On Core Sub-Criticality as a Tool of Safety Enhancement

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ABSTRACT – Core sub-criticality can play an important role if the safety enhancement of a nuclear system is necessary, in particular, when minor actinides submitted for transmutation cause an essential degradation of the reactivity feedback effects or/and a significant reduction of the delayed neutron fraction. The present work shows that that a core sub-criticality together with a thermo-hydraulics optimization can compensate the possible degradation of the Doppler-effect and the reduction of the delayed neutron fraction. The particular dependence of the spallation neutron yield allows the creation of a supplementary negative feedback effect in case of accelerator coupled hybrid systems. A number of quantitative examples are provided in this context.

Keywords: safety, ADS, actinide transmutation, spallation reaction.

I. INTRODUCTION

It is known that core sub-criticality can be quite helpful for safety improvement, in particular, when feedback effects, the delayed neutron fraction or other safety related parameters are degraded due to the presence of long-lived actinides subjected to transmutation. There are multiple modes of operation of a sub-critical cores combined with an external neutron source (Refs. 1-3). For example, this source can be independent from the neutron production in the core (as in Accelerator Driven Systems – ADS), or it can be "coupled to" or "coordinated by" the core power level (e.g., Accelerator Coupled Systems – ACS [Ref. 2] or Delayed Enhanced Neutronics systems – DEN [Ref. 3]) and some others. Each combination opens some new opportunities for safety improvement which are discussed below. Regarding the corresponding engineering design, it could be:

- an *independent* mechanism of supplementary neutron production (as in the ADS) to achieve the desired power level, or
- a "source of artificial delayed neutrons" (DEN-system) consuming part of the in-core released energy for supplementary neutron production. The external neutron source is directly "coupled" with the core power level. As a result, the supplementary neutron creation will be delayed to the time required for a fission energy transfer from the core to a special neutron production mechanism (spallation, *bremsstrahlung*-photonuclear, nuclear fusion, etc.). The physical background of this concept is rather simple: an intermediate process "hides" neutrons (of some neutron generation) temporarily to recover them later. This allows the neutron life time be artificially increased and to slow-down dangerous transients.

In order to illustrate in more detail the potential of hybrid systems of different types for safety enhancement, three examples[†] will be discussed below:
(a) an increase of the total fraction of delayed neutrons in DEN systems (Section II);

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[*] Presently retired.
[†] *It should be noted that these examples have rather theoretical and illustrative character.*

(b) an improvement of the dynamics of a sub-critical system with degraded Doppler-effect (Section III);
(c) the design of an additional feedback effect to improve reactor safety (Section IV).

II. ON "ARTIFICIAL" GROUP OF DELAYED NEUTRONS AND ITS DELAY TIME

In some cases critical systems suffer from the decrease of the delayed neutron fraction due to the specific fuel properties (e.g., in actinide transmuter cores, in systems with circulating fuels, etc.). It may lead to undesirable acceleration of transients, to the necessity of the limitation of admissible reactivity variation and, finally, to the significant deterioration of safety.

A DEN system consists of a sub-critical core coupled to an external neutron source producing a quantity of neutrons, necessary to sustain a chain reaction in the core (Refs. 2-3). As mentioned earlier, this neutron production can be considered "delayed" if compared with prompt neutrons of fission. *Such a system with sub-critical core operates in the "critical mode" having the increased total fraction of delayed neutrons.* This fraction consists of the delayed neutrons of two kinds:

- originating from fission product decay (so-called "natural" delayed neutrons as in traditional critical reactors) and
- originating from a supplementary neutron production mechanism with their particular neutron spectrum and spatial characteristics. Unlike the first kind, their delay depends on the engineering design of the installation and can be optimized by the designer. This group can be considered as a group of "artificially" created delayed neutrons.

This particular property of the DEN system, if compared with the conventional critical reactors, can improve the reactor dynamics significantly. Moreover, the DEN system operates in a critical mode but, unlike the ADS, it takes advantage of favorable temperature feedbacks, existing in these systems.

Point kinetics equations of a DEN-system can be presented in the classical form (Ref. 4):

$$\begin{aligned} \frac{dP(t)}{dt} &= \frac{\rho(t)-\beta}{\Lambda}P(t)+\sum_{i=1}^{6}\lambda_i C_i(t)+Q(t), \\ \frac{dC_i(t)}{dt} &= \frac{\beta_i}{\Lambda}P(t)-\lambda_i C_i(t). \end{aligned} \tag{1}$$

where $\rho(t) = -r_0 + \Delta\rho(t)$ is the reactivity of core, $r_0 = \left(1-k_{eff}^0\right)/k_{eff}^0$ is the nominal sub-criticality level and $\Delta\rho(t)$ is the possible reactivity variation; P is power; variable C_i (having the dimensions of the specific power) reflects the contribution of delayed neutrons of i^{th}-group with the fraction β_i and the corresponding decay constant λ_i (note, that C_i is equivalent the concentration of delayed neutron precursors); $\beta = \sum_{i=1}^{6}\beta_i$ is the total fraction of delayed neutrons; Λ is the neutron generation time; the term $Q(t)$ describes an external source of neutrons.

