Measurement of the γ Angle at the B Factories: Status and Prospects

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On Behalf of the BaBar and Belle Experiments

Abstract — The methods to measure the angle γ of the CKM unitarity triangle at the B factories are presented. Special emphasis is given to the measurement of $\sin(2\beta+\gamma)$ using the $B^0 \to D^{*\pm}\pi^{\mp}$ decays which has already produced results providing an interesting constraint in the $\rho-\eta$ plane. Various methods using $B \to DK$ decays are also presented.

1 Introduction

The aim of the B factory experiments is to measure precisely the sides and the angles of the unitarity triangle in order to overconstrain its parameters. In this way stringent tests of the Standard Model in the sector of CP violation can be performed. From the current measurements of CKM related quantities ($\sin(2\beta$,$|V_{ub}|$,etc.) using the method described in Ref. [1], the range $37^\circ < \gamma < 80^\circ$ would appear to be favoured. However the direct measurement of this angle is a crucial ingredient in the high precision tests mentioned before.

The measurement of the γ angle is difficult because γ is in the Wolfenstein phase convention the weak phase between V_{ub} and V_{cb}. The small ratio V_{ub}/V_{cb} enters therefore in all the possible interference terms sensitive to γ and as a rule of thumb the experimentalist is confronted to the constant term $Br \times A \simeq 10^{-5}$, where Br and A are the relevant branching fraction and the CP asymmetry.

Until recently it was thought that the measurement of the angle γ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix was beyond the reach of present day B factories such as BaBar and Belle. A number of experimental developments has resulted in a change in attitude. The speed at which the angles β has been measured has confirmed that the current experiments can extract the maximum information from the data. At the same time, both Belle and BaBar are aiming to exceed their design luminosities by considerable amounts; BaBar expects to take 500 fb^{-1} by 2006 and 1000 fb^{-1} by the end of the decade.

In the following sections, a number of possible methods are discussed with no aim at an exhaustive presentation, stressing the methods that have already produced results or will do so in the near future.

2 Experimental Techniques

A description of the BaBar detector is given in Ref. [2]. The Belle detector is described in Ref.[3]. All the following measurements contain some or more of the following elements. Beam constraints are used to define a signal region. Background from continuum events are suppressed by using a series of event shape variables often in the form of Fisher discriminants or neural nets. Particle Identification is performed using energy loss in the tracking detectors and the calorimeter and the response in dedicated detectors (DIRC, aerogel, TOF). If required, the two B decay vertices are reconstructed and flavour tagging performed. Finally a global maximum likelihood fit is used to achieve the greatest sensitivity. Using the exclusive reconstruction the signature for the signal is a narrow peak in m_{ES}, the beam energy constrained mass of the reconstructed B. An alternative signature is given by ΔE the difference between the energy of the reconstructed B candidate and the beam energy in the center of mass system. The partial reconstruction yields m_{miss}, the missing mass recoiling against the fast and slow pions in the decays chain $B^0 \to D^{*\pm}\pi^{\mp}$ followed by $D^{*+} \to D^0\pi^+$: it peaks at the D^0 mass for signal events.

3 $\sin(2\beta+\gamma)$ with $B^0 \to D^{(*)\pm}\pi^\mp$

$\sin(2\beta+\gamma)$ can be measured in the decays $B^0 \to D^{(*)\pm}\pi^\mp$. As B^0 and $\bar{B}^0$ can decay to the same final state (Fig.1), CP violation arises in the interference between mixing and decay, giving a term proportional to the combination $2\beta+\gamma$, where 2β is the weak phase from mixing and γ from the decay amplitudes.

This method, proposed in [4], is theoretically clean because there is no penguin contribution and the strong phase difference $\delta^{(*)}$ between the two amplitudes is measurable. The time-dependent CP violating asymmetries are proportional to $r^{(*)} = A(\bar{B}^0 \to D^{(*)-}\pi^+)/A(B^0 \to D^{(*)-}\pi^+) \simeq 0.02$, where the amplitude at the numerator is suppressed by a factor λ^2 relative to the favored amplitude.

Figure 1: Feynman diagram for the Cabibbo-favored decay $B^0 \to D^{*-}\pi^+$ (left) and the Cabibbo-suppressed decay $\bar{B}^0 \to D^{*-}\pi^+$ (right).

