{\chi}_2 \sigma_y \vec{\sigma} \cdot \hat{k} \chi_1),
\end{aligned} \tag{3}$$

where \hat{k} is the unit vector along the three-momentum of the initial neutron beam, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the standard set of Pauli matrices, and $g_1 - g_3$ are the partial amplitudes for the np -interaction. Due to the isotopic invariance of the strong interaction the following relation holds:

$$g_1 = \frac{1}{2}g, \tag{4}$$

i.e. the triplet-triplet amplitudes g_2 and g_3 are present only in np -collisions. Therefore polarization phenomena in $n + p \rightarrow n + p + S^0$ collisions are more complicated than in pp -collisions. There is no universality here, polarization phenomena are different for the different scalar mesons. There is, however one general feature: all one-spin observables vanish, for $n + p \rightarrow n + p + S^0$, in particular the T-odd polarization of the final nucleons, emitted in the collisions of unpolarized nucleons and the analyzing powers in $\vec{n} + p$ - or $n + \vec{p}$ -collisions.

The dependence of the differential cross section for $\vec{n} + \vec{p}$ -collisions can be written in the following general form, which is correct near threshold:

$$\frac{d\sigma}{d\omega}(\vec{P}_1, \vec{P}_2) = \left(\frac{d\sigma}{d\omega} \right)_0 \left(1 + \mathcal{A}_1 \vec{P}_1 \cdot \vec{P}_2 + \mathcal{A}_2 \hat{k} \cdot \vec{P}_1 \hat{k} \cdot \vec{P}_2 \right), \tag{5}$$

where the real spin correlation coefficients \mathcal{A}_1 and \mathcal{A}_2 are determined by the following formulas (in terms of the partial threshold amplitudes g_i , $i = 1 - 3$):

$$\begin{aligned}
\mathcal{A}_1 \left(\frac{d\sigma}{d\omega} \right)_0 &= -|g_1|^2 + |g_3|^2, \\
\mathcal{A}_2 \left(\frac{d\sigma}{d\omega} \right)_0 &= 2(|g_2|^2 - |g_3|^2).
\end{aligned} \tag{6}$$

The amplitudes g_i are normalized in such way that:

$$\left(\frac{d\sigma}{d\omega} \right)_0 = |g_1|^2 + 2|g_2|^2 + |g_3|^2, \tag{7}$$

where $(d\sigma/d\omega)_0$ is the differential cross section with unpolarized particles. So, from Eqs. (6) and (7) one can find:

$$\begin{aligned} 4|g_1|^2 &= (1 - 3\mathcal{A}_1 - \mathcal{A}_2) \left(\frac{d\sigma}{d\omega} \right)_0, \\ 4|g_2|^2 &= (1 + \mathcal{A}_1 + \mathcal{A}_2) \left(\frac{d\sigma}{d\omega} \right)_0, \\ 4|g_3|^2 &= (1 + \mathcal{A}_1 - \mathcal{A}_2) \left(\frac{d\sigma}{d\omega} \right)_0, \end{aligned} \quad (8)$$

which shows that the moduli of the three threshold amplitudes for $n + p \rightarrow n + p + S^0$ can be determined by the measurement of \mathcal{A}_1 , \mathcal{A}_2 , and $(d\sigma/d\omega)_0$. Taking into account the isotopic relation (4), one obtains:

$$\mathcal{R} = \frac{(d\sigma_{np}/d\omega)_0}{(d\sigma_{pp}/d\omega)_0} = 1 - \mathcal{A}_1 - \mathcal{A}_2.$$

The dependence of the polarization \vec{P}_3 of the final neutron on the polarization \vec{P}_1 of the initial neutron can be parametrized as:

$$\vec{P}_3 = p_1 \vec{P}_1 + p_2 \hat{k} (\hat{k} \cdot \vec{P}_1), \quad (9)$$

where p_1 and p_2 are the spin transfer coefficients, which can be expressed in terms of the partial amplitudes g_i :

$$p_1 = -\frac{2\mathcal{R}e g_2 (g_1 - g_3)^*}{|g_1|^2 + 2|g_2|^2 + |g_3|^2}, \quad p_2 = \frac{2|g_2|^2 - 2\mathcal{R}e [g_1 g_3^* - g_2 (g_1 - g_3)^*]}{|g_1|^2 + 2|g_2|^2 + |g_3|^2}. \quad (10)$$

The dependence of the polarization \vec{P}_4 of the final proton on the polarization \vec{P}_1 of the initial neutron can be also be parametrized in terms of two real coefficients:

$$\vec{P}_4 = p_3 \vec{P}_1 + p_4 \hat{k} (\hat{k} \cdot \vec{P}_1) \quad (11)$$

with the following formulas for p_3 and p_4 :

$$p_3 = \frac{2\mathcal{R}e g_2 (g_1 + g_3)^*}{|g_1|^2 + 2|g_2|^2 + |g_3|^2}, \quad p_4 = \frac{2|g_2|^2 + 2\mathcal{R}e [g_1 g_3^* - g_2 (g_1 + g_3)^*]}{|g_1|^2 + 2|g_2|^2 + |g_3|^2}. \quad (12)$$

Comparing Eqs. (6), (7), (10), and (12), one can find the following relation between these polarization observables:

$$p_1 + p_2 + p_3 + p_4 = 1 + \mathcal{A}_1 + \mathcal{A}_2, \quad (13)$$

which has to be verified by any model describing this process.

