Model independent properties of two-photon exchange in elastic electron proton scattering

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Abstract

We derive from first principles, as the C-invariance of the electromagnetic interaction and the crossing symmetry, the general properties of two-photon exchange in electron-proton elastic scattering. We show that the presence of this mechanism destroys the linearity of the Rosenbluth separation but does not affect the terms related to the electromagnetic form factors.

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I. INTRODUCTION

Recent developments in the field of hadron electromagnetic form factors (FFs) are due to the very precise and surprising data obtained at the Jefferson Laboratory (JLab), in $\vec{e}+p \to e+\vec{p}$ elastic scattering, based on the polarization transfer method [1, 2], which show that the electric and magnetic distributions in the proton are different.

The application of the polarization transfer method, proposed about 30 years ago [3] has been possible only recently, as it needs high intensity polarized beams, large solid angle spectrometers and advanced techniques of polarimetry in the GeV range. Experiments have been performed at JLab up to $Q^2 = 5.6$ GeV^2 and an extension up to 9 GeV^2 is in preparation [4].

The existing data show a discrepancy between the Q^2-dependence of the ratio $R = \mu_p G_{Ep}/G_{Mp}$ of the electric to the magnetic proton form factors (Q^2 is the momentum transfer squared, μ_p=2.79 is the proton magnetic moment), whether derived with the standard Rosenbluth separation [5] or with the polarization method.

Therefore a careful experimental and theoretical analysis of this problem is necessary. The important point here is the calculation of radiative corrections to the differential cross section and to polarization observables in elastic ep-scattering. If these corrections are large (in absolute value) for the differential cross section [6], in particular for high resolution experiments, a simplified estimation of radiative corrections to polarization phenomena [7] shows that radiative corrections are small for the ratio P_L/P_T of longitudinal to transverse polarization of the proton emitted in the elastic collision of polarized electrons with an unpolarized proton target.

For this reaction, the one-photon exchange is considered to be the main mechanism. In the standard calculations [6], the two-photon exchange mechanism is only partially taken into account considering the special part of the integral, where one photon carries all the momentum transfer and the second photon is almost real. This contribution allows to overcome the problem of the 'infrared' divergence. But it has been pointed out [8] that, at large momentum transfer, the role of another mechanism, where the momentum transfer is shared between the two photons, can be relatively increased, due to the steep decreasing of the electromagnetic form factors with Q^2. This effect can eventually become so large that the traditional description of the electron-hadron interaction in terms of electromagnetic currents (and electromagnetic form factors) can become incorrect.

Numerous tests of the validity of the one-photon mechanism have been done in the past, using different methods: test of the linearity of the Rosenbluth formula for the differential cross section, comparison of the e^+p and e^-p-cross sections, attempts to measure various T-odd polarization observables.

Note that the two-photon exchange should appear at smaller Q^2 for heavier targets: d, ${}^3He$, ${}^4He$, because the corresponding form factors decrease faster with Q^2 in comparison with protons. In [9] the possible effects of 2γ-exchange have been estimated from the precise data on the structure function $A(Q^2)$, obtained at Jlab in electron deuteron elastic scattering, up to $Q^2 = 6$ GeV^2 [10, 11]. The possibility of 2γ-corrections has not been excluded by this analysis, starting from $Q^2 = 1$ GeV^2, and the necessity of dedicated experiments was pointed out. From this kind of consideration, one would expect to observe the two-photon contribution in eN-scattering at larger momentum transfer, for $Q^2 \simeq 10$ GeV^2.

The exact calculation of the 2γ-contribution to the amplitude of the $e^{\pm}p \to e^{\pm}p$-process requires the knowledge of the matrix element for the double virtual Compton scattering, $\gamma^* + N \to \gamma^* + N$, in a large kinematical region of colliding energy and virtuality of both photons, and can not be done in a model independent form.

However general properties of the hadron electromagnetic interaction, as the C-invariance and the crossing symmetry, give rigorous prescriptions for different observables for the elastic scattering of electrons and positrons by nucleons, in particular for the differential cross section and for the proton polarization, induced by polarized electrons. These concrete prescriptions help in identifying a possible manifestation of the two-photon exchange mechanism. For example, an attempt [12] of resolving the discrepancy between the existing data on the ratio R, conserving the linear ϵ-dependence of the elastic cross section in presence of 2γ-corrections is in contradiction with the C-invariance of the electromagnetic interaction (ϵ is the degree of polarization for the virtual photon).

