Precise method for the determination of the neutron electric form factor based on a relativistic analysis of the process d(e, e'n)p

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(Dated: August 25, 2003)

Abstract

We generalize the recoil polarization method for the determination of the proton form factor to the case of the disintegration of vector polarized deuterons by longitudinally polarized electrons, $\vec{d}(\vec{e}, e'n)p$. We suggest to measure for this reaction, in the kinematics of quasi-elastic en-scattering, the ratio $R_{xz} = A_x/A_z$ of the asymmetries induced by the x- and z-components of the deuteron vector polarization. In the framework of the relativistic impulse approximation the ratio R_{xz} is sensitive to G_{En} in a wide interval of momentum transfer squared, whereas it depends weakly on the details of the np-interaction and on the choice of the deuteron wave function. Moreover, in the range $0.5 \le Q^2 \le 1.5 \text{ GeV}^2$, the ratio R_{xz} shows a smooth dependence on Q^2 , making the analysis simpler.

PACS numbers: 12.20.-m, 13.40.-f, 13.60.-Hb, 13.88.+e

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I. INTRODUCTION

In the present work we are interested in the deuteron as a source of information on the neutron, in particular on its electromagnetic interaction, where the isotopic invariance is strongly violated. The most evident example is the neutron electric form factor, G_{En} , which shows a very different behavior when compared with G_{Ep} . Note in this respect the deviation of the last precise data on G_{Ep} [1, 2] from the 'standard' dipole dependence on the momentum transfer squared. For a better understanding of the physics of the nucleon electromagnetic structure, more precise data about G_{En} are required in a wide region of momentum transfer squared. The non-triviality of the neutron electromagnetic structure appears also in the time-like region of momentum transfer [3–6], where an unusual behavior of the cross section of the e^+e^- -annihilation processes has been observed:

$$\sigma(e^+ + e^- \to \bar{n} + n) > \sigma(e^+ + e^- \to \bar{p} + p) \text{ for } 2E = 1.88 \div 2.45 \text{ GeV}$$
.

The present work is devoted to the theoretical analysis of specific polarization phenomena in the deuteron electrodisintegration, $e+d \rightarrow e+n+p$, which are sensitive to G_{En} at relatively large momentum transfer. Two kinds of polarization experiments for $e+d \rightarrow e+n+p$ are especially interesting for this aim: the scattering of longitudinally polarized electrons by a vector polarized deuteron target, $\vec{d}(\vec{e},en)p$, with detection of the scattered electron and neutron [7, 8], and the scattering of longitudinally polarized electrons by an unpolarized deuteron target, with measurement of the neutron polarization, $d(\vec{e},e\vec{n})p$ [9–11]. These experiments have been performed or are planned at Mainz and JLab, at momentum transfer squared spanning from $Q^2 = 0.5$ to 2 GeV².

A correct and effective extraction of G_{En} from such data needs an adequate theoretical interpretation of the reaction mechanism in $e+d \to e+n+p$. First of all, the main symmetry properties of electromagnetic hadron interaction have to be taken into account, such as the conservation of the hadronic electromagnetic current (for the subprocess $\gamma^* + d \to n + p$, where γ^* is a virtual photon), i.e. the gauge invariance of the electromagnetic interaction and the relativistic invariance. At these values of momentum transfer relativistic effects cannot be considered as corrections, as the three-momentum of the elastically scattered nucleon is comparable with its mass.

The description of the final state np-interaction (FSI) has also to be properly taken into account. In order to decrease the model dependence of FSI, one should avoid approaches

based on the concept of non-relativistic NN-potentials. Instead of NN-potentials, a model independent description of FSI in $\gamma^* + d \rightarrow n + p$ can be derived from the phases of NN-scattering, which are available from the phase-shift analysis of the huge amount of data about the NN-interaction.

The relativistically invariant impulse approximation (RIA) [12], with subsequent unitarization of the corresponding multipole amplitudes [13, 14], seems the most appropriate model for the description of $\gamma^* + d \rightarrow n + p$.

But what about the relativistic description of the deuteron structure, which can be done in terms of wave functions? Note, in this respect, that only the kinematical region for $e^- + d \rightarrow e^- + n + p$, which corresponds to quasi-elastic $e^- + n^* \rightarrow e^- + n$ scattering $(n^*$ is a virtual neutron), is especially sensitive to neutron form factors. This region corresponds to the emission of the neutron along the three-momentum of the virtual photon, when $Q^2 \simeq W^2 - M_d^2$, where M_d is the deuteron mass and W is the effective mass of the produced np-system. In such conditions the virtuality of the neutron is small, therefore the argument (p) of the deuteron wave function (in impulse representation) is also small. In conditions of evidently nonrelativistic momentum p, the standard S- and D-components of the deuteron wave function (DWF), derived from the existing NN-potentials, can be safely used. Therefore, the possible four-components of the relativistic DWF can be related with good accuracy to the nonrelativistic S- and D-components [15].

