

General analysis of polarization phenomena in $e^+ + e^- \rightarrow N + \bar{N}$ for axial parametrization of two-photon exchange

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Abstract

General properties of the differential cross section and of the polarization observables are derived for the $e^+ + e^- \rightarrow N + \bar{N}$ reaction, in presence of two-photon exchange. Polarization effects are investigated for longitudinally polarized electron and polarized antinucleon and/or polarized nucleon in the final state. The single-spin asymmetry induced by the transverse polarization of the electron beam is also discussed.

1 Introduction

The measurement of the nucleon electromagnetic form factors (FFs) in the space-like (SL) region of momentum transfer squared has a long history [1]. The electric and magnetic FFs were traditionally determined using the Rosenbluth separation [2], based on the measurement of the unpolarized differential cross section, for different angles, at the same momentum transfer. More recently, the polarization transfer method [3], which requires longitudinally polarized electron beam and the measurement of the recoil proton polarization, could be applied [4]. It turned out that the ratio of the proton electric and

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magnetic FFs determined by the two methods, which are based on the same physics (one-photon exchange mechanism), lead to appreciably different results, and this difference is increasing with Q^2 , the four-momentum transfer squared [5]. One possible explanation for this discrepancy is the presence of a two-photon exchange (TPE) contribution to the elastic electron-nucleon scattering [6–8]. Model independent properties of TPE in elastic electron-proton scattering have been derived in Ref. [9], where it was shown that the presence of such mechanism destroys the linearity of the Rosenbluth fit. No deviation from linearity is observed in the available data (in the limit of their precision) [10]. A previous study on deuteron target [11] focussed already the attention to this problem, following a discrepancy observed in two precise experiments on electron deuteron elastic scattering [12,13].

Estimates of the TPE contribution to the elastic electron-deuteron scattering were made in Refs. [14,15] within the framework of the Glauber theory. It was shown [14] that this contribution decreases very slowly with momentum transfer squared q^2 and may dominate the cross section at high q^2 values. Since the TPE amplitude is essentially imaginary, the difference between positron and electron scattering cross sections depends upon the small real part of the TPE amplitude [14]. Recoil polarization effects may be substantial, in the region where the one- and two-photon exchange contributions are comparable. If the TPE mechanism become sizeable, the straightforward extraction of FFs from the experimental data would no longer be possible [14]. It is known that double scattering dominates in collisions of high-energy hadrons with deuterons at high q^2 values [15], and in this paper it was predicted that the TPE effect in the elastic electron-deuteron scattering represents a 10 % effect at $q^2 \cong 1.3 \text{ GeV}^2$. At the same time the importance of the TPE mechanism was considered in Ref. [16]. The fact that the TPE mechanism, where the momentum transfer is shared between the two virtual photons, can become important with increasing q^2 value was already indicated more than thirty years ago [14–16].

Experimental investigation of nucleon FFs in the time-like (TL) region could shed new light on this problem and bring additional valuable information on the internal nucleon structure. In TL region the nucleon FFs can be accessed through the reactions $e^+ + e^- \rightarrow N + \bar{N}$ or $\bar{N} + N \rightarrow e^+ + e^-$.

The data on nucleon FFs in TL region are scarce and the determination of the individual electric and magnetic FFs is prevented by the statistics that it is possible to achieve [17]. A short review of the present status in this field of investigations can be found in Ref. [18]. Unexpected results have been observed in the measurements of the nucleon FFs in the TL region (although the accuracy of the data is poor): the proton magnetic FF is smaller than the neutron one in the kinematical region covered by the experiments and the TL proton magnetic FF is considerably larger than the corresponding SL

quantity.

The neutron FFs were measured by the FENICE collaboration [19], using the ADONE e^+e^- collider in Frascati up to $q^2 \approx 6 \text{ GeV}^2$. The proton FFs were measured in a more broad region of q^2 values. The differential cross section for $\bar{p} + p \rightarrow e^+ + e^-$ was measured by the Fermilab experiment E835 in the region of large q^2 , up to $q^2=18.22 \text{ GeV}^2$ [17].

Recent and planned experiments focus on the TL region. Preliminary data from BaBar [20], using the radiative return, show surprising features as, for example, that the ratio G_E/G_M is larger than one, in contradiction with polarization data from the SL region, and with previous TL data [21]. Moreover, TL FFs, still extracted under the hypothesis $|G_E| = |G_M|$, do not follow smoothly the asymptotic behavior predicted by QCD but show a few structures as a function of a q^2 . At the future antiproton facility, at GSI, a precise separation of FFs is planned [22] as well as polarization measurements [23]. These ones will firstly allow to measure the relative phase of FFs, which are complex in the TL region. New measurements are also planned at Frascati, after an upgrade of *DAΦNE* [24], in particular for neutrons. The possibility of accessing polarization observables is under study.

The reactions $e^+ + e^- \rightarrow N + \bar{N}$ and $N + \bar{N} \rightarrow e^+ + e^-$ are the crossing channel reactions of elastic electron–nucleon scattering. FFs describing the annihilation channel are assumed to be the analytical continuation of the SL ones. From crossing symmetry one expects that the reaction mechanisms are common for the scattering and the annihilation channels. This concerns, in particular, the problem of the TPE contribution.

Theoretically the reaction $e^+ + e^- \rightarrow N + \bar{N}$ was studied in a few papers. The dependence of the polarization of the emitted baryons in $e^+ + e^- \rightarrow B + \bar{B}$ on the polarization of colliding $e^+ + e^-$ -beams was firstly discussed in Ref. [25]. All polarization effects for baryons with spin 1/2 were calculated, assuming one-photon exchange approximation. Numerical estimates of polarization effects were given for the nucleons, on the basis of two different versions of a VMD model. Polarization effects appear to be very sensitive to the choice of the nucleon FFs parametrization and are rather large in absolute value.

The existence of the T-odd single-spin asymmetry normal to the scattering plane requires a non-zero phase difference between the electric and magnetic FFs. The measurement of the polarization of one of the outgoing nucleons allows to determine the phase of the ratio G_E/G_M . In Ref. [26] it was also shown that measurements of the proton polarization in $e^+ + e^- \rightarrow p + \bar{p}$ reaction strongly discriminate between the analytic forms of different models suggested to describe the proton data in the SL region. In Ref. [27] it was shown that this statement holds for other models, for double-spin polarization observables as

well, and for the angular asymmetry, defined with respect to the cross section at 90° .