The purpose of this paper is to derive the correct ϵ-dependence of the 2γ-contribution to the differential cross section and to find a 'model independent' parametrization of these additional terms. The experimental test of the predicted ϵ-dependence of the differential cross section will be a signature of the presence of the 2γ-contribution and allow to estimate its role.

The standard expression of the matrix element for elastic ep-scattering, in framework of one-photon exchange is:

$$\mathcal{M}_1 = \frac{e^2}{Q^2}\overline{u}(k_2)\gamma_\mu u(k_1)\overline{u}(p_2)\left[F_{1N}(Q^2)\gamma_\mu - \frac{\sigma_{\mu\nu}q_\nu}{2m}F_{2N}(Q^2)\right]u(p_1), \tag{1}$$

where k_1 (p_1) and k_2 (p_2) are the four-momenta of the initial and final electron (nucleon), m is the nucleon mass, $q = k_1 - k_2$, $Q^2 = -q^2 > 0$. F_{1N} and F_{2N} are the Dirac and Pauli nucleon electromagnetic form factors, which are real functions of the variable Q^2 - in the space-like region of momentum transfer. The same form factors describe also the one-photon mechanism for the scattering of positrons. From Eq. (1) one can find the following expression for the differential cross section (in the laboratory system (Lab)):

$$\frac{d\sigma}{d\Omega_e} = \sigma_M\left[G_{MN}^2(Q^2) + \frac{\epsilon}{\tau}G_{EN}^2(Q^2)\right], \tag{2}$$

$$\tau = \frac{Q^2}{4m^2},\ G_{MN} = F_{1N} + F_{2N},\ G_{EN} = F_{1N} - \tau F_{2N}$$

where σ_M is the Mott cross section, for the scattering of unpolarized electrons by a point charge particle (with spin 1/2), ϵ is an independent kinematical variable, which, together with Q^2, fully determines the kinematics of elastic ep-scattering and can be written, the limit of $m_e = 0$, as:

$$\epsilon = \frac{1}{1 + 2(1+\tau)\tan^2\dfrac{\theta_e}{2}}, \tag{3}$$

where θ_e is the electron scattering angle in Lab system. Therefore $0(\theta_e = \pi) \leq \epsilon \leq 1(\theta_e = 0)$.

If one takes into account the two-photon mechanism, the expression of the differential cross section, Eq. (2), is essentially modified.

It requires, first of all, a generalization of the spin structure of the matrix element, which can be done, in analogy with elastic np-scattering [13], using the general properties of the electron-hadron interaction such as the P-invariance and the relativistic invariance. Taking into account the identity of the initial and final states and the T-invariance of the electromagnetic interaction, the process $e^\pm p \to e^\pm p$, in which four particles with spin 1/2 participate, is characterized by six independent products of four-spinors, describing the initial and final fermions. The corresponding parametrization of the matrix element can be done in many different but equivalent forms, in terms of six invariant complex amplitudes, $\mathcal{A}_i(s, Q^2)$, where $s = (k + p_1)^2$ is the square of the total energy of the colliding particles. In the physical region of the $e^\pm p \to e^\pm p$-reaction the relations: $Q^2 \geq 0$ and $s \geq (m+m_e)^2 \simeq m^2$, apply.

Previously another set of variables, ϵ and Q^2, which is equivalent to s and Q^2 (in Lab system) was considered. These variables are well adapted to the description of the properties of one-photon exchange, because, in this case, only the Q^2-dependence of the form factors has a dynamical origin, whereas the linear ϵ-dependence in Eq. (2) is a trivial consequence of the one-photon mechanism. On the other hand, the variables s and Q^2 are better suited to the analysis of crossing symmetry.

The conservation of the lepton helicity, which is a general property of the electromagnetic interaction in electron-hadron scattering, reduces the number of invariant amplitudes, in general complex, from six to three.