In DEN-systems the intensity of the external neutron source $Q(t)$ is assumed to be proportional to the output power $P^{out}(t)$ of the reactor (here we do not take into account feedback delay, related to the transformation of the output energy into external neutrons). Then the intensity of the external source can be expressed as follows: $Q(t) = \xi P^{out}(t)$, where ξ is the corresponding normalization coefficient. For the neutron self-consistency, in nominal conditions the external source has to be equal to $Q_0 = r_0 P_0^{out} / \Lambda$ (where $P_0^{out} = P_0$, i.e. in steady state the system has to evacuate all generated heat). Therefore, we find that $\xi = r_0 / \Lambda$ and we obtain for the external neutron source

$$Q(t) = \frac{r_0}{\Lambda} P^{out}(t) . \tag{2}$$

To take into account the dependence $P^{out}(t)$ explicitly, the fission energy transfer from the core to the energy producing mechanism should be described. The simplest one-point thermo-hydraulic scheme of such a heat transfer can be presented by the Newton cooling model (Ref. 5)

$$C_p \frac{dT_c(t)}{dt} = P(t) - K \Delta T_{c \to k}(t) .$$

where T_c is the core temperature and C_p is the heat capacity of the core. The first term in the right part of the above Eq. describes the rate of the energy production (i.e. thermal power of the core) and the second one presents the rate of the thermal energy evacuation, which is assumed to be proportional to the core/environment temperature difference, i.e.

$$P^{out}(t) \propto \Delta T_{c \to k} ,$$

while K is the corresponding coefficient. Note that in present model the coefficient K includes all delays related to energy transfer from core to energy production device. Therefore, the source term in Eq. (1) can be rewritten as follows:

$$Q(t) = r_0 K \Delta T_{c \to k}(t) / \Lambda .$$

With the notations: $C^+ \equiv r_0 C_p \Delta T_{c \to k} / \Lambda$, $\beta^+ \equiv r_0$, $\lambda^+ \equiv K / C_p$, one obtains the following system of equations:

$$\begin{cases} \dfrac{dP}{dt} = \dfrac{\Delta\rho - (\beta + \beta^+)}{\Lambda} P + \sum_{i=1}^{6} \lambda_i C_i + \lambda^+ C^+ , \\ \dfrac{dC_i}{dt} = \dfrac{\beta_i}{\Lambda} P - \lambda_i C_i , \\ \dfrac{dC^+}{dt} = \dfrac{\beta^+}{\Lambda} P - \lambda^+ C^+ . \end{cases} \tag{3}$$

After comparison of Eqs. (3) with Eqs. (1), one may note that C^+ and λ^+ play the role of the effective concentration of neutron precursors and of the effective decay constant in the kinetics of the coupled system (of DEN type). Both parameters take into account the heat removal process. The sub-criticality level plays the role of the fraction β^+ of the “artificial” delayed neutrons: $\beta^+ = r_0$. It means that the system has a larger total fraction of delayed neutrons $\beta_{eff} = \beta + \beta^+$ which, together with parameters λ_i and λ^+, defines the “characteristic” transient time in the system. The effective neutron generation time Λ_{eff} is defined now by the following expression:

$$\Lambda_{eff} = \Lambda\left(1 - \beta^+ + \sum_{i=1}^{6}\beta_i\right) + \frac{\beta^+}{\lambda^+} + \sum_{i=1}^{6}\frac{\beta_i}{\lambda_i}. \tag{4}$$

This neutron generation time could be increased significantly compared with the corresponding critical cores both by variation of sub-criticality level (β^+) and by adjustment of thermo-hydraulic parameters (λ^+).

Certainly, the model of the heat transfer, discussed above is rather rough. More realistic models would be preferred to take into account complex heat transfer phenomena with their time dependences. On the other hand, in this case the effective “decay constant” λ^+ would be rather difficult to assess analytically. Nevertheless, λ^+ in any case *should correspond to the inverse characteristic time of heat removal in the reactor*. Therefore, the artificial group of delayed neutrons provides some unique properties with respect to the safety improvement:

- due to the “integral nature” of neutron production, discussed in Appendix, sharp perturbations of reactivity or thermo-hydraulic parameters do not cause dangerous power oscillations (consequently dangerous temperature oscillations);
- there is the ability to optimize the time characteristics of transients by choosing both the effective “decay constant” λ^+ and the fraction of the supplementary group of delayed neutrons β^+, i.e. the sub-criticality level of the system;
- it does not lead to an undesirable growth of reactivity during loss of flow events in systems with circulating fuels;
- penalties related to the supplementary neutron production can be relatively modest because a small level of core sub-criticality, consequently rather weak intensity of the external neutron source (when compared with conventional ADS) would be sufficient.

III. ON BEHAVIOR OF ADS WHEN FEEDBACK EFFECTS ARE DEGRADED

Let us imagine that a degradation of safety characteristics of a critical core leads to decrease (down to zero-level) of the negative Doppler effect (or a similar rapid negative feedback effect) which usually plays the most important stabilization role in the standard safety related situations. One could ask *if a reasonably chosen sub-criticality level in the case of ADS could compensate the degraded Doppler effect so that the asymptotic power after insertion of the positive reactivity $\Delta\rho_{TOP}$ would be equal to the asymptotic power of the “non-degraded” critical core.*

The answer to the above question depends on the particularities of a system and the analysis for the ADS with mobile fuel core will be presented below as an example. In this case point kinetics equations have to be slightly changed to take into account the fuel circulation, namely:

$$\begin{aligned}&\frac{dP(t)}{dt}=\frac{\rho(t)-\beta^*}{\Lambda}P(t)+\sum_{i=1}^{6}\lambda_i C_i(t)+Q(t);\\&\frac{dC_i(t)}{dt}=\frac{\beta_i}{\Lambda}P(t)-\lambda_i^* C_i(t);\end{aligned}\qquad(5)$$

where $\lambda_i^*=\lambda_i\,\vartheta_i^{-1}(t)$ and $\beta^*=\sum_{i=1}^{6}\beta_i\vartheta_i$ are modified parameters of λ_i and β. The correcting factors $\vartheta_i(t)=\left\{1+\left[1-\exp(-\lambda_i\tau_{out})\right]/\lambda_i\tau_{in}\right\}^{-1}$ take into account decay of precursors of delayed neutrons beyond active core region (Ref. 3). Here τ_{out} and τ_{in} are periods of time, that circulating fuel spends out- and in active core region correspondingly.

