The associated photoproduction of positive kaons and $\pi^0\,\Lambda$ or $\pi\,\Sigma$ pairs in the region of the $\Sigma(1385)$ and $\Lambda(1405)$ resonances

Matthias F. M. Lutz

GSI, Planckstrasse 1, D-64291 Darmstadt, Germany
Institut für Kernphysik, TU Darmstadt, D-64289 Darmstadt, Germany

Madeleine Soyeur

Département d'Astrophysique, de Physique des Particules,
de Physique Nucléaire et de l'Instrumentation Associée,
Service de Physique Nucléaire,
Commissariat à l'Energie Atomique/Saclay,
F-91191 Gif-sur-Yvette Cedex, France

Abstract

The $\gamma p \rightarrow K^+\pi^0\Lambda$ and $\gamma p \rightarrow K^+\pi\Sigma$ reactions are studied in the kinematic region where the $\pi^0\Lambda(1116)$ and $\pi\Sigma(1192)$ pairs originate dominantly from the decay of the $\Sigma(1385)$ and $\Lambda(1405)$ resonances. We consider laboratory photon energies around 2 GeV, significantly above the threshold for producing the $K^+\,\Sigma(1385)$ and $K^+\,\Lambda(1405)$ final states. We compute for both reactions the process in which the ingoing photon dissociates into a real K^+ and a virtual K^-, the off-shell K^- scattering subsequently off the proton target to produce the $\pi^0\,\Lambda$ or $\pi\,\Sigma$ pair. The $K^-p \rightarrow \pi^0\Lambda$ and $K^-p \rightarrow \pi\Sigma$ amplitudes are calculated in the framework of a chiral coupled-channel effective field theory of meson-baryon scattering. The structure of the amplitudes reflects the dominance of the $\Lambda(1405)$ in the $\pi\Sigma$ channel and of the $\Sigma(1385)$ in the $\pi\Lambda$ channel. The full pion-hyperon final state interaction is included in these amplitudes. We extract from the calculated cross section the gauge-invariant double kaon pole term. We found this term to be large and leading to sizeable cross sections for both the $\gamma p \rightarrow K^+\pi^0\Lambda$ and $\gamma p \rightarrow K^+\pi\Sigma$ reactions, in qualitative agreement with the scarce data presently available. Accurate measurements of these cross sections should make it possible to extract the contribution of the double kaon pole and hence to assess the possibility of studying kaon-nucleon dynamics just below threshold through these reactions.

Key words: Strangeness photoproduction; $\Lambda(1405)$; $\Sigma(1385)$

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1 Introduction

The simplest process leading to strange particle creation in photon-nucleon interactions is the associated production of a kaon and a hyperon. The hyperon decays subsequently into specific channels. We consider the interaction of 2 GeV photons (in the laboratory reference frame) with proton targets. We select final states consisting of a K^+ and a $\pi^0\,\Lambda$ or a $\pi\,\Sigma$ pair respectively and restrict the invariant mass of these pairs to the mass range of the $\Sigma^0(1385)$ and the $\Lambda(1405)$. The $\Sigma^0(1385)$ decays primarily into the $\pi^0\,\Lambda$ channel $(88 \pm 2\,\%)$ and less importantly $(12 \pm 2\,\%)$ into the $\pi\,\Sigma$ channel [1]. The $\Lambda(1405)$ decays entirely into the $\pi\,\Sigma$ channel [1]. The $\Sigma^0(1385)$ and the $\Lambda(1405)$ are therefore expected to dominate the production of $\pi^0\,\Lambda$ and $\pi\,\Sigma$ pairs in the corresponding mass range.

These resonances overlap in mass and have to be separated experimentally by distinctive decays. A particularly interesting idea is to study the $\Lambda(1405)$ in the $\pi^0\,\Sigma^0$ channel [2]. The transition between the $\Sigma^0(1385)$ and the $\pi^0\,\Sigma^0$ channel is forbidden because the isospin Clebsch-Gordan coefficient vanishes. The $\pi^0\,\Sigma^0$ decay is therefore a unique signature of the $\Lambda(1405)$ channel. Such a measurement has not yet been performed but is intended at ELSA (Bonn) where the $\pi^0\,\Sigma^0$ pair could be detected through a multi-photon final state $(\pi^0\,\Sigma^0 \;\rightarrow\; \pi^0\,\Lambda(1116)\,\gamma \;\rightarrow\; \pi^0\,n\,\pi^0\,\gamma)$ with the Crystal Barrel [2]. The photoproduction of $\Lambda(1405)$ resonances is also presently studied in the charged decay channels, $\pi^-\,\Sigma^+$ and $\pi^+\,\Sigma^-$, at SPring-8/LEPS with incident photon energies in the range $1.5 < E_\gamma^{Lab} < 2.4$ GeV [3] and at ELSA with the SAPHIR detector at 2.6 GeV [4]. These channels are expected to be dominated by the $\Lambda(1405)$ with some contribution from the $\Sigma^0(1385)$. There are no published cross sections yet. The only data presently available in the kinematics of interest [5] were obtained at DESY thirty years ago with space-like photons, in electroproduction experiments where the scattered electron and the produced K^+ are detected in coincidence. With this method the $\Sigma^0(1385)$ and $\Lambda(1405)$ channel could not be separated at all. The differential cross sections for the $e\,p \rightarrow e\,K^+\,Y$ reaction obtained in these measurements characterize globally strangeness production processes for missing masses ranging from 1.35 GeV until 1.45 GeV. An interesting trend of these data is that the t-dependence of the cross section, for given photon energy and virtuality, seems to show a sharp drop as would be expected if the dynamics were dominated by t-channel exchanges. The Mandelstam variable t is defined as the square of the 4-momentum transfer from the proton target to the $\Sigma^0(1385)$ or $\Lambda(1405)$.