We derive here the expressions for the differential cross section and various polarization observables for the case when the matrix element of the reaction $e^+ + e^- \rightarrow B + \bar{B}$ contains TPE contribution. The parametrization of the TPE term follows from the analytic continuation to the TL region of the approach used in the SL region in Refs. [9,28,29].

The spin structure of the matrix element of the $e^+ + e^- \rightarrow N + \bar{N}$ reaction is parametrized in terms of three independent complex functions depending on two variables (energy and angle) rather than in terms of the standard nucleon FFs.

This analysis follows the same steps as in Ref. [30], where model independent properties of TPE were studied for the inverse reaction $p + \bar{p} \rightarrow e^+ + e^-$ and polarization observables were derived using tensor parametrization for the TPE term. The purpose of this paper is to give explicit formulae for the observables, which will be useful for future experiments, as planned, for example, in Frascati [24] and Novosibirsk [31]. We analyze the properties of the single- and double-spin observables derived for the axial parametrization of the TPE term.

2 Differential cross section

The matrix element of the reaction $e^+ + e^- \rightarrow N + \bar{N}$ can be obtained by analytic continuation of the matrix element for the elastic electron-nucleon scattering [9,32], parametrizing the TPE contribution in axial form:

$$\begin{aligned} \mathcal{M} = & -\frac{e^2}{q^2} \left\{ \bar{u}(-k_2) \gamma_\mu u(k_1) \bar{u}(p_2) \left[F_{1N}(q^2, t) \gamma_\mu - \frac{1}{2m} F_{2N}(q^2, t) \sigma_{\mu\nu} q_\nu \right] u(-p_1) + \right. \\ & \left. + \bar{u}(-k_2) \gamma_\mu \gamma_5 u(k_1) \bar{u}(p_2) \gamma_\mu \gamma_5 u(-p_1) A_N(q^2, t) \right\}, \end{aligned} \quad (1)$$

where k_1 (k_2) and p_1 (p_2) are the four-momenta of the electron (positron) and antinucleon (nucleon), respectively; $q = k_1 + k_2 = p_1 + p_2$, m is the nucleon mass.

The three complex amplitudes, $F_{1N,2N}(q^2, t)$ and $A_N(q^2, t)$, which generally are functions of two independent kinematical variables, q^2 and $t = (k_1 - p_1)^2$, fully describe the spin structure of the matrix element for the reaction $e^+ + e^- \rightarrow N + \bar{N}$ - for any number of exchanged virtual photons.

This expression (1) holds under assumption of the P-invariance of the electromagnetic interaction and conservation of lepton helicity, which is correct for standard QED at the high energy, i.e., in zero electron mass limit. Note, however, that expression (1) is one of the many equivalent representations of the $e^+ + e^- \rightarrow N + \bar{N}$ reaction matrix element.

In the Born (one-photon-exchange) approximation these amplitudes reduce to:

$$F_{1N}^{Born}(q^2, t) = F_{1N}(q^2), \quad F_{2N}^{Born}(q^2, t) = F_{2N}(q^2), \quad A_N^{Born}(q^2, t) = 0, \quad (2)$$

where $F_{1N}(q^2)$ and $F_{2N}(q^2)$ are the Dirac and Pauli nucleon electromagnetic FFs, respectively, and they are complex functions of the variable q^2 . The complexity of FFs arises from the final-state strong interaction of the produced $N\bar{N}$ -pair. In the following we use the standard magnetic $G_{MN}(q^2)$ and charge $G_{EN}(q^2)$ nucleon FFs which are related to FFs $F_{1N}(q^2)$ and $F_{2N}(q^2)$ as follows

$$G_{MN} = F_{1N} + F_{2N}, \quad G_{EN} = F_{1N} + \tau F_{2N}, \quad \tau = \frac{q^2}{4m^2}, \quad N = p, n. \quad (3)$$

By analogy with these relations, let us introduce a linear combinations of the $F_{1N,2N}(q^2, t)$ amplitudes which in the Born approximation correspond to the Sachs FFs G_{MN} and G_{EN} :

$$\begin{aligned} \tilde{G}_{MN}(q^2, t) &= F_{1N}(q^2, t) + F_{2N}(q^2, t), \\ \tilde{G}_{EN}(q^2, t) &= F_{1N}(q^2, t) + \tau F_{2N}(q^2, t). \end{aligned} \quad (4)$$

To separate the effects due to the Born and TPE contributions, let us single out the dominant contribution and define the following decompositions of the amplitudes

$$\begin{aligned} \tilde{G}_{MN}(q^2, t) &= G_{MN}(q^2) + \Delta G_{MN}(q^2, t), \\ \tilde{G}_{EN}(q^2, t) &= G_{EN}(q^2) + \Delta G_{EN}(q^2, t). \end{aligned} \quad (5)$$

$\Delta G_{MN}(q^2, t)$, $\Delta G_{EN}(q^2, t)$, and $A_N(q^2, t)$ are of the order of $\sim \alpha$, while $G_{MN}(q^2)$ and $G_{EN}(q^2)$ are of the order of $\sim \alpha^0$. Since the terms ΔG_{MN} , ΔG_{EN} and A_N are small in comparison with the dominant ones, we neglect in the following the bilinear combinations of these terms.

We can rewrite the matrix element (1) explicating the vector and the axial terms as:

$$M = -\frac{e^2}{q^2} \left(j_\mu^{(v)} J_\mu^{(v)} + j_\mu^{(a)} J_\mu^{(a)} \right), \quad (6)$$

where

$$j_\mu^{(v)} = \bar{u}(-k_2)\gamma_\mu u(k_1), \quad J_\mu^{(v)} = \bar{u}(p_2) \left[\tilde{G}_{MN}\gamma_\mu - \frac{\tilde{G}_{MN} - \tilde{G}_{EN}}{m(1-\tau)} P_\mu \right] u(-p_1),$$

$$j_\mu^{(a)} = \bar{u}(-k_2)\gamma_\mu\gamma_5 u(k_1), \quad J_\mu^{(a)} = \bar{u}(p_2)\gamma_\mu\gamma_5 u(-p_1)A_N,$$

and $P = (p_2 - p_1)/2$.