The term $Q(t)$ in Eqs. (5), describing an external neutron source of ADS, is proportional to the sub-criticality level r_0 under nominal conditions and takes into account eventual changes of the proton current $\Delta I_p(t)$:

$$Q(t)=\frac{r_0}{\Lambda}P_0\left(1+\frac{\Delta I_p(t)}{I_{p0}}\right),$$

where subscript “0” denotes the nominal values of the corresponding variables.

Let us employ a simplified two-point scheme for the heat transfer (see Fig. 1) in a circulating-fuel system. The following equations describe transients of the system:

$$\begin{aligned}&c_pM_c\frac{dT_c(t)}{dt}=P(t)-c_p\dot{M}(t)\left[T_c(t)\text{-}T_h(t\text{-}\tau_c)\right];\\&c_pM_h\frac{\partial T_h(t)}{\partial t}=-K\left[T_h(t)-T_k\right]-c_p\dot{M}(t)\left[T_h(t)-T_c(t-\tau_h)\right].\end{aligned}\qquad(6)$$

In Eqs. (6), T_c, T_h and T_k are the average temperatures of the core, heat-exchanger and condenser correspondingly; $\dot{M}$ is the fuel mass flow; M_c and M_h are the core and heat-exchanger masses; c_p is the specific heat capacity; τ_h and τ_c are the fuel transit time in the circuits of “core → heat-exchanger” and of “heat-exchanger → core” correspondingly.

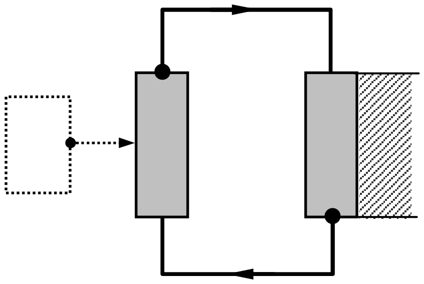

Figure 1. Two-point heat exchange scheme for the nuclear system with circulating fuel.

Let us compare the asymptotic power levels after reactivity insertion in two systems: in the critical system with “normal” thermal feedback (α) and in the ADS with degraded feedback (α'), where $(\alpha < 0) \rightarrow (\alpha' \leq 0)$, $|\alpha| > |\alpha'|$. Here a linear model of feedback effects is assumed, i.e. reactivity variations due to feedback in the standard and degraded core are given by:

$$\begin{Bmatrix} \Delta\rho_{feedback} \\ \Delta\rho'_{feedback} \end{Bmatrix} = \begin{Bmatrix} \alpha \\ \alpha' \end{Bmatrix} \Delta T_c$$

In fact, for a detailed analysis of the safety potential of any nuclear system, one is obliged to take into account all important feedbacks inherent to this system. However, for illustration of particularities of new feedbacks, it seems sufficient to use the model with only one “generalized" traditional feedback as it is presented below.

The stationary solution for the reactor power can be obtained from Eqs. (5) and (6) with zero time derivatives. In the case of ADS the following expression describes the asymptotic value of power variations:

$$\Delta P^{ADS} = \frac{P_0}{2|A'|}\left[-\left(|A'| + r_0 - \Delta\rho_{TOP}\right) + \sqrt{\left(|A'| + r_0 - \Delta\rho_{TOP}\right)^2 + 4\Delta\rho_{TOP}|A'|}\right], \qquad (7)$$

where the following notations are used:

$$A' \equiv \alpha' P_0 / K_{tot}; \quad K_{tot}^{-1} \equiv K^{-1} + (c_p \dot{M})^{-1}. \qquad (8)$$

In the case of a critical system with a “non-degraded” feedback ($A \equiv \alpha P_0 / K_{tot}$) the asymptotic response of the system to a reactivity insertion is given by

$$\Delta P^{Crit} = P_0 \Delta\rho_{TOP} / |A|. \qquad (9)$$

Comparing Eq. (7) with Eq. (9) one can evaluate the sub-criticality level, necessary to reach the same asymptotic power level in both cases:

$$r_0 = \left(1 - |A'|/|A|\right)\left(|A| + \Delta\rho_{TOP}\right), \qquad (10a)$$

which in the case of a total loss of feedback effects ($\alpha' = A' = 0$) becomes

$$r_0 = |A| + \Delta\rho_{TOP} \qquad (10b)$$

The sub-criticality level, required to compensate the degraded feedback effect, consists of two parts (Eqs. [10b]): the first one compensates the positive reactivity which would appear due to the core cooling from the temperature T_c to T_k and the second compensates the inserted reactivity $\Delta\rho_{TOP}$.

