Fixed Field Alternating Gradient Synchrotrons

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Abstract. This paper gives a brief history and a general description of fixed field alternating gradient (FFAG) accelerators.

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Introduction

The concept of fixed field alternating gradient (FFAG) accelerators dates from the early 1950's [1]. They were seen as a way to apply the principles of strong focusing and synchrotron stability and yielded high intensity machines, at a time where fixed orbit strong focusing synchrotrons eventually took over, while cyclotrons were limited to lower energies.

Only 5 FFAGs have been constructed and operated up to now :
- 3 electron machines in the 1950's, by MURA (Midwestern Universities Research Association), which saw many tasks of accelerator physics first tackled [1],
- 2 proton machines by KEK recently, in a context of determining technological progress regarding magnetism, acceleration and other beam manipulation equipments.

Nevertheless, FFAGs have regularly been proposed as an alternative solution to Linac, RCS and other cyclotron, for the production of proton beams. More recently, the neutrino factory studies triggered strong R&D activity in the field, and on the other hand the emergence of new concepts as well as modern technologies have revived the interest in the method, and pushed to re-exploring potential applications [2].

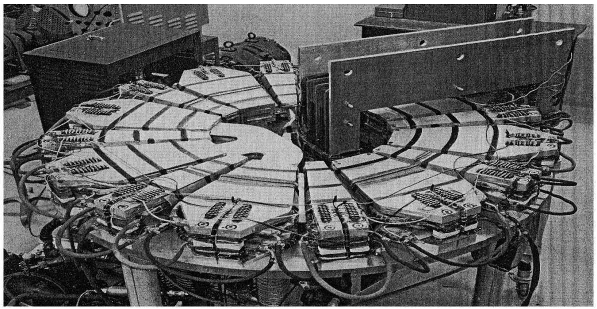
FFAGs are one of the most active fields in accelerator research today, with 9 workshops from Dec. 1999 till Oct. 2004.

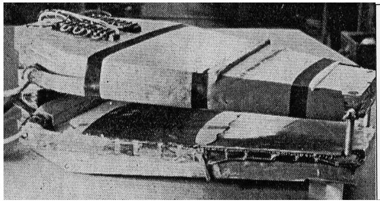
All these aspects will be addressed in the following, briefly though. A large amount of References will however, we hope, be of some help to the interested reader for digging into the subject.

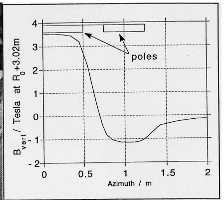
The MURA electron FFAGs

First model, radial sector FFAG, Mark II

Work on "Mark II" began in 1955, 2 years after the invention of the concept. The machine (photos below) [3] was first operated in March 1956, at the University of Michigan. The model was to be a proof of the FFAG principle, it eventually had a rich history of demonstrating experiments regarding effects of resonances, RF acceleration, beam stacking, RF KO, etc. The magnetic field is by principle fixed in time, with mid-plane form $B(r,\theta) = B_0(\frac{r}{r_0})^K \mathscr{F}(\theta)$ ($K > 0$ a constant, r_0 a reference radius, $\mathscr{F}(\theta)$ an axial form factor) (see plot below), fast increasing with radius, from lower energy, larger gap, on inner orbit (where the beam is injected) to largest energy, smallest gap, on outer orbit.







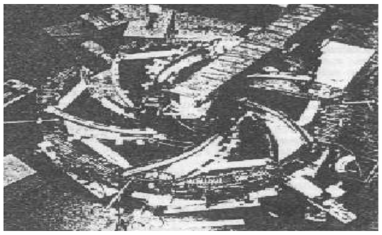
Part of the r^K shape of B is due to gap size decreasing with r, and the rest to coil winding arrangement thus allowing K (and hence tunes) to be varied. The ring is built from an alternance (hence the $\mathscr{F}(\theta)$ form factor) of positive dipoles which yield radial focusing ($\frac{\rho(s)}{B(s)}\frac{dB}{d\rho} > 0$) and shorter, negative dipoles which yield radial defocusing ($\frac{\rho(s)}{B(s)}\frac{dB}{d\rho} < 0$), thus insuring AG strong focusing. The radial dependence $B = B_0(r/r_0)^K$ determines the "scaling" property (also known as the "zero-chromaticity condition") : tunes are independent of the orbit (hence, of energy), closed orbits are similar wrt. geometrical center (they have a scalloped shape, due to the alternating curvature). Series of basic properties ensue, like a large circumference factor $\mathscr{C}/2\pi\rho$, momentum compaction $\alpha = 1/(1+K)$, $\gamma_{tr} = \sqrt{1+K}$