Therefore we can write the following parametrization of the spin structure of the matrix element following the formalism of [13]:

$$\mathcal{M} = \frac{e^2}{Q^2}\overline{u}(k_2)\gamma_\mu u(k_1)\overline{u}(p_2)\left[\mathcal{A}_1(s,Q^2)\gamma_\mu - \mathcal{A}_2(s,Q^2)\frac{\sigma_{\mu\nu}q_\nu}{2m} + \mathcal{A}_3(s,Q^2)\gamma\cdot K\mathcal{P}_\mu\right]u(p_1), \quad (4)$$

$$K = \frac{k_1+k_2}{2},\ \mathcal{P} = \frac{p_1+p_2}{2},$$

where $\mathcal{A}_1 - \mathcal{A}_3$ are the corresponding invariant amplitudes. In case of one-photon exchange

$$\mathcal{A}_1(s,Q^2) \to F_{1N}(Q^2),\ \mathcal{A}_2(s,Q^2) \to F_{N2}(Q^2),\ \mathcal{A}_3 \to 0.$$

But in the general case (with multi-photon exchange) the situation is more complicated, because:

- The amplitudes $\mathcal{A}_i(s,Q^2)$, $i = 1-3$, are complex functions of two independent variables, s, and Q^2.

- The set of amplitudes $\mathcal{A}_i^{(-)}(s,Q^2)$ for the process $e^- + p \to e^- + p$ is different from the set $\mathcal{A}_i^{(+)}(s,Q^2)$ of corresponding amplitudes for positron scattering, $e^+ + p \to e^+ + p$, which means that the properties of the scattering of positrons can not be derived from $\mathcal{A}_i^{(-)}(s,Q^2)$, as in case of the one-photon mechanism.

- The relation of the amplitudes $\mathcal{A}_i(s,Q^2)$ with the nucleon electromagnetic form factors is non-trivial and includes many other quantities, as, for example, the form factors of the Δ-excitation - through the amplitudes of the virtual Compton scattering.

In this framework, the standard phenomenology of electron-hadron physics does not hold anymore, and in particular, it would not be anymore possible to express the internal structure of a hadron in terms of form factors, which are real functions of one variable.

In the following text, we will show that the situation is not so involved, and that even in case of two-photon exchange, one can still use the formalism of form factors, if one takes into account the C-invariance of the electromagnetic interaction of hadrons.

A deeper analysis of Eq. (4) shows that the spin structure of $\mathcal{A}_1$ and $\mathcal{A}_2$ corresponds to exchange by vector particle (in t-channel), whereas the spin structure for the amplitude $\mathcal{A}_3$ corresponds to tensor exchange. This means that the amplitudes $\mathcal{A}_1$ and $\mathcal{A}_2$ contain only C-odd t-exchanges, whereas the $\mathcal{A}_3$ describes only C-even $t-$channel exchanges.

We can therefore conclude that the amplitudes $\mathcal{A}_1$ and $\mathcal{A}_2$ are determined by exchanges of odd numbers of photons and the amplitude $\mathcal{A}_3$ is determined by exchanges of even numbers of photons, and corresponds to a different dynamics. Therefore the relation:

$$\mathcal{A}_{1,2}(s,Q^2) \to F_{1,2N}(Q^2), \tag{5}$$

holds up to 3γ-corrections, and the amplitudes $\mathcal{A}_{1,2}$ can not contain any contribution from 2γ-exchange. This means also that, contrary to the assumption of [12], the phases of $\mathcal{A}_{1,2}$ are very small, $\delta_{1,2} \simeq \alpha^2$, ($\alpha \simeq 1/137$). The possible s-dependence of $\mathcal{A}_{1,2}$ is of the same order of magnitude.

In summary, the physics of the electromagnetic form factors in the electron-hadron interaction holds even beyond the one-photon mechanism. The complications, which arise from two-photon exchange, enter only in the amplitude $\mathcal{A}_3$.

Another consequence is that the difference of the amplitudes $\mathcal{A}_{1,2}^{(+)}$ and $\mathcal{A}_{1,2}^{(-)}$ is of the order of α^2 and therefore the difference of cross sections $d\sigma(e^-p)/d\Omega_e$ - $d\sigma(e^+p)/d\Omega_e$ can be parametrized as the product of a definite combination of form factors, G_{Ep}, G_{Mp} and $Re\mathcal{A}_3(s,Q^2)$, i.e. can be predicted on the bases of the data on e^-p-scattering.