The following example illustrates these evaluations. Fig. 2 presents transients of core power (a) and of core temperature (b) in the *critical* molten salt thermal reactor (so called AMSTER concept, Ref. 6) with the “standard” feedback coefficient $\alpha = -1.95$ pcm/°C ($|A| = 487.5$ pcm, $P_0 / K_{tot} = 250$ °C), and the inserted reactivity $\Delta\rho_{TOP} = 175$ pcm. This result can be compared with the corresponding “fully degraded” ($\alpha' = A' = 0$) *sub-critical*

($r_0 = 665\,\text{pcm}$) system, where sub-criticality level was chosen in accordance with Eq. (10b). Transients for the critical system as well as for ADS are calculated in one-group approximation for delayed neutrons ($\beta = 350$ pcm, $\lambda = 0.08\ \text{s}^{-1}$) as indicated in the Ref. 6.

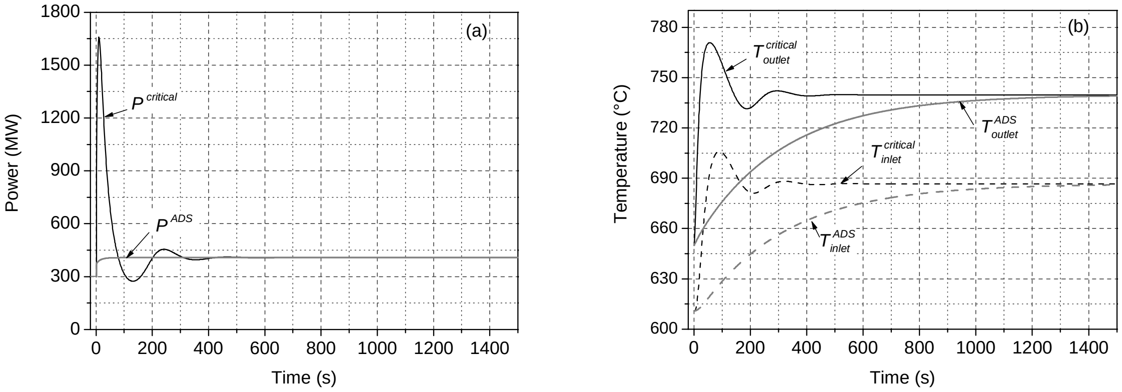

Figure 2. Unprotected TOP transients in the critical reactor with the standard feedback effect and in the corresponding sub-critical system with fully degraded feedback effect.

In brief, both critical system with the standard feedback effect and sub-critical system without feedback have similar asymptotic values with respect to the core power and temperature. However, transients are quite different: there are considerable power and temperature oscillations in critical reactors while, transient in sub-critical systems is rather monotonic. It is important to note that the monotony of power and temperature curves during both reactivity and thermo-hydraulic transients have already been reported earlier (Ref. 3).

IV. ON A NEW FEEDBACK EFFECT IN COUPLED HYBRID SYSTEMS (Y_n-EFFECT)

IV.A. Two approaches for "core - source" coupling in DEN-systems: I-mode vs E-mode.

It is known, that ADS is more favorable (compared with the similar critical reactor) regarding unprotected reactivity accidents (e.g. Ref. Schikorr, 2001), where core sub-criticality mitigates consequences of the reactivity insertion. On the other hand, a system functioning in a critical regime (including DEN system) is safer in the case of unprotected thermo-hydraulic type of transients. It would be rather attractive to combine these inherent advantages of both ADS and DEN.A possible solution to merge the above advantages will be presented. For this purpose, the operation of the DEN system has to be modified. In hybrid systems, at least two modes of coupling between external neutron source and core can be envisaged, i.e.:

1. When it is supposed to modify the intensity of an external neutron source Q by varying proton beam current I_p at a fixed nominal value of the proton energy, namely

$$I_p = I_{p0} P^{out} / P_0^{out} . \tag{11}$$

Here P^{out} is output power of the installation, and here and latter a subscript (or a superscript) “0” denotes nominal values of the corresponding variables. Hereafter this method of “accelerator-core” coupling will be designated as “***I*-mode**” coupling.

2. When any *change of output power leads to proportional change of proton energy* ε_p at the fixed nominal value of the proton current, namely

$$\varepsilon_p = \varepsilon_{p0} P^{out} / P_0^{out} . \tag{12}$$

This coupling method will be denoted “***E*-mode**” coupling.

Difference between the *E*- and *I*-modes, which we propose to make use of, is based on a non-linear behavior of the neutron yield Y_n with respect to the proton energy ε_p variations (hereafter “**Y_n-effect**”). In fact, experimental studies (Refs. 7-9) as well as Monte Carlo simulations (Ref. 10) demonstrated some features of this dependence: after proton energy passed the threshold of reaction [zone (1’) in Fig. 3], the neutron yield grows rather rapidly with energy [zone (1”) in Fig. 3]; above a certain value of ε_p dependence $Y_n(\varepsilon_p)$ has a moderated quasi-linear behavior [zone (2) in Fig. 3]. So, there is a value of proton energy $\varepsilon_p^{optimum}$ which is optimal with respect to neutron economy (i.e. maximum neutron production per one incident proton and per consumed energy). Therefore, it is generally considered that there is no reason to increase the energy of protons further than $\varepsilon_p^{optimum}$ since the production of neutrons in the spallation target becomes less efficient when compared with the increase of proton current.