Note that the isovector a_0 -meson can be produced in the following processes of the NN -interaction:

$$p + p \rightarrow p + p + a_0^0,$$

$$p + p \rightarrow n + p + a_0^+,$$

$$n + p \rightarrow p + p + a_0^-,$$

$$n + p \rightarrow n + p + a_0^0.$$

It is possible to show that the production of the charged a_0^\pm -mesons -in the near threshold regime is characterized by a single amplitude, with the spin structure as in Eq. (2), where the following relations hold:

$$g(pp \rightarrow npa_0^+) = -g(np \rightarrow ppa_0^-) = \frac{1}{\sqrt{2}}g(np \rightarrow ppa_0^0).$$

Therefore the previous statements on polarization phenomena for $p + p \rightarrow p + p + S^0$ apply also to the production of the charged a_0^\pm -mesons.

III. THE DYNAMICS FOR THE t -CHANNEL

The standard dynamics for different processes of meson production in NN -collisions, $N + N \rightarrow N + N + V$, $N + N \rightarrow N + N + P$, and $N + N \rightarrow N + N + S^0$, (where P , S , and V denote pseudoscalar, scalar and vector mesons, respectively) is based on the consideration of mesonic exchanges in t -channel, such as π , η , σ , a_0 , ρ , ω , etc. In such approach one has to know the meson-nucleon coupling constants, PNN , SNN , and VNN and the amplitudes of different subprocesses, such as :

$$P^* + N \rightarrow N + S,$$

$$S^* + N \rightarrow N + S, \quad (14)$$

$$V^* + N \rightarrow N + S,$$

where the index '**' denotes virtual mesons - with space-like four-momenta. In principle some information exist on the coupling constants, but the threshold amplitudes for the processes (14) are poorly known, in particular for those processes that can not be experimentally studied. Therefore model calculations have to be done, in order to find these amplitudes. In case of complex amplitudes not only the absolute values are important, but also the relative signs and phases.

In this section we analyze different t -channel exchanges for the processes of scalar meson production in NN -collisions, and give, when it is possible, model independent predictions. In other words, we will find expressions for the polarization phenomena in $n + p \rightarrow n + p + S^0$, which depend only on the quantum numbers, spin and parity \mathcal{J}^P , and isospin \mathcal{I} , of the exchanged meson, but not on the corresponding coupling constants, hadronic form factors and threshold amplitudes. Let us consider the spin structure of the matrix element (4) for different exchanged particles.

A. Scalar exchange, $\mathcal{J}^P = 0^+$, $\mathcal{I}=0$

Taking into account the identity of initial and final protons in $p + p \rightarrow p + p + S^0$, the σ -exchange here is characterized by four different t -channel diagrams (Fig. 1). Each diagram has a different spin structure, with different order of the two-component spinors χ_i , $i = 1 - 4$. Only the sum of all these diagrams, which satisfies the Pauli principle for the initial and final protons, generates the correct spin structure (2), with a single amplitude g . In threshold conditions the spin propagator for the considered diagrams is written as:

$$\frac{1}{t - m_\sigma^2} = -\frac{1}{mm_s + m_\sigma^2},$$

where m , m_σ , and m_s are the masses of the nucleon, of the virtual σ -meson, and of the produced scalar meson, correspondingly. Such equality is correct at the reaction threshold, and it follows from the assumption of S -wave production of final particles. The difference in these propagators, which appears far from threshold, generates P - and higher waves of produced particles.

The σ -exchange for $n+p \rightarrow n+p+S^0$ is described by two diagrams, only (Fig. 2). Taking into account the isotopic invariance of the strong interaction, one can write the following threshold matrix element for the process $n+p \rightarrow n+p+S^0$, corresponding to σ -exchange:

$$\mathcal{M} = \mathcal{M}_{1\sigma} + \mathcal{M}_{2\sigma} = 2\mathcal{M}_{1\sigma},$$

$$\mathcal{M}_{1\sigma} = \frac{g_{\sigma NN}}{t - m_\sigma^2} \mathcal{N} A(\sigma^* p \rightarrow Sp) (\chi_4^\dagger I \chi_2) (\chi_3^\dagger I \chi_1), \quad (15)$$

where $g_{\sigma NN}$ is the σNN coupling constant, \mathcal{I} is the unit 2×2 matrix, $A(\sigma^* p \rightarrow Sp)$ is the partial amplitude for the subprocess $\sigma^* + p \rightarrow S + p$ - in the S -state, $\mathcal{N} = 2m(E_1 + m) = m(4m + m_s)$ is the normalization factor, related to the transformation from four-component Dirac spinors to two-component Pauli spinors.

In order to find the partial amplitudes g_i , corresponding to σ -exchange, it is necessary to apply the Fierz-transformation, in its two-component form, to the spinor construction in Eq. (15):

$$(\chi_4^\dagger I \chi_2) (\chi_3^\dagger I \chi_1) = -\frac{1}{2} (\chi_4^\dagger \sigma_y \tilde{\chi}_3^\dagger) (\tilde{\chi}_2 \sigma_y \chi_1) + \frac{1}{2} (\chi_4^\dagger \sigma_a \sigma_y \tilde{\chi}_3^\dagger) (\tilde{\chi}_2 \sigma_y \sigma_a \chi_1).$$

Comparing this with the general parametrization of the spin structure for the threshold matrix element of the process $n+p \rightarrow n+p+S^0$, one can find the following expressions for the particle amplitudes g_i :

$$\begin{aligned} g_{1\sigma}(np \rightarrow npS^0) &= -A_\sigma, \\ g_{2\sigma}(np \rightarrow npS^0) &= A_\sigma, \\ g_{3\sigma}(np \rightarrow npS^0) &= A_\sigma, \\ g_\sigma(pp \rightarrow ppS^0) &= -2A_\sigma, \end{aligned} \quad (16)$$

where

$$A_\sigma = \mathcal{N} A(\sigma p \rightarrow pS^0) \frac{g_{\sigma NN}}{t - m_\sigma^2}.$$

Eqs. (16) allow to predict all polarization phenomena in $n+p \rightarrow n+p+S^0$, independently on the details of the considered model, such as the value of the constant $g_{\sigma NN}$ and the amplitude $A(\sigma p \rightarrow pS^0)$.

