New possibility for further measurements of nucleon form factors at large momentum transfer in time-like region: $\overline{p}+p\to\ell^{+}+\ell^{-}$, $\ell=e$ or μ.

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Abstract

We briefly summarize the status with electromagnetic nucleon form factors, and give in this framework, arguments to study the angular dependence of the differential cross section and single-spin polarization phenomena (polarized target or polarized beam) in $\overline{p}+p\to\ell^{+}+\ell^{-}$, in view of the availability of future antiproton beams.

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Nucleon electromagnetic form factors (FFs) are fundamental quantities for the understanding of the nucleon structure. They can be experimentally measured as well as theoretically calculated and therefore constitute a privileged background for the test of theoretical models. Since their first determination, which valued the Nobel Prize to R. Hofstadter in 1961, new and exciting developments in this field have been recently done, thanks to the advent, of high intensity, highly polarized electron beams, hadron polarimeters and polarized targets, in the space-like region and high energy antiproton beams, at Fermi-Lab.

Recent experimental data on nucleon electromagnetic form factors (FFs) in time-like (TL) [1] and space-like (SL) [2, 3] regions of momentum transfer square q^2, and many new theoretical developments show the necessity of a global description of form factors in the full region of q^2.

Form factors are analytical functions of q^2, being real functions in the SL region (due to the hermiticity of the electromagnetic Hamiltonian) and complex functions in the TL region. The Phràgmen-Lindelöf theorem [4] gives a rigorous prescription for the asymptotic behavior of analytical functions: $\lim_{q^2\to -\infty} F^{(SL)}(q^2) = \lim_{q^2\to\infty} F^{(TL)}(q^2)$. This means that, asymptotically, the FFs, have the following constrains: 1) the time-like phase vanishes and 2) the real part of the FFs, $\mathcal{R}eF^{(TL)}(q^2)$, coincides with the corresponding value $F^{(SL)}(q^2)$.

The existing experimental data about the electromagnetic FFs of charged pion or proton in the time-like region do not allow a complete test of the Phràgmen-Lindelöf theorem, especially concerning the vanishing phase, as the cross section depends on the square of the modulus of the form factor. Only the study of more complicated reactions such as $\pi^- + p \to n + \ell^+ + \ell^-$ [5] or $\overline{p} + p \to \pi^0 + \ell^+ + \ell^-$ [6] allows, in principle: to determine the nucleon FFs in the unphysical region of TL momentum transfer, for $4m_e^2 \le q^2 \le 4m^2$, where m_e is the leptonic mass, and to determine the relative phase of pion and nucleon form factors.

The cross section for the process $\overline{p}+p \to \ell^+ + \ell^-$, for the one-photon mechanism, neglecting the leptonic mass, can be expressed as a function of FFs according to the following formula [7]:

$$\frac{d\sigma}{d(cos\theta)} = \frac{\pi\alpha^2}{8m^2\tau\sqrt{\tau(\tau-1)}}\left[\tau|G_M|^2(1+\cos^2\theta) + |G_E|^2\sin^2\theta\right], \tag{1}$$

where θ is the angle between the lepton and the antiproton in the center of mass frame, $\tau = q^2/(4m^2)$, m is the nucleon mass, $\alpha = e^2/(4\pi) = 1/137$.

The Rosenbluth separation of the $|G_E|^2$ and $|G_M|^2$ contributions, in TL region, which is equivalent here to the linearity of $d\sigma/d\cos\theta$, with respect to $\cos^2\theta$, has not been realized yet.

In order to determine the form factors, due to the poor statistics in the existing data [1], it is necessary to integrate the differential cross section over a wide angular range. One typically assumes that the G_E-contribution plays a minor role in the cross section at large q^2 and the experimental results are usually given in terms of $|G_M|$, under the hypothesis that $G_E = 0$ or $|G_E| = |G_M|$. The first hypothesis is arbitrary. The second hypothesis is strictly valid at threshold only, i.e. for $\tau = 1$, but there is no theoretical argument which justifies its validity at any other momentum transfer, where $q^2 \neq 4m^2$.