The probability that a state produced at time 0 as a B^0 or $\bar{B}^0$ decays into the final state $D^{(*)\mp}\pi^\pm$ at time t is

$$Prob(B^0 \to D^{(*)\mp}\pi^\pm)(t) = \frac{1}{4\tau}e^{-|t|/\tau}\left[1 \pm C\cos(\Delta m t) + S^\mp \sin(\Delta m t)\right], \quad (1)$$

$$Prob(\bar{B}^0 \to D^{(*)\mp}\pi^\pm)(t) = \frac{1}{4\tau}e^{-|t|/\tau}\left[1 \mp C\cos(\Delta m t) - S^\mp \sin(\Delta m t)\right], \quad (2)$$

where τ is the B^0 lifetime, Δm is the $B^0-\bar{B}^0$ mixing frequency, and we have defined

$$\begin{aligned} C &= \frac{1-r^{(*)2}}{1+r^{(*)2}}, \\ S^\pm &= \frac{2r^{(*)}}{1+r^{(*)2}}\sin(2\beta+\gamma\pm\delta^{(*)}). \end{aligned} \quad (3)$$

In order to increase the size of the selected signal sample, BaBar uses also the partial reconstruction technique for the $B^0 \to D^{*\pm}\pi^\mp$, where only the fast pion and the slow pion from the $D^{*+} \to D^0\pi^+$ are reconstructed [5]. Based on a sample of 81 fb^{-1} the selected signal sample consists of 5200 (4700) fully reconstructed $B^0 \to D^\pm\pi^\mp$ ($B^0 \to D^{*\pm}\pi^\mp$) events (Fig. 2), and 6400 (25100) partially reconstructed $B^0 \to D^{*\pm}\pi^\mp$ events with a lepton (kaon) tag (Fig. 3).

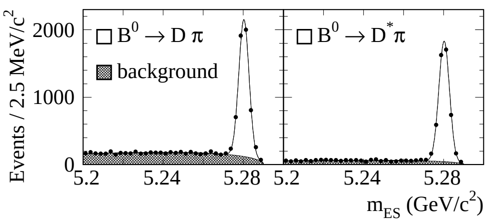

Figure 2: BaBar. The m_{ES} distributions for exclusively reconstructed $B^0 \to D^\pm\pi^\mp$ and $B^0 \to D^{*\pm}\pi^\mp$ events [7].

The fit of the Δt distributions (Fig. 4) yields the CP violation parameters [6, 7]

$$\begin{aligned} 2r^*\sin(2\beta+\gamma)cos(\delta^*) &= -0.063 \pm 0.024\,(stat.) \pm 0.014\,(syst.) \\ 2r^*\sin(2\beta+\gamma)cos(\delta^*) &= -0.068 \pm 0.038\,(stat.) \pm 0.020\,(syst.) \\ 2r\sin(2\beta+\gamma)cos(\delta) &= -0.022 \pm 0.038\,(stat.) \pm 0.020\,(syst.) \end{aligned} \quad (4)$$

where the first result is obtained with the partial reconstruction. Belle has also performed this measurement but with lower precision [8].

A time-dependent CP-violating asymmetry can be defined from the numbers of events observed at time t with specific combinations of flavor tag and reconstructed final state:

$$\mathcal{A}_{CP} = \frac{N(\text{tag } B^0, D^{*\pm}\pi^\mp)(t) - N(\text{tag } \bar{B}^0, D^{*\pm}\pi^\mp)(t)}{N(\text{tag } B^0, D^{*\pm}\pi^\mp)(t) + N(\text{tag } \bar{B}^0, D^{*\pm}\pi^\mp)(t)}. \quad (5)$$

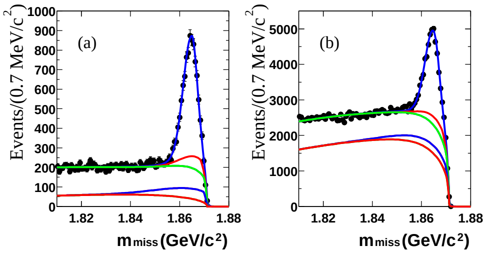

Figure 3: BaBar. The m_{miss} distributions for (a) lepton-tagged and (b) kaon-tagged events for the partial reconstruction of $B^0 \to D^{\pm}\pi^{\mp}$. The curves show from bottom to top the cumulative distributions of various backgrounds and of the $B^0 \to D^{\pm}\pi^{\mp}$ signal [6].