Both the $\Sigma(1385)$ and $\Lambda(1405)$ resonances are located close and below the $\bar{K}N$ threshold. These resonances seem to be of rather different nature. The $\Sigma(1385)$ belongs to the large N_c ground state baryons and appears well-described by quark models [6,7]. The $\Lambda(1405)$ is a complex baryonic state. Its mass, in particular the large splitting between the $\Lambda_{1/2^-}(1405)$ and the $\Lambda_{3/2^-}(1520)$, cannot be understood in the constituent quark model with residual quark-quark interactions fitting the other low-lying baryonic states [6,7]. There are many indications that the quark model description of the $\Lambda(1405)$, if valid at all, requires a sizeable $q^4\bar{q}$ component [8–10]. This observation is closely related to the early idea that the $\Lambda(1405)$ can be viewed as a bound kaon-nucleon system [11–18] and to the later picture of the $\Lambda(1405)$ as a kaon-soliton bound state [19,20]. The $\bar{K}N$ nature of the $\Lambda(1405)$ was also inferred from the SU(3) cloudy bag model description [21,22]. Extensive studies of the $\Lambda(1405)$ based on chiral Lagrangians [23–31] suggest that this resonance is generated by meson-baryon interactions.

The different nature of the $\Sigma(1385)$ and of the $\Lambda(1405)$ resonances is built in the chiral coupled-channel approach of kaon-nucleon scattering developed in Ref. [27] and which will be used in this work. The baryon resonances belonging to the large N_c ground state baryon mutiplets (hence the $\Sigma(1385)$) are introduced explicitly as fundamental fields of the effective Lagrangian. The other baryon resonances (in particular the $\Lambda(1405)$) are generated dynamically by meson-baryon coupled-channel dynamics.

The $\Sigma(1385)$ and $\Lambda(1405)$ resonances have also different spectral properties. The $\Sigma(1385)$ mass distribution is very close to a Breit-Wigner form [32]. The spectral shape of the $\Lambda(1405)$ departs from a Breit-Wigner [33,34]. It depends strongly on the initial and final states through which it is measured, emphasizing the need for a full understanding of the coupling of the $\Lambda(1405)$ to its different decay channels.

We study the $\gamma p \to K^+\pi^0\Lambda$ and $\gamma p \to K^+\pi\Sigma$ reactions with the idea of using future accurate data on these processes (mainly t-distributions) to gain understanding of the $K^-p \to \pi^0\Lambda$ and of the $K^-p \to \pi\Sigma$ amplitudes below the $\bar{K}N$ threshold, where they are dominated by the $\Sigma(1385)$ and $\Lambda(1405)$ resonances. This procedure requires that these reactions be significantly driven by the process in which the ingoing photon dissociates into a real K^+ and a virtual K^-, the off-shell K^- scattering subsequently off the proton target to produce the $\pi^0\,\Lambda$ or $\pi\,\Sigma$ pair. Such dynamics would show in a sharp drop of the differential cross sections $d\sigma/dt$ with increasing $|t|$. This drop has both a double pole component behaving like $1/(m_K^2 - t)^2$ and a single pole dependence going like $1/(m_K^2 - t)$. The amplitude associated with the K^- t-channel exchange alone is not gauge-invariant. There are many other terms of order α building up the gauge-invariant amplitude. We argue that the contributions from all these other terms to the cross section will not affect the double pole term, which is made gauge-invariant and entirely fixed by our calculation. This issue is of interest because the double kaon pole term contribution to the $\gamma p \to K^+\pi\,Y$ cross section is large (contrary to what is found in other meson photoproduction processes as discussed below). Our hope is that very accurate data will make it possible to isolate this term by expressing the differential cross sections $d\sigma/dt$ as a superposition of double and single K$^-$ pole terms with less singular contributions. Such an information would be most useful in constraining $\bar{K}N$ dynamics at low and subthreshold energies. The K^-p inelastic cross sections close to threshold are indeed poorly known [27].

We compute the t-channel K$^-$-exchange contribution to the $\gamma p \to K^+\pi^0\Lambda$ and $\gamma p \to K^+\pi\Sigma$ reactions for photon laboratory energies around 2 GeV using the $K^-p \to \pi^0\Lambda$ and $K^-p \to \pi\Sigma$ amplitudes obtained in Ref. [27] and extract the gauge-invariant double K^- pole contribution for both processes. The steps of that calculation are outlined in Section 2. Our numerical results for the $\gamma\, p \to K^+\, \pi^0\, \Lambda(1116)$ and $\gamma\, p \to K^+\, \pi\, \Sigma(1192)$ reactions based on the double K$^-$ pole term are displayed and discussed in Section 3. We conclude by a few remarks in Section 4.

2 The $\gamma\, p \to K^+\, \pi^0\, \Lambda(1116)$ and $\gamma\, p \to K^+\, \pi\, \Sigma(1192)$ reaction cross sections

The t-channel K$^-$-exchange contributions to the amplitudes for the $\gamma\, p \to K^+\, \pi^0\, \Lambda(1116)$ and $\gamma\, p \to K^+\, \pi\, \Sigma(1192)$ reactions are displayed in Figs. 1 and 2.

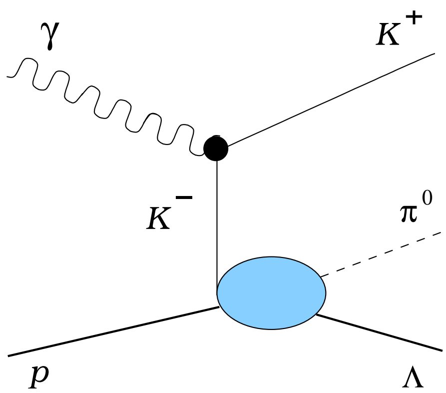

Fig. 1. K$^-$-exchange contribution to the $\gamma\, p \to K^+\pi^0\Lambda$ amplitude.