One can find:

$$p_{1\sigma} = 1, \quad A_{1\sigma} = A_{2\sigma} = p_{2\sigma} = p_{3\sigma} = p_{4\sigma} = 0, \quad (17)$$

$$\mathcal{R}_\sigma = \frac{\sigma(np \rightarrow npS^0)}{\sigma(pp \rightarrow ppS^0)} = 2. \quad (18)$$

In (18) we took into account the identity of the produced protons in the reaction $p + p \rightarrow p + p + S^0$.

B. η -exchange, $\mathcal{J}^P = 0^-, \mathcal{I} = 0$

As in the case of σ -exchange, the η -exchange generates four different diagrams for pp -interaction and two diagrams for np -interaction. Let us consider, therefore, the simplest case of $n + p \rightarrow n + p + S^0$ -processes, the matrix element of which can be written as:

$$\mathcal{M} = \mathcal{M}_{1\eta} + \mathcal{M}_{2\eta} = 2\mathcal{M}_{1\eta},$$

$$\mathcal{M}_{1\eta} = -A_\eta(\chi_4^\dagger \vec{\sigma} \cdot \hat{k} \chi_2)(\chi_3^\dagger \vec{\sigma} \cdot \hat{k} \chi_1), \quad (19)$$

$$A_\eta = \frac{\mathcal{N}k}{E + m} \frac{g_{\eta NN}}{t - m_\eta^2} A(\eta^* N \rightarrow NS^0),$$

where $g_{\eta NN}$ is the coupling constant for the ηNN -vertex, $A(\eta^* N \rightarrow NS^0)$ is the threshold amplitude for $\eta + N \rightarrow N + S^0$, which describes the partial transition: $\ell_\eta = 1 \rightarrow j^P = 1/2^+$. The kinematical factor $\frac{k}{E + m} = \sqrt{\frac{m_s}{4m + m_s}}$ (at threshold) results from the transformation of the pseudoscalar vertex $\bar{u}\gamma_5 u$ to its two-component equivalent $\chi^\dagger \vec{\sigma} \cdot \hat{k} \chi$.

After applying the Fierz transformation to the spin structure (19) to the standard form, Eq. (4), one can find the following formulas for the partial amplitudes $g_{i\eta}$, $i = 1 - 3$, corresponding to η -exchange:

$$\begin{aligned} g_{1\eta}(np \rightarrow npS^0) &= -A_\eta, \\ g_{2\eta}(np \rightarrow npS^0) &= -A_\eta, \\ g_{3\eta}(np \rightarrow npS^0) &= A_\eta, \\ g_\eta(pp \rightarrow ppS^0) &= -2A_\eta, \end{aligned} \quad (20)$$

and similarly, to σ -exchange, one can predict all polarization observables:

$$A_{1\eta} = A_{2\eta} = p_{3\eta} = p_{4\eta} = 0, \quad p_{1\eta} = -1, \quad p_{2\eta} = 2, \quad \mathcal{R} = 2. \quad (21)$$

Comparing Eq. (17) and Eq.(21), one can see that the polarization transfer coefficients, characterizing a change of neutron polarization, must be very sensitive to the quantum number of the isoscalar exchange in t -channel, other polarization observables, such as \mathcal{A}_1 , \mathcal{A}_2 , p_3 and p_4 vanish for both exchanges $\mathcal{J}^P = 0^+$ and $\mathcal{J}^P = 0^-$.

C. Pion exchange, $\mathcal{J}^P = 0^-$, $\mathcal{I}=1$

Any isovector exchange for the process $n + p \rightarrow n + p + S^0$ is characterized by a set of four Feynman diagrams Fig. (3) with exchange of neutral and charged pion.

Considering the contribution of all these diagrams, and taking into account the isotopic invariance predictions for the two vertices of the considered diagrams:

$$g_{\pi^0 pp} = -g_{\pi^0 nn} = \frac{1}{\sqrt{2}}g_{\pi^+ np} = \frac{1}{\sqrt{2}}g_{\pi^- pn},$$

one can find (after applying the Fierz transformation) the following expressions for the partial amplitudes $g_{i\pi}$, $i = 1 - 3$, corresponding to π -exchange:

$$\begin{aligned} g_{1\pi}(np \rightarrow npS^0) &= A_\pi, \\ g_{2\pi}(np \rightarrow npS^0) &= -3A_\pi, \\ g_{3\pi}(np \rightarrow npS^0) &= 3A_\pi, \\ g_\pi(pp \rightarrow ppS^0) &= 2A_\pi, \end{aligned} \quad (22)$$

where $A_\pi = \mathcal{N} \sqrt{\frac{m_s}{4m + m_s}} \frac{g_{\pi NN}}{t - m_\pi^2} A(\pi^* N \rightarrow NS)$. In principle, the amplitude $A(\pi^* N \rightarrow NS)$ for threshold S -production in πN -interaction can be experimentally measured through the corresponding differential cross section, but this method will not determine its sign.

However numerical values of the polarization phenomena for the $n + p \rightarrow n + p + S^0$ processes can be predicted exactly, without information on $A(\pi^* N \rightarrow NS)$:

$$A_{1\pi} = 2/7, A_{2\pi} = 0, p_{1\pi} = -3/7, p_{2\pi} = 6/7, p_{3\pi} = -6/7, p_{4\pi} = 12/7, \quad (23)$$

and they are very different from the case of isoscalar exchange.

Another interesting result concerns the large difference in cross section, for pp - and np -interaction: $\mathcal{R}_\pi = 14$, i.e. for scalar meson production (in NN -interaction) we have large isotopic dependence.

