F_2 and NLO BFKL Kernels

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We propose a new method to test the resummation schemes of the next-to-leading order (NLO) BFKL evolution kernels using the Mellin transformed j-moments of the proton structure function F_2.

1 Introduction

The Balitsky Fadin Kuraev Lipatov (BFKL) evolution equation [1], based on the summation of leading logarithms of energy in the perturbative QCD expansion gives valuable tools for the investigation of deep inelastic scattering at small x_{Bj} with a clear evidence for sizable higher order corrections. At NLO these corrections [2] appeared to be so large that they overshoot the expected phenomenological effect. However, it was realized [3] that the main problem came from the existence of spurious singularities which ought to be cancelled by an appropriate resummation at all orders of the perturbative expansion, as required by the QCD renormalization group. Indeed, various resummation schemes have been proposed for the NLO BFKL kernels [3, 4] and we present in this paper a method for testing the (resummed) BFKL predictions for the proton structure functions, *via* a transformation to Mellin space.

2 BFKL prediction in Mellin space

The formulation of the proton structure functions in the leading order (LO) BFKL approximation can be expressed as follows [5]:

$$\begin{pmatrix} F_T \\ F_L \\ G \end{pmatrix} (x_{Bj}, Q^2) = \int \frac{d\gamma}{2i\pi} \left(\frac{Q^2}{Q_0^2}\right)^{\gamma} e^{\frac{\alpha_s N_c}{\pi}\chi^{LO}(\gamma)\ln(1/x_{Bj})} \begin{pmatrix} h_T(\gamma) \\ h_L(\gamma) \\ 1 \end{pmatrix} \Omega(\gamma) \tag{1}$$

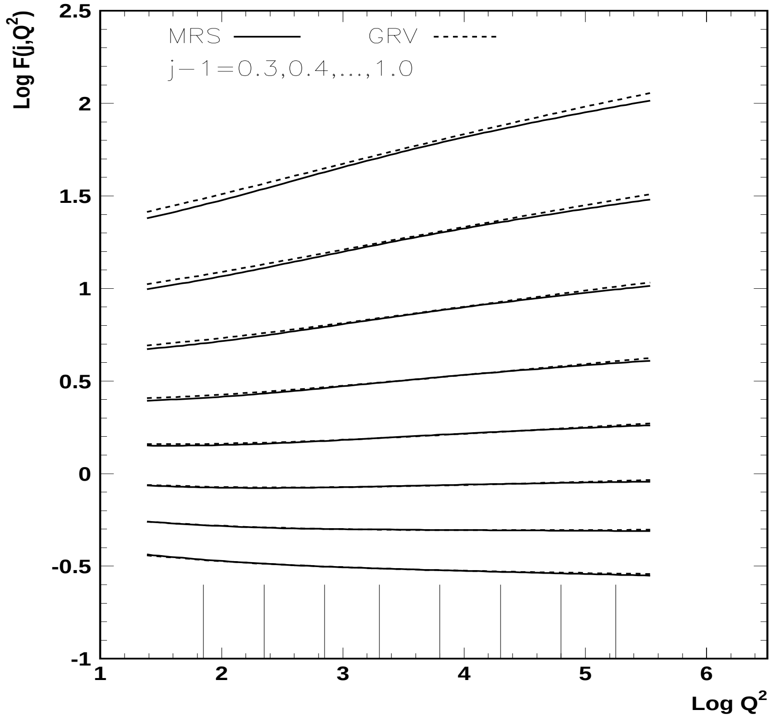

Figure 1: Values of the Mellin transforms of F_2 scaled by the average quark charges squared : $\frac{1}{<e^2>}\int_0^1 x^{j-2}F_2(x,Q^2)dx$ as a function of $\ln Q^2$ for different values of $j-1$ ($j-1=0.3$ is the lowest curve of the plot).

where the BFKL kernel is : $\chi^{LO}(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$. In formula (1), F_T, F_L and G stand respectively for transverse, longitudinal and gluon structure functions ; α_s is the (fixed) coupling constant ; $\omega(\gamma)$ is an (unknown) non-perturbative coupling to the proton while $h_T(\gamma)$ and $h_L(\gamma)$ correspond to the known perturbative couplings to the photon, usually called LO impact factors. The the Mellin transform of the relation (1) leads to a series of poles in j. Retaining the right most pole $\gamma^*(j)$, two model independent predictions can be drawn :

(i) $\partial \ln \tilde{F}_2(j,Q^2)/\partial \ln Q^2 = \gamma^*(j)$ with $F_2 = F_T + F_L$

(ii) $\chi^{LO}(\gamma^*(j)) = (j-1)/\bar{\alpha}$

Interestingly, up to a factorization assumption, these predictions remain formally valid for the NLO BFKL resummed kernels [6]. We get :