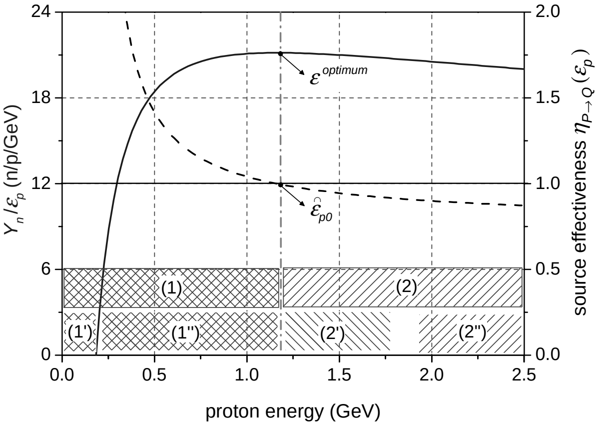

Figure 3. The dependence of the Y_n/ε_p - function (solid line) and of the source effectiveness $\eta_{P\to Q}$ (dash line) on incident proton energy ε_p.

Here, contrary to the above argumentation, we propose to take advantage of this decrease in neutron production efficiency in order to accentuate the self-stabilizing behavior of the E-coupled system. Detailed description of a hybrid system based on the *E*-mode coupling is outside the scope of this paper. However, in order to give some quantitative illustration of the main principle, a simplified model of a system with the *E*-coupling is presented in the next Section.

IV.B. Example of E-mode coupled system (DENNY)

Let us consider a DEN system with a predefined sub-criticality level r_0, where a part of the produced energy feeds an external neutron source. The DEN system with an accelerator is coupled to the core in the *E*-mode will be called "DENNY" (Delayed Enhanced Neutronics with Non-linear neutron Yield).

External neutrons are supposed to be created in the lead spallation target by incident protons accelerated up to the energy ε_{p0}. As it will be shown later, the nominal proton energy ε_{p0} is preferable to be greater than $\varepsilon_p^{optimum}$. The proton current I_p *is fixed* and its value is chosen to sustain the nominal power level P_0 in nominal state: $I_{p0} \propto r_0 P_0$. In the frame of the point-kinetics model employed in Section II [Eqs. (1-2)], the source term Q has to be revised in order to describe the *E*-mode coupling:

$$Q = \frac{r_0 P_0}{\Lambda} \frac{Y_n\left(\varepsilon_p\right)}{Y_n\left(\varepsilon_{p0}\right)}, \quad \text{where} \quad \varepsilon_p = \varepsilon_{p0} \frac{P^{out}}{P_0^{out}} = \varepsilon_{p0} \frac{\Delta T_{c\to k}}{\Delta T_{c\to k}^0}, \tag{13}$$

Let us study the response of the DENNY system on a reactivity insertion. A new equilibrium power level ($\bar{P}$) of the system after insertion of the reactivity $\Delta\rho_{TOP}$ follows from the stationary kinetic Eqs. (1) with the source term described by Eq. (13):

$$\left(\Delta\rho_{TOP} - r_0 + \Delta\rho_{feedback}\right)\bar{P} + r_0 P_0 Y_n\left(\bar{\varepsilon}_p\right)/Y_n\left(\varepsilon_{p0}\right) = 0, \quad \bar{\varepsilon}_p = \varepsilon_{p0} P / P_0. \tag{14}$$

In Ref. 8, the authors propose an empirical formula for neutron yield per one incident proton in a thick lead target (diameter 200 mm, length 600 mm). This dependence as a function of proton energy is given by

$$Y_n\left(\varepsilon_p\right) = -a + b\varepsilon_p^{3/4} \tag{15}$$

with the following empirical parameters: $a = 8.2$, $b = 29.3$.

Eqs. (14-15) together with the feedback model describe the equilibrium states of the DENNY system after reactivity transients[*].After reactivity transients a new power level $\bar{P}$ will be determined not only by the core feedbacks, but also by the capability of an external source to produce sufficient neutrons to sustain this power. A non-linear neutron production influences the equilibrium power level, and its effectiveness will depend on the choice of the nominal proton energy ε_{p0}. The Y_n-effect increases the asymptotic power if $\varepsilon_{p0} < \varepsilon_p^{optimum}$ [region (1) in Fig. 3]. Contrary, it reduces the power growth if $\varepsilon_{p0} \geq \varepsilon_p^{optimum}$ [region (2) in Fig. 3]. In fact, if the condition $\left(\delta Q / \delta P\right) < \left(r_0 / \Lambda\right)$ is fulfilled, the external neutron source is not able to support the increasing power, what will limit the power growth $\Delta\bar{P} = \bar{P} - P_0$.

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[*] Existence of these states (as well as their stability analysis) is not discussed in this work. Here we suppose that inherent properties of the system (i.e., negative feedback, Y_n-effect, etc.) will ensure these stable equilibrium states. Presented analysis is not valid for systems which do not match these conditions.

Let us introduce the function $\mathrm{H}_{P\to Q}\left(\varepsilon_p,\varepsilon_{p0}\right)$ describing the effectiveness of neutron production by an external source:

$$\mathrm{H}_{P\to Q}\left(\varepsilon_p,\varepsilon_{p0}\right)\equiv\frac{\Lambda}{r_0}\frac{dQ}{dP}. \tag{16}$$

In the frame of the above model it can be re-expressed as

$$\mathrm{H}_{P\to Q}\left(\varepsilon_p,\varepsilon_{p0}\right)=\frac{\Lambda}{r_0}\left(\frac{dQ}{dY_n}\right)\left(\frac{dY_n}{d\varepsilon_p}\right)\left(\frac{d\varepsilon_p}{dT_h}\right)\left(\frac{dT_h}{dP}\right), \tag{17a}$$

or, using Eqs. (13) and after some simplifications, it becomes

$$\mathrm{H}_{P\to Q}\left(\varepsilon_p,\varepsilon_{p0}\right)=\left(\frac{\varepsilon_{p0}}{Y_n\left(\varepsilon_{p0}\right)}\right)\left(\frac{dY_n\left(\varepsilon_p\right)}{d\varepsilon_p}\right). \tag{17b}$$