Then the differential cross section of the reaction $e^+ + e^- \rightarrow N + \bar{N}$ can be written as follows in the centre of mass system (CMS):

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4q^6} \left[L_{\mu\nu}^{(v)} H_{\mu\nu}^{(v)} + 2\text{Re}(L_{\mu\nu}^{(i)} H_{\mu\nu}^{(i)}) \right], \quad (7)$$

$$L_{\mu\nu}^{(v)} = j_\mu^{(v)} j_\nu^{(v)*}, \quad L_{\mu\nu}^{(i)} = j_\mu^{(a)} j_\nu^{(a)*}, \quad H_{\mu\nu}^{(v)} = J_\mu^{(v)} J_\nu^{(v)*}, \quad H_{\mu\nu}^{(i)} = J_\mu^{(a)} J_\nu^{(a)*},$$

where β is the nucleon velocity in the reaction CMS, $\beta = \sqrt{1 - 4m^2/q^2}$, and we neglected the terms proportional to A_N^2 .

The leptonic tensors for the case of longitudinally polarized electrons have the forms

$$\begin{aligned} L_{\mu\nu}^{(v)} &= -q^2 g_{\mu\nu} + 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu}) + 2i\lambda_e \langle \mu\nu q k_2 \rangle, \\ L_{\mu\nu}^{(i)} &= -2i \langle \mu\nu k_1 k_2 \rangle + \lambda_e [q^2 g_{\mu\nu} - 2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu})], \end{aligned} \quad (8)$$

where $\langle \mu\nu ab \rangle = \varepsilon_{\mu\nu\rho\sigma} a_\rho b_\sigma$ and λ_e is the degree of the electron longitudinal polarization. The other components of the electron polarization lead to a suppression by a factor m_e/m , where m_e is the electron mass.

Taking into account the polarization states of the produced baryon and antibaryon, the hadronic tensors can be written as a sum of three contributions:

$$H_{\mu\nu}^{(k)} = H_{\mu\nu}^{(k)}(0) + H_{\mu\nu}^{(k)}(1) + H_{\mu\nu}^{(k)}(2), \quad k = v, i, \quad (9)$$

where the tensor $H_{\mu\nu}^{(k)}(0)$ describes the production of unpolarized particles, the tensor $H_{\mu\nu}^{(k)}(1)$ describes the production of polarized nucleon or antinucleon and the tensor $H_{\mu\nu}^{(k)}(2)$ corresponds to the production of both polarized particles, N and \bar{N} .

Let us firstly consider the production of unpolarized $N\bar{N}$ - pair as a result of annihilation of unpolarized e^+e^- - pair. In this case the general structure of the hadronic tensors can be written as

$$H_{\mu\nu}^{(v)}(0) = H_1 \tilde{g}_{\mu\nu} + H_2 P_\mu P_\nu, \quad H_{\mu\nu}^{(i)}(0) = -4iA_N G_{MN}^* \langle \mu\nu p_1 p_2 \rangle, \quad (10)$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2$. One can get the following expressions for these structure functions when the hadronic current is given by Eq. (1)

$$\begin{aligned} H_1 &= -2q^2(|G_{MN}|^2 + 2\text{Re}G_{MN}\Delta G_{MN}^*), \\ H_2 &= \frac{8}{\tau - 1} \left[|G_{EN}|^2 - \tau |G_{MN}|^2 + 2\text{Re}G_{EN}\Delta G_{EN}^* - 2\tau \text{Re}G_{MN}\Delta G_{MN}^* \right]. \end{aligned} \quad (11)$$

The differential cross section of the reaction $e^+ + e^- \rightarrow N + \bar{N}$, for the case of unpolarized particles, has the form

$$\frac{d\sigma_{un}}{d\Omega} = \frac{\alpha^2 \beta}{4q^2} D, \quad (12)$$

with

$$\begin{aligned} D &= (1 + \cos^2 \theta)(|G_{MN}|^2 + 2\text{Re}G_{MN}\Delta G_{MN}^*) + \frac{1}{\tau} \sin^2 \theta (|G_{EN}|^2 + \\ &\quad + 2\text{Re}G_{EN}\Delta G_{EN}^*) - \frac{4}{\tau} \sqrt{\tau(\tau - 1)} \cos \theta \text{Re}G_{MN}A_N^*, \end{aligned}$$

where θ is the angle between the momenta of the electron and the detected antinucleon, in the $e^+ + e^- \rightarrow N + \bar{N}$ reaction CMS. Note that Eq. (12) was obtained neglecting the terms of the order of α^2 compared to the dominant (Born approximation) terms. In the one-photon-exchange limit the expression (12) coincides with the result obtained for the differential cross section in Ref. [25]. The TPE contribution brings three new terms which are all of the order of α compared to the Born contribution.

At the threshold of the reaction, $q^2 = 4m^2$, the equality $G_{MN} = G_{EN} = G_N$ holds (this relation follows from the definition (3)) and the Eq. (12) reduces to

$$\begin{aligned} \frac{d\sigma_{un}^{th}}{d\Omega} &= \frac{\alpha^2 \beta}{2q^2} D^{th}, \\ D^{th} &= |G_N|^2 + \text{Re}G_N(\Delta G_{MN}^* + \Delta G_{EN}^*) + \cos^2 \theta \text{Re}G_N(\Delta G_{MN}^* - \Delta G_{EN}^*). \end{aligned}$$

As it was shown in Ref. [29], symmetry properties of the amplitudes with respect to the $\cos \theta \rightarrow -\cos \theta$ transformation can be derived from the C invariance of the considered $1\gamma \otimes 2\gamma$ mechanism:

$$\Delta G_{MN,EN}(\cos \theta) = -\Delta G_{MN,EN}(-\cos \theta), \quad A_N(\cos \theta) = A_N(-\cos \theta). \quad (13)$$

Let us consider the situation when the experimental apparatus does not distinguish the nucleon from the antinucleon. Then we measure the following sum of the differential cross sections

$$\frac{d\sigma_+}{d\Omega} = \frac{d\sigma}{d\Omega}(\cos \theta) + \frac{d\sigma}{d\Omega}(-\cos \theta).$$

We can stress, using the properties (13), that this quantity does not depend on the TPE terms. This statement agrees with the conclusion of the paper [33]: for processes of the type $e^+ + e^- \rightarrow a^+ + a^-$, if the apparatus which detects the final particles does not distinguish the particle a^+ from the particle a^- , then the interference term between the matrix elements corresponding to the one-photon and two-photon exchange diagrams does not contribute to the cross section.

