Search for evidence of two photon contribution in elastic electron proton data.

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Abstract

We reanalyze the most recent data on elastic electron proton scattering. We look for a deviation from linearity of the Rosenbluth fit to the differential cross section, which would be the signature of the presence of two photon exchange. The present data does not show evidence for such deviation.

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I. INTRODUCTION

Form factors (FFs) characterize the internal structure of composite particles. Since the first measurements [1], electromagnetic probes are traditionally preferred to the hadronic beams, since the electromagnetic leptonic interaction is exactly calculable in QED, and one can safely extract the dynamical information about the hadronic vertex. Unfortunately, it is not a direct procedure, because, one has to introduce radiative corrections, which become very large for experiments with good energy resolution. Radiative corrections for elastic electron-hadron scattering were firstly calculated by Schwinger [2] and are important for any discussion of the experimental determination of the differential cross section. They are also calculable in QED, within some assumptions concerning the hadronic interaction. There are standard procedures applied to elastic eN scattering data, but question may arise if the necessary accuracy has been reached [3].

Assuming form factor scaling, $G_{Ep} = G_{Mp}/\mu_p$, where G_{Ep} and G_{Mp} are the electric and magnetic proton FFs and $\mu_p = 2.79$ is the magnetic moment of the proton, G_{Mp} has been extracted up to a value of the four momentum squared, $Q^2 \simeq 31$ GeV^2 [4] and fall with Q^2 according to a dipole form:

$$G_D(Q^2) = (1 + Q^2[\mathrm{GeV}^2]/0.71)^{-2}. \tag{1}$$

Recently, new developments, due to the very precise and surprising data obtained at the Jefferson Laboratory (JLab), in $\vec{e}+p \to e+\vec{p}$ elastic scattering [5, 6], based on the polarization transfer method show that the electric and magnetic distributions in the proton are different.

The application of the polarization transfer method, proposed about 30 years ago, [7] has been possible only recently, as it needs high intensity polarized beams, large solid angle spectrometers and advanced techniques of polarimetry in the GeV range. Experiments have been performed at JLab up to $Q^2 = 5.6$ GeV^2 and an extension up to 9 GeV^2 is in preparation [8].

The following parametrization for the ratio R of the electric and magnetic form factors well describes these experimental data [9]:

$$R = \mu_p G_{Ep}/G_{Mp} = 1 - 0.13(Q^2\ [\mathrm{GeV}^2] - 0.04) \tag{2}$$

which implies that the ratio monotonically decreases and deviates from unity, as Q^2 increasing, reaching a value of $\simeq 0.3$ at $Q^2 \simeq 5.5$ GeV^2.

Therefore, a clear discrepancy appears between the Q^2-dependence of the ratio R of the electric to the magnetic proton form factors, whether derived with the standard Rosenbluth separation or with the polarization transfer method. This statement is confirmed by a reanalysis of the existing data [10] and by recent measurements [11, 12]. This discrepancy is very puzzling, as no evident experimental bias has been found in the data or in the method used, and it has been sources of different speculations.

One has to stress, at this point, that FFs extracted with the polarization method, are not incompatible with the measured cross section: it has been shown that, constraining the ratio R from polarization measurements and extracting G_{Mp} from the measured cross section, leads to a renormalization of 2-3% only, with respect to the Rosenbluth data [9], well inside the error bars.

Instead, the problem is related to the slope of the ϵ dependence of the reduced cross section, (ϵ is the polarization of the virtual photon, $\epsilon = [1 + 2(1+\tau)\tan^2(\theta/2)]^{-1}$) which is directly related to G_{Ep}. The difference of such slope especially appears with respect to the last, precise data [12].

A possible question arises on the validity of the one-photon mechanism at large Q^2, and, generally, on the radiative corrections to the differential cross section and to polarization observables in elastic eN-scattering. If these corrections are large (in absolute value) for the differential cross section [13], in particular for high resolution experiments, a simplified estimation of the radiative corrections to polarization phenomena [14] shows that they are small for the ratio P_L/P_T of longitudinal to transverse polarization of the proton emitted in the elastic collision of longitudinally polarized electrons with an unpolarized proton target.

