Two-photon exchange and electromagnetic proton form factors

Egle Tomasi-Gustafsson[a]

[a]DAPNIA/SPhN, CEA/Saclay, 91191 Gif-sur-Yvette Cedex, France

This talk is dedicated to the memory of Professor Michail P. REKALO

The presence of 2γ exchange in electron proton elastic scattering is discussed. From C-invariance and crossing symmetry, the 2γ contribution induces a specific dependence of the reduced cross section on the variable ϵ. No evidence of such dependence exists in the available experimental data.

1. INTRODUCTION

In 1999 M. P. Rekalo and myself investigated the possible presence of 2γ exchange [1] in the precise data on the structure function $A(Q^2)$, obtained at the Jefferson Laboratory (JLab) in *electron* $-$ *deuteron* elastic scattering, up to a value of momentum transfer squared, $-t = Q^2 = 6$ GeV^2 [2,3]. A prescription for the differential cross section was derived from general properties of the hadron electromagnetic interaction, as the C-invariance and the crossing symmetry. The discrepancy from the two experiments [2,3], which had been performed in different kinematical conditions, could not be explained in terms of a 2γ contribution, but the possibility of 2γ corrections was not excluded, starting from $Q^2 = 1$ GeV^2 and the necessity of dedicated experiments was pointed out [1].

The relative contribution of 2γ exchange, through its interference with the main (i.e., one-photon) mechanism is expected to be of the order of the fine structure constant, $\alpha = e^2/4\pi \simeq 1/137$. But, more than thirty years ago, it was observed [4] that the relative role of two-photon exchange can essentially increase in the region of high momentum transfer, due to the steep decreasing of the form factors (FFs). This effect can be observed in particular in *ed*-elastic scattering where it would appear already at momentum transfer squared of the order of 1 GeV^2.

The main consequence of the presence of 2γ exchange is that the traditional description of the electron-hadron interaction in terms of electromagnetic currents (and electromagnetic FFs) can become incorrect. In one-photon exchange, two real amplitudes (functions of one variable, Q^2) fully describe elastic *ep* scattering. If the 2γ exchange is present, one has to deal with three complex amplitudes, which are functions of two kinematical variables, Q^2 and the polarization of the virtual photon ϵ ($\epsilon = [1 + 2(1+\tau)\tan^2(\theta_e/2)]^{-1}$, θ_e is the electron scattering angle in the Lab system, $\tau = Q^2/(4m^2)$, m is the nucleon mass).

The presence of 2γ exchange leads to very complicated analysis of polarization effects. It destroys the linearity in the variable ϵ of the differential cross section for elastic eN

scattering [5], and the relatively simple dependence of the ratio P_x/P_z (the components of the final nucleon polarization in the scattering of longitudinally polarized electrons by an unpolarized nucleon target) on the ratio of the electric and magnetic FFs, G_E/G_M, which holds for the one-photon mechanism [6]. It can be shown that the situation is not so involved, and that even in case of two-photon exchange, one can still use the formalism of FFs, if one takes into account the C-invariance of the electromagnetic interaction of hadrons. In this case, a model independent determination of FFs can be obtained either with a specific combination of three T-odd (or five T-even) polarization observables or with positron and electron beams in the same kinematical conditions [7].

The exact calculation of the 2γ contribution to the amplitude of the $e^{\pm}p \to e^{\pm}p$-process requires the knowledge of the matrix element for the double virtual Compton scattering, $\gamma^* + N \to \gamma^* + N$, in a large kinematical region of colliding energy and virtuality of both photons, and cannot be done in a model independent form. Therefore we follow another approach: general properties of the hadron electromagnetic interaction, as the C-invariance and the crossing symmetry, give rigorous prescriptions for different observables for the elastic scattering of electrons and positrons by nucleons, in particular for the differential cross section and for the proton polarization, induced by polarized electrons. These concrete prescriptions help in identifying a possible manifestation of the two-photon exchange mechanism and to avoid unjustified assumptions. For example, symmetry properties appear in the spin structure of the amplitudes, with respect to the change $x \to -x$ with $x = \sqrt{(1+\epsilon)/(1-\epsilon)}$.

2. MODEL INDEPENDENT ANALYSIS OF EXPERIMENTAL DATA

Crossing symmetry allows to connect the matrix elements for the cross-channels: $e^- + N \to e^- + N$, in s–channel, and $e^+ + e^- \to N + \overline{N}$, in t–channel.

The C-invariance of the electromagnetic hadron interaction and the corresponding selection rules can be applied to the annihilation channel and this allows to find specific properties for one and two-photon exchanges.