Note also that the TPE terms do not contribute to the total cross section of the reaction $e^+ + e^- \rightarrow N + \bar{N}$, which can be written as

$$\sigma_t(q^2) = \frac{4\pi \alpha^2 \beta}{3} \frac{1}{q^2} \left[|G_{MN}(q^2)|^2 + \frac{1}{2\tau} |G_{EN}(q^2)|^2 \right]. \quad (14)$$

On the other hand, the relative contribution of TPE mechanism is enhanced in the following angular asymmetry

$$A(q^2, \theta_0) = \frac{\sigma(q^2, \theta_0) - \sigma(q^2, \pi - \theta_0)}{\sigma(q^2, \theta_0) + \sigma(q^2, \pi - \theta_0)}, \quad (15)$$

where the quantities $\sigma(q^2, \theta_0)$ and $\sigma(q^2, \pi - \theta_0)$ are defined as follows

$$\sigma(q^2, \theta_0) = \int_0^{\theta_0} \frac{d\sigma}{d\Omega}(q^2, \theta) d\Omega, \quad \sigma(q^2, \pi - \theta_0) = \int_{\pi - \theta_0}^{\pi} \frac{d\sigma}{d\Omega}(q^2, \theta) d\Omega.$$

Using the symmetry relations (13) one can obtain for the asymmetry $A(q^2, \theta_0)$ the following expression

$$\begin{aligned} A(q^2, \theta_0) = \frac{2}{d} \int_0^{\theta_0} d\cos\theta \left[(1 + \cos^2\theta) \operatorname{Re} G_{MN}(q^2) \Delta G_{MN}^*(q^2, \cos\theta) \right. \\ \left. + \frac{\sin^2\theta}{\tau} \operatorname{Re} G_{EN}(q^2) \Delta G_{EN}^*(q^2, \cos\theta) \right. \\ \left. - \frac{2}{\tau} \sqrt{\tau(\tau - 1)} \cos\theta \operatorname{Re} G_{MN}(q^2) A_N^*(q^2, \cos\theta) \right], \quad (16) \end{aligned}$$

where the quantity d is

$$d = \frac{1 - x_0}{3} \left[(4 + x_0 + x_0^2) |G_{MN}|^2 + \frac{1}{\tau} (2 - x_0 - x_0^2) |G_{EN}|^2 \right], \quad x_0 = \cos\theta_0.$$

The TPE contributions can be removed considering the sum of the quantities $\sigma(q^2, \theta_0)$ and $\sigma(q^2, \pi - \theta_0)$. As a result we have

$$\Sigma(q^2, \theta_0) = \sigma(q^2, \theta_0) + \sigma(q^2, \pi - \theta_0) = \frac{\pi\alpha^2}{q^2}\beta d. \quad (17)$$

The terms of the order of α^2 are everywhere neglected.

3 Single-spin polarization observables

Let us consider single-spin observables and calculate the hadronic tensors when the produced antinucleon is polarized. One finds

$$\begin{aligned} H_{\mu\nu}^{(v)}(1) &= \frac{2}{m} \frac{1}{\tau - 1} \left[im^2(\tau - 1) |\tilde{G}_{MN}|^2 \langle \mu\nu q s_1 \rangle + \right. \\ &\quad \left. i \text{Re} \tilde{G}_{MN} (\tilde{G}_{EN} - \tilde{G}_{MN})^* (\langle \nu p_2 p_1 s_1 \rangle P_\mu - \langle \mu p_2 p_1 s_1 \rangle P_\nu) + \right. \\ &\quad \left. Im \tilde{G}_{MN} \tilde{G}_{EN}^* (\langle \nu p_2 p_1 s_1 \rangle P_\mu + \langle \mu p_2 p_1 s_1 \rangle P_\nu) \right], \\ H_{\mu\nu}^{(i)}(1) &= mA_N \left[(\tilde{G}_{MN} + \tilde{G}_{EN})^* (s_{1\mu} p_{2\nu} + s_{1\nu} p_{2\mu}) + \right. \\ &\quad (\tilde{G}_{MN} - \tilde{G}_{EN})^* (s_{1\mu} p_{1\nu} + s_{1\nu} p_{1\mu}) - \\ &\quad (\tilde{G}_{MN} + \tilde{G}_{EN})^* (s_{1\mu} p_{1\nu} - s_{1\nu} p_{1\mu}) - \\ &\quad (\tilde{G}_{MN} - \tilde{G}_{EN})^* (s_{1\mu} p_{2\nu} - s_{1\nu} p_{2\mu}) - \\ &\quad \left. 2q \cdot s_1 \tilde{G}_{MN}^* g_{\mu\nu} - \frac{2}{m^2} \frac{q \cdot s_1}{1 - \tau} (\tilde{G}_{MN} - \tilde{G}_{EN})^* p_{1\mu} P_\nu \right], \end{aligned} \quad (18)$$

where $s_{1\mu}$ ($s_1 \cdot p_1 = 0$) is the antinucleon polarization four-vector.

The polarization four-vector of a particle, of mass m and energy E , in the system where its momentum is \vec{p} , is connected with the polarization vector $\vec{\chi}$ in its rest frame by a Lorentz boost

$$\vec{s} = \vec{\chi} + \frac{\vec{p} \cdot \vec{\chi} \vec{p}}{m(E + p)}, \quad s^0 = \frac{1}{m} \vec{p} \cdot \vec{\chi}.$$

Let us define a coordinate frame in CMS of the reaction $e^+ + e^- \rightarrow N + \bar{N}$ where the z axis is directed along the momentum of the antinucleon (\vec{p}), the y axis is orthogonal to the reaction plane and directed along the vector $\vec{k} \times \vec{p}$, (\vec{k} is the electron momentum), and the x axis forms a left-handed coordinate system. Therefore, the components of the unit vectors are: $\hat{\vec{p}} = (0, 0, 1)$ and $\hat{\vec{k}} = (-\sin\theta, 0, \cos\theta)$ with $\hat{\vec{p}} \cdot \hat{\vec{k}} = \cos\theta$.