In the standard calculations of the radiative corrections [13], the two-photon exchange mechanism is only partially taken into account considering the special part of the complicated loop integral, where one virtual photon carries all the momentum transfer and the second virtual photon is almost real. This contribution allows to overcome the problem of the 'infrared' divergence. But it has been pointed out [15] that, at large momentum transfer, the role of another part of the integral, where the momentum transfer is shared between the two photons, can be relatively increased, due to the steep decreasing of the electromagnetic form factors with Q^2. This effect can eventually become so large (especially at large Q^2) that the traditional description of the electron-hadron interaction in terms of electromagnetic currents (and electromagnetic form factors) can become incorrect.

Numerous tests of the possible 2γ contribution for elastic ep scattering have been done in the past, using different methods: test of the linearity of the Rosenbluth formula for the differential cross section, comparison of the e^+p and e^-p-cross sections, attempts to measure various T-odd polarization observables, but no effect was visible beyond the precision of the experimental data. Only recently the non-zero T-odd asymmetry in the scattering of transversally polarized electrons, $\vec{e}^- + p \to e^- + p$ has been detected [16].

Note that the two-photon exchange should appear at smaller Q^2 for heavier targets: d, ${}^3He$, ${}^4He$, because the corresponding form factors decrease faster with Q^2 in comparison with protons. From these considerations, one would expect to observe the two-photon contribution in eN-scattering at larger momentum transfer, for $Q^2 \simeq 10$ GeV^2. In Ref. [17] the possible effects of 2γ-exchange have been estimated from the precise data on the structure function $A(Q^2)$, obtained at JLab in electron deuteron elastic scattering, up to $Q^2 = 6$ GeV^2 [18, 19]. The possibility of a 2γ-contribution has not been excluded by this analysis, starting from $Q^2 = 1$ GeV^2, and the necessity of dedicated experiments was pointed out.

No complete calculations has been done yet, but on the basis of ad-hoc assumptions, one can - semiempirically- reconcile the data, assuming that the radiative corrections are negligible for polarization data, and contribute to the cross section linearly in ϵ [20, 21], or in framework of other model-dependent assumptions [22].

An analysis of the existing data on electron and positron elastic scattering on proton, seems to show that such effect would be more pronounced at low Q^2-value [23], adding confusion to a possible evidence of a two-photon contribution.

We previuosly showed [17, 24–26], in a model independent way, that the presence of two-photon exchange destroys the linearity of the Rosenbluth fit, inducing a specific dependence of the differential cross section on the variable ϵ, which is especially large for $\epsilon \leq 1$.

The purpose of this paper is to re-analyse the data about the differential ep scattering, with respect to the deviation of linearity of the Rosenbluth fit, according to this predicted dependence.

II. PARAMETRIZATION OF THE 2γ CONTRIBUTION

The traditional way to measure electromagnetic proton form factors consists in the measurement of the ϵ dependence of the reduced elastic differential cross section σ_{red}, at fixed Q^2. Assuming that the interaction occurs through the exchange of one photon, one can write:

$$\sigma_{red}(Q^2,\epsilon)=\epsilon(1+\tau)[1+(2E/m)\sin^2(\theta/2)]\frac{1}{\sigma_{Mott}}\frac{d\sigma}{d\Omega}=\tau G_{Mp}^2(Q^2)+\epsilon G_{Ep}^2(Q^2), \quad (3)$$

with $\tau=Q^2/(4m^2)$, m is the proton mass, E and θ are the incident electron energy and the scattering angle of the outgoing electron, respectively.