In the absence of background and experimental effects, $\mathcal{A}_{CP} = -2r\sin(2\beta+\gamma)\cos\delta\sin(\Delta m\Delta t)$. The asymmetry plots obtained with the partial reconstruction in the signal region are shown in Fig. 5.

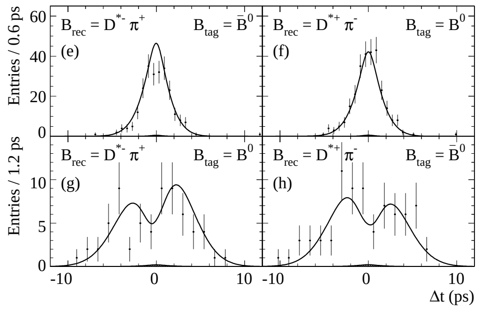

Figure 4: BaBar. The Δt distributions for exclusively reconstructed $B^0 \to D^{*\pm}\pi^{\mp}$ events with the fitted curve overlayed [7].

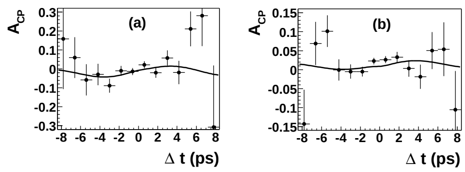

Figure 5: BaBar. The asymmetry A_{CP} with the partial reconstruction for (a) lepton- and (b) kaon-tagged events. The curves show the projection of the fit function [6].

In order to interpret these results in terms of $\sin(2\beta+\gamma)$, the value of r and r^* are needed. They can be estimated from the measurement of $B^0 \to D_s^{(*)+}\pi^-$ and the SU(3) symmetry relation [9] : $r = 0.019 \pm 0.004$ and $r^* = 0.017^{+0.005}_{-0.007}$. An additional error of 30% is introduced to account for uncertainties in the SU(3) symmetry breaking. Using the Feldman-Cousins method, the combined limit $|\sin(2\beta+\gamma)| > 0.87(0.56)$ at 68% (95%) CL is obtained [6]. The corresponding constraint in the $\rho - \eta$ plane is shown in Fig.6. It can be seen that despite the limited statistics an interesting constraint can be obtained, which is complementary to the measurement of $\sin(2\beta)$ and excludes two branches of the $\sin(2\beta)$ solutions.

The limit on $\sin(2\beta+\gamma)$ from $B^0 \to D^{*\pm}\pi^{\mp}$ together with the measurements of $\sin(2\beta)$ and $\cos(2\beta)$ [10] can be used to set a combined constraint in the $\rho - \eta$ plane presented in figure 7 [11]. It shows that it is possible to constrain the apex of the unitarity triangle using only measurements related to CP violation in the B sector. This is the first step of a more ambitious program, the reconstruction of the CKM matrix using only measurements related to the weak phases [12]. En passant we note that this method will provide a very interesting measurement of $|V_{ub}|$, free from the usual form factor uncertainties.

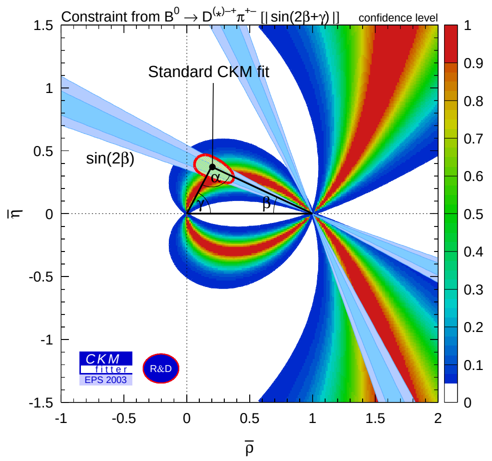

Figure 6: Constraint in the $\rho - \eta$ plane from the BABAR measurement of CP violating parameters in the $B^0 \to D^{(*)+}\pi^-$ decay modes.