Note that, unlike elastic electron–nucleon scattering in the Born approximation, the hadronic tensor $H_{\mu\nu}^{(v)}(1)$ in the TL region contains a symmetric part even in the Born approximation due to the complexity of the nucleon FFs. Taking into account the TPE contribution leads to antisymmetric terms in the $H_{\mu\nu}^{(i)}(1)$ tensor. So, these terms lead to non–zero polarization of the outgoing antinucleon (the initial state is unpolarized), which can be written as

$$P_y = \frac{2\sin\theta}{\sqrt{\tau}D} \left\{ \cos\theta [ImG_{MN}G_{EN}^* + Im(G_{MN}\Delta G_{EN}^* - G_{EN}\Delta G_{MN}^*)] - \sqrt{\frac{\tau-1}{\tau}} ImG_{EN}A_N^* \right\}. \quad (19)$$

In the one–photon–exchange (Born) approximation this expression gives the well known result for the polarization P_y obtained in Ref. [25]. One can see also that

- The polarization of the outgoing antinucleon in this case is determined by the polarization component which is perpendicular to the reaction plane.
- The polarization, being T–odd quantity, does not vanish even in the one–photon–exchange approximation due to the complexity of the nucleon FFs in the TL region (to say more exactly, due to the non–zero difference of the phases of these FFs). This is principal difference with the elastic electron–nucleon scattering.
- In the Born approximation this polarization becomes equal to zero at the scattering angle $\theta = 90^0$ (as well at $\theta = 0^0$ and 180^0). The presence of the TPE contributions leads to a non–zero value of the polarization at this angle and it is determined by a simple expression

$$P_y(90^0) = -2\frac{\sqrt{\tau-1}}{\tau\bar{D}} ImG_{EN}A_N^*, \quad \bar{D} = D(\theta = 90^0).$$

Here the function A_N is also taken at the value $\theta = 90^0$. This quantity expected to be small due to the fact that it is determined by the interference of the one–photon and two–photon exchange amplitudes and may be of the order of α . The measurement of this polarization at $\theta = 90^0$ can give information about the TPE contribution and its behaviour as a function of q^2 .

In the threshold region we can conclude that in the Born approximation this polarization vanishes, due to the relation $G_{EN} = G_{MN}$ which is valid at the threshold. The TPE contributions induces a non zero polarization, which is determined by a simple formula

$$P_y^{th}(\theta) = \frac{\sin 2\theta}{D^{th}} ImG_N(\Delta G_{EN} - \Delta G_{MN})^*.$$

Note that, at threshold, this polarization can still vanish if $\Delta G_{EN} = \Delta G_{MN}$. In this case the differential cross section does not contain any explicit dependence on the angular variable θ . In the general case, the amplitudes $\Delta G_{EN, MN}$ depend on the θ variable. The effect of the TPE contributions for the polarization at an arbitrary scattering angle is expected to increase as q^2 increase, as the TPE amplitudes decrease more slowly with q^2 in comparison with the nucleon FFs.

Using the properties of the TPE amplitudes with respect to the $\cos\theta \rightarrow -\cos\theta$ transformation, we can remove the contributions of the TPE effects by constructing the following quantities. Let us introduce the terms $P_y(q^2, \theta_0)$ and $P_y(q^2, \pi - \theta_0)$, which are integrals of the polarization $P_y(q^2, \theta)$ over the angular regions connected by the above mentioned transformation

$$P_y(q^2, \theta_0) = \int_0^{\theta_0} P_y(q^2, \theta) d\Omega, \quad P_y(q^2, \pi - \theta_0) = \int_{\pi - \theta_0}^{\pi} P_y(q^2, \theta) d\Omega.$$

Let us calculate the sum and the difference of these two quantities. At first order of the coupling constant α , we obtain

$$\begin{aligned} D^P(q^2, \theta_0) &= P_y(q^2, \theta_0) - P_y(q^2, \pi - \theta_0) = \\ &= -\frac{8\pi R}{\sqrt{\tau}} \left(1 - \frac{R^2}{\tau}\right)^{-(3/2)} \sin(\delta_{MN} - \delta_{EN}) \left[\sqrt{z} + \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{z} - \sqrt{2}}{\sqrt{z} + \sqrt{2}} \right| \right], \end{aligned} \quad (20)$$

where

$$R = \frac{|G_{EN}|}{|G_{MN}|}, \quad z = (1 - x_0^2) \left(1 - \frac{R^2}{\tau}\right),$$

and $\delta_{MN}(\delta_{EN})$ is the phase of the complex FF $G_{MN}(G_{EN})$. We can see that, in this approximation, the quantity D^P does not depend on the TPE contribution. So, the phase difference of FFs can be correctly determined from this quantity, if the ratio R is known.

Let us consider the ratio of the function $\Sigma(q^2, \theta_0)$, Eq. (17), calculated at two values of θ_0 :

$$\frac{\Sigma(q^2, \theta_1)}{\Sigma(q^2, \theta_2)} = \frac{1 - x_1}{1 - x_2} \cdot \frac{4 + x_1 + x_1^2 + \frac{1}{\tau}(2 - x_1 - x_1^2)R^2}{4 + x_2 + x_2^2 + \frac{1}{\tau}(2 - x_2 - x_2^2)R^2}, \quad x_i = \cos\theta_i, \quad i = 1, 2$$

This ratio allows to determine R , minimizing systematic errors.

The magnitude of the TPE contribution to the polarization P_y , integrated over the considered angular region, can be obtained from the sum of the quantities introduced above

$$\Sigma^P(q^2, \theta_0) = P_y(q^2, \theta_0) + P_y(q^2, \pi - \theta_0) =$$

$$\begin{aligned}
= & \frac{8\pi}{\sqrt{\tau}} \int_0^{\theta_0} d\cos\theta \frac{\sin\theta}{D_B} \left\{ \cos\theta \operatorname{Im}(G_{MN}\Delta G_{EN}^* - G_{EN}\Delta G_{MN}^*) - \right. \\
& 2\frac{\cos\theta}{D_B} \operatorname{Im}G_{MN}G_{EN}^* \left[(1 + \cos^2\theta) \operatorname{Re}G_{MN}\Delta G_{MN}^* + \right. \quad (21) \\
& \left. \left. \frac{\sin^2\theta}{\tau} \operatorname{Re}G_{EN}\Delta G_{EN}^* \right] - \right. \\
& \left. \sqrt{\frac{\tau-1}{\tau}} \left[\operatorname{Im}G_{EN}A_N^* + 4\frac{\cos^2\theta}{D_B} \operatorname{Im}G_{MN}G_{EN}^* \operatorname{Re}G_{MN}A_N^* \right] \right\},
\end{aligned}$$