The Rosenbluth separation [27] allows to extract the electric and the magnetic form factors, from a linear fit to the reduced cross section, as a function of ϵ, at a fixed Q^2. The slope is directly related to G_{Ep} and the intercept to G_{Mp}. However, due to the coefficient τ, the weight of the magnetic contribution becomes larger, as the momentum transfer increases, reducing the sensitivity of the elastic cross section to the electric contribution. As an example, at $Q^2 \simeq 4$ GeV^2, the term related to the electric FF contributes for less than 10% to the reduced cross section, assuming the dipole scaling, whereas it would be as low as 2%, if one assumes the Q^2 dependence of R from Eq. 2. One can estimate the level of precision required to extract the electric FFs from the experimental cross section at large Q^2.

In presence of 2γ exchange, Eq. (3) can be rewritten in the following general form:

$$\sigma_{red}(Q^2,\epsilon)=\epsilon G_E^2(Q^2)+\tau G_M^2(Q^2)+\alpha F(Q^2,\epsilon), \quad (4)$$

where $\alpha=e^2/(4\pi)$, and $F(Q^2,\epsilon)$ is a real function (of both independent variables Q^2 and ϵ), describing the effects of the $1\gamma\bigotimes 2\gamma$ interference.

Note that, in the general case, $F(Q^2,\epsilon)$ contains two different contributions, one due to the effect of the two-photon contribution on the form factors $G_{E,M}$, and another one due to the third spin structure in the matrix element of eN scattering, induced by 2γ exchange. Both contributions can be calculated only in framework of some model, considering different intermediate states for the 2γ box diagrams.

In such situation, any model independent statement concerning the function $F(Q^2,\epsilon)$ is important, in particular concerning its ϵ dependence, due to the large sensitivity of the extraction of G_E^2 to the additional contribution $F(Q^2,\epsilon)$, at large Q^2.

We proved earlier [24], that the C-invariance and the crossing symmetry of hadron electromagnetic interaction, result in the following symmetry properties of $F(Q^2, \epsilon)$ with respect to the variable $x = \sqrt{\frac{1+\epsilon}{1-\epsilon}}$:

$$F(Q^2, x) = -F(Q^2, -x). \tag{5}$$

This means that the $1\gamma \bigotimes 2\gamma$ contribution $F(Q^2, \epsilon)$ has an essential non linear ϵ dependence, at any Q^2. For example, the additional (third) spin structure is generating the following ϵ-dependence:

$$F(Q^2, x) \to \epsilon\sqrt{\frac{1+\epsilon}{1-\epsilon}} f^{(T)}(Q^2, \epsilon) \text{ or } F(Q^2, x) \to \sqrt{\frac{1+\epsilon}{1-\epsilon}} f^{(A)}(Q^2, \epsilon), \tag{6}$$

where the upper index, (T) or (A), corresponds to the tensor [25] or axial [26] parametrization for the third amplitude.

In order to estimate the upper limit for a possible 2γ contribution to the differential cross section and the corresponding changing to $G_{E,M}(Q^2)$, we analyzed four sets of data [3, 11, 12, 28], applying Eq. (4) with the following parametrization for $F(Q^2, x)$:

$$F(Q^2, x) \to \epsilon\sqrt{\frac{1+\epsilon}{1-\epsilon}} f^{(T)}(Q^2). \tag{7}$$

It is important to stress that Eq. (7) is a simple expression which contains the necessary symmetry properties of the $1\gamma \bigotimes 2\gamma$ interference, through a specific (and non linear) ϵ dependence.

We checked that the parametrization:

$$F(Q^2, x) \to \sqrt{\frac{1+\epsilon}{1-\epsilon}} f^{(A)}(Q^2)$$

gives qualitatively similar results.

For the Q^2 dependence of $f^{(T,A)}$ we take:

$$f^{(T,A)}(Q^2) = \frac{C}{(1 + Q^2[\text{GeV}]^{\ 2}/0.71)^2(1 + Q^2[\text{GeV}]^2/m_{T,A}^2)^2}, \tag{8}$$

where C is a fitting parameter, $m_{T,A}$ is the mass of a tensor or vector meson with positive C-parity. For $m_{T,A} \simeq 1.5$ GeV (typical value of the corresponding mass), one can predict that the relative role of the 2γ contribution should increase with Q^2.