D. Scalar exchange, $\mathcal{J}^P = 0^+, \mathcal{I}=1$

In this case, both processes $p + p \rightarrow p + p + S^0$ and $n + p \rightarrow n + p + S^0$ is characterized by a set of four Feynman diagrams, the neutral a_0 -exchange for pp -collisions and the neutral+charged a_0 -exchanges for np -collisions.

The partial amplitudes g_{ia} , $i = 1 - 3$, corresponding to a_0 -exchange are determined by the following formulas:

$$\begin{aligned} g_{1a}(np \rightarrow npS^0) &= A_a, \\ g_{2a}(np \rightarrow npS^0) &= 3A_a, \\ g_{3a}(np \rightarrow npS^0) &= 3A_a, \\ g_a(pp \rightarrow ppS^0) &= 2A_a, \end{aligned} \quad (24)$$

and $A_a = \mathcal{N} \frac{g_{aNN}}{t - m_a^2} A(a^*N \rightarrow NS)$. The threshold amplitude $A(a^*N \rightarrow NS)$ for the exotic subprocess $a^* + N \rightarrow N + S^0$ has to be determined in framework of a model.

From these formulas, independently from the values of the coupling constant g_{aNN} and the threshold amplitude $A(a^*N \rightarrow NS)$, one can predict the following values for the polarization observables:

$$A_{1a} = 2/7, A_{2a} = 0, p_{1a} = 3/7, p_{2a} = 0, p_{3a} = 6/7, p_{4a} = 0, \mathcal{R}_a = 14. \quad (25)$$

They are different from the case of isoscalar exchange and from (25). Due to isovector exchange, the isotopic ratio \mathcal{R} is large also in this case.

E. Correlation of spin and isospin structures for the threshold regime

Combining the previously derived contributions, one can find for the threshold amplitudes g_i the following expressions:

$$\begin{aligned}
 g_1(np \rightarrow npS^0) &= -A_\sigma + A_a - A_\eta + A_\pi, \\
 g_2(np \rightarrow npS^0) &= A_\sigma + 3A_a - A_\eta - 3A_\pi, \\
 g_3(np \rightarrow npS^0) &= A_\sigma + 3A_a + A_\eta + 3A_\pi, \\
 g(pp \rightarrow ppS^0) &= 2g_1(np \rightarrow npS^0).
 \end{aligned}
 \tag{26}$$

Let us note that the S-channel parametrization of the spin structure for the threshold matrix elements allows (after applying the Fierz transformation) to unify different t -channel contributions- with different \mathcal{J}^P - in universal and transparent form, which is well adapted to the analysis of the sensitivity of the polarization phenomena, in $n + p \rightarrow n + p + S^0$, to the quantum numbers of the t -channel meson. Such unification allows to simplify all calculations: we can express the matrix element for any exchange in terms of three amplitudes only, whereas, for example, a model with $\eta + \pi + \sigma + a$ contains $2 + 4 + 2 + 4 = 12$ different Feynman diagrams - with different spin structures. All these twelve contributions to the total matrix elements, in general, interfere, so it is in principle necessary to calculate $12 \times 12 = 144$ terms, instead than $3 \times 3 = 9$ terms, for the g_i parametrization of the matrix elements. Moreover, these nine terms are the same for any model, whereas, for example, adding vector exchanges will increase essentially the number of t -contributions. Another advantage of the $t \rightarrow s$ Fierz transformation is the explicit dependence of the observables on a definite combination of coupling constants, hadronic form factors and elementary amplitudes. This helps in finding out which contributions play the most important role in the t -channel dynamics, and gives a feedback on the coupling constants by comparison with the experimental data.

Another important property of the partial amplitudes g_i for np -processes, Eq. (26), concerns a strong correlation of the spin and isospin structure of the matrix element in

the threshold region. The amplitudes $g_2(np \rightarrow npS^0)$ and $g_3(np \rightarrow npS^0)$, describing the np -interaction in the isotopic singlet state, are in general different, even in case of a definite isospin in t -channel, if the P-parity of the t -channel is not fixed. However, for definite \mathcal{J}^P -exchanges, we have:

$$\begin{aligned} g_2(np \rightarrow npS^0) &= g_3(np \rightarrow npS^0), \text{ if } \mathcal{J}^P = 0^+, \mathcal{I} = 0 \text{ and } \mathcal{I} = 1, \\ g_2(np \rightarrow npS^0) &= -g_3(np \rightarrow npS^0), \text{ if } \mathcal{J}^P = 0^-, \mathcal{I} = 0 \text{ and } \mathcal{I} = 1. \end{aligned} \quad (27)$$

The relations (27) hold also for any combination of isoscalar and isovector exchanges.

The total cross section for $n + p \rightarrow n + p + S^0$ is the incoherent sum of isosinglet and isotriplet contributions:

$$\begin{aligned} \sigma(np \rightarrow npS^0) &= \sigma_0(np \rightarrow npS^0) + \sigma_1(np \rightarrow npS^0), \\ \sigma_0(np \rightarrow npS^0) &= 2|g_2(np \rightarrow npS^0)|^2 + |g_3(np \rightarrow npS^0)|^2, \\ \sigma_1(np \rightarrow npS^0) &= |g_1(np \rightarrow npS^0)|^2 = \frac{1}{2}\sigma(pp \rightarrow ppS^0). \end{aligned} \quad (28)$$