Due to the C-invariance of the electromagnetic hadron interaction, instead of three complex functions, depending on two variables $\mathcal{A}_i(s,Q^2)$, $i=1-3$, the matrix element for elastic electron-nucleon scattering is determined by two real nucleon form factors, $G_{E,MN}(Q^2)$, which are functions of one variable only, and two real functions, $Re\mathcal{A}_3$ and $Im\mathcal{A}_3$, which are essentially smaller in absolute value, $\mathcal{A}_3(s,Q^2) \simeq \alpha$. Moreover, the same set of functions describes the elastic positron-nucleon scattering, too.

In addition to C-invariance, the crossing symmetry also bring important information on the properties of two-photon exchange, by relating the matrix elements for the cross-channels: $e^- + N \to e^- + N$, in s-channel, and $e^+ + e^- \to N + \overline{N}$, in t-channel. The transformation from s- to t-channel can be realized by the following substitution:

$$k_2 \to -k_2,\ p_1 \to -p_1.$$

and for the invariant variables:

$$s = (k_1 + p_1)^2 \to (k_1 - p_1)^2,\ Q^2 = -(k_1 - k_2)^2 \to -(k_1 + k_2)^2 = -t.$$

The crossing symmetry states that the same amplitudes $\mathcal{A}_i(s, Q^2)$ describe the two channels, when the variables s and Q^2 scan the physical region of the corresponding channels. So, if $t \geq 4m^2$ and $-1 \leq \cos\theta \leq 1$ (θ is the angle of the proton production with respect to the electron three-momentum, in the center of mass (CMS) for $e^+ + e^- \to N + \overline{N}$), the amplitudes $\mathcal{A}_i(t, \cos\theta)$, $i = 1 - 3$, describe the process $e^+ + e^- \to p + \overline{p}$.

The C-invariance of the electromagnetic hadron interaction and the corresponding selection rules can be also applied to the annihilation channel and allow to describe in a transparent way the properties of the different observables for the eN-elastic scattering, using the crossing symmetry.

To illustrate this, let us consider firstly the one-photon mechanism for $e^+ + e^- \to p + \overline{p}$. The conservation of the total angular momentum $\mathcal{J}$ allows one value, $\mathcal{J} = 1$, and the quantum numbers of the photon: $\mathcal{J}^P = 1^-$, $C = 1$. The selection rules with respect to the C and P-invariances allow two states for e^+e^- (and $p\overline{p}$):

$$S = 1,\ \ell = 0 \text{ and } S = 1,\ \ell = 2 \text{ with } \mathcal{J}^P = 1^-, \tag{6}$$

where S is the total spin and ℓ is the orbital angular momentum. As a result the θ-dependence of the cross section for $e^+ + e^- \to p + \overline{p}$, in the one-photon exchange mechanism is:

$$\frac{d\sigma}{d\Omega}(e^+ + e^- \to p + \overline{p}) \simeq a(t) + b(t)\cos^2\theta, \tag{7}$$

where $a(t)$ and $b(t)$ are definite quadratic contributions of $G_{Ep}(t)$ and $G_{Mp}(t)$, $a(t),\ b(t) \geq 0$ at $t \geq 4m^2$.

Using the kinematical relations:

$$\cos^2\theta = \frac{\epsilon + 1}{\epsilon - 1} = \frac{\cot^2\theta_e/2}{1 + \tau} \tag{8}$$

between the variables in the CMS of $e^+ + e^- \to p + \overline{p}$ and in the LAB system for $e^- + p \to e^- + p$, it appears clearly that the one-photon mechanism generates a linear ϵ-dependence (or a $\cot^2\theta_e/2$) of the Rosenbluth differential cross section for elastic eN-scattering in Lab system.

Let us consider now the $\cos\theta$-dependence of the $1\gamma\otimes 2\gamma$-interference contribution to the differential cross section of $e^+ + e^- \to p + \overline{p}$. The spin and parity of the 2γ-states is not fixed, but only a positive value of C-parity, $C(2\gamma) = +1$, is allowed. An infinite number of states with different quantum numbers can contribute, and their relative role is determined by the dynamics of the process $\gamma^* + \gamma^* \to p + \overline{p}$, with both virtual photons.