The importance of the contribution of the K$^-$-exchange term to the $\gamma\, p \to K^+\, \pi^0\, \Lambda(1116)$ and $\gamma\, p \to K^+\, \pi\, \Sigma(1192)$ cross sections can but be assessed by accurate $d\sigma/dt$ measurements for both reactions. As discussed earlier, such data are expected in the near future but not yet available.

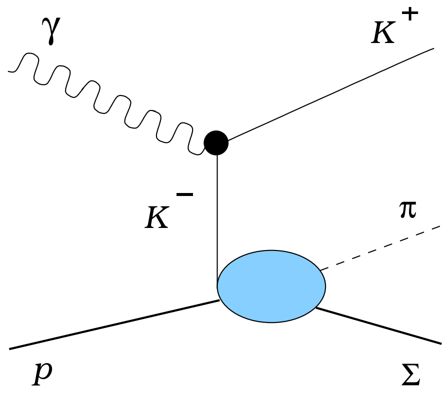

Fig. 2. K^--exchange contribution to the $\gamma\, p \to K^+ \pi\Sigma$ amplitude. The $\pi\Sigma$ symbol stands for $\pi^-\Sigma^+$, $\pi^0\Sigma^0$ or $\pi^+\Sigma^-$.

In order to get nevertheless a rough feeling for the t-dependence of the cross section, we have used the old data of Ref. [5] characterizing the sum of the $\gamma_v\, p \to K^+\, \Sigma^0(1385)$ and $\gamma_v\, p \to K^+\, \Lambda(1405)$ processes measured with space-like photons, in electroproduction experiments. These data are displayed in Fig. 3.

We plot the sum of the unpolarized transverse and longitudinal cross sections which dominate the process (ε is the transverse photon polarization) [5]. The points at $t = -0.18$ GeV^2 and $t = -0.23$ GeV^2 are direct measurements. The points corresponding to $t = -0.27$ GeV^2 and $t = -0.37$ GeV^2 have been measured at $q^0_\gamma \simeq 3.5$ GeV and extrapolated to $q^0_\gamma = 2.5$ GeV by rescaling the differential cross section according to the energy dependence predicted by our model (i.e. an increase of about a factor of 2). There is clearly a large uncertainty associated with this procedure. We consider therefore the data displayed in Fig. 3 only as a tenuous indication that the reaction could proceed through a t-channel exchange in qualitative agreement with the rapid fall-off expected from a dominant double K^--pole term.

It should be noted however that there are no good reasons to expect significant s-channel contributions to the $\gamma p \to K^+\pi^0\Lambda$ and $\gamma p \to K^+\pi\Sigma$ reactions at $E_\gamma \simeq 2$ GeV. The corresponding total center of mass energy in these kinematics is $\sqrt{s} = 2.15$ GeV. There are no baryon resonances in that mass range known to decay into the $K^+\pi^0\Lambda$ or $K^+\pi\Sigma$ channels. It is interesting to recall that, for the $\gamma p \to K^+\Lambda$ and $\gamma p \to K^+\Sigma^0$ reactions at $E_\gamma \simeq 2$ GeV, the dominance of the K^--exchange at low t could be inferred from photo- and electroproduction data [35]. The importance of the $\Sigma^0(1385)$ and $\Lambda(1405)$ resonances in the $\bar{K}N$ dynamics close to threshold provides additional grounds for expecting a similar picture to hold for the $\gamma p \to K^+\Sigma^0(1385)$ and $\gamma p \to K^+\Lambda(1405)$ reactions.

We calculate the cross section for the $\gamma\, p \to K^+\, \pi\, Y$, where Y denotes either the $\Lambda(1116)$ or the $\Sigma(1192)$. The 4-momenta of the photon, the proton, the

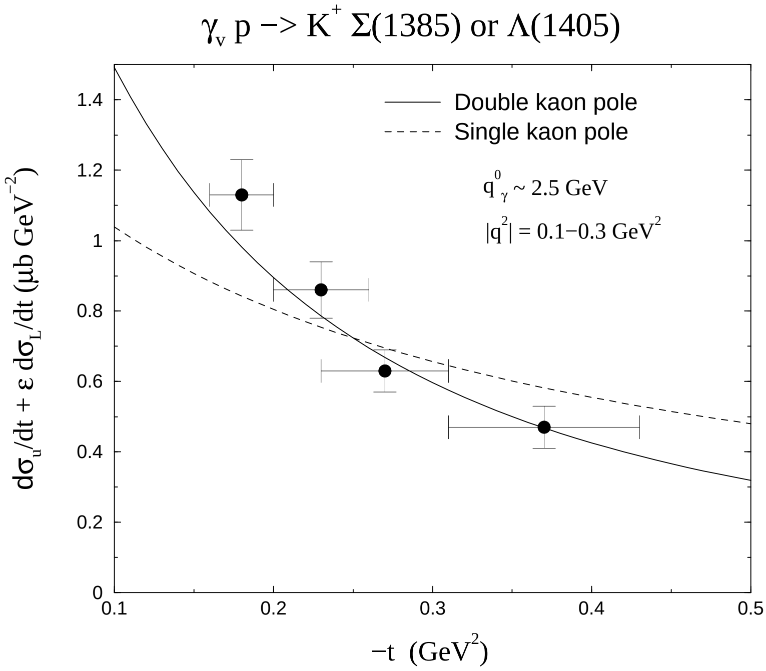

Fig. 3. Differential cross section for the production of $K^+\Lambda(1405)$ and $K^+\Sigma^0(1385)$ final states induced by the scattering of virtual photons from proton targets. The data are from Ref. [5]. The quantity q^2 is the photon virtuality and q^0_γ is the (virtual) photon energy. The two points at t=-0.27 GeV^2 and -0.37 GeV^2 are extrapolated from data taken at higher energy according to a prescription explained in the text. The full and dashed lines show the shape of the t-dependence of the cross section as expected from double and single K$^-$-pole terms respectively (with arbitrary normalizations).