IV. A POSSIBLE MODEL FOR $N + N \rightarrow N + N + S^0$

We analyze here in detail a model for $N + N \rightarrow N + N + S^0$, based on $\sigma + \pi$ -exchange. In order to justify such model, let us mention that the η -contribution can be neglected, in (26), due to the fact that the coupling constant $g_{\eta NN}$ is presently poorly known, being in the range $1 \div 7$ [44–47]. Concerning the a_0 -exchange, both the ingredients of such contribution, the coupling constant $g_{a_0 NN}$ and the amplitude $A(aN \rightarrow NS^0)$ are presently not known. The mass of the a_0 -meson is larger in comparison with the pion mass. In any case, the $\sigma + \pi$ -model can be considered a realistic starting point for the analysis, containing the exchange of mesons with different P-parities and isospin. As a result, all three partial amplitudes are different:

$$\begin{aligned} g_1(np \rightarrow npS^0) &= A_\pi(1 - r), \\ g_2(np \rightarrow npS^0) &= A_\pi(-3 + r), \end{aligned} \quad (29)$$

$$g_3(np \rightarrow npS^0) = A_\pi(3 + r),$$

$$g(pp \rightarrow ppS^0) = 2A_\pi(1 - r),$$

where r is the ratio of the corresponding contributions:

$$r = \frac{g_{\sigma NN}}{g_{\pi NN}} \left(\frac{t - m_\sigma^2}{t - m_\pi^2} \right) \frac{A(\sigma^* N \rightarrow NS^0)}{A(\pi^* N \rightarrow NS^0)} \sqrt{\left(1 + 4 \frac{m}{m_s}\right)}.$$

One can see that all the physics of this model (with eight different Feynman diagrams) is contained in a single parameter r , which is basically the ratio of the coupling constants and the elementary amplitudes. In the general case, the ratio r is a complex parameter, which depends on the excitation energy of the produced NNS -system. Therefore the polarization phenomena for the process $n + p \rightarrow n + p + S^0$ and the ratio of cross section for np - and pp -collisions, can be expressed in terms of two parameters $|r|^2$ and $\mathcal{R}er$:

$$\mathcal{R} = \frac{\sigma(np \rightarrow npS^0)}{\sigma(pp \rightarrow ppS^0)} = 2 \frac{7 - 2\mathcal{R}er + |r|^2}{1 - 2\mathcal{R}er + |r|^2} = 2 \left(7 - 6 \frac{|r|^2 - 2\mathcal{R}er}{1 - 2\mathcal{R}er + |r|^2} \right). \quad (30)$$

The coefficients \mathcal{A}_1 and \mathcal{A}_2 , which characterize the polarized $\vec{n}\vec{p}$ -collisions, can be written as follows:

$$\mathcal{A}_1 = 2 \frac{1 + \mathcal{R}er}{7 - 2\mathcal{R}er + |r|^2}, \quad \mathcal{A}_2 = \frac{3\mathcal{R}er}{7 - 2\mathcal{R}er + |r|^2}. \quad (31)$$

The ratio r can be found in framework of a model for the elementary subprocesses $\pi^*(\sigma^*) + N \rightarrow N + S^0$, under several assumptions concerning coupling constants, cut-off parameters, form factors... To avoid the uncertainties related to these choices, we assume that r is real. This is the case, for example, of the effective Lagrangian approach, which gives real amplitudes and, therefore, real values for the ratio r . Another possibility is the saturation of the $A(\sigma^* N \rightarrow NS^0)$ and $A(\pi^* N \rightarrow NS^0)$ amplitudes by a single nucleon resonance with $\mathcal{J}^P = 1/2^+$. Such mechanism produce complex amplitudes with zero relative phase, and r is a real parameter, too.

In this case we can predict the r -dependence of \mathcal{R} , \mathcal{A}_1 , and \mathcal{A}_2 (Figs. 4-6). The ratio \mathcal{R} of the corresponding cross sections can be very large in the region $r \simeq 1$ (Fig. 4). The

experimental determination of R will allow to find two solutions for r :

$$r_{\pm} = 1 \pm 2\sqrt{\frac{3}{R-2}}$$

The function $A_1(r)$ has two extrema: the maximum at $r = -1 + \sqrt{10}$, where $A_1^{max} = (2 + \sqrt{10})/\sqrt{6}$ and a minimum at $r = -1 - \sqrt{10}$, where $A_1^{min} = (2 - \sqrt{10})/\sqrt{6}$, see Fig. 5.

The function $A_2(r)$ has a very similar behavior, with a the maximum at $r = +\sqrt{7}$, where $A_2^{max} = (1 + \sqrt{7})/4$ and a minimum at $r = -\sqrt{7}$, where $A_2^{min} = (1 - \sqrt{7})/4$, see Fig. 6.

V. COMMENTS ON THE NN FINAL STATE INTERACTION (FSI)

In order to illustrate possible effects of FSI for the considered reaction, let us make an oversimplified estimation, using only the NN -scattering length approximation, which will result in an upper limit for the NN-FSI. Let us consider, for illustration, the case of pion exchange. One finds the following correction for some observables which have been previously discussed:

- The ratio of np and pp -cross sections:

$$R^{(\pi)} = \frac{1}{2} \left[\frac{a_s(np)}{a_s(pp)} \right]^2 + \frac{27}{2} \left[\frac{a_t(np)}{a_s(pp)} \right]^2.$$

- Polarization effects for np -collisions:

$$C_{nn}^{(\pi)} = \frac{9 - R_{st}^2}{27 + R_{st}^2}, \quad D_{nn}^{(\pi)} = \frac{6(R_{st} - 3)}{27 + R_{st}^2},$$

where $R_{st} = [a_s(np)/a_t(np)]$, $a_s(NN)$ is the singlet NN -scattering length and $a_t(np)$ is the triplet np -scattering length (for S-state).

Using the following values for the scattering lengths [48]: $a_s(np) = -23.768$ fm, $a_t(np) = 5.424$ fm, and $a_s(pp) = -7.8098$ fm, one can find: $R^{(\pi)} \simeq 11$ (instead of 14), $C_{nn}^{(\pi)} \simeq -0.22$ (instead of 0.29), and $D_{nn}^{(\pi)} \simeq -0.96$ (instead of -0.43), i.e. the final state np -interaction can change the sign of $C_{nn}^{(\pi)}$, but does not affect much the ratio $R^{(\pi)}$.