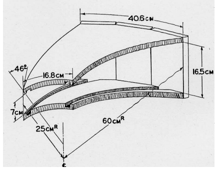
easily put beyond top energy, feasibility of arbitrary RF programs : no need to track B, and so forth. In the linear approximation the motion about a closed orbit satisfies Hill's equations $x'' + \frac{1-n}{\rho^2}x = 0$, $z'' + \frac{n}{\rho^2}z = 0$ with $n(s) \approx -\frac{\rho}{B}\frac{dB}{dr} = -K/\mathscr{C}$, thus amenable to regular optical treatment, working point gymnastics, defect analysis, etc. The longitudinal motion in presence of RF obeys, as in synchrotrons, $\ddot{\phi} + \frac{\Omega^2}{\cos\phi_s}(\sin\phi - \sin\phi_s) = 0$. The Table below gives the main parameters of Mark II.

$E_{inj} - E_{max}$	*keV*	25 - 400
orbit radius	*m*	0.34 - 0.50
lattice		$\mathbf{\frac{D}{2}F\frac{D}{2}}$
number of cells		8
field index K, tunable		≈3.4
ν_r / ν_z, tunable		2.2-3 / 1-3
Magnet		radial sector
F, D sectors	*deg*	25.74, 10.44
gap, max.-min.	*cm*	6 - 4
Injection	continuous or pulsed	
Acceleration	betatron core, at first, ...	
swing	*Gauss*	40 - 150
rep. rate	*Hz*	a few 10's
		... RF, next
freq. swing	*MHz*	10 in [35, 75]
gap voltage	*V*	50

Second model, spiral sector FFAG, Mark V

Work on "Mark V" began in 1955, a year after the invention of the concept. The machine (photo below) [4] was first operated in August 1957 in the MURA Lab., Madison. Objectives were to validate the strong focusing spiral optics with its advantage of a smaller circumference, and perform beam physics, accelerator studies.





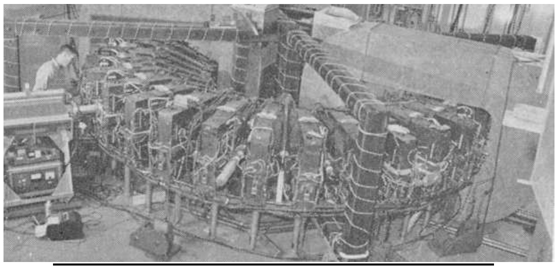
Magnet yoke

The idea in the spiral FFAG was to superpose a positive field on top of the alternating sign of the radial sector case, so as to always have the right curvature and hence decrease the circumference factor, which yields the "Thomas focusing" of cyclotrons. Yet by doing so the vertical focusing is weakened and needs be recovered by spiraling the poles. Appropriate field form for insuring scaling property and constant closed orbit to spiral edge angle, is $B(r,\theta)|_{z=0} = B_0\,(r/r_0)^K\ \mathscr{F}\left((\ln\frac{r}{r_0})/w - N\theta\right)$, The axial modulation $\mathscr{F}$ is called the "flutter", it has the approximate form $\mathscr{F} = 1 + f\sin(\ln\frac{r}{r_0}/w - N\theta)$. Expansion of the equations of motion around the closed orbit in the linear approximation (or as well a hard edge matrix model) yields the tunes $\nu_r \approx \sqrt{1+K}$, $\nu_z \approx \sqrt{-K + (f/Nw)^2/2}$. The Table below gives the main parameters of Mark V.

