FIXED FIELD ALTERNATING GRADIENT SYNCHROTRONS

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Abstract

This paper gives a brief history and a general description of fixed field alternating gradient (FFAG) accelerators, from the early years till the most recent proton accelerator projects.

INTRODUCTION

The concept of fixed field alternating gradient (FFAG) accelerators dates from the early 1950's [1]. They were seen as a way to apply the principles of strong focusing and synchrotron stability and yielded high intensity machines, at a time where fixed orbit strong focusing synchrotrons eventually took over, while cyclotrons were limited to lower energies.

Only 5 FFAGs have been constructed and operated up to now :
- 3 electron machines in the 1950's, by MURA (Midwestern Universities Research Association), which saw many crucial tasks of accelerator physics first tackled [1],
- 2 proton machines by KEK recently, in a context of determining technological progress regarding magnetism, acceleration and other beam manipulation equipments.

Nevertheless, FFAGs have regularly been proposed as an alternative solution to Linac, RCS and other cyclotron, for the production of proton beams. More recently, the neutrino factory studies triggered strong R&D activity in the field, and on the other hand the emergence of new concepts as well as modern technologies have revived the interest in the method, and pushed to re-exploring potential applications [2].

FFAGs are one of the most active fields in accelerator research today, with 9 workshops from Dec. 1999 till Oct. 2004, and again two planned for 2005.

All these aspects will be addressed in the following, briefly though. A large amount of References will however, we hope, be of some help to the interested reader for digging into the subject.

THE MURA ELECTRON FFAGS

First model, radial sector FFAG, Mark II

Work on "Mark II" began in 1955, 2 years after the invention of the concept. The machine (Fig. 1) [3] was first operated in March 1956, at the University of Michigan. The model was to be a proof of the FFAG principle, it eventually had a rich history of demonstrating experiments regarding effects of resonances, RF acceleration, beam stacking, RF KO, etc. The magnetic field is by principle fixed in time, with mid-plane form $B(r,\theta) = B_0(\frac{r}{r_0})^K \mathcal{F}(\theta)$ ($K > 0$ a constant, r_0 a reference radius, $\mathcal{F}(\theta)$ an axial form factor) (see plot below), fast increasing with radius, from lower energy, larger gap, on inner orbit (where the beam is injected) to largest energy, smallest gap, on outer orbit.

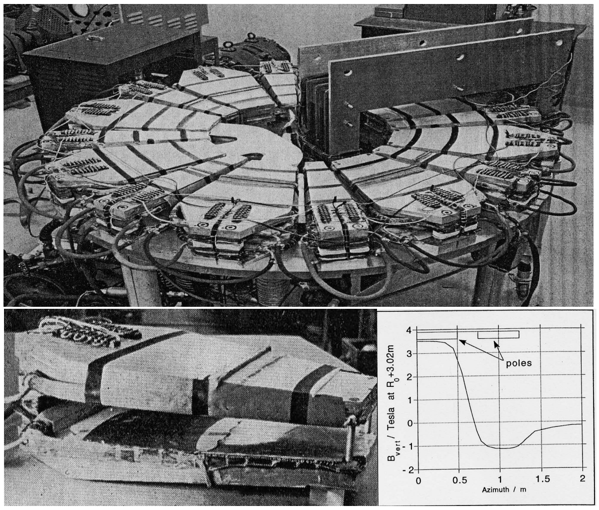


Figure 1: The first MURA FFAG, the F magnet and field.

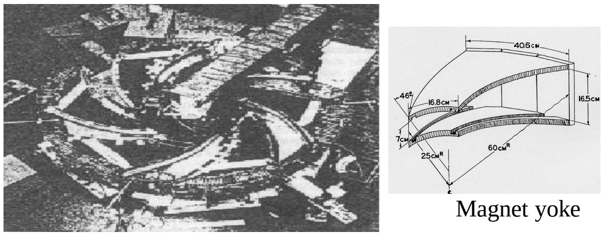
The r^K shape of B is due for part to the gap size decreasing with r, and for the rest to coil winding arrangement thus allowing K (and hence tunes) to be varied.