III. RESULTS AND DISCUSSION

In presence of 2γ, the dependence of the reduced cross section con ϵ can be parametrized as a function of three parameters, G_E^2, G_M^2 and C, according to Eqs. (4) and (8).

In Fig. 1, we show σ_{red} as a function of the variable ϵ for different Q^2 values, together with the results of the three parameters fit, for the data from Ref. [28]. Fits of similar quality can be obtained for all the considered sets of data.

In Fig. 2, from top to bottom, the electric and magnetic FFs, as well as the two photon parameter C, are shown as a function of Q^2 (solid symbols). The previously published data, derived from the traditional Rosenbluth fit are also shown (open symbols).

The numerical values of FFs and of the 2γ-coefficient for the data set analyzed here are reported in Table I. The resulting parameter C is compatible with zero. The effect of including a third fitting parameter, the 2γ term, is to increase the error on FFs. For the three points of Ref. [11], at higher Q^2, the coefficient C becomes quite large, but is still compatible with zero within the error. For this reason, these points are reported in the table but not in Fig. 2.

The present analysis does not give any evidence of a 2γ contribution, as the possible 2γ term is always vanishes, within the error bars. The new values of G_E and G_M are compatible with the published values, deduced in frame of a standard ϵ fit. Furthermore, there is no systematic effect that could allow, at least, to guess the sign of the 2γ coefficient. If one could determine at least the sign of the interference contribution, it would be possible to predict the relative value of the differential cross section for e^-p and e^+p-scattering. The following relation holds:

$$\frac{\sigma(e^+p)-\sigma(e^-p)}{\sigma(e^+p)+\sigma(e^-p)}=\frac{\alpha F(Q^2,\epsilon)}{\epsilon G_E^2(Q^2)+\tau G_M^2(Q^2)}.$$

Another approach - introducing $ad-hoc$ a linear ϵ contribution to the differential cross section - results in a corresponding decreasing of the G_E^2 contribution, bringing the results in agreement with the polarization measurements. This can not be considered a proof of the presence of the 2γ contribution, but shows that a small correction to the slope of the reduced cross section is sufficient to solve the discrepancy. Such correction may arise, for example, from a revision of the standard procedure of the calculation of radiative corrections, especially at $\epsilon\to 1$, which would produce a large effect on the value of G_E^2, as extracted from the Rosenbluth fit.

$Q^2[GeV^2]$	$\frac{G_E}{G_D} \pm \Delta\left(\frac{G_E}{G_D}\right)$	$\frac{G_M}{G_D} \pm \Delta\left(\frac{G_M}{G_D}\right)$	$C \pm \Delta C$	Ref.
1	0.7± 0.4	0.11± 0.04	2.6± 3.0	[3]
2	1.1± 0.2	0.1± 0.03	0.68± 1.2	[3]
2.5	0.7± 0.4	1.06± 0.03	2.5± 1.5	[3]
3	1.1± 0.5	1.02± 0.07	1.5± 3.9	[3]
1.75	1.0± 0.1	1.05± 0.01	-1±2	[28]
2.5	0.5± 0.3	1.07± 0.02	2±3	[28]
3.25	1.0± 0.3	1.04± 0.03	-3± 6	[28]
4.0	0.8± 0.3	1.04± 0.03	1± 3	[28]
5.0	0.9± 0.4	1.02± 0.05	0.1± 5	[28]
0.65	1.0± 0.2	0.98± 0.03	0.4± 2	[11]
0.9	1.1± 0.1	0.99 ± 0.02	-2± 2	[11]
2.2	1.2 ±0.3	1.03 ± 0.04	-3± 3	[11]
2.75	0.8 ±0.3	1.06 ± 0.03	0.6± 4	[11]
3.75	1.6 ±0.5	1.0 ±0.1	-17± 15	[11]
4.25	1.8± 0.6	1.0 ±0.2	-19± 26	[11]
5.2	4.1± 2.3	0.8 ±1.5	-166± 195	[11]
2.64	0.9 ±0.1	1.05 ±0.01	0.4± 2	[12]
3.2	0.8± 0.2	1.05 ±0.02	4± 4	[12]
4.1	0.99± 0.4	1.03 ±0.05	5± 12	[12]

TABLE I: Form factors and 2γ-coefficient, Eq. (8).