$E_{inj} - E_{max}$	*keV*	35 - 180
orbit radius	*m*	0.34 - 0.52
number of sectors		6
field index K, tunable		0.7
flutter F_{eff}, rms		1.1
ν_r / ν_z, tunable		1.4 / 1.2
β_r / β_z	*m*	0.45-1.3 / 0.6-1.4
Magnet		spiral sector
edge/radius angle, $Atg(Nw)$	*deg*	46
$r_{min} - r_{max}$	*m*	0.25 - 0.61
gap, max.-min.	*cm*	16.5 - 7
Injection	continuous or pulsed	
Acceleration	betatron cores & RF	
RF voltage	*V*	150

A 50 MeV, two-way, electron FFAG

Work on the 50 MeV electron FFAG began in 1957 [5]. The machine (photo below) was first operated in 1959 with two 27 MeV beams stored in opposing directions, as made possible by the radial sector optics using identical dipoles in a FODO arrangement. 51 MeV energy, one-way, was reached in 1960 after modifications in the magnets. Colliding beams, once envisaged, a hot task in the mid-50's, need intensity, RF stacking allowed it, 10 amperes intensity was obtained that way. The Table above gives the FFAG parameters.

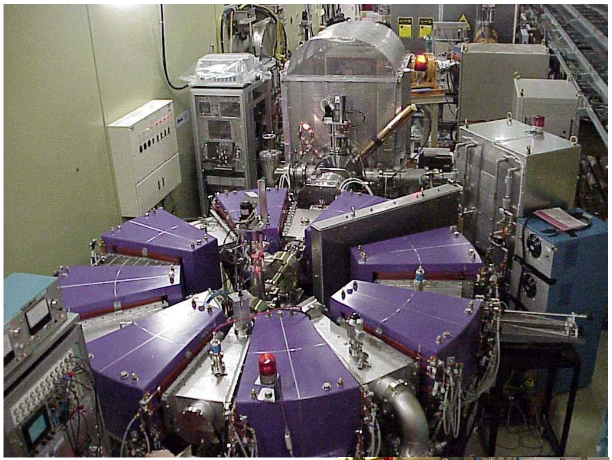


$E_{inj} - E_{max}$	*MeV*	0.1 - 50
orbit radius	*m*	1.20 - 2.00
lattice		FODO
number of cells		16
K		9.25
ν_r / ν_z		4.42 / 2.75
Magnet		radial sector
sector angle	*deg*	6.3
peak field	*T*	0.52
gap, max.-min.	*cm*	8.6 - 8.0
Acceleration	betatron & RF	
betatron range	*MeV*	0.1 - 2
RF swing	*MHz*	20 - 23
voltage p-to-p	*kV*	1.3 - 3
cycle rep. rate	*Hz*	60

The KEK proton machines

POP

KEK POP (proof of principle) machine (photos below) [6] is the first proton FFAG, first operated in 2000.



		POP	**150 MeV**
$E_{inj} - E_{max}$	MeV	0.05 - 0.5	12 - 150
orbit radius	m	0.8 - 1.14	4.7 - 5.2
lattice		DFD	
number of cells		8	12
K		2.5	7.6
β_r / β_z max.	m	0.7 / 0.7	3.8 / 1.3
ν_r / ν_z		2.2 / 1.25	3.7 / 1.2
Magnet		radial sector	
D, F sectors	deg	2.8 / 14	3.43 / 10.24
B_D/B_F max-min	T	.04-.13/.14-.32	.3-.8 / .5-1.6
gap, max.-min.	cm	30 - 9	20 - 4
Acceleration			
swing	MHz	0.6 - 1.4	1.5 - 4.6
voltage p-to-p	kV	1.3 - 3	19
cycle time	ms	1	4
rep. rate	Hz	10^3	250
$\dot{B}$ equivalent	T/s	180	280

Its design has strongly benefited from modern magnet computation tools and sophisticated tracking codes. The DFD lattice allows comfortable drifts, it is based on a radial sector triplet (two negative dipoles at both ends and a larger, positive one in between, in a common yoke, see photo) with gap shape $g_0(r_0/r)^K$ producing the scaling radial field dependence $B_0(r/r_0)^K$. The acceleration uses high gradient, broad band "FINEMET" technologies yielding a narrow cavity (see photo) and a potential 1 kHz rep. rate [7]. Injection is on inner radius, via an electrostatic inflector, either single-turn (using a chopper in the injection line) or multi-turn (using two bump electrodes). Tunes are adjustable via the B_F/B_D ratio. The Table above, col. "POP", gives the main parameters of the POP FFAG.