Note that the considered kinematical regime in $\gamma^* + d \to n + p$, which is the most convenient for the determination of G_{En} , corresponds to nonperturbative QCD at any value of momentum transfer squared Q^2 , so that all the prescriptions of QCD such as helicity conservation, quark counting rules, formalism of reduced deuteron form factors or reduced nuclear matrix elements cannot be applied here. Moreover, existing experimental data, including polarization effects, concerning different processes with deuteron target: $e^- + d \to e^- + d$ [16], $\gamma + d \to d + \pi^0$ [17], and $\gamma + d \to n + p$ [18] do not confirm the pQCD predictions at JLab energies.

The main aim of this paper is to show that, in the reaction $d(\vec{e}, e'n)p$ (with a vector polarized deuteron target and longitudinally polarized e^- beam), the information on G_{En} can be obtained in a simple and precise way from the ratio A_x/A_z , where $A_{x,z}$ are the corresponding asymmetries for the x- and z-components of the deuteron polarization (in the $\gamma^* + d \rightarrow n + p$ reaction plane). A similar procedure, firstly suggested in [19], has

been recently realized for the processes $\vec{e} + p \rightarrow e + \vec{p}$ [1, 2], $d(\vec{e}, e'\vec{n}p)$ [10, 11], where the ratio of the x- and z-components of the nucleon polarization has been measured, and for the ${}^3\vec{H}e(\vec{e}, e'n)pp$ -process as well (for the ratio of the x- and z- components of the ${}^3\vec{H}e$ polarization [20].

The above mentioned ratios (in impulse approximation) for d and ${}^{3}He$ targets, is essentially determined by the ratio of the electric and magnetic form factors G_{En}/G_{Mn} . From the experimental point of view its measurement is very convenient as many systematic errors essentially cancel and the analysis is simplified.

It is evident that in the case of polarized nuclear targets, such as \vec{d} or ${}^{3}\vec{H}e$, the ratios of target asymmetries (or final neutron polarizations) contain various nuclear effects, as well as other corrections (FSI, etc).

In the present work we update and adapt the formalism developed in [12]. Such formalism takes rigorously into account important nuclear ingredients such as the Fermi motion of the nucleons in the target and the possible Wigner rotation of the vectors of the nucleon polarization. All numerical estimations are based on a relativistic treatment of the nucleon electromagnetic structure and of the FSI effects.

The paper is organized as follows: after a description of the formalism of polarization phenomena in Sec. II, the results are presented in Sec. III, using different DWF and for different values of G_{En} , with and without FSI. In the Conclusion we summarize the results.

II. FORMALISM

The main observable discussed here is the ratio of two T-even asymmetries for $\vec{d}(\vec{e}, e'n)p$, A_x and A_z , induced by the P_x and P_z components of the deuteron vector polarization (in the $\gamma^* + d \to n + p$ reaction plane, with the z-axis along the direction of the γ^* three-momentum \vec{q}). This ratio does not depend on the electron helicity. In conditions of quasi-elastic enscattering, i.e. for $Q^2 = W^2 - M_d^2$ and $\vartheta^* \simeq 180^\circ$ (ϑ^* is the angle of the emitted proton with respect to \vec{q} in the center of mass system of $\gamma^* + d \to n + p$), we show that the ratio $R_{xz} = A_x/A_z$ is sensitive to the ratio of the neutron form factors, G_{En}/G_{Mn} . This has been analytically proved for elastic ep-scattering [19]. In the case of a deuteron target the situation is more complicated due, on one side, to the Fermi motion of bound nucleons, and from another side, to the Wigner rotation of the nucleonic spin. The final np-interaction

plays also a role. All these aspects of the deuteron physics should be carefully taken into account for the extraction of G_{En} from polarization observables in $e + d \rightarrow e + n + p$.

The asymmetries ratio can be written as follows [21]:

$$R_{xz} = \frac{A_x}{A_z} = \frac{\sqrt{1 + \epsilon} A_x^{(0)} + \sqrt{2\epsilon} A_x^{(1)} \cos \phi}{\sqrt{1 + \epsilon} A_z^{(0)} + \sqrt{2\epsilon} A_z^{(1)} \cos \phi},$$

$$\epsilon^{-1} = 1 + \frac{2\vec{q}^2}{Q^2} \tan^2 \left(\frac{\theta_e}{2}\right),$$
(1)

where θ_e is the electron scattering angle in the laboratory (Lab) system, and ϕ is the angle between the electron scattering plane and the reaction plane.