The radiative corrections are taken into account in the data analysis, according to the prescription of Ref. [13]. Typically, the experimental results on the measured elastic cross section, are corrected by a global factor δ_R:

$$\sigma^{red} = \sigma^{red}_{meas} e^{\delta_R} \simeq \sigma^{red}_{meas}(1+\delta_R). \tag{9}$$

This factor *contains a large ϵ dependence*, and a smooth Q^2 dependence, and it is common for the electric and magnetic part. At the largest Q^2 considered here, it can reach 30-40%. A linear ϵ dependence can be considered a good approximation for the radiative corrections:

$\delta_R = \epsilon\delta'$, but our conclusions hold for any monotonically increasing function.

Therefore, Eq. (3) can be rewritten as:

$$\sigma^{red}_{meas}(Q^2,\epsilon) = \sigma^{red}(Q^2,\epsilon)[1-\delta_R(Q^2,\epsilon)] = G_M^2(Q^2)\left(\tau + \epsilon\frac{G_E^2(Q^2)}{G_M^2(Q^2)}\right)[1-\epsilon\delta'(Q^2)]$$
$$= G_M^2(Q^2)\left[\tau + \epsilon\left(\frac{G_E^2(Q^2)}{G_M^2(Q^2)} - \tau\delta'(Q^2)\right)\right]. \tag{10}$$

At a fixed Q^2 (dropping the Q^2-dependence) we can see that the slope of the measured cross section has also a linear behavior with ϵ, with an effective slope a:

$$a = \frac{G_E^2}{G_M^2} - \tau\delta'. \tag{11}$$

This shows that the slope of the *measured* reduced cross section, as a function of ϵ (which is related to the electric FF squared), can vanish, and, for $\delta' \geq \dfrac{G_E^2}{\tau G_M^2}$ can even be negative. In Fig. 3 we show σ_{red} as a function of ϵ, with and without radiative corrections. One can see that the slope is compatible with zero, for the uncorrected data, starting from $Q^2 \leq 2$ GeV^2, and, then, becomes negative.

The extraction of G_E from the data requires a large precision in the calculation of radiative corrections, and in the procedure used to apply to the data. In particular at large ϵ, an overestimate of the radiative corrections, can be source of a large change in the slope.

A similar study has been done from the other sets of data and leads to similar results.

IV. CONCLUSIONS

From this analysis it appears that the available data on *ep* elastic scattering does not show any evidence of deviation from the linearity of the Rosenbluth fit, and hence of the presence of the two photon contribution, when parametrized according to Eq. (7).

A careful study of radiative corrections when applied to the experiment seems necessary. Radiative corrections at large Q^2 are huge, and have a large influence on the slope of the reduced cross section, changing even its sign.

The new generation of experiments performed at large Q^2 makes use of large acceptance detectors and requires huge corrections of the raw data for acceptance and efficiency. The factor of radiative corrections is obtained after integration over the acceptance. It appears as a global factor on the reduced cross section, equally affecting the electric and the magnetic term. Its (important) ϵ dependence is the main factor determining the slope of the reduced cross section, which is directly related to the extracted electric form factor.