where

$$D_B = (1 + \cos^2\theta)|G_{MN}|^2 + \frac{\sin^2\theta}{\tau}|G_{EN}|^2.$$

Let us consider the single-spin asymmetry induced by the transverse polarization of the electron or positron beam. The leptonic tensors corresponding to an arbitrary polarized electron beam have the form

$$\begin{aligned}
L_{\mu\nu}^{(v)}(s_e) &= 2im_e \langle \mu\nu s_e q \rangle, \\
L_{\mu\nu}^{(i)}(s_e) &= -2m_e [-k_2 \cdot s_e g_{\mu\nu} + k_{2\mu} s_{e\nu} + k_{2\nu} s_{e\mu} + k_{1\mu} s_{e\nu} - k_{1\nu} s_{e\mu}], \quad (22)
\end{aligned}$$

where $s_{e\mu}$ ($s_e \cdot k_1 = 0$) is the electron polarization four-vector.

Then the asymmetry can be written as

$$A_e = 4\frac{m_e}{\sqrt{q^2}} \frac{\beta}{D_B} \operatorname{Im}G_{MN}A_N^* (\hat{k} \times \hat{p}) \cdot \vec{\xi}, \quad (23)$$

where $\vec{\xi}$ is the unit vector along the polarization of the electron in its rest system. A_e is determined by the electron spin vector component which is perpendicular to the reaction plane. It vanishes in the Born approximation as it is determined by the imaginary part of the product $G_{MN}A_N^*$, i.e., by the spin structure induced by the TPE contribution. The same term defines the magnitude of the P_{xy} and P_{yx} components of the polarization correlation tensor of the nucleon and antinucleon. Moreover, A_e is proportional to the electron mass and vanishes for scattering angles $\theta = 0^0$ and 180^0 .

4 Double-spin polarization observables

If one of the colliding beam is longitudinally polarized then the antinucleon acquires x - and z -components of the polarization, which lie in the $e^+e^- \rightarrow$

$N\bar{N}$ reaction plane. These components can be written as (we assume 100 % polarization of the electron beam)

$$\begin{aligned}
P_x &= -\frac{2 \sin \theta}{\sqrt{\tau} D} \left[\text{Re}(G_{MN} G_{EN}^* + G_{MN} \Delta G_{EN}^* + G_{EN} \Delta G_{MN}^*) - \right. \\
&\quad \left. - \sqrt{\frac{\tau - 1}{\tau}} \cos \theta \text{Re} G_{EN} A_N^* \right], \\
P_z &= \frac{2}{D} \left[\cos \theta (|G_{MN}|^2 + 2 \text{Re} G_{MN} \Delta G_{MN}^*) + \sqrt{\frac{\tau - 1}{\tau}} \sin^2 \theta \text{Re} G_{EN} A_N^* \right].
\end{aligned} \tag{24}$$

These polarization components are T-even observables and they are non-zero also for the elastic electron-nucleon scattering in the Born approximation. Note that in the Born approximation we obtained the result of Ref. [25].

Transversally polarized electron beams lead to a polarization for the antinucleon, which is a factor (m_e/m) smaller than for the case of the longitudinal polarization of the electron beam.

The polarization component P_z vanishes when the proton is emitted at an angle $\theta = 90^\circ$ in the Born approximation. But the presence of the TPE term A_N in the electromagnetic hadron current may lead to non-zero value of this quantity if the amplitude $A_N(\theta = 90^\circ)$ is not zero, since the value of this component is determined by the term $\text{Re} G_{EN} A_N^*$.

Let us consider the case when the produced antinucleon and nucleon are both polarized. The corresponding hadronic tensors can be written as

$$\begin{aligned}
H_{\mu\nu}^{(v)}(2) &= C_1 g_{\mu\nu} + C_2 P_\mu P_\nu + C_3 (P_\mu s_{1\nu} + P_\nu s_{1\mu}) + C_4 (P_\mu s_{2\nu} + P_\nu s_{2\mu}) + \\
&\quad C_5 (s_{1\mu} s_{2\nu} + s_{1\nu} s_{2\mu}) + i C_6 (P_\mu s_{1\nu} - P_\nu s_{1\mu}) + i C_7 (P_\mu s_{2\nu} - P_\nu s_{2\mu}), \\
H_{\mu\nu}^{(i)}(2) &= i A_N \left[\frac{2}{\tau - 1} (G_{MN} - G_{EN})^* P_\nu \langle \mu s_1 s_2 P \rangle + \right. \\
&\quad G_{MN}^* (m^2 \langle \mu \nu s_1 s_2 \rangle + q \cdot s_1 \langle \mu \nu s_2 p_1 \rangle + q \cdot s_2 \langle \mu \nu s_1 p_2 \rangle - \\
&\quad p_{1\mu} \langle \nu s_1 s_2 p_2 \rangle - p_{2\nu} \langle \mu s_1 s_2 p_1 \rangle - \\
&\quad \left. s_{2\mu} \langle \nu p_2 s_1 p_1 \rangle - s_{1\nu} \langle \mu p_2 s_2 p_1 \rangle) \right],
\end{aligned} \tag{25}$$

where $s_{2\mu}$ is the nucleon polarization 4-vector ($p_2 \cdot s_2 = 0$). We omitted the terms proportional to q_μ or q_ν , since they do not contribute to the cross section and to the polarization observables due to the conservation of the leptonic current. The antisymmetrical part of the tensor $H_{\mu\nu}^{(v)}(2)$ arises from the fact that nucleon FFs in the TL region are complex quantities.