K$^+$, the pion and the hyperon are denoted by q, p, $\bar{q}_K$, $\bar{q}_\pi$ and $\bar{p}_Y$ respectively. The photon, proton and hyperon polarizations are indicated by the symbols λ_γ, λ and $\bar{\lambda}_Y$. The total cross section reads

$$\sigma_{\gamma\, p \to K^+\, \pi\, Y} = \frac{1}{|\vec{v}_\gamma - \vec{v}_p|}\, \frac{1}{2\, q^0}\, \frac{m_p}{p^0} \int \frac{d^3\vec{\bar{q}}_K}{(2\pi)^3}\, \frac{1}{2\, \bar{q}^{\,0}_K} \int \frac{d^3\vec{\bar{q}}_\pi}{(2\pi)^3}\, \frac{1}{2\, \bar{q}^{\,0}_\pi} \int \frac{d^3\vec{\bar{p}}_Y}{(2\pi)^3}\, \frac{m_Y}{\bar{p}^{\,0}_Y}$$
$$(2\,\pi)^4\, \delta^4(q + p - \bar{q}_K - \bar{q}_\pi - \bar{p}_Y) \sum_{\lambda_\gamma, \lambda, \bar{\lambda}_Y} \frac{1}{4}\, |M_{\gamma\, p \to K^+\, \pi\, Y}|^2. \quad (1)$$

The first step of the calculation is to factorize the full amplitude $M_{\gamma\, p \to K^+\, \pi\, Y}$ into the photon-kaon vertex and the $K^-\, p \to \pi\, Y$ amplitude in accordance with the reaction mechanism depicted in Figs. 1 and 2. We have

$$\sum_{\lambda_\gamma, \lambda, \bar{\lambda}_Y} \frac{1}{4}\ |\ M_{\gamma\, p \to K^+\, \pi\, Y}\ |^2 = -e^2\, \frac{(t + m_K^2)}{(t - m_K^2)^2}\, \frac{1}{2} \sum_{\lambda, \bar{\lambda}_Y} |M_{K^-\, p \to \pi\, Y}|^2. \quad (2)$$

We work in the photon-proton center of mass reference frame where the total energy of the reaction is denoted by $\sqrt{s}$. In that reference frame, the photon 3-momentum is -$\vec{p}$ and the 3-momentum of the $\pi\, Y$ pair is -$\vec{\bar{q}}_K$. It is useful to define the invariant mass $\sqrt{\bar{w}^2}$ of the final $\pi\, Y$ pair by

$$\bar{w}^2 = (p + q - \bar{q}_K)^2 = s + m_K^2 - 2\,\sqrt{s}\,\sqrt{m_K^2 + \vec{\bar{q}}_K^{\,2}} \tag{3}$$

and to express the 4-momentum transfer $t = (q - \bar{q}_K)^2$ as function of that variable,

$$t\;(s, \bar{w}^2, \cos\theta) = m_K^2 - \frac{1}{2s}(s - m_p^2)\,(s + m_K^2 - \bar{w}^2) \\ \Big(1 - \sqrt{1 - \frac{4\,m_K^2\,s}{(s + m_K^2 - \bar{w}^2)^2}}\,\cos\theta\Big), \tag{4}$$

where θ is the angle between the initial photon and the produced kaon.

Using these variables, the total cross section (1) can be rewritten as

$$\sigma_{\gamma\, p \to K^+\, \pi\, Y} = \frac{\alpha\, m_p}{16\,\pi\, s\, |\vec{p}\,|^2}\, \frac{(t + m_K^2)}{(t - m_K^2)^2} \int\limits_{(M_Y + m_\pi)^2}^{(\sqrt{s} - m_K)^2} d\bar{w}^2 \int\limits_{t_+(s,\bar{w}^2)}^{t_-(s,\bar{w}^2)} dt \int \frac{d^3\vec{\bar{q}}_\pi}{(2\pi)^3} \\ \frac{1}{2\,\bar{q}^{\,0}_{\,\pi}} \int \frac{d^3\vec{\bar{p}}_Y}{(2\pi)^3}\, \frac{m_Y}{\bar{p}^{\,0}_{\,Y}}\, (2\,\pi)^4\, \delta^4(\bar{w} - q_\pi - \bar{p}_Y)\, \frac{1}{2} \sum_{\lambda, \bar{\lambda}_Y} |M_{K^- p \to \pi\, Y}|^2, \tag{5}$$

with $t_-(s, \bar{w}^2) = t\;(s, \bar{w}^2, \cos\theta = +1)$ and $t_+(s, \bar{w}^2) = t\;(s, \bar{w}^2, \cos\theta = -1)$.

It is convenient to express Eq. (5) in terms of the total $K^- p \to \pi\, Y$ cross section, $\sigma_{K^- p \to \pi\, Y}$. This quantity is frame independent. Its expression in the $K^- p$ center of mass reads

$$\sigma_{K^-\, p \to \pi\, Y} = \frac{1}{\sqrt{\bar{w}^2}\, |\vec{q}_{K^- p}|}\, \frac{m_p}{2} \int \frac{d^3\vec{\bar{q}}_\pi}{(2\pi)^3}\, \frac{1}{2\,\bar{q}^{\,0}_{\,\pi}} \int \frac{d^3\vec{\bar{p}}_Y}{(2\pi)^3}\, \frac{m_Y}{\bar{p}^{\,0}_{\,Y}} \\ (2\,\pi)^4\, \delta^4(\bar{w} - \bar{q}_\pi - \bar{p}_Y)\, \frac{1}{2} \sum_{\lambda, \bar{\lambda}_Y} |M_{K^- p \to \pi\, Y}|^2, \tag{6}$$

in which $q_{K^- p}$ is the K^- momentum in the $K^- p$ center of mass. Its value as function of the total center of mass energy $\sqrt{\bar{w}^2}$ of the $K^- p$ system is given by

$$|\vec{q}_{K^- p}|^2 = \frac{1}{4\,\bar{w}^2}\, \{\bar{w}^4 - 2\,\bar{w}^2\, (m_p^2 + m_K^2) + (m_p^2 - m_K^2)^2\}. \tag{7}$$