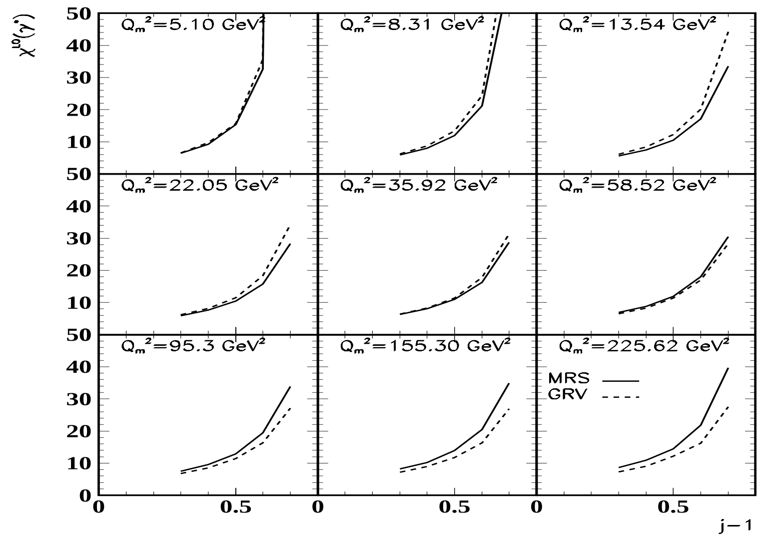

Figure 2: BFKL kernel at LO $\chi^{LO}(\gamma^*(j))$ for MRST [7] and GRV [8].

(i) $\partial \ln \tilde{F}_2(j, Q^2)/\partial \ln Q^2 = \gamma^*(j, Q^2)$
(ii) $\chi^{NLO}(\gamma^*(j, Q^2), j) = (j-1)/\bar{\alpha}(Q^2)$

In the following, we present the results at NLO for one particular scheme [3] but the method can be extended to other ones.

3 Experimental analysis

In this section, we check the reliability of the Mellin space method by a study of precise parametrizations of the data, MRST [7] and GRV [8]. In addition, we can easily transform these parametrizations in Mellin space by calculating with a high accuracy the integral $\int_0^1 x^{j-2} F_2(x, Q^2) dx$ for different values of j (see Fig. 1). Note that the DGLAP evolution is automatically satisfied by the input functions (MRST [7] and GRV [8]) and we want to compare them with the BFKL evolution using relations (i) and (ii) at LO and NLO. Indeed, one question we ask in our analysis is whether or not there exist a difference between the solutions of DGLAP and BFKL evolution equations.

It is clear from Fig. 1 that for $0.3 \leq j-1 \leq 1$, the slope of $\ln \tilde{F}_2$ vs $\ln Q^2$ is almost constant in the 9 ranges of Q^2. Hence, condition (i) is roughly

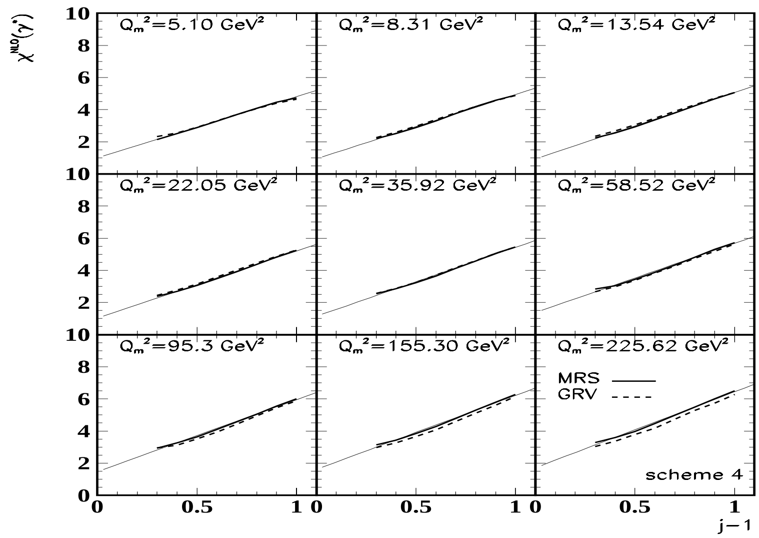

Figure 3: BFKL kernel at NLO $\chi^{NLO}(\gamma^*(j, Q^2))$. The linear fit of $\chi^{NLO}(\gamma^*(j))$ vs $j-1$ is also displayed for the MRST parametrization.

satisfied at LO and better at NLO. Then, it is straightforward to extract the effective anomalous dimensions $\gamma^*(j, Q^2)$. Taking into account these values, we can test the relation (ii). Results are displayed on Fig. 2 (for LO) and Fig. 3 (for NLO). For the relation (ii) to be satisfied, the functions $\chi(\gamma^*(j, Q^2))$ should give points aligned on a straight line extrapolating form $j-1=0.3$ to $j-1=1$ in the different ranges of Q^2 and the slope should give the average value of $1/\bar{\alpha}$ for the Q^2 range considered. We see on Fig. 2 that the LO completly fails in shape and magnitude whereas the NLO test of relation (ii) is satisfactory, as illustrated by Fig. 3. More precisely, Fig. 3 illustrates that the following relation is obeyed : $\chi^{NLO}(\gamma^*(j, Q^2), j) = (j-1)/\bar{\alpha}_{eff}(Q^2) + b$, where $\bar{\alpha}_{eff}(Q^2) \sim \bar{\alpha}(Q^2)$ and $b \neq 0$, arising probably from higher order terms not taken into account in this analysis.

4 Conclusion

We have proposed a method which allows sensitive tests of resummed BFKL kernels at NLO. We have presented results for one correct scheme and in a more detailed analysis we will discuss other ones [6]. Moreover, our study corroborates the proximity between DGLAP and NLO BFKL evolutions [3, 9].

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