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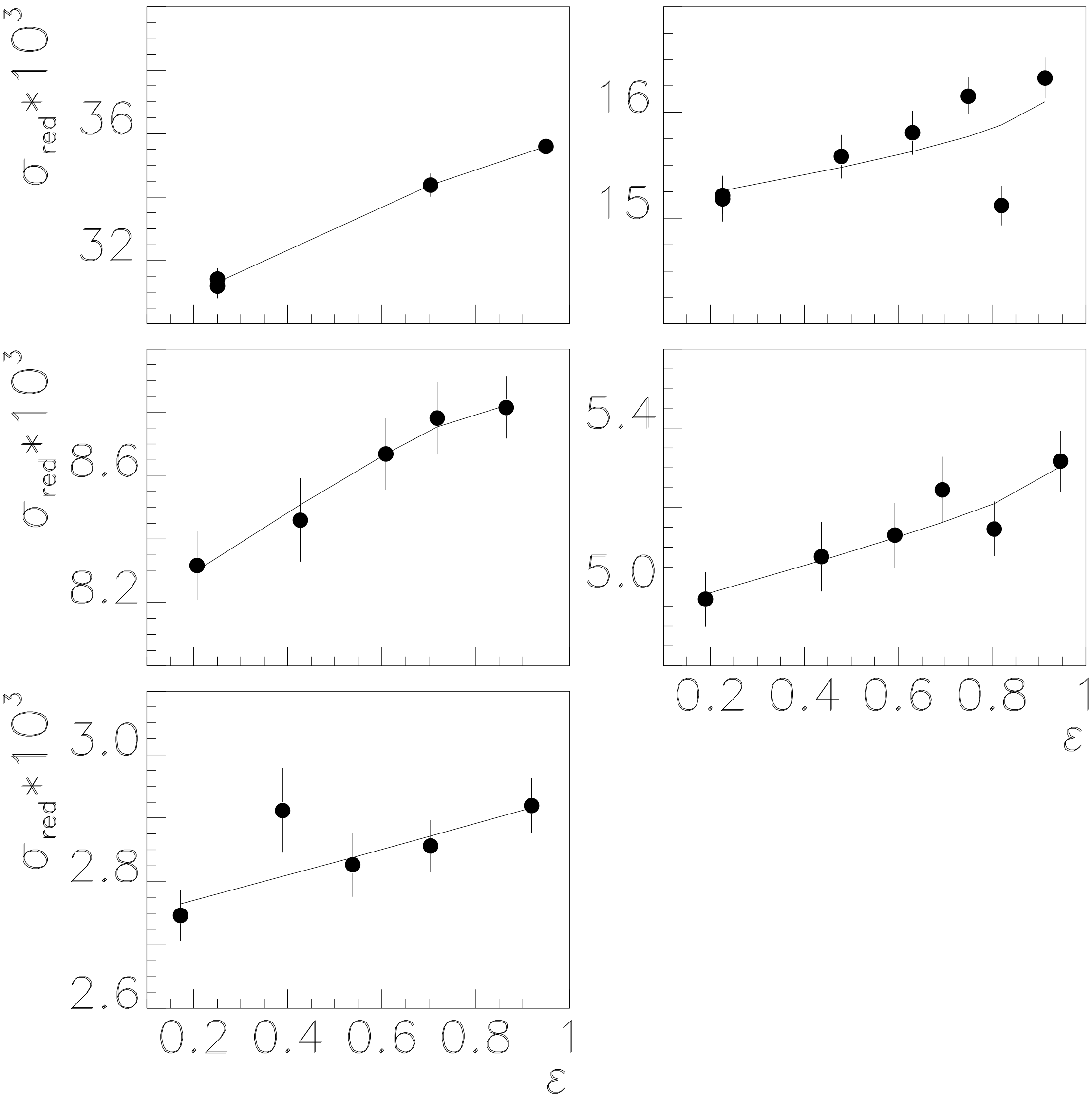

FIG. 1: Reduced cross section from at Q^2=1.75, 2.5, 3.25, 4, and 5 GeV^2. Data are from [28]. The lines are three-parameters fits according to 4.