The $|G_M|^2$ values depend, in principle, on the kinematics where the measurement was performed and the angular range of integration, however it turns out that these two assumptions for G_E lead to comparable values for $|G_M|$.

In the SL region the situation is different. The cross section for the elastic scattering of electron on protons is sufficiently large to allow the measurements of angular distribution and/or of polarization observables. The existing data on G_M show a dipole behavior up to the highest measured value, $-q^2 \simeq 31$ GeV^2 [9] according to

$$G_M(q^2)/\mu_p = G_d, \text{ with } G_d = \frac{1}{\left[1 - \dfrac{q^2}{m_d^2}\right]^2}, \; m_d^2 = 0.71 \text{ GeV}^2. \tag{2}$$

It should be noticed that the independent determination of both G_M and G_E FFs, from the unpolarized $\ell^- + p$-cross section, has been done up to $-q^2 = 8.7$ GeV^2 [8], and the further extraction of G_M [9] assumes $G_E = G_M/\mu_p$. The behavior of G_E, deduced from polarization experiment $p(\vec{e}, e'\vec{p})$ differs from G_M/μ_p, with a deviation from G_d up to 70% at $-q^2$=5.6 GeV^2 [3]. This is the maximum momentum at which new, precise data are available. Extension of this measurement at $-q^2$=9 GeV^2 is under way at JLab [10].

The main experimental results, which have to be understood, are the following:

- It has been found that the electric and the magnetic distribution in the proton, at small distances, are not equal, and the the electric charge distribution do not follow a dipole behavior, as a function of the momentum transfer, as previously assumed.
- The values of G_M in the TL region, obtained under the assumption that $|G_E| = |G_M|$, are larger than the corresponding SL values. This has been considered as a proof of

the non applicability of the Phràgmen-Lindelöf theorem, (up to s=18 GeV^2, at least, or as an evidence that the asymptotic regime is not reached [12]. Note that in TL region, one uses the notation $s = q^2$, $s = 2m^2+2Em$, E is the energy of the antiproton beam (in the LAB system of $\overline{p}+p$-collisions).

One can express the angular dependence of the differential cross section for $\overline{p}+p \to \ell^+ + \ell^-$ as a function of the angular asymmetry $\mathcal{R}$ as:

$$\frac{d\sigma}{d(\cos\theta)} = \sigma_0 \left[1+\mathcal{R}\cos^2\theta\right],\ \mathcal{R} = \frac{\tau|G_M|^2 - |G_E|^2}{\tau|G_M|^2 + |G_E|^2} \tag{3}$$

where σ_0 is the value of the differential cross section at $\theta = \pi/2$. These quantities are very sensitive to the different underlying assumptions about the s-dependence of the FFs [13], therefore a precise measurement of the ratio $\mathcal{R}$ would be very interesting.

The measurement of the differential cross section for the process $\overline{p}+p \to \ell^+ + \ell^-$ at a fixed value of s and for two different angles θ, allowing the separation of the two FFs, $|G_M|^2$ and $|G_E|^2$, is equivalent to the well known Rosenbluth separation for the elastic ep-scattering. However in TL, this procedure is simpler, as it requires to change only one kinematical variable, $\cos\theta$, whereas, in SL it is necessary to change simultaneously two kinematical variables: the energy of the initial electron and the electron scattering angle, fixing the momentum transfer square, q^2.

The angular dependence of the cross section, Eq. 3, results directly from the assumption of one-photon exchange, where the spin of the photon is equal 1 and the electromagnetic hadron interaction satisfies the $C-$invariance. Therefore the measurement of the differential cross section at three angles (or more) would also allow to test the presence of 2γ exchange [14].