To illustrate this, let us consider firstly the one-photon mechanism for $e^+ + e^- \to p + \overline{p}$. The conservation of the total angular momentum, $\mathcal{J}$, allows only $\mathcal{J} = 1$ and $\mathcal{J}^P = 1^-$, $C = -1$ (the quantum numbers of the photon). The selection rules with respect to C and P-invariances allow two states for e^+e^- (and $p\overline{p}$): $S = 1$, $\ell = 0$ and $S = 1$, $\ell = 2$ with $\mathcal{J}^P = 1^-$, where S is the total spin and ℓ is the orbital angular momentum. As a result, the θ-dependence of the cross section (θ is the angle of the detected proton in the CMS system) in the one-photon exchange mechanism is:

$$\frac{d\sigma}{d\Omega}(e^+ + e^- \to p + \overline{p}) \simeq a(t) + b(t)\cos^2\theta \tag{1}$$

where $a(t)$ and $b(t)$ are definite quadratic contributions of $G_E(t)$ and $G_M(t)$, $a(t)$, $b(t) \geq 0$ for $t \geq 4m^2$.

Let us consider now the $\cos\theta$-dependence of the $1\gamma \bigotimes 2\gamma$-interference contribution to the differential cross section of $e^+ + e^- \to p + \overline{p}$. The spin and parity of the 2γ-states is not fixed, in general, but, as C-parity is a multiplicative quantum number, in this case only a positive value of C-parity, $C(2\gamma) = +1$, is allowed. Therefore, the $\cos\theta$-dependence of the contribution to the differential cross section for the $1\gamma\bigotimes 2\gamma$-interference is C-odd:

$$\frac{d\sigma}{d\Omega}^{(int)}(e^+ + e^- \to p + \overline{p}) = \cos\theta[c_0(t) + c_1(t)\cos^2\theta + c_2(t)\cos^4\theta + ...] \qquad (2)$$

where $c_i(t)$, $i = 0, 1..$ are real coefficients, which are functions of t only. An infinite number of states with different quantum numbers can contribute, and their relative role is determined by the dynamics of the process $\gamma^* + \gamma^* \to p + \overline{p}$, with both virtual photons. But the odd $\cos\theta$ (or x)-dependence is essentially different from the even $\cos\theta$-dependence of the cross section for the one-photon approximation. Therefore the interference contribution to the differential cross section (2) cannot be reduced to a linear function in $\cos^2\theta$, because this is in contradiction with the C-invariance of hadronic electromagnetic interaction.

The following correspondence holds between kinematical variables in the crossed channels: $\cos^2\theta \leftrightarrow (1+\epsilon)/(1-\epsilon)$. It follows that, in presence of 2γ exchange, the reduced elastic ep cross section can be rewritten in the following general form [8]:

$$\sigma_{red}(Q^2,\epsilon) = \epsilon G_E^2(Q^2) + \tau G_M^2(Q^2) + \alpha F(Q^2,\epsilon) \qquad (3)$$

where $F(Q^2,\epsilon)$ is a real function describing the effects of the $1\gamma\bigotimes 2\gamma$ interference. In order to estimate the upper limit for a possible 2γ contribution to the differential cross section and the corresponding change in $G_{E,M}(Q^2)$, we analyzed four sets of data [9], applying Eq. (3) with a simple parametrization for $F(Q^2,\epsilon)$, which is consistent with C-invariance [8]:

$$F(Q^2,\epsilon) \to \epsilon\sqrt{\frac{1+\epsilon}{1-\epsilon}}f(Q^2),\ f(Q^2) = \frac{C}{(1+Q^2[\mathrm{GeV}]^2/0.71)^2(1+Q^2[\mathrm{GeV}]^2/m^2)^2} \qquad (4)$$

where C is a fitting parameter, m is the mass of a tensor or an axial meson with positive C-parity. For $m \simeq 1.5$ GeV, one can predict that the relative role of the 2γ contribution should increase with Q^2. It is important to stress that Eq. (4) is a simple expression which contains the necessary symmetry properties of the $1\gamma\bigotimes 2\gamma$ interference, through a specific (and non linear) ϵ-dependence. Therefore, in presence of 2γ, the dependence of the reduced cross section on ϵ can be parametrized as a function of three parameters, G_E^2, G_M^2 and C, according to Eqs. (3) and (4). In Fig. 1, from top to bottom, the electric and magnetic FFs, as well as the two-photon parameter C, are shown as a function of Q^2 (solid symbols).

The previously published data, derived from the traditional Rosenbluth fits, are also shown (open symbols). Including a third fitting parameter, C, increases the errors on the extracted FFs. The resulting parameter C is compatible with zero.