$E_{inj} - E_{max}$	*keV*	25 - 400
orbit radius	*m*	0.34 - 0.50
lattice		$\frac{\mathbf{D}}{\mathbf{2}}\mathbf{F}\frac{\mathbf{D}}{\mathbf{2}}$
number of cells		8
field index K, tunable		≈3.4
ν_r / ν_z, tunable		2.2-3 / 1-3
Magnet	radial sector	
F, D sectors	*deg*	25.74, 10.44
gap, max.-min.	*cm*	6 - 4
Injection	continuous or pulsed	
Acceleration	betatron core, at first, ...	
swing	*Gauss*	40 - 150
rep. rate	*Hz*	a few 10's
	... RF, next	
freq. swing	*MHz*	10 in [35, 75]
gap voltage	*V*	50

The ring is built from an alternance (hence the $\mathcal{F}(\theta)$ form factor) of positive dipoles which yield radial focusing ($\frac{\rho(s)}{B(s)}\frac{dB}{d\rho} > 0$) and shorter, negative dipoles which yield radial defocusing ($\frac{\rho(s)}{B(s)}\frac{dB}{d\rho} < 0$), thus insuring AG strong focusing. The radial dependence $B = B_0(r/r_0)^K$ determines the "scaling" property (also known as the "zero-chromaticity condition") : tunes are independent of the orbit (hence, of energy), closed orbits are similar wrt. geometrical center (they have a scalloped shape, due to the alternating curvature). Series of basic properties ensue, like a large circumference factor $\mathcal{C}/2\pi\rho$, momentum compaction $\alpha = 1/(1+K)$, $\gamma_{tr} = \sqrt{1+K}$ easily put beyond top energy, feasibility of arbitrary RF programs : no need to track B, and so forth. In the linear approximation the motion about a closed orbit satisfies Hill's equations $x'' + \frac{1-n}{\rho^2}x = 0$, $z'' + \frac{n}{\rho^2}z = 0$ with $n(s) \approx -\frac{\rho}{B}\frac{dB}{dr} = -K/\mathcal{C}$, thus amenable to regular optical treatment, working point gymnastics, defect analysis, etc. The longitudinal motion in presence of RF obeys, as in synchrotrons, $\ddot{\phi} + \frac{\Omega^2}{\cos\phi_s}(\sin\phi - \sin\phi_s) = 0$. The Table above gives the main parameters of Mark II.

Second model, spiral sector FFAG, Mark V

Work on "Mark V" began in 1955, a year after the invention of the concept. The machine (Fig. 2) [4] was first operated in August 1957 in the MURA Lab., Madison. Objectives were to validate the strong focusing spiral optics with its advantage of a smaller circumference, and perform beam physics, accelerator studies.



Magnet yoke

Figure 2: The second, spiral, MURA FFAG

$E_{inj} - E_{max}$	*keV*	35 - 180
orbit radius	*m*	0.34 - 0.52
number of sectors		6
field index K, tunable		0.7
flutter F_{eff}, rms		1.1
ν_r / ν_z, tunable		1.4 / 1.2
β_r / β_z	*m*	0.45-1.3 / 0.6-1.4
Magnet		spiral sector
edge/radius angle, $Atg(Nw)$	*deg*	46
$r_{min} - r_{max}$	*m*	0.25 - 0.61
gap, max.-min.	*cm*	16.5 - 7
Injection	continuous or pulsed	
Acceleration	betatron cores & RF	
RF voltage	*V*	150

The idea in the spiral FFAG was to superpose a positive field on top of the alternating sign of the radial sector case, so as to always have the right curvature and hence decrease the circumference factor, which yields the "Thomas focusing" of cyclotrons. Yet by doing so the vertical focusing is weakened and needs be recovered by spiraling the poles. Appropriate field form for insuring the scaling property and constant closed orbit to spiral edge angle, is $B(r,\theta)|_{z=0} = B_0\,(r/r_0)^K\ \mathcal{F}\left((\ln\frac{r}{r_0})/w - N\theta\right)$, The axial modulation $\mathcal{F}$ is called the "flutter", it has the approximate form $\mathcal{F} = 1 + f\sin(\ln\frac{r}{r_0}/w - N\theta)$. Expansion of the equations of motion around the closed orbit in the linear approximation (or as well a hard edge matrix model) yields the tunes $\nu_r \approx \sqrt{1+K}$, $\nu_z \approx \sqrt{-K + (f/Nw)^2/2}$. The Table above gives the main parameters of Mark V.