But the $\cos\theta$-dependence of the contribution to the differential cross section for the $1\gamma\otimes 2\gamma$-interference can be predicted on the basis of its C-odd nature:

$$\frac{d\sigma}{d\Omega}(e^+ + e^- \to p + \overline{p}) = \cos\theta[c_0(t) + c_1(t)\cos^2\theta + c_2(t)\cos^4\theta + ...] \tag{9}$$

where $c_i(t)$, $i = 0, 1..$ are real coefficients, which are functions of t, only. This odd $\cos\theta$-dependence is essentially different from the even $\cos\theta$-dependence of the cross section for the one-photon approximation, Eq. (7).

From C-invariance it follows also that:

$$\mathcal{A}_3(t, -\cos\theta) = \mathcal{A}_3(t, +\cos\theta) \tag{10}$$

It is therefore incorrect to approximate the interference contribution to the differential cross section (9) by a linear function in $\cos^2\theta$, because it is in contradiction with the C-invariance of hadronic electromagnetic interaction. Such approximation can be done only when all coefficients $c_i(t)$ vanish, i.e. in absence of $1\gamma\otimes 2\gamma$-interference!

Using Eq. (9), the crossing symmetry allows to predict the non-trivial ϵ-dependence of the interference contribution to the differential cross section of ep-scattering, in the Lab system, $\Delta\sigma$:

$$\Delta\sigma(e^- p \to e^- p) \simeq x f(x^2, Q^2), \tag{11}$$

$$f(x^2, Q^2) = c_0(Q^2) + c_1(Q^2)x^2 + c_2(Q^2)x^4 + ..., \ x = \sqrt{\frac{\epsilon + 1}{\epsilon - 1}}.$$

Again the C-invariance does not allow to approximate the function $xf(x^2, Q^2)$ by a linear ϵ-dependence:

$$xf(x^2, Q^2) \neq d_0(Q^2) + d_1(Q^2)\epsilon,$$

which would make (11) compatible with the Rosenbluth formula.

Note that the relation:

$$\Delta\sigma(e^-p \to e^-p) = -\Delta\sigma(e^+p \to e^+p) \tag{12}$$

holds with relative accuracy α, (due to the 3γ-contribution to the form factors $G_{E,MN}$.

Let us now analyze the $\cos\theta$-dependence of the interference terms for the lowest possible values $\mathcal{J}^P$ for the 2γ-system, in order to get a hint of the relative values of the coefficients $c_i(t)$ in Eq. (9). Taking into account the conservation of the leptonic and the nucleonic electromagnetic currents, $q \cdot \ell = q \cdot \mathcal{J} = 0$, the CMS spin structure of the one-photon amplitude for the annihilation process $e^+ + e^- \to p + \overline{p}$ can be written as:

$$\mathcal{M}_1 = \frac{e^2}{t}\ell \cdot \mathcal{J} = -\frac{e^2}{t}\vec{\ell} \cdot \vec{\mathcal{J}}, \tag{13}$$

with

$$\vec{\ell} = \sqrt{t}\phi_2^\dagger(\vec{\sigma} - \hat{\vec{k}}\vec{\sigma} \cdot \hat{\vec{k}})\phi_1, \tag{14}$$

$$\vec{\mathcal{J}} = \sqrt{t}\chi_2^\dagger \left[G_M(t)(\vec{\sigma} - \hat{\vec{p}}\vec{\sigma} \cdot \hat{\vec{p}}) + \frac{1}{\sqrt{\tau}}G_E(t)\hat{\vec{p}}\vec{\sigma} \cdot \hat{\vec{p}}\right]\chi_1, \tag{15}$$

where ϕ_1 and ϕ_2 (χ_1 and χ_2) are the two-component spinors of the electron and positron (proton and antiproton), $\hat{\vec{k}}$ ($\hat{\vec{p}}$) is the unit vector along the three momentum of the electron (proton) in CMS.

Note that the term $G_{Mp}(t) - \dfrac{1}{\sqrt{\tau}}G_{Ep}(t)$, describes the $\overline{p}p-$production with ℓ=2. Therefore, at threshold, $\tau \to 1$, where the finite radius of the strong interaction allows the $\overline{p}p$-production only in S-state, the following relation:

$$G_{EN}(t) = G_{MN}(t),\ t \to 4m^2 \tag{16}$$

holds and it is the physical background of this so particular relation between the nucleon electromagnetic form factors at threshold.