3. CONCLUSIONS

From the present analysis it appears that the available data on ep elastic scattering do not show any evidence of deviation from the linearity of the Rosenbluth fit, and hence of the presence of the 2γ contribution, when parametrized according to Eq. (4).

Besides the deviation from the linearity of the Rosenbluth fit, other possible methods to test the presence of 2γ exchange in elastic electron-hadron scattering can be listed:

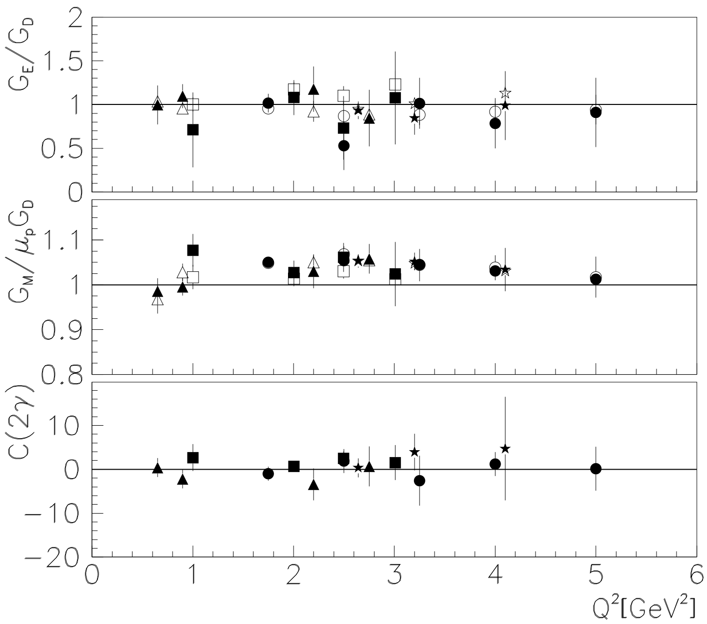

Figure 1. From top to bottom: G_E/G_D, $G_M/\mu G_D$ and the two-photon contribution, C. $\mu = 2.79$ is the proton magnetic moment, $G_D = (1 + Q^2[\mathrm{GeV}^2]/0.71)^{-2}$. Published data are shown as open symbols, the present results which include the 2γ contribution as solid symbols.

comparison of the cross section for scattering of unpolarized electrons and positrons (by protons or deuterons) in the same kinematical conditions; specific polarization phenomena such as the appearance of T-odd polarization observables; violation of definite relations between T-even polarization observables and structure functions [1,7]. The experimental evidence of the presence of the 2γ-exchange and its quantitative estimation are very important. If this effect appears in elastic *ep* scattering already in the range of momentum transfer investigated at JLab, the findings based on the one-photon assumption will have to be reanalyzed at the light of a new and complicated formalism. In this case, as indicated already long ago [4] most of the advantages related to the electromagnetic probe would be lost.

The results quoted here would not have been obtained without a fruitful collaboration and enlightning discussions with Professor M. P. Rekalo.

Thanks are due to G.I. Gakh for a careful reading of the manuscript.

REFERENCES

1. M. P. Rekalo, E. Tomasi-Gustafsson and D. Prout, Phys. Rev. C 60 (1999) 042202 .
2. L. C. Alexa *et al.*, Phys. Rev. Lett. 82 (1999) 1375.
3. D. Abbott *et al.*, Phys. Rev. Lett. 82 (1999) 1379.
4. J. Gunion and L. Stodolsky, Phys. Rev. Lett. 30 (1973) 345;V. Franco, Phys. Rev. D 8 (1973) 826 ; V. N. Boitsov, L.A. Kondratyuk and V.B. Kopeliovich, Sov. J. Nucl. Phys. 16 (1973) 237; F. M. Lev, Sov. J. Nucl. Phys. 21 (1973) 45.
5. M. N. Rosenbluth, Phys. Rev. 79 (1950) 615.
6. A. Akhiezer and M. P. Rekalo, Dokl. Akad. Nauk USSR, 180 (1968) 1081; [Sov. J. Part. Nucl. 4 (1974) 277].
7. M. P. Rekalo and E. Tomasi-Gustafsson, Eur. Phys. J. A 22 (2004) 331; Nucl. Phys. A 740 (2004) 271; Nucl. Phys. A 742 (2004) 322.
8. E. Tomasi-Gustafsson and G. I. Gakh, arXiv:hep-ph/0412137.
9. R. C. Walker *et al.*, Phys. Rev. D 49 (1994) 5671; L. Andivahis *et al.*, Phys. Rev. D 50 (1994) 5491; M. E. Christy *et al.* [E94110 Collaboration], Phys. Rev. C 70 (2004) 015206; I. A. Qattan *et al.*, arXiv:nucl-ex/0410010.