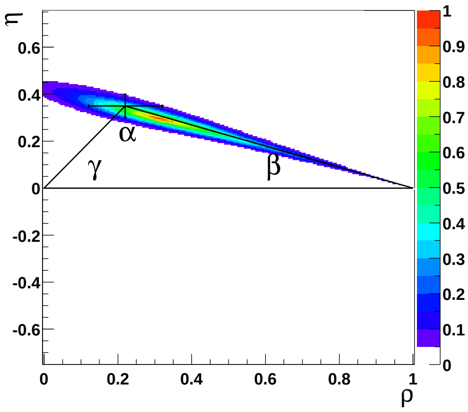

Figure 7: Constraint in the $\rho - \eta$ plane from the BABAR measurement of CP violating parameters in the $B^0 \to D^{(*)+}\pi^-$ decay modes, using also the measurement of $\sin(2\beta)$ and $\cos(2\beta)$ [11].

4 Methods using $B \to DK$ Decays

Several proposed methods for measuring γ exploit the interference between $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$ (Fig. 8) which occurs when the D^0 and the $\bar{D}^0$ decay to a common final state f.

We will present preliminary results for three methods:

1. f can be a CP eigenstate (GLW method);
2. f is doubly CKM suppressed for D^0 and CKM allowed for $\bar{D}^0$ (ADS method);
3. f is a three body final state and γ can be measured with a Dalitz plot analysis.

Figure 8: Feynman diagrams for $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$. The latter is CKM and color-suppressed with respect to the former.

4.1 The GLW Method : $B^\pm \to D_{CP} K^\pm$

The CP eigenstates $\mid D^0_\pm\rangle$ of the neutral D meson system with CP eigenvalues ± 1 are given by:

$$\mid D^0_\pm\rangle = \frac{1}{\sqrt{2}}\left(\mid D^0\rangle \pm \mid \bar{D}^0\rangle\right) \tag{6}$$

so that the $B^\pm \to D^0_+ K^\pm$ transition amplitudes can be expressed as:

$$\begin{aligned} \sqrt{2}A(B^+ \to D^0_+ K^+) &= A(B^+ \to D^0 K^+) + A(B^+ \to \bar{D}^0 K^+) \\ \sqrt{2}A(B^- \to D^0_+ K^-) &= A(B^- \to D^0 K^-) + A(B^- \to \bar{D}^0 K^-) \end{aligned} \tag{7}$$

These relations are exact, originate from pure tree decays and receive no contributions from penguins. They can be represented by 2 triangles in the complex plane. Since the transition amplitude A($B^+ \to \bar{D}^0 K^+$) = A($B^- \to D^0 K^-$) and the difference in CP-violating weak phase between the $B^+ \to D^0 K^+$ and the $B^- \to \bar{D}^0 K^-$ amplitudes is proportional to $e^{2i\gamma}$, these triangles allow a determination of γ by measuring the six amplitudes. A complementary method uses $B^0 \to D^0_+ K^{*0}$, $B^0 \to \bar{D}^0 K^{*0}$ and $B^0 \to D^0 K^{*0}$.

In practice, the following measurables are defined

$$\begin{aligned} R_\pm &= 2\frac{\Gamma(B^- \to D^0_\pm K^-) + \Gamma(B^+ \to D^0_\pm K^-)}{\Gamma(B^- \to D^0 K^-) + \Gamma(B^+ \to \bar{D}^0 K^-)} = 1 + r_B^2 \pm 2 r_B \cos\delta\cos\gamma \\ A_\pm &= \frac{\Gamma(B^- \to D^0_\pm K^-) - \Gamma(B^+ \to D^0_\pm K^-)}{\Gamma(B^- \to D^0_\pm K^-) + \Gamma(B^+ \to D^0_\pm K^-)} = \frac{\pm 2 r_B \cos\delta \sin\gamma}{R_\pm}, \end{aligned} \tag{8}$$

where

$$r_B \equiv \left|\frac{A(B^- \to \bar{D}^0 K^-)}{A(B^- \to D^0 K^-)}\right| \tag{9}$$

and δ is the strong phase difference between these two amplitudes. The measurement of these four quantities allows to determine r_B, δ and γ.

The ΔE distributions obtained by BaBar for the D^0 decaying to flavor modes and CP eigenstates are shown in figure 9. The results obtained by BaBar with a sample of 80 fb^{-1} and by Belle with a sample of 78 fb^{-1} are presented in Table 1.