VI. CONCLUSIONS

Let us summarize the main results concerning the production of scalar mesons in nucleon-nucleon collisions near threshold.

- We parametrize the spin structure of the threshold matrix element for the processes $p + p \rightarrow p + p + S^0$, where S^0 is a neutral scalar meson, $S = \sigma, f_0$, or a_0 , in terms of a single spin structure, corresponding to a singlet-singlet transition in the pp -system. The same structure describes the production of the charged a_0^\pm -mesons in the processes: $p + p \rightarrow p + n + a_0^+$ and $n + p \rightarrow p + p + a_0^-$.
- The process $n + p \rightarrow p + p + S^0$ is characterized by a more complicated spin structure, where the matrix element contains, in the general case, three contributions with different spin structures. One contribution coincides with the $\mathcal{M}(pp \rightarrow ppS^0)$ matrix element -due to isotopic invariance, and two additional contributions describe the triplet-triplet transition in the np -system, which are forbidden by the Pauli principle, in case of the pp -system.
- The suggested model-independent parametrization of the spin structure, which is based on the general symmetry properties of the strong interaction (such as P-invariance, isotopic invariance, conservation of angular momentum and Pauli principle) allows to analyze polarization effects for pp - and np -collisions in a transparent way. These effects are very peculiar for threshold conditions, where all final particles are emitted in relative S -states.
- The standard t -channel dynamics for the process $N + N \rightarrow N + N + S^0$ is generated by different meson exchanges with $\mathcal{J}^P = 0^+$ and $\mathcal{J}^P = 0^-$ and isotopic spin $\mathcal{I}=0$ and $\mathcal{I}=1$. The dependence of the polarization phenomena in $n + p \rightarrow n + p + S^0$ on the quantum number of the exchanged mesons can be described in terms of the corresponding S-channel partial amplitudes, with the help of the Fierz transformation in its two-component form.

- As an example of a 'realistic' model in the threshold region, we considered the $\pi + \sigma$ -exchange. An attractive property of such model is that the polarization phenomena in np -collisions depend on a single complex parameter r , which characterizes the relative role of σ - and π -exchange. The polarization observables, which do not vanish, show a large sensitivity to r .
- The processes $p + p \rightarrow p + p + S^0$ and $n + p \rightarrow n + p + S^0$ show large isotopic effects, i.e. S-state production of scalar mesons is larger in np -collisions than in pp -collisions, and polarization phenomena are different for these processes.

Note that these results do not depend on the structure of the f_0 and a_0 -mesons ($q\bar{q}$ -states, four-quark state, $K\bar{K}$ -molecule etc.) and on the mechanisms of their decays.

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FIGURES

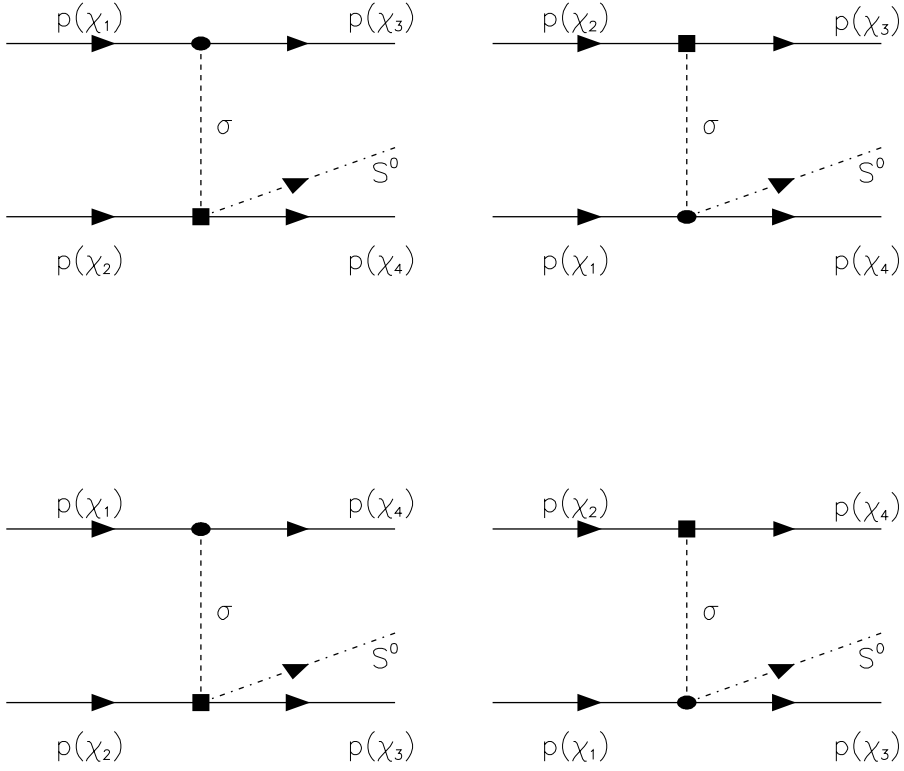


FIG. 1. Feynman diagrams for σ -exchange, for $p + p \rightarrow p + p + S^0$, where $p(\chi_i)$, $i=1-4$, means that the corresponding proton (in initial or final state) is described by the 2-component spinor χ_i .