Polarization phenomena will be especially interesting in $\overline{p}+p \to \ell^+ + \ell^-$. For example, the transverse polarization, P_y of proton target (or transverse polarization of antiproton beam) results in nonzero analyzing power [7, 12]:

$$\frac{d\sigma}{d\Omega}(P_y) = \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1+\mathcal{A}P_y\right],$$

$$\mathcal{A} = \frac{\sin 2\theta Im G_E^* G_M}{D\sqrt{\tau}},\ D = |G_M|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E|^2\sin^2\theta$$

This analyzing power characterizes the T-odd correlation $\vec{P}\cdot\vec{k}\times\vec{p}$, where $\vec{k}(\vec{p})$ is the three momentum of the $\overline{p}$ beam (produced lepton). It is important to note that the τ-dependence of $\mathcal{A}$ is very sensitive to existing models of the nucleon FFs, which reproduce equally well the data in SL region [15].

The same information can be obtained from the final polarization in $\ell^+ + \ell^- \to \vec{p} + \overline{p}$, but in this case one has to deal with the problem of hadron polarimetry, in conditions of very small cross sections.

The main problems in view of a global interpretation of the four nucleon FFs (electric and magnetic, for neutron and proton) in TL and SL momentum transfer region that can be solved by future measurements with a polarized antiproton beam (or with unpolarized antiproton beam on a polarized proton target) are:

- The separation of the electric and magnetic FFs, through the angular distribution of the produced leptons: the measurement of the asymmetry $\mathcal{A}$ (from the angular dependence of the differential cross section for $\overline{p} + p \leftrightarrow \ell^+ + \ell^-$) is sensitive to the relative value of G_M and G_E.

- The presence of a large relative phase of magnetic and electric proton FFs in the TL region, if experimentally proved at relatively large momentum transfer, can be considered a strong indication that these FFs have a different behavior.

- The study of the processes $\overline{p} + p \to \pi^0 + \ell^+ + \ell^-$ and $\overline{p} + p \to \pi^+ + \pi^- + \ell^+ + \ell^-$, will allow to measure proton FFs in the TL region, for $s \leq 4m^2$, where the vector meson contribution plays an important role.

Evaluation of counting rates

In the hypothesis $|G_M| = |G_E|$, the measured form factors, in the TL region [1], can be fitted by the QCD-inspired function:

$$|G_M| = \frac{56}{s^2}\left(ln\frac{s}{\Lambda^2}\right)^{-2}, \text{s is expressed in GeV}^2$$

with $\Lambda = 0.3$ GeV. After $\cos\theta$ integration, Eq. (1) gives:

$$d\sigma(p\overline{p} \to \ell^+\ell^-) = \frac{4\pi\alpha^2}{3\beta_p s}|G_M|^2\left(1 + \frac{2m^2}{s}\right), \tag{4}$$

and $\beta_p = \sqrt{1 - 4m^2/s}$.

So, assuming a luminosity $\mathcal{L} = 2 \cdot 10^{32}$ cm^2/s [16], we find the number of events per day (1 day=10^5 s) as in Table 1.

s [GeV ^{2}]	E [GeV]	σ [pb]	Events/day
5	2.2	830	16600
10	4.9	8.	161
15	7.5	0.7	13
20	10.2	0.12	2.4
25	12.9	0.03	0.64
30	15.5	0.01	0.22

TABLE I: Expected counting rates, for $\overline{p} + p \to \ell^+ + \ell^-$.

These numbers show that it will be possible, at the future GSI facility, to separe the electric and magnetic FFs in a wide region of s and to extend the measurement of FFs up to the largest available energy, corresponding to $s \simeq 30$ GeV^2.

In conclusion, let us summarize the main points of this note concerning new possibilities opened by the future facility with antiproton beams at GSI:

- extension of measurements of proton FFs, in the TL region, up to $s = 30$ GeV^2 (comparable to the maximum value of $-q^2$, achieved in SL region);
- measurement of the angular dependence of the differential cross section for $\overline{p} + p \to \pi^+ + \pi^- + \ell^+ + \ell^-$, which will firstly allow the separation of electric and magnetic FFs, in TL region, in a wide region of s;
- using polarized target (or beam) the measurement of the analyzing power will allow to measure for the first time the s-dependence of the relative phase of electric and magnetic FFs.

This program is especially interesting with respect to the important problem of the transition to the asymptotic behaviour of nucleon electromagnetic FFs predicted by QCD, which actually gives rise to many discussions and speculations.

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