Figure 9: BaBar. ΔE distributions for $B^{\pm} \to D^0 K^{\pm}$ where the D^0 decays in a flavor mode $D^0 \to K^-\pi^+$, $D^0 \to K^-\pi^+\pi^+\pi^-$ and $D^0 \to K^-\pi^+\pi^0$ (left), and in the CP eigenstates $\pi^+\pi^-$ and K^+K^- (right) [14].

Table 1: BaBar and Belle measurements of $R_{\pm}$ and $A_{\pm}$.

	R_+	A_+	R_-	A_-
Belle	$1.21 \pm 0.25 \pm 0.14$	$0.06 \pm 0.19 \pm 0.04$	$1.41 \pm 0.27 \pm 0.15$	$-0.19 \pm 0.17 \pm 0.05$
BaBar	$1.06 \pm 0.19 \pm 0.06$	$0.07 \pm 0.17 \pm 0.06$	-	-

4.2 The ADS Method

In this method f is doubly CKM suppressed for D^0 and CKM allowed for $\bar{D}^0$. Large asymmetries are anticipated however the number of events for the doubly CKM suppressed mode is expected to be quite small.

BaBar searches for $B^- \to \bar{D}^0 K^-$ followed by $\bar{D}^0 \to K^+\pi^-$, as well as the charge conjugate sequence. Here the favored B decay followed by the doubly CKM-suppressed D decay interferes with the suppressed B decay followed by the CKM-favored D decay. We use the notation $B^- \to [h_1^+ h_2^-]_D h_3^-$ (with each $h_i = \pi$ or K) for the decay chain $B^- \to \bar{D}^0 h_3^-$, $\bar{D}^0 \to h_1^+ h_2^-$. We can define

$$\mathcal{R}_{K\pi}^{\pm} \equiv \frac{\Gamma([K^{\mp}\pi^{\pm}]_D K^{\pm})}{\Gamma([K^{\pm}\pi^{\mp}]_D K^{\pm})} = r_B^2 + r_D^2 + 2 r_B r_D \cos(\pm\gamma + \delta), \tag{10}$$

where

$$r_B \equiv \left| \frac{A(B^- \to \bar{D}^0 K^-)}{A(B^- \to D^0 K^-)} \right|, \quad \delta \equiv \delta_B + \delta_D, \tag{11}$$

$$r_D \equiv \left| \frac{A(D^0 \to K^+\pi^-)}{A(D^0 \to K^-\pi^+)} \right| = 0.060 \pm 0.003, \tag{12}$$

and δ_B and δ_D are strong phase differences between the two B and D decay amplitudes, respectively.

Using a sample of 109 fb^{-1}, BaBar measures $\mathcal{R}_{K\pi} = (\mathcal{R}_{K\pi}^+ + \mathcal{R}_{K\pi}^-)/2 = (4 \pm 12) \times 10^{-3}$ [17], consistent with zero. This can be translated in an upper limit $r_B < 0.22$ at 90 % CL (Fig. 10).

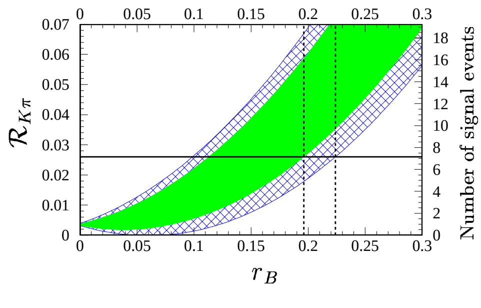

Figure 10: BaBar. Expectations for $\mathcal{R}_{K\pi}$ and N_{sig} *vs.* r_B for the ADS method. Filled-in area: allowed region for any value of δ, with a $\pm 1\sigma$ variation on r_D, and $48° < \gamma < 73°$. Hatched area: additional allowed region with no constraint on γ. The horizontal line represents the 90% C.L. limit $\mathcal{R}_{K\pi} < 0.026$. The dashed lines are drawn at $r_B = 0.196$ and $r_B = 0.224$. They represent the 90% C.L. upper limits on r_B with and without the constraint on γ [17].

This result excludes the most favorable scenarios for the sensitivity of the methods using $B \to D^0 K$ decays. A study [18] of the expected sensitivity of the GLW and ADS methods concludes that even with 500 fb^{-1}, if $r_B = 0.1$, the constraint on γ will be very weak.