The specific dependence of R_{xz} on ϵ and ϕ is a model independent result, which is based on the following properties of the hadron electromagnetic interaction:

- the validity of the one-photon-exchange mechanism for $e + d \rightarrow e + n + p$;
- the conservation of the electromagnetic current for $\gamma^* + d \rightarrow n + p$ (the gauge invariance);
- the P-invariance of hadron electrodynamics;
- the validity of QED for the description of the γee -vertex.

The quantities $A_x^{(0,1)}$ and $A_z^{(0,1)}$ (the structure functions) are real functions of three independent kinematical variables, Q^2 , W, and ϑ^* . Their dependence on the nucleon electromagnetic form factors can be predicted in the framework of a definite model for the process $\gamma^* + d \to n + p$.

Note that $A_x^{(0)}$ and $A_z^{(0)}$ are determined by quadratic combinations of the transversal components of the electromagnetic current for $\gamma^* + d \to n + p$, whereas $A_x^{(1)}$ and $A_z^{(1)}$ are driven by the interference of the longitudinal and the transversal components of this current. Moreover, $A_x^{(0)}$ and $A_z^{(1)}$ vanish at $\vartheta^* = 0^\circ$ and 180° , due to the helicity conservation for collinear kinematics in $\gamma^* + d \to n + p$. These results are also model independent.

However, considering the one-nucleon contributions of the electromagnetic current for $\gamma^* + d \to n + p$, one can expect that its longitudinal component is determined by G_{En} and the transversal one by G_{Mn} , at least in the Breit system. Therefore, the contributions $A_{x,z}^{(1)}$, in Eq. (1), should contain terms proportional to the product $G_{En}G_{Mn}$.

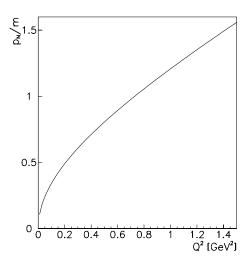


FIG. 1: Q^2 -dependence of the nucleon momentum in d(e, e'n)p, in quasi-elastic kinematics (Lab system).

The presence of two contributions in both asymmetries, A_x and A_z , results from the specific character of the deuteron dynamics for $\gamma^* + d \rightarrow n + p$ and their relative role is determined by the corresponding model.

A realistic model has to be relativistic, due to the fast rising of the emitted nucleon momentum, p_N , with increasing Q^2 , which can be written in the Lab system as:

$$\frac{p_N}{m} = 2\sqrt{\tau(1+\tau)}, \ \tau = \frac{Q^2}{4m^2},$$

in quasi-elastic kinematics (see Fig. 1), where m is the nucleon mass. Therefore, the corresponding impulse approximation can be realized by means of the Feynman technique, in which all particles are treated relativistically, namely: the nucleons are described by the four-components Dirac spinors, and deuteron - by the polarization 4-vector U_{μ} . The diagrams illustrated in Fig. 2 determine the amplitude in RIA. Comparing with a nonrelativistic approach, the diagrams (a) and (b) represent the relativized description of the one–nucleon–exchange mechanism (which are equally important in both models). The deuteron–exchange (c) as well as the contact diagram (d) insure the electromagnetic current conservation in $\gamma^* + d \rightarrow n + p$. The contact diagram can be related to the contribution of the meson–exchange currents. Of course, this diagram is not comprehensive of all variety of these currents, but for its structure and origin it falls into this class. The deuteron diagram can be also related to the np-interaction in definite states, with the quantum numbers of the

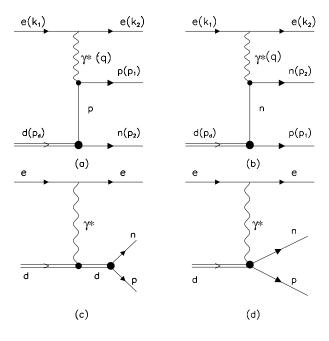


FIG. 2: Feynman diagrams describing RIA for the process $e + d \rightarrow e + n + p$.

deuteron. The deuteron structure (which, in the nonrelativistic approach, corresponds to DWF) is described here by the relativistic form factors of the dnp- vertex with one virtual nucleon [15]. In order to calculate the dependence of these form factors on the nucleon virtuality we use the relation between them and nonrelativistic DWF.