A 50 MeV, two-way, electron FFAG

Work on the 50 MeV electron FFAG began in 1957 [5]. The machine (Fig. 3) was first operated in 1959 with two 27 MeV beams stored in opposing directions, as made possible by the radial sector optics using identical dipoles in a FODO arrangement. 51 MeV energy, one-way, was reached in 1960 after modifications in the magnets. Colliding beams, once envisaged, a hot task in the mid-50's, need intensity, RF stacking was developed and allowed it, 10 amperes intensity was obtained that way. The Table above gives the FFAG parameters.

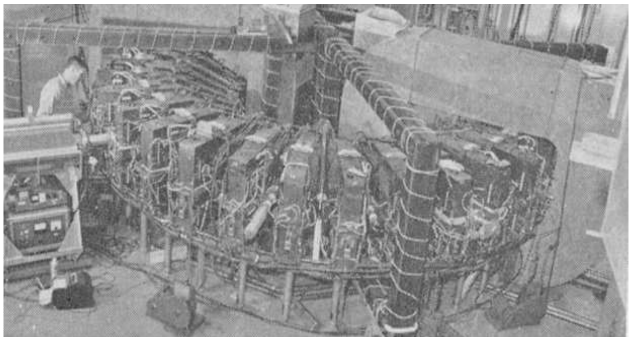


Figure 3: Third, 50 MeV electron FFAG.

$E_{inj} - E_{max}$	*MeV*	0.1 - 50
orbit radius	*m*	1.20 - 2.00
lattice		FODO
number of cells		16
K		9.25
ν_r / ν_z		4.42 / 2.75
Magnet		radial sector
sector angle	*deg*	6.3
peak field	*T*	0.52
gap, max.-min.	*cm*	8.6 - 8.0
Acceleration	betatron & RF	
betatron range	*MeV*	0.1 - 2
RF swing	*MHz*	20 - 23
voltage p-to-p	*kV*	1.3 - 3
cycle rep. rate	*Hz*	60

THE KEK PROTON MACHINES

POP

KEK POP (proof of principle) machine (Fig. 4) [6] is the first proton FFAG, first operated in 2000.

Its design has strongly benefited from modern magnet computation tools and sophisticated tracking codes. The DFD lattice allows comfortable drifts, it is based on a radial sector triplet (two negative dipoles at both ends and

Figure 4: POP FFAG, FDF dipole triplet with scaling gap, broad band accelerating cavity.

a larger, positive one in between, in a common yoke with so-called "scaling" gap shape $g_0(r_0/r)^K$ producing the radial field dependence $B_0(r/r_0)^K$. The acceleration uses high gradient, broad band "FINEMET" technologies yielding a narrow cavity (see Fig. 4) and a potential 1 kHz rep. rate [7]. Injection is on inner radius, via an electrostatic inflector, either single-turn (using a chopper in the injection line) or multi-turn (using two bump electrodes). Tunes are adjustable via the B_F/B_D ratio. The Table below, col. "POP", gives the main parameters of the POP FFAG.

		POP	**150 MeV**
$E_{inj} - E_{max}$	MeV	0.05 - 0.5	12 - 150
orbit radius	m	0.8 - 1.14	4.7 - 5.2
lattice		DFD	
number of cells		8	12
K		2.5	7.6
β_r / β_z max.	m	0.7 / 0.7	3.8 / 1.3
ν_r / ν_z		2.2 / 1.25	3.7 / 1.2
Magnet		radial sector	
D, F sectors	deg	2.8 / 14	3.43 / 10.24
B_D/B_F max-min	T	.04-.13/.14-.32	.3-.8 / .5-1.6
gap, max.-min.	cm	30 - 9	20 - 4
Acceleration			
swing	MHz	0.6 - 1.4	1.5 - 4.6
voltage p-to-p	kV	1.3 - 3	19
cycle time	ms	1	4
rep. rate	Hz	10^3	250
$\dot{B}$ equivalent	T/s	180	280