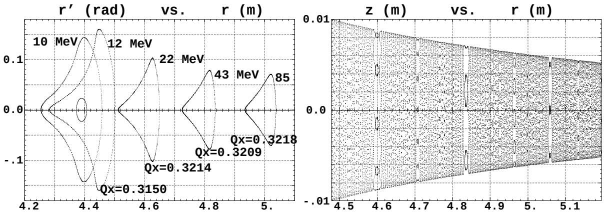
150 MeV proton FFAG

This second, higher energy, proton FFAG was first operated in 2003 [8]. The structure is similar to POP, it uses a 10 MeV cyclotron injector.

One distinguishing feature is the return-yoke free dipole triplet (see lower-left corner in the photo) which facilitates beam injection and extraction. Due to the extending fringe fields and to saturation effects, the zero-chromaticity condition is not fully fulfilled, so that tunes slightly vary over the energy span. The project has various goals, as investigating applications to cancer proton therapy, accelerator driven systems, and includes R&D related to high repetition rate, fast extraction, etc. The Table above, col. "150 MeV", gives the main parameters of the machine.

Tracking

A remark arises from experience : FFAG design *must* resort to tracking, possibly using field maps [9], as early as the first order design stages, in order to access optical functions, tunes, and other first order machine parameters.



Horizontal motion limits, 12→150 MeV acceleration in the KEK 150 MeV FFAG [10]

Analytic or matrix approach can only yield approximate values of zero-th and first order parameters, only good as starting guidelines [3]. That specificity of FFAG design was already clear in the early years, were digital computation was abundantly used in field and trajectory calculations [1]. In addition, tracking is the only way one can access transverse stability limits (left Figure above), amplitude or momentum detuning, 6-D acceleration (right Figure), etc.

Precision 6-D tracking is of prime importance for instance when comparing muon FFAGs (see last Section) based either on "scaling" optics (strongly non-linear transverse motion) or on "non-scaling" optics (strongly non-linear longitudinal motion).

From 1964 to today's R&D

After the MURA years, some activity kept going on FFAG, usually in devising alternatives to Linac or synchrotron designs in (high power) proton beam based projects, with such possible advantages as their allowing low circulating current, or lower investment cost. Let us mention for illustration, the European spallation neutron source project (ESS) [11], based on a MW range pulsed proton beam, that lead to the FFAG parameters below (col. A in the Table) [12], and the Fermilab proton driver, a 8 GeV project that lead to two FFAG designs, a spiral sector one, and a radial sector one : parameters above (col. B in the Table) [13],

		A	B
beam power	MW	5	0.5
top E	GeV	3	8
p/pulse		$2\,10^{14}$	$3.6\,10^{12}$
rep. rate	Hz	50	105
radius	m	140	474
injection E	MeV	430	600
DFD sectors		20	32
K		21	120
magnet width	m	2.5	4.5
RF freq./voltage	MHz/kV	1.6-2 / 200	7.5 / -

Drawbacks in these types of tentatives were of various nature, concerning generally magnet size, insertion in an existing installation, the question of high power beam injection, operation costs, etc. In a general manner, large apertures that characterize scaling FFAGs entail massive magnets, radial sector optics entails large circumference.

Today's trends

However, fixed field allows high repetition rate and high average intensity, whereas large aperture entails large geometrical acceptance and the zero-chromaticity condition yields large momentum acceptance. As a consequence, the FFAG method is still actively considered, with mostly two kinds of arguments.

On the one hand, possible interest of FFAGs, either using classical technologies or moreover benefiting from modern ones like high gradient RF [7], SC magnets [14], etc., remains problem-dependent. As a matter of fact, many contemporary Japan constructions were launched in this context : protontherapy machine, ADS proton driver, muon beam manipulation, etc., while other recent projects also rely on that solution [15], like the large acceptance, fast acceleration of muons in the neutrino factory - the subject of the next Section. On the other hand, new concepts have arisen these last years, which contribute to the "rebirth" [2], and in addition, the domain strongly takes profit from the power of nowadays computers and computing tools.