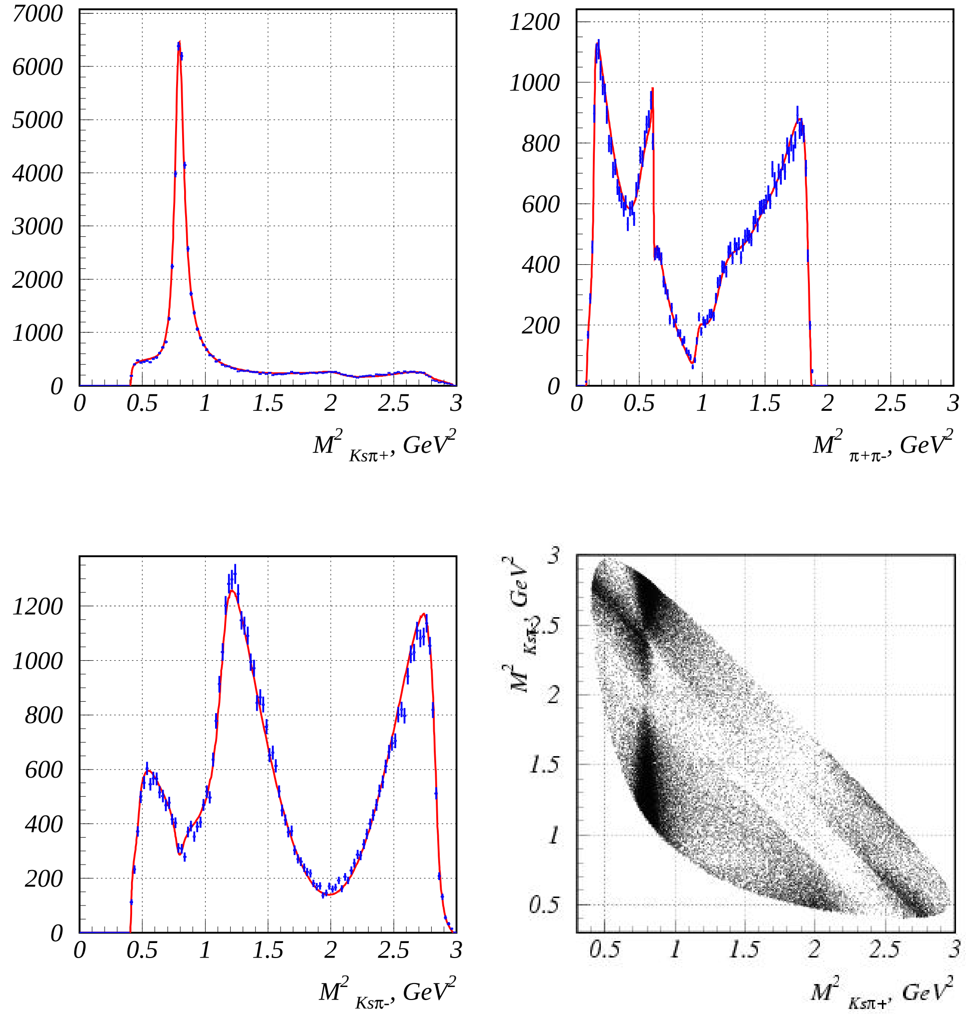

Figure 11: Belle. $M^2_{K\pi^+}$, $M^2_{K\pi^-}$, $M^2_{\pi^+\pi^-}$ distributions and Dalitz plot of $D^0 \to K_s\pi^+\pi^-$ decay from $D^{*+} \to D^0\pi^+$ process [20].

4.3 The Dalitz Plot Method

In this method [19] the decay to a three body final state common to D^0 and $\bar{D}^0$ like $K_s\pi^+\pi^-$ is considered. The amplitude of the B^+ decay can be written as

$$M_+ = g(m_+^2, m_-^2) + r_B e^{i\gamma+i\delta} g(m_-^2, m_+^2), \tag{13}$$

where m_+^2 and m_-^2 are the squared invariant masses of the $K_s\pi^+$ and $K_s\pi^-$ combinations, respectively, and g is the complex amplitude of the decay $\bar{D}^0 \to K_S\pi^+\pi^-$. Similarly, the amplitude of the charge conjugate B^- decay is

$$M_- = g(m_-^2, m_+^2) + r_B e^{-i\gamma+i\delta} g(m_+^2, m_-^2). \tag{14}$$

Once the functional form of g is fixed by a choice of a model for $D^0 \to K_s\pi^+\pi^-$ decays, the Dalitz distributions for B^+ and B^- decays can be fitted simultaneously by the above expressions for M_+ and M_-, with r, γ, and δ as free parameters. Belle determines $|g|$ with a study of $D^0 \to K_s\pi^+\pi^-$ decay from $D^{*+} \to D^0\pi$ (Fig. 11): 57800 events are selected.