Summing over the polarizations of the $p\overline{p}$-system and averaging over the polarizations of the initial e^+e^--system, one can find with the help of Eqs. (14,15):

$$|\vec{\ell} \cdot \vec{\mathcal{J}}|^2 = \frac{t}{2}\left[(1 + \cos^2\theta)|G_{Mp}(t)|^2 + \frac{1}{\tau}\sin^2\theta|G_{Ep}(t)|^2\right] \tag{17}$$

with the standard θ-dependence of the differential cross section for $e^+ + e^- \to \overline{p} + p$ [15].

After substituting $t \to Q^2$ and $\cos\theta^2 \to (\epsilon + 1)/(\epsilon - 1)$ in Eq. (17), one can find the linear ϵ-dependence for the Rosenbluth formula for the differential cross section of elastic ep-scattering in terms of $|G_{Ep}|^2$ and $|G_{Mp}|^2$ in Lab system.

In the same way one can find the spin structure of the 2γ-contributions to the matrix element for $e^+ + e^- \to \overline{p} + p$, using Eq. 4:

$$\overline{u}(-k_2)\hat{\mathcal{P}}u(k_1)\overline{u}(p_2)\hat{\mathcal{K}}u(-p_1) = \mathcal{L}\mathcal{N} \tag{18}$$

$$\mathcal{L} = \frac{m}{2}\sqrt{t(\tau-1)}\phi_2^\dagger(\vec{\sigma}\cdot\hat{\vec{p}} - \cos\theta\vec{\sigma}\cdot\hat{\vec{k}})\phi_1, \tag{19}$$

$$\mathcal{N} = -\frac{t}{2}\chi_2^\dagger(\vec{\sigma}\cdot\hat{\vec{k}} - \cos\theta\vec{\sigma}\cdot\hat{\vec{p}} + \frac{1}{\sqrt{\tau}}\cos\theta\vec{\sigma}\cdot\hat{\vec{p}})\chi_1. \tag{20}$$

The corresponding interference term can be written as:

$$\overline{\vec{\ell}\cdot\vec{\mathcal{J}}\mathcal{L}^*\mathcal{N}^*} \simeq Re\left[G_M(t) - \frac{1}{\tau}G_E(t)\right]\cos\theta\sin^2\theta \tag{21}$$

with a specific θ dependence. Applying the crossing symmetry, the corresponding θ dependence of the interference contribution to the differential cross section of the eN-scattering in Lab system takes the form:

$$\frac{2\epsilon}{1-\epsilon}\sqrt{\frac{1+\epsilon}{1-\epsilon}}, \tag{22}$$

which is not linear, as it was assumed in order to justify the considerations in [12] and [14]. The assumption of a linear ϵ-dependence of the interference term implies that the product $\sqrt{\frac{\epsilon+1}{\epsilon-1}}f(x^2)$, with $x^2 = \frac{\epsilon+1}{\epsilon-1}$, is ϵ-independent, again in contradiction with the C-invariance of hadronic electromagnetic interaction and with crossing symmetry.

Let us discuss now how unique is the $\cos\theta\sin^2\theta$-dependence for $e^+ + e^- \to p + \overline{p}$. One can show, on the basis of Eqs. (19) and (20), that such term arises from a definite superposition of states of the 2γ-system with quantum numbers $\mathcal{J}^P = 1^+$ and 2^+, when the e^+e^--system has $S = \ell = 1$. The individual states have different structures:

$$\mathcal{J}^P = 1^+, \ell = 1 \ \to \cos\theta Re G_M(t),$$
$$\mathcal{J}^P = 2^+, \ell = 1 \ \to \cos\theta[Re G_M(t) + sin^2\theta\frac{1}{\tau}Re G_E(t)].$$

The simplest linear $\cos\theta$-dependence corresponds to the exchange by the axial state with $\mathcal{J}^P = 1^+$, $\ell = S = 1$. It is therefore possible to use, for the discussion of interference phenomena instead of (4), another equivalent parametrization of 2γ-exchange:

$$\mathcal{M}_2 \simeq \tilde{\mathcal{A}}_3(s, Q^2)\overline{u}(k_2)\gamma_\mu\gamma_5 u(k_1)\overline{u}(p_2)\gamma_\mu\gamma_5 u(p_1).$$

In conclusion, the general symmetry properties of electromagnetic interaction, such as the C-invariance and the crossing symmetry, allow to obtain rigorous results concerning two-photon exchange contributions for elastic ep-scattering and to analyze the effects of this mechanism in eN-phenomenology.