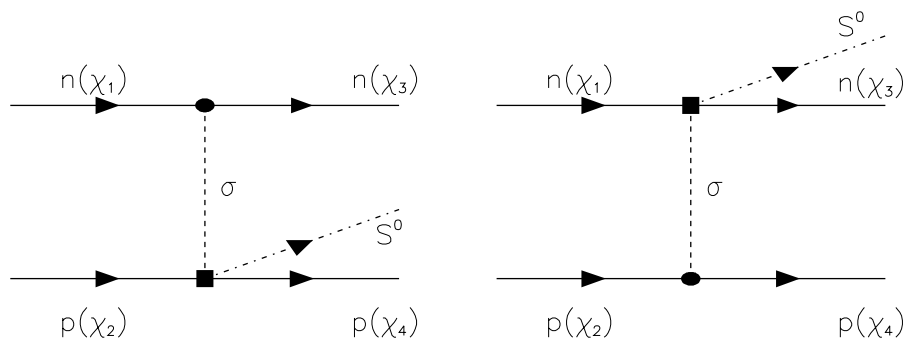


FIG. 2. Feynman diagrams for σ -exchange for $n + p \rightarrow n + p + S^0$. Notations as in Fig. 1.

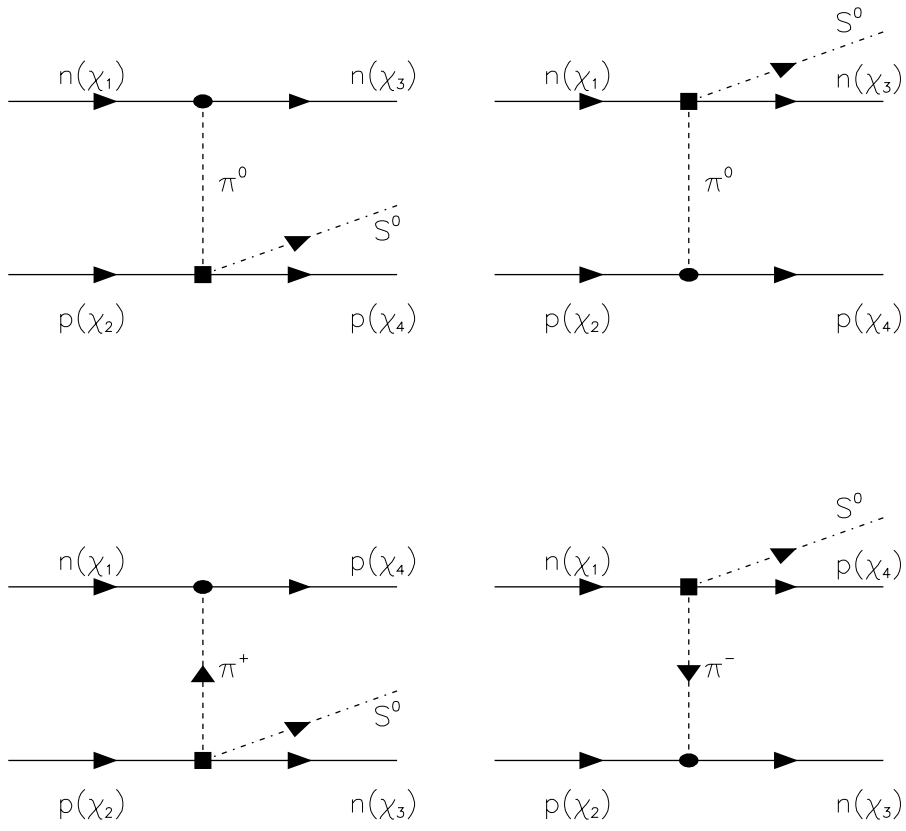


FIG. 3. Feynman diagrams for π -exchange for $n + p \rightarrow n + p + S^0$. Notations as in Fig. 1.