The function $\eta_{P\to Q}\left(\varepsilon_{p0}\right)\equiv\mathrm{H}_{P\to Q}\left(\varepsilon_{p0},\varepsilon_{p0}\right)=dQ/dP$ is a measure of the local effectiveness of the neutron source, i.e. a source response due to an infinitesimal power change in a nominal state. Eq. (14) is non-linear with respect to the variable $\overline{P}$ and can be solved numerically. However, linearization of Eq. (14) allows us to characterize the Y_n-effect analytically with respect to the infinitesimal power fluctuation. Introducing normalized power reactivity coefficients

$$A\equiv P_0\left(d\rho_{feedback}(P_0)/dP_0\right) \text{ and } B\equiv P_0\left(d\rho_{source}(P_0)/dP_0\right) \tag{18}$$

we rewrite Eq. (14) in the linear form:

$$\delta\rho_{ext}+\delta\overline{P}\left(A+B\right)/P_0=0. \tag{19}$$

Taking into account that $Q(P_0)=P_0$ (being the initial condition) and after some modifications, one obtains the following expression for the parameter B:

$$B=r_0P_0\left[\frac{d}{dP_0}\left(\frac{Q(P_0)}{P_0}\right)\right]=-r_0\left(1-\eta_{P\to Q}(P_0)\right). \tag{20}$$

With respect to the global neutron balance in the DENNY system, Eq. (19) demonstrates that the parameter B may be considered as a measure of supplementary neutron production feedback and arising in the system due to Y_n-effect. As it follows from Eq. (20), coefficient B is proportional to the nominal sub-criticality level r_0 and depends on the $\eta_{P\to Q}(P_0)$ functional behaviour.

At $\widehat{\varepsilon}_{p0}=\left(4a/b\right)^{4/3}=1.16\text{ GeV}$ the function $\eta_{P\to Q}(\widehat{\varepsilon}_{p0})=1$ and this defines the limit between the “destabilizing” area of the Y_n-effect (amplification of $\Delta\overline{P}$, similar to positive feedback) at $\varepsilon_{p0}<\widehat{\varepsilon}_{p0}$ and the “stabilizing” area of the Y_n-effect (suppression of $\Delta\overline{P}$, similar to negative feedbacks) at $\varepsilon_{p0}\geq\widehat{\varepsilon}_{p0}$ (Fig. 3.). Note that $\widehat{\varepsilon}_{p0}$ is equal to the optimum energy with respect to the neutron economy defined by $\varepsilon_p^{optimum}$.

The effectiveness of the Y_n-effect for safety improvement can be described by the transient suppression parameter D, defined as a ratio of asymptotic power values of *E*-coupled and *I*-coupled systems after a certain reactivity insertion transient, namely

$$D = \overline{P}^{(E-mode)} / \overline{P}^{(I-mode)} . \tag{21}$$

If $D < 1$, it signifies that the Y_n-effect stabilizes the system.

Fig. 4 presents the D-values at different r_0 and $\Delta\rho_{TOP}$ for the linear model of in-core feedback (non-degraded core) and the thermo-hydraulic parameters used in Section III. Three nominal energy values have been chosen: the optimal $\hat{\varepsilon}_{p0}$ = 1.16 GeV (a), as well as higher ε_{p0} = 1.60 GeV (b) and lower ε_{p0} = 0.80 GeV (c).

The following points are worth mentioning:

- the stabilizing role of the Y_n-effect increases when both r_0 and $\Delta\rho_{TOP}$ increase. This effect can be quite significant (up to 30 % at $r_0 = 15\beta$). A further growth of $\Delta\rho_{TOP}$ leads to the saturation of such a tendency;
- the augmentation of the nominal proton energy ε_{p0} accentuates the stabilizing impact of Y_n-effect due to reduction of the source effectiveness $\eta_{P\to Q}(\varepsilon_{p0})$.