The doubly differential cross section $d\sigma/dtd\bar{w}^2$ can be written as

$$\frac{d\sigma_{\gamma p\to K^+\pi Y}}{dt\,d\bar{w}^2} = \frac{\alpha}{4\pi}\,\frac{(\bar{w}^4 - 2\,\bar{w}^2\,(m_p^2+m_K^2)+(m_p^2-m_K^2)^2)^{1/2}}{(s-m_p^2)^2} \frac{(t+m_K^2)}{(t-m_K^2)^2}\,\sigma_{K^-\,p\to\pi\,Y}(\bar{w}^2). \tag{8}$$

We remark first that the amplitude $M_{\gamma p\to K^+\pi Y}$ obtained by calculating the graph of Fig. 1 (or Fig. 2) is not gauge-invariant. To obtain the full gauge-invariant amplitude, all the other diagrams of order α leading to the same final state should be added. In the energy range under consideration ($E_\gamma \simeq 2$ GeV), there are many possible diagrams involving poorly known couplings and hence large uncertainties.

Instead of attempting to calculate these graphs, we resort to the pole scheme method [36]. This technique was used to derive gauge-invariant results in the vicinity of a pole for electroweak processes involving radiative corrections [37]. The idea of the method is to decompose the amplitude according to its pole structure and to expand it around the pole. To any order in perturbation theory, the residues of the poles are gauge-invariant. The expansion provides therefore subsets of gauge-invariant expressions associated with a given pole structure.

We apply this method to derive the gauge-invariant cross section corresponding to the double K$^-$-pole term to first order in α. We expect this term to play a significant role in the $\gamma\,p \to K^+\,\pi\,Y$ process. The key point is that the graph of Fig. 1 (or Fig. 2) is the only process which can contribute to the double K$^-$-pole term. We will therefore decompose the corresponding cross section according to its pole structure, keep only the double K$^-$-pole term and extract the gauge-invariant cross section associated with that pole structure by calculating the residue at the pole.

According to this procedure, the gauge-invariant cross section corresponding to the double K$^-$-pole term reads

$$\frac{d\sigma_{\gamma p\to K^+\pi Y}}{dt\,d\bar{w}^2} = \frac{\alpha}{2\pi}\,\frac{(\bar{w}^4 - 2\,\bar{w}^2\,(m_p^2+m_K^2)+(m_p^2-m_K^2)^2)^{1/2}}{(s-m_p^2)^2} \frac{m_K^2}{(t-m_K^2)^2}\,\sigma_{K^-\,p\to\pi\,Y}(\bar{w}^2). \tag{9}$$

We stress that the double pole term is the only one which can be determined this way, because it does not get contributions from any other graph but the t-channel kaon-exchange diagram. In contrast, the single pole term is built up as a sum of many processes, in particular the interference of the t-channel kaon-exchange diagram with s- and u-channel terms.

In order to be able to extract the double pole term from accurate t-distributions, it has to be reasonably large. We speculate so in view of the numerical results discussed in the next section. As shown earlier, it is also compatible with the few data points available.

We emphasize that this would be a specific behaviour of the $\gamma\, p \to K^+\, \pi\, Y$ reactions. The situation is indeed different in other meson photoproduction processes, even when they are dominated by t-channel exchanges. It is interesting to consider for example the pole structure for the photoproduction of ω-mesons in the energy range $1.4 < E_\gamma < 1.6$ GeV. In these kinematics, the $\gamma p \to \omega p$ reaction is expected to be well described by a simple pion-exchange diagram for small momentum transfers and with standard form factors [38]. We use the specific model of Ref. [38]. Neglecting the t-dependence of the form factors and concentrating on the pole structure of the differential cross section in the limit of local couplings, we have

$$\begin{aligned}\frac{d\sigma_{\gamma\, p\to\omega\, p}}{dt} &\propto \frac{-t}{4\, m_p^2}\, \frac{(m_\omega^2 - t)^2}{(m_\pi^2 - t)^2} \\ &= \frac{-m_\pi^2}{4\, m_p^2}\, \frac{(m_\omega^2 - m_\pi^2)^2}{(m_\pi^2 - t)^2} + \frac{(m_\omega^2 - m_\pi^2)}{4\, m_p^2}\, \frac{(m_\omega^2 - 3m_\pi^2)^2}{(m_\pi^2 - t)} \\ &\qquad + \frac{1}{4\, m_p^2}\, (2m_\omega^2 - 2m_\pi^2 - t). \qquad (10)\end{aligned}$$

We see that the differential cross section behaviour is dominated by the single pion pole term. The double pion pole is small and even negative. It acts as a minor correction to the cross section, as a consequence of the smallness of the pion mass. The pole scheme technique used above is not predictive in this case and the dominant single pion pole term has to be determined phenomenologically. As a consequence of this observation, the effective πNN and $\pi\gamma\omega$ couplings are not constrained directly by the data and depend on further assumptions such as the form factors assigned to the vertices. We remark that the particular form of the couplings chosen in Ref. [38] makes the full pion-exchange diagram gauge-invariant, irrespectively of its pole structure.