The RIA amplitude for $\gamma^* + d \to n + p$ reaction can be unambiguously calculated for any values of the kinematic variables Q^2 , W and ϑ^* .

In the case of the nonrelativistic description of the deuteron electrodisintegration the situation is somewhat different. The standard nuclear approach to the investigation of the electron scattering by deuterons and other nuclei requires the knowledge of the operator of the electron–nucleon interaction. This operator cannot contain the antinucleon contribution which is inevitably present in the covariant description of the eN- scattering (with one virtual nucleon). The method based on the Foldy-Wouthoysen transformation is usually applied. It allows to obtain this operator in the form of the expansion over the powers of $\sqrt{Q^2}/m$. Naturally, this method is not valid at $Q^2 \geq m^2$, but such values of momentum transfer are currently accessible. Therefore, a relativistic description of the $e^- + d \rightarrow e^- + n + p$ reaction is more appropriate. FSI have also to be taken properly into account. Instead of the nonrelativistic concept of NN-potential (in any form), which is typically assumed for nuclear processes, even at large Q^2 , it is more straightforward to use the phases of NN-

scattering, which are available now up to E=0.8 GeV for np-scattering. Generalizing the Fermi-Watson approach, we will use the following unitarization procedure:

$$f_{[A]}^{(0)}(Q^2, W) \longrightarrow f_{[A]}^{(0)}(Q^2, W) exp(i\delta_{[A]}(W)),$$
 (2)

where $f_{[A]}^{(0)}(Q^2, W)$ is an RIA multipole amplitude for the process $\gamma^* + d \to n + p$ leading to the production of the np system in a state with quantum numbers [A] = J (total angular momentum), L (orbital angular momentum), S (total spin), and I (total isotopic spin), while $\delta_{[A]}(W)$ is the NN phase shift in the state with quantum numbers [A]. This expression is strictly valid in the energy range $2m \le W \le (2m + m_{\pi})$ at any value of Q^2 in the region of space-like momentum transfers.

The substitution given by Eq. (2) is performed only for those multipole amplitudes that describe the production of the np system with nonzero phase shift $\delta_{[A]}(W)$. This means that, at each energy W, there exist the maximum angular-momentum value $J = J_m(W)$ (more precisely, maximum value of L for the np system) that limits the number of those multipole amplitudes that are modified by the unitarization. A similar "cut" on L is imposed by a finite (and small) range of NN interaction. Even despite this constrain, it is necessary to consider and modify a rather large number of multipole amplitudes. At J = 0(J = 1), there are 3 (14) independent transitions, so that it is necessary to modify 18(J-1) + 14 + 3 = 18J - 1 independent multipole amplitudes. For $J_m \geq 6$ their number exceeds one hundred. The unitarization procedure is carried out in a relativistic approach, without any restrictions on Q^2 .

The nucleon electromagnetic current is described in terms of the Dirac (F_1) and Pauli (F_2) form factors. For the np phase shifts, we took the results from [22], where the phases have been found up to $W \simeq 2.24$ GeV.

$Q^2[GeV^2]$	E [GeV]	E' [GeV]	$\theta_e \ [\mathrm{deg}]$	ε
0.5	2.721	2.460	15.8	0.958
1.0	4.232	3.698	14.5	0.960
1.5	4.232	3.376	19.26	0.923

TABLE I: Table 1: Kinematical parameters for the JLab experiment [23].

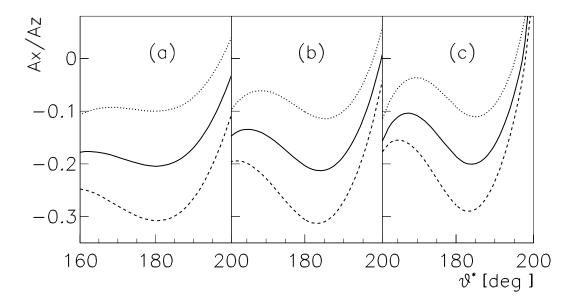


FIG. 3: ϑ^* -dependence of the ratio $R_{xz} = A_x/A_z$ for different values of the neutron electric form factor: Galster parametrization (solid line), Galster parametrization scaled by a factor 0.5 (dashed line), Galster parametrization scaled by a factor 1.5 (dotted line) at Q^2 =0.5 GeV² (a); Q^2 =1 GeV² (b); Q^2 =1.5 GeV² (c); ϑ^* < 180° (> 180°) correspond to ϕ = 0 (180°).