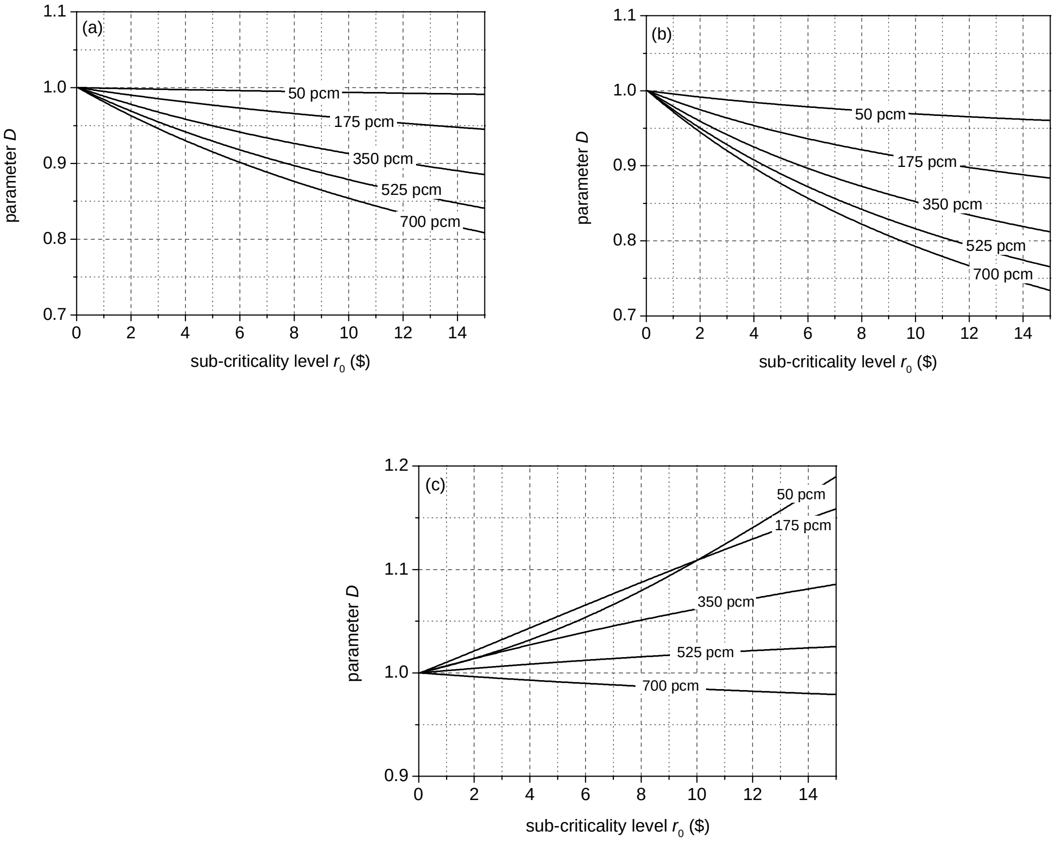

Fig.4. Reactivity transient suppression parameter D as a function of the sub-criticality level r_0 at different values of the inserted reactivity $\Delta\rho_{TOP}$. Nominal proton energies are: (a) $\varepsilon_{p0} = 1.16$ GeV; (b) $\varepsilon_{p0} = 1.60$ GeV; (c) $\varepsilon_{p0} = 0.80$ GeV. The feedback parameter A equals to -487.5 pcm.

Parameter D depends also on the power reactivity coefficients A [see Eq. (18)], explicitly introduced in Section III [Eq. (8)]. It should be reminded that this parameter reflects both the in-core reactivity feedback effects and thermo-hydraulics of the system. Figs. 5(a,b) illustrates that the impact of Y_n-effect on the power stabilization increases when the absolute value of the feedback coefficient A decreases. This dependence of the transient suppression parameter D on parameter A is expectable: indeed, if $A \to 0$ the Y_n-effect remains the only feedback effect of the system.

In the above study the choice of the nominal energy of protons was based on the analysis of *positive* reactivity insertions. However, if the value $\varepsilon_{p0} = \varepsilon_p^{optimum}$ the DENNY system becomes unstable with respect to *negative* reactivity insertions ($\Delta\rho_{TOP} < 0$), what could be unwanted for its control. To avoid this effect it is preferable to choose the proton energy at a nominal state as follows $\widehat{\varepsilon}_{p0} \to \varepsilon'_{p0} = \widehat{\varepsilon}_{p0} + \Delta\varepsilon_m$ (region 2" on Fig. 3), where the margin $\Delta\varepsilon_m$ [zone (2') on Fig. 3] makes the system more stable with respect to negative reactivity insertions, if during system functioning the proton energy stays above the optimal energy, i.e. $\varepsilon_p \geq \widehat{\varepsilon}_{p0}$.