150 MeV proton FFAG

This second, higher energy, proton FFAG was first operated in 2003 [8]. The structure is similar to POP, it uses a 10 MeV cyclotron injector. One distinguishing feature is the return-yoke free dipole triplet (see lower-left corner in the photo, Fig. 5) which facilitates beam injection and extraction. Due to the extending fringe fields and to saturation effects, the zero-chromaticity condition is not fully satisfied, so that tunes slightly vary over the energy span. The project has various goals, as investigating applications to cancer proton therapy, accelerator driven systems, and includes R&D related to high repetition rate, fast extraction, etc. The Table above, col. "150 MeV", gives the main parameters of the machine.

Figure 5: KEK 150 MeV proton FFAG

Tracking

A remark arises from experience : the design of FFAG machines *must* resort to tracking, possibly using field maps [9], as early as the first order design stages, in order to access closed orbits, tunes, and other optical functions. Analytic or matrix approach can only yield approximate values of zeroth and first order parameters, only good as starting guidelines [3]. That specificity of FFAG design was already clear in the early years, were digital computation was abundantly used in field and trajectory calculations [1]. In addition, tracking is the only way one can access transverse stability limits (Fig. 6 left), amplitude or momentum detuning, 6-D acceleration (Fig. 6 right), etc.

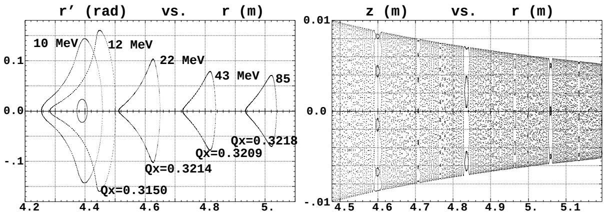


Figure 6: Horizontal motion limits (left) and 12→150 MeV acceleration (right) in the KEK 150 MeV FFAG [10].

Precision 6-D tracking is of prime importance for instance when comparing muon FFAGs (see last Section) based either on “scaling” optics (strongly non-linear transverse motion) or on “non-scaling” optics (strongly non-linear longitudinal motion).

FROM 1964 TO TODAY’S R&D

After the MURA years, some activity kept going on FFAG, usually in devising alternatives to Linac or synchrotron designs in (high power) proton beam based projects, with such possible advantages as their allowing low circulating current, or lower investment cost. Let us mention for illustration, the European spallation neutron source project (ESS) [11], based on a MW range pulsed proton beam, that lead to the FFAG parameters below (col. A in the Table) [12], and the Fermilab 8 GeV proton driver project that lead to two FFAG alternative proposals, a spiral sector and a radial sector design (col. B in the Table) [13],

		A	B
beam power	MW	5	0.5
top E	GeV	3	8
p/pulse		$2\,10^{14}$	$3.6\,10^{12}$
rep. rate	Hz	50	105
radius	m	140	474
injection E	MeV	430	600
DFD sectors		20	32
K		21	120
magnet width	m	2.5	4.5
RF freq./voltage	MHz/kV	1.6-2 / 200	7.5 / -

Drawbacks in these types of tentatives were of various nature, concerning generally magnet size, insertion in an existing installation, high power beam injection in short drifts, operation costs, etc. In a general manner, large apertures that characterize scaling FFAGs entail massive magnets, radial sector optics entails large circumference.

Today’s trends

However, fixed field allows high average intensity, whereas large apertures entail large geometrical acceptance and the zero-chromaticity condition yields large momentum acceptance. As a consequence, the scaling FFAG method is still actively considered, benefiting in particular from modern technologies as stressed earlier, including as well scaling field SC magnet developments [14]. As a matter of fact, many contemporary Japan constructions were launched in this context : ADS proton driver, muon beam manipulation, protontherapy machine, high power electron beams, etc. [15], as well as the large acceptance, fast acceleration of muons in the neutrino factory [16]. As to proton beams, a recent table of comparative performance of FFAG, cyclotron and synchrotron machines can be found in Ref. [17].

On the other hand, new concepts have arisen these last years, in particular that of “non-scaling” FFAG, which contribute to their “rebirth”, this is the subject of the following.