III. RESULTS

We calculated the ratio R_{xz} for the electron kinematics reported in Table I, which correspond (at $Q^2=0.5$ and 1 GeV² to the kinematical conditions of the experiment E93-026 recently performed at JLab [23]. Note that $\epsilon \simeq 1$ for the quasi-elastic regime.

We consider the ϑ^* -dependence of R_{xz} in the interval $\vartheta^* = 180 \pm 20^0$, and use the relation $W^2 = Q^2 + M_d^2$ to determine the variable W in the quasi-elastic *en*-region. The nucleon electromagnetic form factors are taken as follows:

$$G_{Ep}(Q^2) = G_{Mp}(Q^2)/\mu_p = G_{Mn}(Q^2)/\mu_n = G_D = (1 + Q^2/0.71 \text{ GeV}^2)^{-2},$$

 $G_{En} = G^G = -\tau \mu_n G_D/(1 + 5.6\tau),$

where $\mu_p(\mu_n)$ is the magnetic moment of the proton (neutron). In Fig. 3 the results for R_{xz} are shown at $Q^2 = 0.5$, 1, and 1.5 GeV². The calculation corresponding to the Paris DWF

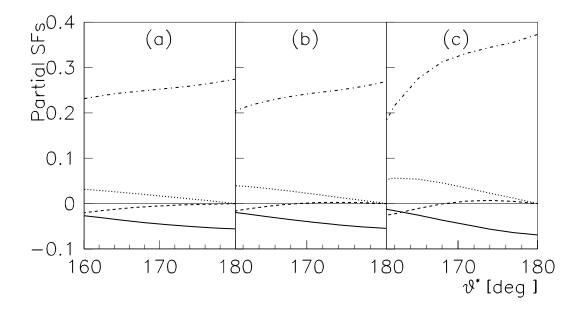


FIG. 4: ϑ^* -dependence of the partial structure functions $A_x^{(0)}$ (dashed line), $A_x^{(1)}$ (solid line), $A_z^{(0)}$ (dash-dotted line), and $A_z^{(1)}$ (dotted line) for Q^2 =0.5 GeV² (a); Q^2 =1 GeV² (b); Q^2 =1.5 GeV² (c)

.

[24] and to the Galster parametrization for G_{En} [25] is shown as solid line. The calculations corresponding to $G_{En}=0.5$ G^G (dashed line) and $G_{En}=1.5$ G^G (dotted line) are also reported, in order to show the large sensitivity of the ratio R_{xz} to G_{En} , in the considered Q^2 and ϑ^* range. Only at $Q^2=0.5$ GeV² a small effect of FSI and DWF appears at $\vartheta^*\simeq 160^\circ$, where the cross section is smaller than in the quasi-elastic region. At larger Q^2 , the effects of FSI and DWF are negligible, making simpler the extraction of G_{En} from experimental data on R_{xz} .

The relative role of the two possible contributions to A_x and A_z (see Eq. (1)) is shown in Fig. 4. For $\vartheta^* \neq 180^\circ$ these contributions are comparable for A_x , being negative for $\vartheta^* < 180^\circ$. For A_z , the transversal contribution $A_z^{(0)}$, related to G_{Mn}^2 , is essentially larger in comparison to $A_z^{(1)}$.

IV. CONCLUSIONS

In conclusion, the suggested method for the determination of G_{En} , at relatively large Q^2 , from the ratio $R_{xz} = A_x/A_z$ of the T-even asymmetries in $\vec{d}(\vec{e}, e'n)p$, measured in the kinematical conditions of quasi-elastic en- scattering, seems promising and may be comparable in accuracy with the measurements of G_{Ep} through the recoil polarization method. The nuclear effects, such as the Fermi motion and the Wigner rotation of the nucleon polarization, can be taken rigorously into account through a formalism based on RIA. The final np-interaction and the choice of the deuteron wave function do not influence the suggested procedure.

This method would be especially useful when the direction of the target polarization can be done rapidly, allowing a simultaneous measurement of the two asymmetries. It is the case of internal gas targets.

For external targets, the procedure of rotating the target polarization may involve complicated manipulations of the magnets and the experimental supplies, making the measurements with the two polarization states of the target sequential in time. Although, in this case some parameters may change, the ratio of the asymmetries seems more reliable for the extraction of G_{En} , as it is less sensitive to the models than the individual asymmetries.

V. ACKNOWLEDGMENTS

Three of us, (G.I.G., A.P.R., and M.P.R.) acknowledge the kind hospitality of Saclay, where this work was done. Thanks are due to D. Day for useful discussions and remarks.

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