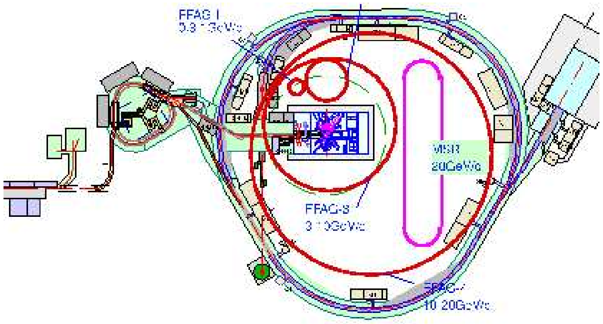
A table of comparative performance of FFAGs, cyclo- and synchrotrons worked out at HB2004 can be found here [16].

The Neutrino Factory

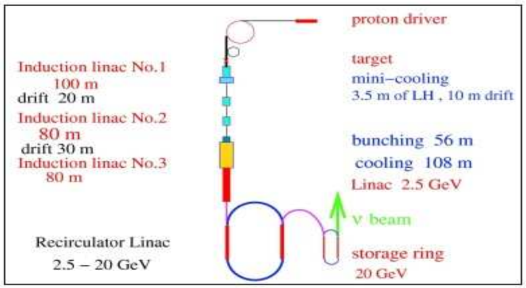
The neutrino factory project (NuFact) has been the ground for emergence of new concepts in FFAG design in the recent years, the most determining one being that of "non-scaling" optics [17].

FFAG based NuFact

The goal of the NuFact facility is to produce high flux neutrino beams by high energy muon decay, at a rate of about 10^{20} per year. Intense muon beams (the tertiary products of the interaction of a MW range proton beam with a high Z target) are accelerated rapidly to 20 GeV or beyond, and stored in a race-track ring where they decay within a few hundred turns along a straight section that points towards a large, distant, physics detector.



The Japan NuFact [18] is based on the acceleration of this muon beam using low frequency RF distributed in a cascade of four FFAG rings inscribed within the J-PARC 50 GeV synchrotron which produces the MW range parent proton beam (see Figure above). The advantage of FFAGs is to be understood by comparison with RLA (recirculating Linac accelerator) based schemes using high frequency, high gradient RF (200 MHz, 15 MV/m elliptical cavities) as in the European [19] and US (Figure below) [20] proposals, as follows. In order to reach the required neutrino rate, the latter require expensive 6-D cooling sections to squeeze the about 300 MeV/c captured muon beam to about 1.5 cm transverse normalised and 1 eV.s longitudinal emittances prior to injection into an RLA for fast acceleration, whereas due to their much larger geometrical and momentum acceptance FFAGs would avoid dedicated cooling sections while yielding the requested neutrino rate in spite of the lower average accelerating gradient. Next, RLAs are complicated and expensive machines, about a third of the cost of the whole facility in the 20 GeV US-Study 2 scheme [20], whereas FFAGs rely on simpler technology.



Non-scaling FFAG optics

NuFact works have entailed a strong activity in the new field of “non-scaling” FFAGs - a specificity of US and Canada [21] - presumed to bring advantages compared to classical “scaling” FFAG as involved in the Japan NuFact design, in particular in terms of lower cost in the higher energy stages of muon acceleration, and in their allowing for high frequency / high gradient RF.

“Non-scaling” optics have the large energy acceptance proper to FFAG, they are based on linear, combined function magnets, therefore yielding large dynamic aperture. By “non-scaling” it is meant that tunes are allowed to vary in the course of acceleration (in practice, a decrease of the cell tune, due to the natural chromaticity, of about a $\frac{1}{2}$integer). In the muon application for instance, acceleration to multi-GeV by 100s cells ring means crossing “forests” of Floquet’s resonances over the few turns in the machine from injection to top energy, fast enough though, not to yield prohibitive constraints on magnet alignment and defects. Other features of “non-scaling” FFAGs are, smaller size magnets compared to scaling FFAGs, due to the reduced beam offset during acceleration, and near-crest acceleration [22].

No “non-scaling” FFAG has ever been built, that motivates a recent proposal for an electron model, in the 10s MeV range [21, 23].

These new concepts are now envisaged as an alternative to “scaling” optics in regular fields of interest of FFAGs, as hadrontherapy, high power proton beams, etc. Besides, they are being extended to non-linear transverse fields which further extends the flexibility of the method and its possible domains of application [24].

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