NON-SCALING FFAGS

NuFact works have entailed a strong activity in the new field of “non-scaling” FFAGs [18, 19] presumed to bring advantages compared to classical “scaling” FFAG as involved in the Japan NuFact design, in particular in terms of lower cost in the higher energy stages of muon acceleration, and in their allowing the use of high frequency / high gradient SC RF.

“Non-scaling” optics has the large energy acceptance proper to FFAG, it was at first based on linear, combined function magnets, therefore prone to large dynamic aperture [20]. By “non-scaling” it is meant that tunes are allowed to vary in the course of acceleration (in practice, a decrease of the cell tune, due to the natural chromaticity, of about a $\frac{1}{2}$integer). In the muon application for instance, multi-GeV acceleration using a hundreds of cells ring means crossing “forests” of Floquet’s resonances over the few turns in the ring from injection to top energy, fast enough though, not to yield prohibitive constraints on magnet alignment and defects. Other features of “non-scaling” optics is, a better circumference factor, smaller aperture magnets compared to scaling FFAGs and in particular smaller dispersion, lower fields, the possibility of near-crest acceleration [21].

The concept has been extended to non-linear magnetic fields, with dramatic consequences on the lattice properties. Non-linear, non-scaling FFAG optics permits such design as isochronous lattice [22], allowing on-crest acceleration, as in cyclotrons. Additional sophistication in the spatial behavior of the magnetic field have also allowed designing weakly non-scaling lattices [23] in which the total tune only varies by a fraction of an integer, making this type of optics a good candidate for various applications [19], amongst which proton drivers as addressed below.

No “non-scaling” FFAG has ever been built, which motivates a recent proposal for an electron model, in the 10s MeV range [24].

These new concepts are now envisaged as an alternative to “scaling” optics in the regular fields of interest of FFAGs, as hadrontherapy, high power proton beams, etc.

An example of a proton booster application

The 1 MW upgrade of the AGS at BNL requires increasing the repetition rate to 2.5 Hz, and the number of particles to 10^{14}ppp. This imposes replacing the 1.5 GeV booster ring, the baseline scenario being based on a superconducting Linac, whereas a non-scaling FFAG appears to be a cost effective alternative.

A non-scaling optics has recently been worked out, based on an “adjusted field profile” that causes the index to be a function of the radial displacement x and of the longitudinal position in the dipoles, that is $n = n(x, \theta)$ (Fig. 7), with the effect of cancelling the momentum dependence of the focusing strength. A consequence of this field shape is a reduced “non-scaling” : the variation of the total tune is only of the order of a fraction of an integer over

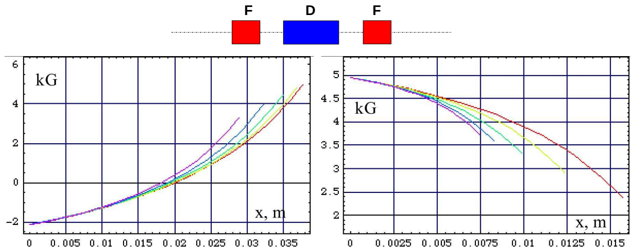


Figure 7: Field profiles vs. radial excursion at some azimuths in the F- (left graph) and D-sector (right) AFP magnets.

the full acceleration cycle (Fig. 8). This method has been applied with dipole triplet cells that have been shown to be advantageous, especially in the FDF configuration that yields low dispersion (Fig. 8). This type of design is believed to yield competitive technology that can allow beam performance at the level of the other accelerator architectures. A main feature is in the compactness of the magnets ensuing from the much reduced beam excursion, compared to scaling FFAG.

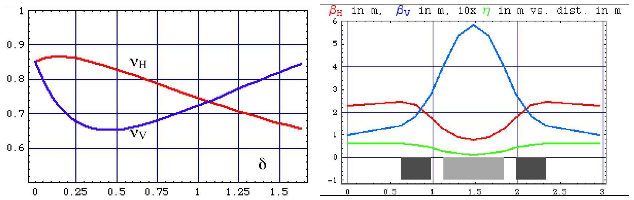


Figure 8: Left : total tune values during acceleration. Right : optical functions in the adjusted field profile non-scaling FDF cell.

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