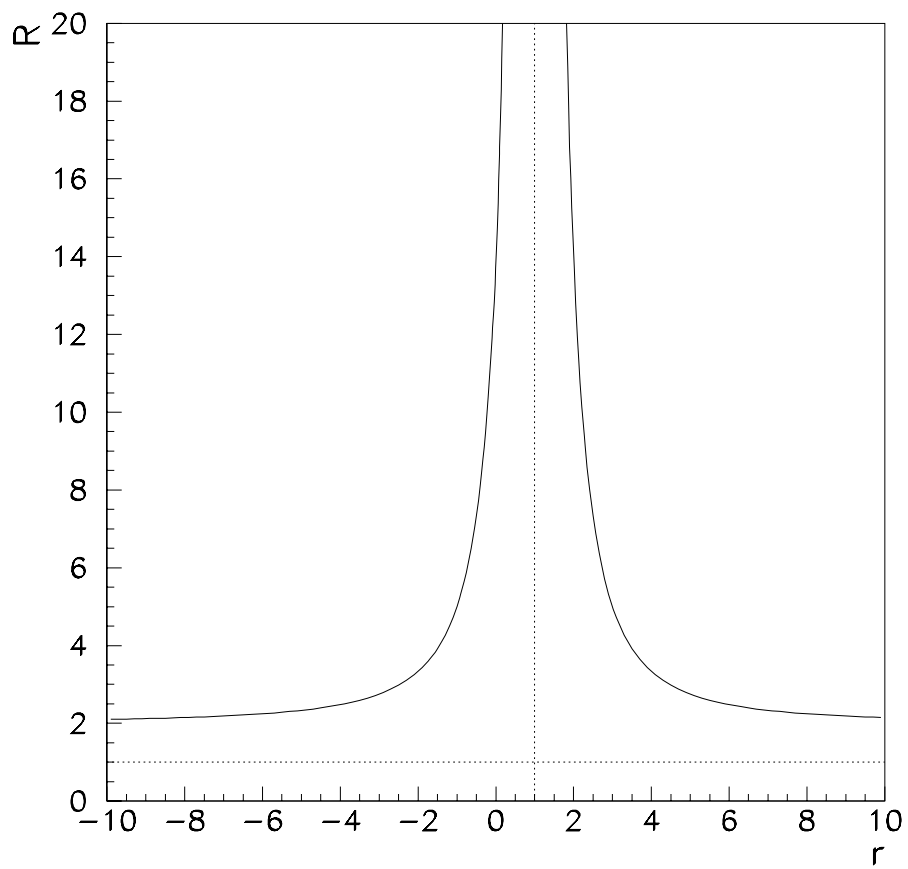


FIG. 4. Dependence of R on r , see Eq. 30

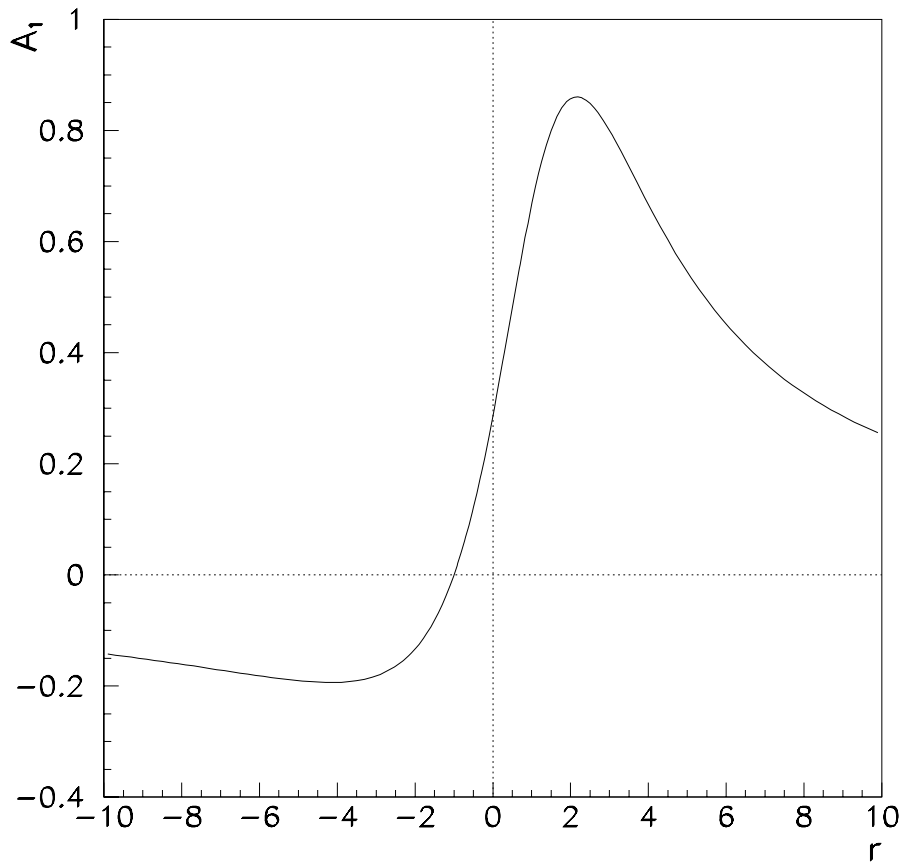


FIG. 5. Dependence of \mathcal{A}_1 on r , see Eq. (31)

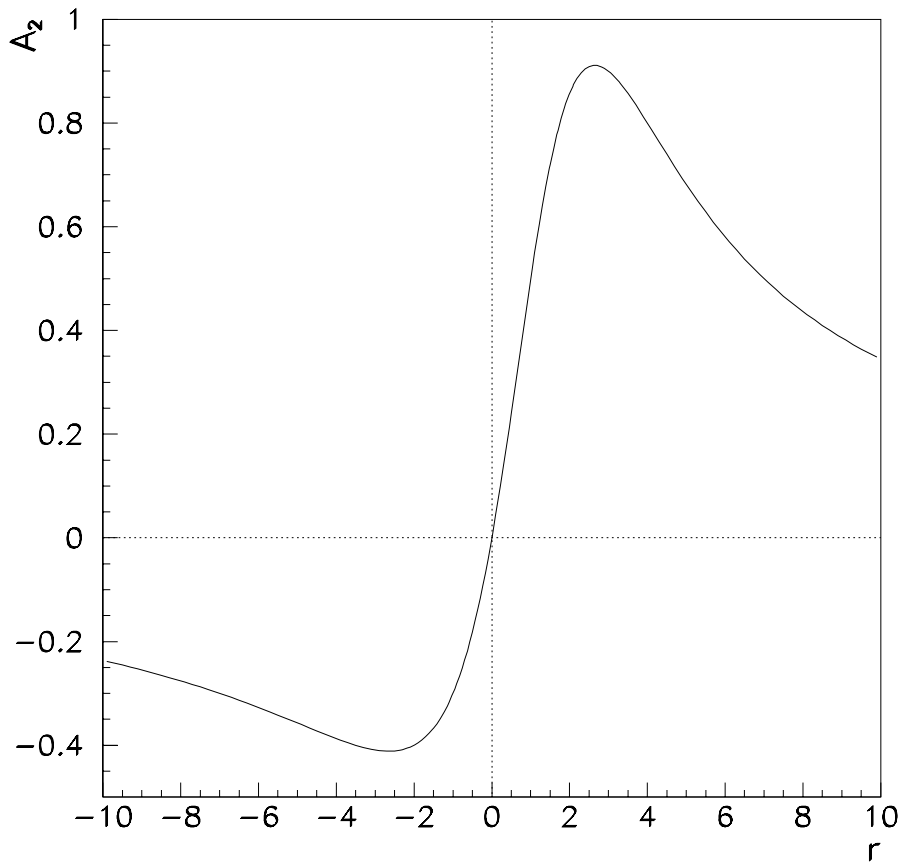


FIG. 6. Dependence of \mathcal{A}_2 on r , see Eq. (31).