The structure functions C_i have the following form

$$\begin{aligned}
C_1 &= \frac{1}{2}(q^2 s_1 \cdot s_2 - 2q \cdot s_1 q \cdot s_2) |\tilde{G}_{MN}|^2, \\
C_2 &= \frac{2}{(\tau - 1)^2} [|\tau \tilde{G}_{MN} - \tilde{G}_{EN}|^2 s_1 \cdot s_2 + \\
&\quad \frac{1}{2m^2} (2q \cdot s_1 q \cdot s_2 - q^2 s_1 \cdot s_2) |\tilde{G}_{EN} - \tilde{G}_{MN}|^2], \\
C_3 &= \text{Re}E_1, \quad C_4 = \text{Re}E_2, \quad C_5 = -\frac{q^2}{2} |\tilde{G}_{MN}|^2, \\
E_1 &= \frac{q \cdot s_2}{\tau - 1} (\tau |\tilde{G}_{MN}|^2 - \tilde{G}_{EN} \tilde{G}_{MN}^*), \quad E_2 = -\frac{q \cdot s_1}{\tau - 1} (\tau |\tilde{G}_{MN}|^2 - \tilde{G}_{EN} \tilde{G}_{MN}^*), \\
C_6 &= \text{Im}E_1, \quad C_7 = \text{Im}E_2. \tag{26}
\end{aligned}$$

It follows that the components of the polarization correlation tensor P_{ik} , ($i, k = x, y, z$), of the nucleon and the antinucleon, in presence of the one-photon-exchange plus two-photon-exchange mechanisms in the $e^+ + e^- \rightarrow N + \bar{N}$ process, have the following expressions:

$$\begin{aligned}
P_{xx} &= \frac{\sin^2 \theta}{\tau D} \left[\tau (|G_{MN}|^2 + 2\text{Re}G_{MN} \Delta G_{MN}^*) + |G_{EN}|^2 + 2\text{Re}G_{EN} \Delta G_{EN}^* \right], \\
P_{yy} &= \frac{\sin^2 \theta}{\tau D} \left[|G_{EN}|^2 + 2\text{Re}G_{EN} \Delta G_{EN}^* - \tau (|G_{MN}|^2 + 2\text{Re}G_{MN} \Delta G_{MN}^*) \right], \\
P_{zz} &= \frac{1}{\tau D} \left[\tau (1 + \cos^2 \theta) (|G_{MN}|^2 + 2\text{Re}G_{MN} \Delta G_{MN}^*) - \sin^2 \theta (|G_{EN}|^2 + \right. \\
&\quad \left. 2\text{Re}G_{EN} \Delta G_{EN}^*) - 4\sqrt{\tau(\tau - 1)} \cos \theta \text{Re}G_{MN} A_N^* \right], \\
P_{xz} = P_{zx} &= -2 \frac{\sin \theta}{\sqrt{\tau} D} \left[\cos \theta \text{Re}(G_{MN} G_{EN}^* + G_{MN} \Delta G_{EN}^* + G_{EN} \Delta G_{MN}^*) - \right. \\
&\quad \left. \sqrt{\frac{\tau - 1}{\tau}} \text{Re}G_{EN} A_N^* \right]. \tag{27}
\end{aligned}$$

For completeness we give also the non-zero coefficients in case of longitudinally polarized electron beam

$$\begin{aligned}
P_{xy} = P_{yx} &= -\frac{1}{D} \sqrt{\frac{\tau - 1}{\tau}} \sin^2 \theta \text{Im}G_{MN} A_N^*, \\
P_{zy} = P_{yz} &= \frac{\sin \theta}{\sqrt{\tau} D} \text{Im}(G_{MN} G_{EN}^* + G_{MN} \Delta G_{EN}^* - G_{EN} \Delta G_{MN}^* + \\
&\quad + \sqrt{\frac{\tau - 1}{\tau}} \cos \theta G_{EN} A_N^*). \tag{28}
\end{aligned}$$

One can easily verify that the following relation holds:

$$P_{xx} + P_{yy} + P_{zz} = 1.$$

Let us note the following properties for these coefficients.

- The components of the tensor describing the polarization correlations P_{xx} , P_{yy} , P_{zz} , P_{xz} , and P_{zx} are T-even observables, whereas the components P_{xy} , P_{yx} , P_{yz} , and P_{zy} are T-odd ones.
- In the Born approximation the expressions for the T-odd polarization correlations coincide with the corresponding components of the polarization correlation tensor of baryon B and antibaryon \bar{B} created by the one-photon-exchange mechanism in the $e^+e^- \rightarrow B\bar{B}$ process [25]³.
- The relative contribution of the interference terms (between one- and two-photon-exchange terms) in these observables will increase as the value q^2 becomes larger since it is expected that the TPE amplitudes decrease more slowly with q^2 compared with the nucleon FFs.

At the reaction threshold, the polarization correlation tensor components have some specific properties:

- All correlation coefficients (both T-odd and T-even) do not depend on the function A_N .
- In the Born approximation the P_{yy} observable is zero, but the presence of the TPE term leads to a non-zero value, determined by the quantity $ReG_N(\Delta G_{EN} - \Delta G_{MN})^*$.
- At the scattering angle $\theta = 90^\circ$ we have the relation $P_{yy} + P_{zz} = 0$.
- The P_{xy} and P_{yx} observables are zero, and P_{yz} and P_{zy} observables are determined by the TPE term only, namely by the quantity $ImG_{MN}(\Delta G_{EN} - \Delta G_{MN})^*$.

³ The expressions in Eq. (24) of Ref. [25] should be:

$$\begin{aligned}
P_{xx} &= \frac{\sin^2\theta}{\tau D} [\tau |G_{MN}|^2 + |G_{EN}|^2], \\
P_{yy} &= \frac{\sin^2\theta}{\tau D} [|G_{EN}|^2 - \tau |G_{MN}|^2], \\
P_{zz} &= \frac{1}{\tau D} [\tau (1 + \cos^2\theta) |G_{MN}|^2 - \sin^2\theta |G_{EN}|^2], \\
P_{xz} = P_{zx} &= -2 \frac{\sin 2\theta}{\sqrt{\tau} D} Re[G_{MN} G_{EN}^*], \\
P_{xy} = P_{yx} &= 0, \\
P_{zy} = P_{yz} &= \frac{\sin\theta}{\sqrt{\tau} D} Im(G_{MN} G_{EN}^*).
\end{aligned}$$

5 Conclusions

Precise measurements of the elastic electron–hadron scattering arised the question of the importance of the TPE mechanism. This problem enters also in the determination of the nucleon FFs in the TL region investigated with the $e^+ + e^- \rightarrow N + \bar{N}$ reactions, since this process is a crossed channel of the elastic electron–nucleon scattering.