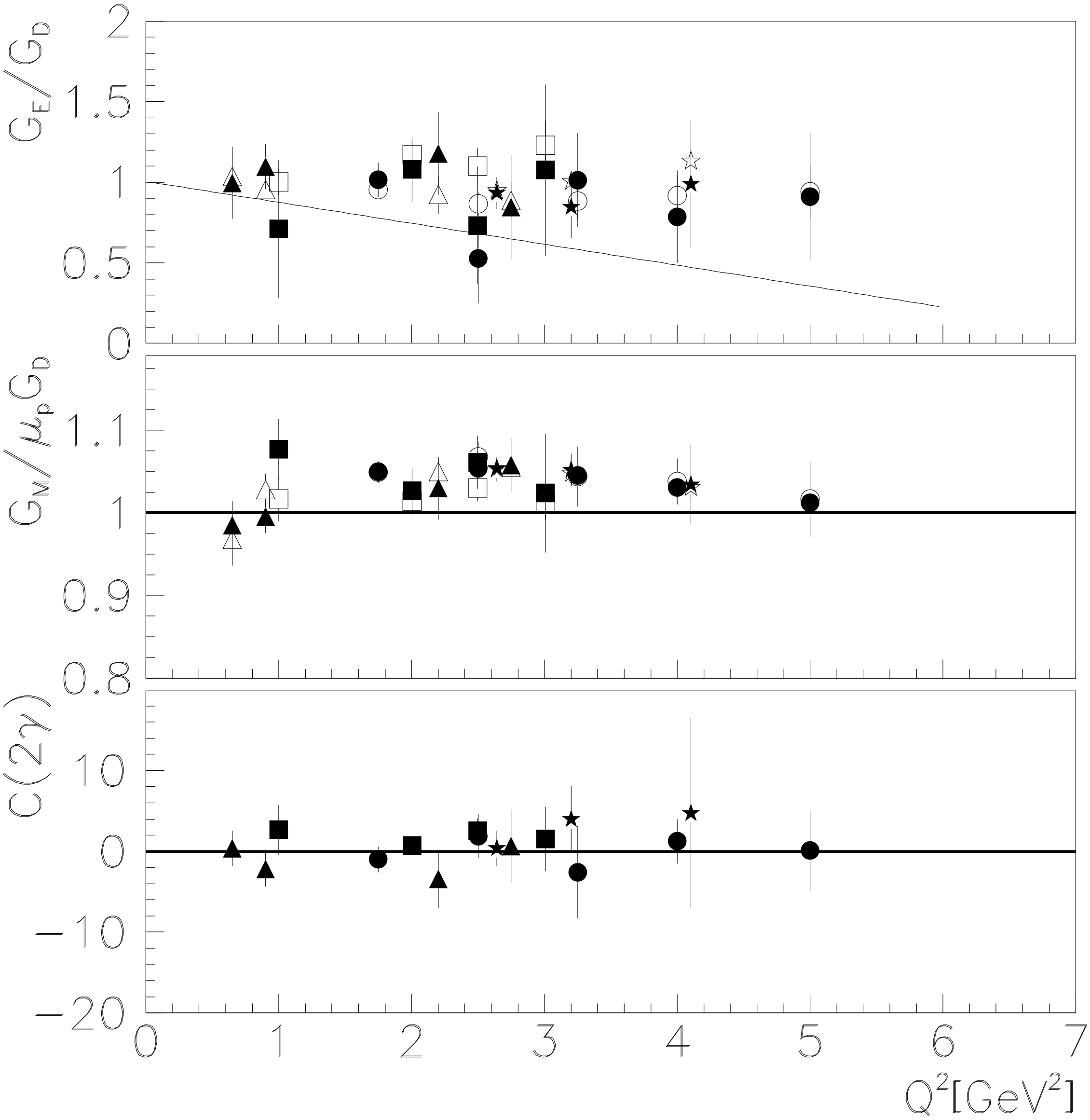

FIG. 2: From top to bottom: G_{Ep}/G_D, $G_{Mp}/\mu G_D$, and two-photon contribution, C according to Eq. (4) with $m_{t,A} = 1.5$. Data are from [3] (squares);[28] (circles); [11] (triangles) (the three points at the highest Q^2 are not shown, due to the large error bar); [12] (stars). The published data, from the traditional Rosenbluth fit are shown as open symbols, the results including the 2γ contribution as solid symbols.