To characterize the momentum transfer dependence of the $K^-\, p \to \pi\, Y$ cross section, one can define the variable $\bar{t} = (p - \bar{p}_Y)^2$ and generalize Eq.(9) to the threefold differential cross section

$$\frac{d\sigma_{\gamma\, p \to K^+\, \pi\, Y}}{dt\, d\bar{w}^2 d\bar{t}} = \frac{\alpha}{2\,\pi}\, \frac{(\bar{w}^4 - 2\,\bar{w}^2\,(m_p^2 + m_K^2) + (m_p^2 - m_K^2)^2)^{1/2}}{(s - m_p^2)^2} \frac{m_K^2}{(t - m_K^2)^2}\, \frac{d\sigma_{K^-\, p \to \pi\, Y}}{d\bar{t}}(\bar{w}^2,\, \bar{t}). \tag{11}$$

We focus on the $K^-\, p \to \pi\, Y$ reaction by integrating over t, using the kinematic boundaries t_- and t_+ defined after Eq. (5). We have

$$\int_{t_+}^{t_-} dt\, \frac{d\sigma_{\gamma\, p \to K^+\, \pi\, Y}}{dt\, d\bar{w}^2 d\bar{t}} = \frac{\alpha}{2\,\pi}\, \frac{(\bar{w}^4 - 2\,\bar{w}^2\,(m_p^2 + m_K^2) + (m_p^2 - m_K^2)^2)^{1/2}}{(s - m_p^2)^2} \frac{m_K^2\,(t_- - t_+)}{(t_+ - m_K^2)\,(t_- - m_K^2)}\, \frac{d\sigma_{K^-\, p \to \pi\, Y}}{d\bar{t}}(\bar{w}^2,\, \bar{t}), \tag{12}$$

or

$$\frac{d\sigma_{\gamma\, p \to K^+\, \pi\, Y}}{d\bar{w}^2 d\bar{t}} = \frac{\alpha}{2\,\pi}\, \frac{|\vec{\bar{q}}_K|\, s^{1/2}}{(s - m_p^2)^3}\, 4\,|\vec{q}_{K^- p}|\sqrt{\bar{w}^2}\, \frac{d\sigma_{K^-\, p \to \pi\, Y}}{d\bar{t}}(\bar{w}^2,\, \bar{t}). \tag{13}$$

The nontrivial dynamical quantity of interest in Eq. (13) is clearly $4\,|\vec{q}_{K^- p}|\sqrt{\bar{w}^2}$ $\frac{d\sigma_{K^-\, p \to \pi\, Y}}{d\bar{t}}(\bar{w}^2,\, \bar{t})$.

To calculate this cross section we use the $\bar{K}p \to \pi Y$ amplitudes derived in Ref. [27] from the chiral SU(3) Lagrangian by solving coupled-channel Bethe-Salpeter equations. We will not repeat here the technical developments involved in this scheme. They are explained and discussed extensively in Ref. [27].

This effective field theory achieves an excellent description of the available data on $K^-\, p$ elastic (direct and charge-exchange) and inelastic ($\pi^0\, \Lambda$, $\pi^+\, \Sigma^-$, $\pi^0\, \Sigma^0$, $\pi^-\, \Sigma^+$) processes up to laboratory K^- momenta of the order of 500 MeV. The interest of the present work is to offer the possibility of testing the amplitudes below the $\bar{K}N$ threshold, in the region where they are dominated by the $\Lambda(1405)$ and the $\Sigma(1385)$. The specific spectral shape of these resonances is a particularly meaningful prediction of the description of Ref. [27]. As mentioned earlier, the shape of the $\Sigma(1385)$ resonance is expected to be close to a Breit-Wigner form. The $\Lambda(1405)$ resonance mass distribution has an asymmetric shape and depends on the initial and final states through which it is observed. The $\pi^0\, \Lambda$ production would also put strong constraints on the $\Sigma(1385)\, \bar{K}N$ coupling which is poorly known.

3 Numerical results

We present our numerical results in three different perspectives. We restrict our calculations to cross sections integrated over $\bar{t}$.

We show first the quantity $4\,|\vec{q}_{K^-p}|\sqrt{\bar{w}^2}\;\sigma_{K^-\,p\to\pi\,Y}(\bar{w}^2)$ as function of the total center of mass energy in the K^-p system, renamed for clarity $\sqrt{s_{K^-p}}$ ($\equiv \sqrt{\bar{w}^2}$). The interest of displaying our results this way is to exhibit the behaviour of the $K^-p \to \pi\,Y$ cross section across threshold. We recall that the $\bar{K}N$ threshold is at $\sqrt{s_{K^-p}} \approx 1.435$ GeV. Our predicted cross sections are displayed in Fig. 4 for the $K^-p \to \pi^-\Sigma^+$ and $K^-p \to \pi^+\Sigma^-$ reactions and in Fig. 5 for the $K^-p \to \pi^0\,\Sigma^0$ and $K^-p \to \pi^0\,\Lambda$ reactions. They are compared to the data available above threshold [39–44].

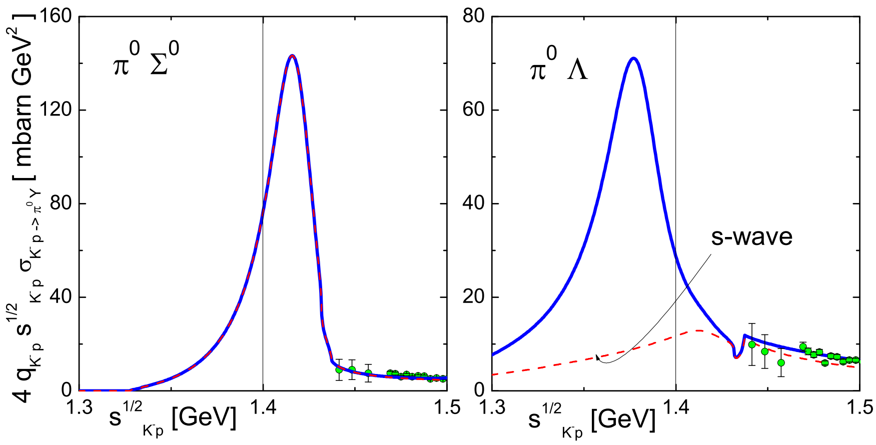

Fig. 4. $K^-p \to \pi^-\Sigma^+$ and $K^-p \to \pi^+\Sigma^-$ cross sections below and above threshold. The length of the K^- 3-momentum is defined by Eq. (7) and $\sqrt{s_{K^-p}}$ ($\equiv \sqrt{\bar{w}^2}$) is the total center of mass energy of the K^-p system. The dashed line represents the contribution from K^-p relative s-wave only. The grey histogram is explained in the text. The data above threshold are from Refs. [39–44].