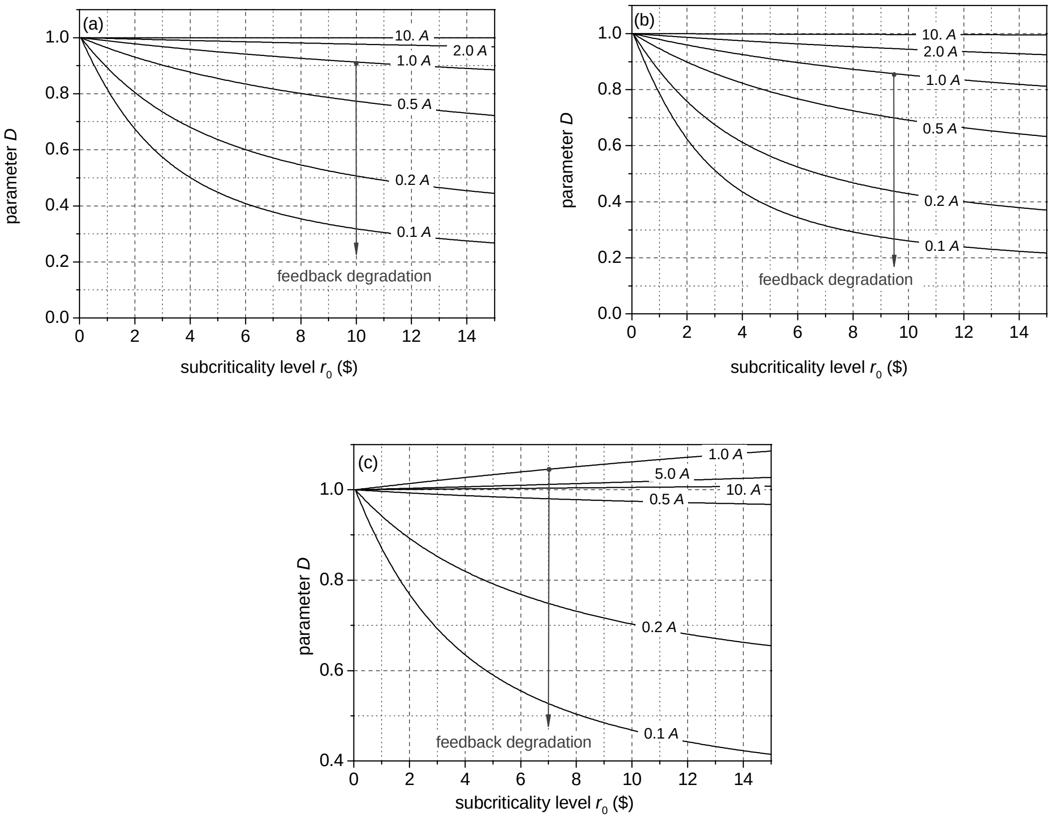

Figure 5. Reactivity transient suppression parameter *D* as a function of the sub-criticality level r_0 at different values of the parameter *A*. Nominal proton energies are: (a) $\varepsilon_{p0} = 1.16$ GeV; (b) $\varepsilon_{p0} = 1.60$ GeV; (c) $\varepsilon_{p0} = 0.80$ GeV . The inserted reactivity $\Delta\rho_{TOP}$ equals to 350 pcm.

In brief, a coupled hybrid system, as mentioned in Section II, may be considered as a *critical* system with two types of neutrons contributing to the neutron balance: "core neutrons" and "source neutrons". Though this separation of neutrons is artificial, it reflects their origin and, therefore, shows the corresponding feedbacks existing in each case. In the same sense, the Y_n-effect *together with core sub-criticality* can be compared with the Doppler feedback effect with respect to the external neutron source.

This last assumption needs more explanations. Indeed, the Doppler effect is the *intrinsic* and the *instantaneous* feedback effect, leading to a *limitation of both the asymptotic reactor power* and *asymptotic* temperatures.

As for the Y_n-effect, it is also based on the *intrinsic* physical phenomenon: dependence of the neutron production in a finite spallation target upon the energy of the incident particle. However, to guarantee that the Y_n-effect is intrinsic, at least two conditions have to be fulfilled:

(A) the system has to be engineered in such a way that it leads to an increase of the proton energy but not of the proton current (as described above);

(B) the system has to remain sub-critical in any situation, i.e. sub-criticality level has to be greater than the maximal value of the inserted reactivity.

For further clarification, let us consider time intervals shorter than the characteristic time of energy transfer from the core to the external neutron producer. During these short periods of time the external source remains unchanged, i.e. the prompt kinetic response of the coupled system on the reactivity insertion is the same, as for an ADS. If (B) is valid then any prompt reactivity insertion will result (similar to Doppler-effect) in a *limited* prompt jump of the reactor power (e.g. Ref. 2, 11) with a magnitude depending only (without any in-core feedbacks) upon the ratio of the “inserted reactivity/margin to core criticality”. In *this* context, the Y_n-effect *together* with core sub-criticality can be considered as an intrinsic and instantaneous feedback.

In fact, the Y_n-effect leads to the moderation of the asymptotic reactor power, if (A) and (B) are valid. It would be quite advantageous for the system safety to have this complementary feedback when “standard” core feedbacks are degraded.

V. CONCLUSIONS

Core sub-criticality can be an important tool for the safety enhancement of a nuclear system. It has been shown that an appropriate choice of the sub-criticality level and of the thermo-hydraulic parameters can compensate both the degradation of the Doppler-effect and the reduction of the delayed neutron fraction. Unlike conventional critical reactors, sub-critical core behavior is able to be more beneficial inherently – it suppresses the power and temperature unprotected transients.

It was suggested that the nonlinear energy dependence of the spallation neutron yield (Y_n-effect) can play the role of a supplementary negative feedback in coupled hybrid systems. The implementation of the Y_n-effect offers the possibility to compensate (to some extent) eventual feedback degradation in the cores dedicated to nuclear waste transmutation

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APPENDIX. ON SOURCE DESCRIPTION IN A DEN-SYSTEM: "HEAT-REMOVAL MODEL" VS "DELAYED POWER MODEL"

The appearance of the new group of delayed neutrons in coupled system has been discussed earlier, for example, by authors of the ACS concept (Ref. 2), where they adopted a "delayed power model" for the source description. There it was assumed that $Q(t) \propto P(t-t_d)$ with t_d being the time delay necessary for energy to transfer from core to neutron production mechanism (accelerator); Q - source term and P - core power.

In our work the more adequate "heat-removal model" (Eqs. 1-2) is applied and as a result another interpretation for the parameters λ^+ and C^+ is given. This modification of the description of the "core power – source intensity" coupling is quite important for the core dynamic behavior. For example, the "delayed power model" by Gandini et al. (Ref. 2) may lead to oscillations of core power in the case of a sharp power increase due to reactivity insertion, while the "heat-removal model" predicts a smooth and monotonous behavior of the transient.

Indeed, taking into account Eq. (3), the explicit expression for the dependence of the external neutron source Q on the core power can be written in the following way:

$$Q(t) = \frac{\lambda^+\beta^+}{\Lambda}\int_{-\infty}^{t} P(t')\exp\left[-\lambda^+\left(t-t'\right)\right]dt'. \tag{A1}$$

As a matter of fact, the artificial neutron production in the DEN-system depends on the thermal energy, accumulated in the core (time integral of core power) as well as on particularities of the heat transfer in the system (parameter λ^+). In other words, the artificial neutron production, caused by a single fission event at any point in time t, will not be only delayed by some characteristic time t_d, but it will be also distributed over the whole period following this event.