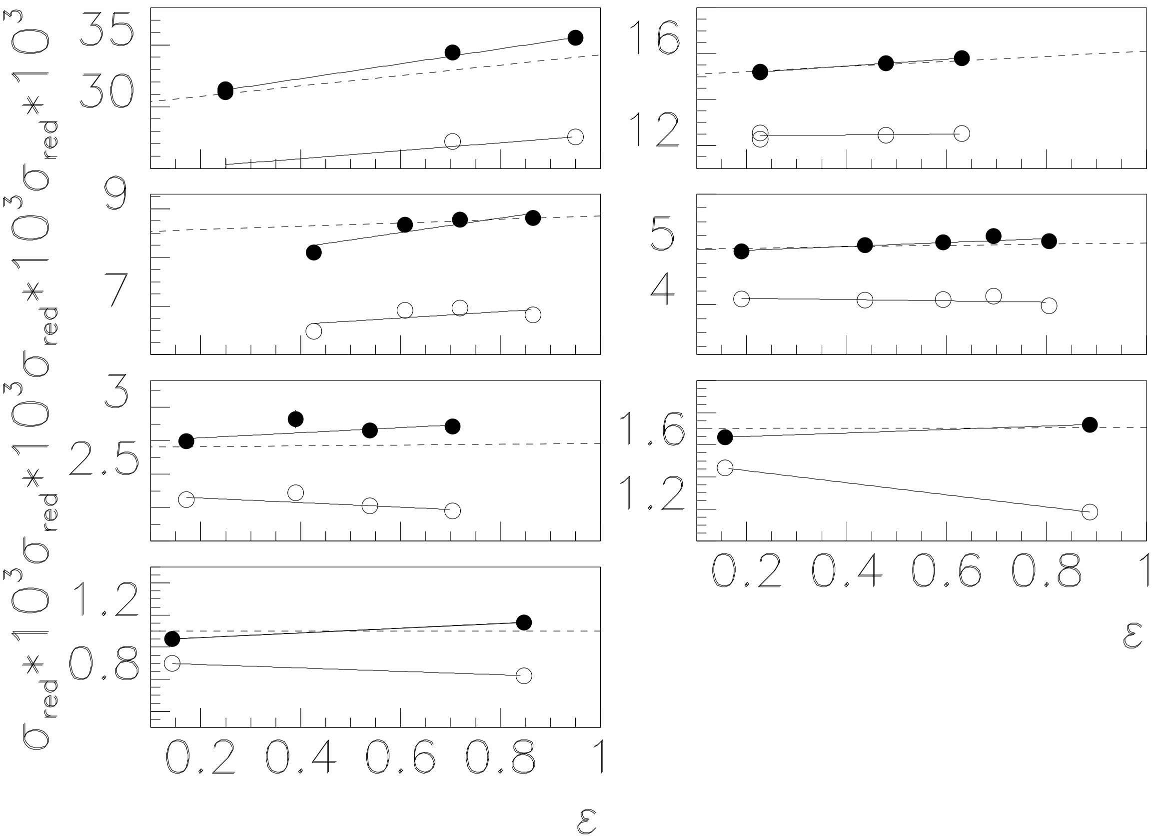

FIG. 3: Reduced cross section with (solid circle) and without (open circles) radiative corrections, for Q^2=1.75, 2.5, 3.25, 4, 5, 6, and 7 GeV^2. Data are from [28]. Two-parameters linear fits are shown as solid lines. The dashed lines show the slopes suggested by the polarization data.