- We showed that the nucleon form factors do not contain any contribution from the 2γ-exchange. Therefore the possible phases of these form factors (in the space-like region) are very small - of the order of α^2 (not as α, [12]). The dependence of these form factors on another, independent kinematical variable, ϵ or s, appears only after including 3γ-exchange.

- The form factors G_{EN} and G_{MN} and the 2γ-amplitude, $\mathcal{A}_3(s, Q^2)$ are the same for e^+p and e^-p elastic scattering. This allows to connect in a rigorous way, the difference of the differential cross sections for $e^{\pm}p$-interaction with the deviations from the ϵ-linearity of the Rosenbluth plot.

- The ϵ-dependence of the interference contribution to the differential cross section of $e^{\pm}p$ elastic scattering is very particular. Any approximation of this term by a linear function is in contradiction with C-invariance and crossing symmetry of the electromagnetic interaction.

- The formal expression of the ϵ-dependence of the interference contribution depends on the quantum numbers of the 2γ-system.

- To have a quantitative estimation of the relative role of two-photon physics in eN-interaction, it is necessary to measure the ϵ-dependence of the differential cross section of eN elastic scattering in several points, and study this behavior in terms of the specific variable $\sqrt{(1+\epsilon)/(1-\epsilon)}$. This will be the unambiguous signature of two-photon contributions.

- The same behavior appears in the difference of the e^+p- and e^-p-differential cross sections.

A similar analysis can be done for polarization phenomena, and it is the object of a future paper.

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[1] M. K. Jones *et al.* [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. **84**, 1398 (2000).

[2] O. Gayou *et al.* [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. **88**, 092301 (2002).

[3] A. Akhiezer and M. P. Rekalo, Dokl. Akad. Nauk USSR, **180**, 1081 (1968); Sov. J. Part. Nucl. **4**, 277 (1974).

[4] Proposal to JLab PAC18: 'Measurement of G_{Ep}/G_{Mp} to Q^2=9 GeV^2 via Recoil Polarization', (Spokepersons: C.F. Perdrisat, V. Punjabi, M.K. Jones and E. Brash), JLab, July 2001.

[5] R.G. Arnold *et al.*, Phys. Rev. Lett. **35**, 776 (1975).

[6] L. W. Mo and Y. S. Tsai, Rev. Mod. Phys. **41**, 205 (1969).

[7] L. C. Maximon and J. A. Tjon, Phys. Rev. C **62**, 054320 (2000);
A. Afanasev, I. Akushevich and N. Merenkov, Phys. Rev. D **64**, 113009 (2001);
A. V. Afanasev, I. Akushevich, A. Ilyichev and N. P. Merenkov, Phys. Lett. B **514**, 269 (2001);

[8] J. Gunion and L. Stodolsky, Phys. Rev. Lett. **30**, 345 (1973);
V. Franco, Phys. Rev. D **8**, 826 (1973);
V. N. Boitsov, L.A. Kondratyuk and V.B. Kopeliovich, Sov. J. Nucl. Phys. **16**, 237 (1973);
F. M. Lev, Sov. J. Nucl. Phys. **21**, 45 (1973).

[9] M. P. Rekalo, E. Tomasi-Gustafsson and D. Prout, Phys. Rev. **C60**, 042202 (1999).

[10] L. C. Alexa *et al.*, Phys. Rev. Lett. **82**, 1374 (1999).

[11] D. Abbott *et al.*, Phys. Rev. Lett **82**, 1379 (1999).

[12] P.A.M. Guichon and M. Vanderhaeghen, arXiv:hep-ph/0306007.

[13] M. L. Goldberger, Y. Nambu and R. Oehme, Annals of Physics **2**,226 (1957).

[14] P. G. Blunden, W. Melnitchouk, and J. A. Tjon, arXiv:hep-ph/0306076.

[15] A. Zichichi, S.M. Berman, N. Cabibbo and R. Gatto, Nuovo Cimento **XXIV**, 170 (1962).