The fit [20] of the Dalitz plot for 146 selected $B^\pm \to D^0K^\pm$ and 39 $B^\pm \to D^{*0}K^\pm$ events based on 140 fb^{-1} gives $\gamma = 77^\circ{}^{+17^\circ}_{-19^\circ} \pm 13^\circ(syst) \pm 11^\circ(model)$ and $r_B = 0.26^{+0.10}_{-0.14} \pm 0.03 \pm 0.04$ for the $B^\pm \to D^0K^\pm$ mode (Fig. 12). This result shows a rather large value for r_B, the expected value being $r_B = 0.13$. The result for γ depends on the model for the functional form of the amplitude in the Dalitz plot.

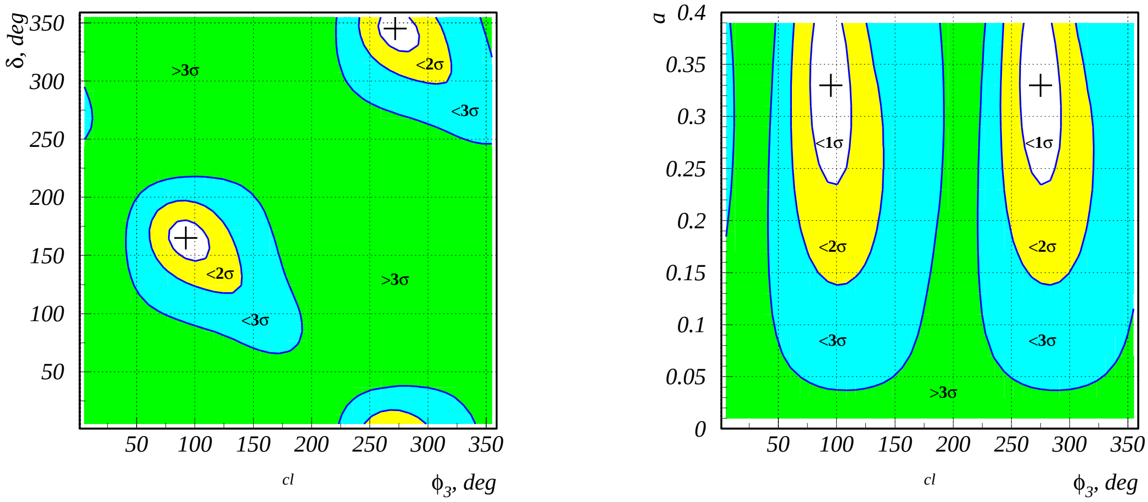

Figure 12: Belle. Constraints on $\gamma = \phi_3$ and δ (left) and $r_B = a$ and $\gamma = \phi_3$ (right) for the Dalitz plot method [20].

5 Conclusion

First interesting results constraining the value of γ have been obtained at the B factories.

- The $\sin(2\beta + \gamma)$ analyses have been already able to measure a small asymmetry at the % level. These analyses will benefit from the large increase in luminosity and by adding new modes like $B^0 \to D^{\pm}\rho^{\mp}$ and $B^0 \to D^{*\pm}\rho^{\mp}$ and have very promising prospects for setting constraints in the $\rho - \eta$ plane.
- The first results show that unfortunately the r_B value is low and that the sensitivity of the GLW and ADS method may be much lower than expected.
- For the Dalitz plot method, the most promising approach using $B \to D^0 K$ decays, the emphasis will be on the control of the model dependence or on a a model independent fit.

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Torino, with its sober neoclassical elegance and its excellent cuisine, was the ideal stage for a fruitful meeting. It is a pleasure to thank the organizers of IFAE 2004 for the impeccable hospitality, the warm atmosphere and the stimulating discussions.

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