The influence of the TPE contribution on the observables of the $e^+ + e^- \rightarrow N + \bar{N}$ reaction was investigated in detail starting from a general parametrization of the relevant matrix element. The expressions for the differential cross section and various polarization observables (such as the antinucleon polarization, the polarization correlation coefficients of the produced $N\bar{N}$ pair, the polarization of the antinucleon due to the electron longitudinal polarization, and the single-spin asymmetry caused by the transverse polarization of the electron beam) were derived in terms of three complex amplitudes.

Using the properties of these amplitudes with respect to the $\cos\theta \rightarrow -\cos\theta$ transformation, it was shown how one can remove or enhance the TPE contribution from various observables. The TPE effects have been singled out in particular observables, when possible.

We showed that the real part of the TPE matrix element can be accessed through the difference between the differential cross sections measured at angles θ and $\pi - \theta$, whereas for the imaginary part of the TPE matrix element one needs the measurement of the single–spin observables (the asymmetry caused by a transversely polarized electron beam or by the nucleon or antinucleon polarization when all the other particles are unpolarized).

Single–spin observables such as the polarization of the produced proton in e^+e^- annihilation may be more difficult to measure than in the electron–proton elastic scattering, due to luminosity. On the other hand, angular distribution is easier to achieve as it is necessary to make measurements at different angles, with fixed beam energy, whereas in scattering reactions measurements should be done at fixed momentum transfer squared, which requires to change simultaneously beam energy and scattering angle.

The present analysis of the TPE effects in the $e^+ + e^- \rightarrow N + \bar{N}$ reaction should be helpful for the experimental investigation of the TL nucleon FFs planned in near future at Frascati and Novosibirsk.

These results complete our previous work on model independent derivation of polarization observables in the electron–proton elastic scattering and in the crossing channels, $e^+ + e^- \rightarrow N + \bar{N}$ and $N + \bar{N} \rightarrow e^+ + e^-$.

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References

- [1] R. Hofstadter, F Bumiller and M. Yearian, *Rev. Mod. Phys.* 30 (1958) 482.
- [2] M. N. Rosenbluth, *Phys. Rev.* 79 (1950) 615.
- [3] A. Akhiezer and M. P. Rekalo, *Dokl. Akad. Nauk USSR* 180 (1968) 1081; *Sov. J. Part. Nucl.* 4 (1974) 277.
- [4] M. K. Jones *et al.*, *Phys. Rev. Lett.* 84 (2000) 1398; O. Gayou *et al.*, *Phys. Rev. Lett.* 88 (2002) 092301; V. Punjabi *et al.*, *Phys. Rev. C* 71 (2005) 055202.
- [5] I. A. Qattan *et al.*, *Phys. Rev. Lett.* 94 (2005) 142301.
- [6] P.A.M. Guichon and M. Vanderhaeghen, *Phys. Rev. Lett.* 91 (2003) 142303.
- [7] P. G. Blunden, W. Melnitchouk, and J. A. Tjon, *Phys. Rev. Lett.* 91 (2003) 142304.
- [8] Y.-C. Chen, A. Afanasev, S. J. Brodsky, C. E. Carlson, and M. Vanderhaeghen, *Phys. Rev. Lett.* 93 (2004) 122301.
- [9] M. P. Rekalo and E. Tomasi-Gustafsson, *Eur. Phys. J. A* 22 (2004) 331.
- [10] E. Tomasi-Gustafsson and G. I. Gakh, *Phys. Rev. C* 72 (2005) 015209.
- [11] M. P. Rekalo, E. Tomasi-Gustafsson and D. Prout, *Phys. Rev. C* 60 (1999) 042202.
- [12] L. C. Alexa *et al.*, *Phys. Rev. Lett.* 82 (1999) 1374.
- [13] D. Abbott *et al.*, *Phys. Rev. Lett.* 82 (1999) 1379.
- [14] J. Gunion, L. Stodolsky, *Phys. Rev. Lett.* 30 (1973) 345.
- [15] V. Franco, *Phys. Rev. D* 8 (1973) 826.
- [16] V. N. Boitsov, L.A. Kondratyuk, and V. B. Kopeliovich, *Sov. J. Nucl. Phys.* 16 (1973) 238; F. M. Lev, *Sov. J. Nucl. Phys.* 21 (1973) 45.
- [17] M. Andreotti *et al.*, *Phys. Lett.* B559 (2003) 20.

- [18] A. De Falco, Workshop on " e^+e^- in the 1-2GeV range: Physics and Accelerator Prospects", 10-13 Sept. 2003, Alghero (Italy); hep-ex/0312012.
- [19] A. Antonelli *et al.*, Nucl. Phys. B517 (1998) 3.
- [20] R. Baldini [for the BaBar collaboration] E.P.S. Conference, Lisbon, July 21-27, (2005) and refs herein.
- [21] B. Bardin *et al.*, Nucl. Phys. B 411 (1994) 3.
- [22] An International Accelerator Facility for Beams of Ions and Antiprotons, *Conceptual Design Report*, <http://www.gsi.de>
- [23] P. Lenisa and F. Rathmann [the PAX Collaboration], arXiv:hep-ex/0505054.
- [24] A. Filippi Workshop on " e^+e^- in the 1-2GeV range: Physics and Accelerator Prospects", 10-13 Sept. 2003, Alghero (Italy).
- [25] A. Z. Dubnickova, S. Dubnicka and M. P. Rekaló, Nuovo Cim. A109 (1996) 241.
- [26] S. J. Brodsky, C. E. Carlson, J. R. Hiller and D. S. Hwang, Phys. Rev. D69 (2004) 054022.
- [27] E. Tomasi-Gustafsson, F. Lacroix, C. Duterte and G. I. Gakh, Eur. Phys. J. A 24 (2005) 419.
- [28] M. P. Rekaló, E. Tomasi-Gustafsson, Nucl. Phys. A740 (2004) 271.
- [29] M. P. Rekaló, E. Tomasi-Gustafsson, Nucl. Phys. A742 (2004) 322.
- [30] G. I. Gakh, E. Tomasi-Gustafsson, Nucl. Phys. A761 (2005) 120.
- [31] D. Nikolenko, private communication.
- [32] D. Flamm and W. Kummer, Nuovo Cim. 28 (1963) 33. S. D. Drell and J. D. Sullivan, Phys. Lett. 19 (1965) 516.
- [33] G. Putzolu, Nuovo Cim. 20 (1961) 542.