We recall that the $\pi\,\Sigma$ channel is dominated by the $\Lambda(1405)$ and the $\pi\,\Lambda$ channel by the $\Sigma(1385)$. The properties of the spectral functions of the $\Sigma(1385)$ and $\Lambda(1405)$ resonances are very apparent in Figs. 4 and 5. The shape of the resonant behaviour of the $K^-p \to \pi^0\,\Lambda$ cross section below threshold is quite symmetric and close to a Breit-Wigner form. The s-wave contribution is small as expected for a process dominated by a p-wave resonance. In contrast, the spectral form of the $K^-p \to \pi\,\Sigma$ cross sections for the three possible $\pi\,\Sigma$ channels is asymmetric and largely given by s-wave dynamics, reflecting the $\Lambda(1405)$ dominance. The grey histograms show (in arbitrary units) the empirical shape of the $\Lambda(1405)$ resonance extracted from the $p\,(\gamma, K^+\pi)\,\Sigma$ reaction at 1.5-2.4 GeV photon energy [3]. It should be emphasized that strictly speaking the comparison implied by Fig. 4 is not yet justified. Only after extracting the double kaon pole contribution from the cross section by a detailed study of t-distributions can this comparison become legitimate. The result of such an analysis is expected to resolve the discrepancy of the histograms and the K^-p scattering data, at least above threshold. Nevertheless, the different line shapes of [3] seems to confirm the prediction of chiral coupled-channel dynamics that the spectral shape of the $\Lambda(1405)$ resonance depends crucially on the initial and final states it is probed with [27,45]. We note that the available K^-p scattering data close to threshold have large error bars, emphasizing the interest of being able to determine from experiment the subthreshold K^-p scattering amplitudes by extracting the double kaon pole contributions to the $\gamma p \to K^+\pi Y$ reactions calculated in this work.

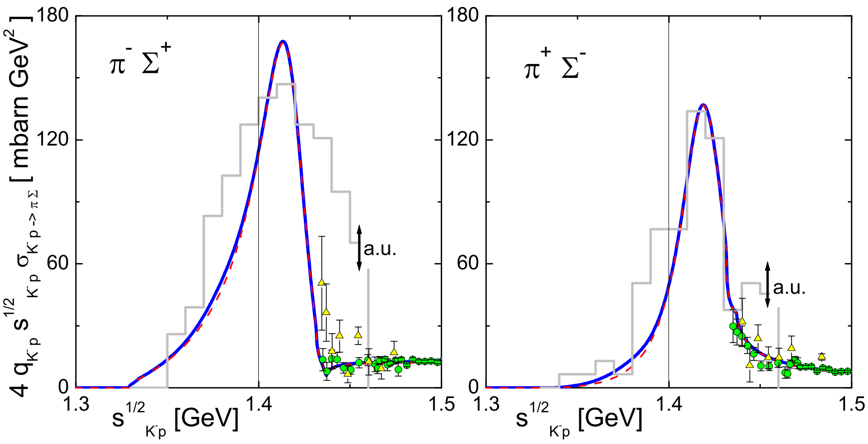

Fig. 5. Same as Fig. 4 for the $K^-\,p \to \pi^0\,\Sigma^0$ and $K^-\,p \to \pi^0\,\Lambda$ channels.

We display in Figs. 6 and 7 the double kaon pole term contributions to the differential cross sections for the $\gamma p \to K^+\pi^0\Lambda$ and $\gamma p \to K^+\pi\Sigma$ reactions as functions of the $\pi\,Y$ total center of mass energy $\sqrt{s_{K^-p}}$ at $E_\gamma = 1.7$ GeV and $E_\gamma = 2.1$ GeV.

They reflect clearly the dynamical features discussed in commenting on Figs. 4 and 5. It is also interesting to note the absolute values of the double kaon pole cross sections. They are large on the scale of what is expected from other theoretical approaches. If we compare our results to the predictions of the model of Ref. [45] at $E_\gamma = 1.7$ GeV, we notice that our calculated cross sections are roughly twice larger for the $\pi\,\Sigma$ channels. It is not easy to trace the origin of this effect. Our gauge-invariant double kaon pole term contains contributions which cannot be mapped easily onto the Feynman diagrams computed in Ref. [45]. The cross section we obtain for the $\pi^0\,\Lambda$ channel is

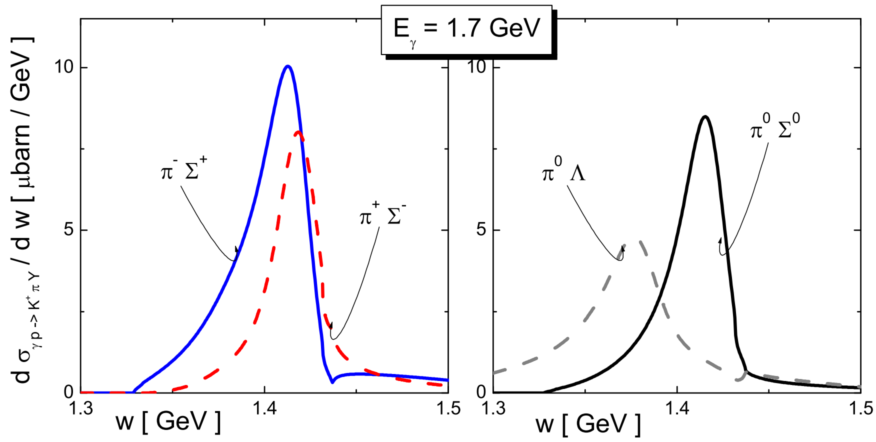

Fig. 6. Double kaon pole term contribution to the differential cross sections for the $\gamma p \to K^+\pi^0\Lambda$ and $\gamma p \to K^+\pi\Sigma$ reactions as function of the $\pi\, Y$ total center of mass energy at $E_\gamma = 1.7$ GeV

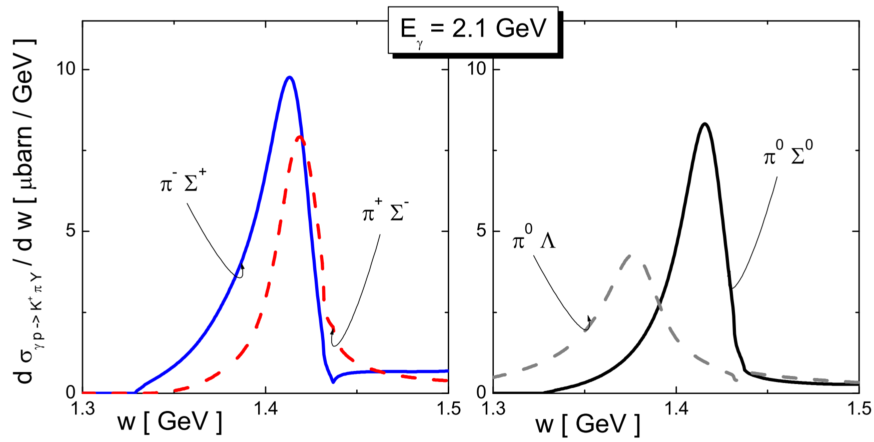

Fig. 7. Same as Fig. 6 at $E_\gamma = 2.1$ GeV

about an order of magnitude larger than the result displayed in Ref. [45]. A substantial part of this effect should be ascribed to the neglect of the $\Sigma(1385)$ resonance in that work.

Finally, we show in Fig. 8 the t-distribution of the double kaon pole term contribution to the $\gamma p \to K^+\pi^0\Sigma^0$ reaction at $E_\gamma = 2.1$ GeV, computed at different values of the $\pi^0\,\Sigma^0$ total center of mass energy. As mentioned in the introduction, this process is a unique signature of the $\Lambda(1405)$ resonance. It is a very nice test of the underlying dynamics of the $\Lambda(1405)$ photoproduction and a clean process to extract the double kaon pole from accurate data.

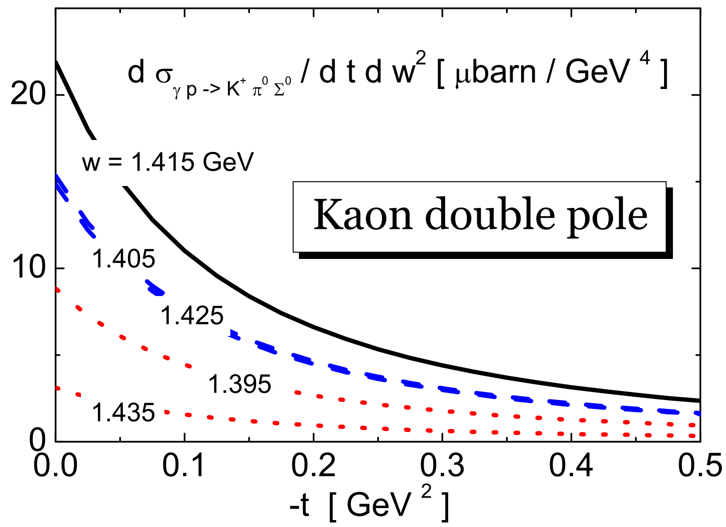

Fig. 8. Double kaon pole term contribution to the t-distribution for the $\gamma p \to K^+\pi^0\Sigma^0$ reaction at $E_\gamma = 2.1$ GeV, computed at different values of the $\pi^0\,\Sigma^0$ total center of mass energy.

4 Conclusion

We have studied the $\gamma p \to K^+\pi^0\Lambda$ and $\gamma p \to K^+\pi\Sigma$ reactions in the kinematic region where the $\pi^0\Lambda(1116)$ and $\pi\Sigma(1192)$ pairs originate dominantly from the decay of the $\Sigma(1385)$ and $\Lambda(1405)$ resonances. We focus on laboratory photon energies around 2 GeV, significantly above the threshold for producing the $K^+\,\Sigma(1385)$ (E_γ^{thresh}=1.41 GeV) and the $K^+\,\Lambda(1405)$ (E_γ^{thresh}=1.45 GeV) final states. We have calculated the t-channel K^--exchange contribution to these reactions using the $K^-p \to \pi Y$ amplitudes of Ref. [27], which have been shown to describe the data available at low kaon momentum. Based on the pole structure of this contribution, we determined the gauge-invariant double kaon pole contribution to the $\gamma p \to K^+\pi Y$ cross sections by calculating the residue at the pole. The relevance of our work stems from the advent of detector systems able to measure exclusively multiparticle final states with great accuracy. Three complementary experiments in the photon energy range considered in this paper are planned with LEPS at SPring-8 [3], SAPHIR at ELSA [4] and the Crystal Barrel at ELSA [2], dealing for the first two with the charged [$\pi^-\Sigma^+$ and $\pi^+\Sigma^-$] channels and for the latter with the neutral [$\pi^0\Sigma^0$ and $\pi^0\Lambda$] final states. These accurate measurements should make it possible to extract the contribution of the double kaon pole and hence to study kaon-nucleon dynamics below threshold.

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