Source Reactivity as an Extra Kinetic Characteristic of Coupled-Source Subcritical Systems

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Abstract - The objective of the present work is to study the kinetic (i.e. in absence of in-core feedbacks) response of the subcritical system to variation of the efficiency of the external neutron source. The particular class of the subcritical system with the intensity of the external source being intrinsically dependent on neutron production in the core (so-called coupled hybrid systems) is considered. The Accelerator Coupled System (ACS) is taken as example, although this analysis may be expanded to other types of the coupled subcritical systems. Within the framework of a simple mathematical model of coupled system, an interpretation of the external coupled source as supplementary group of delayed neutrons is given. An auxiliary quantity – '*source reactivity*' is introduced for convenience and a modified inhour equation for coupled systems is deduced. Analytical solution of the modified inhour equation is obtained in approximation of one group of delayed neutrons. The principal conclusion resulting from this analysis is as follows: the response of the coupled system to 'source reactivity' variation is intrinsically different from the response to core reactivity variation. Namely, there is no equivalent of prompt criticality (accompanied by drastic decrease of the reactor period) in the case of 'source reactivity' variation.

Key words: hybrid system, reactor kinetics

1. Introduction

Hybrid reactor systems, where a subcritical core is associated with an external intensive neutron source (e.g. Accelerator Driven Systems - ADS, fusion-fission hybrids), offer some promising options in resolving the current problems of nuclear power: long-lived radioactive wastes, safety enhancement etc. Diverse operation modes of the external neutron source with the subcritical core were proposed. This source may be fed by the external electric grid and therefore the source is independent on core power level (e.g. ADS) or it may be coordinated with power production in the core (Slessarev *et al.*, 1999; Gandini *et al.*, 2000; D'Angelo *et al.*, 2001; Ridikas *et al.*, 2002; Slessarev and Bokov, 2004).

In their work, Gandini *et al.* (2000) proposed to use a fraction f of the output reactor power to feed the external neutron source (accelerator) with the goal to increase the coupling between ADS neutronics and thermal-hydraulics (so-called Accelerator Coupled System [ACS] concept). The practical realization of this coupling consists in the splitting of the secondary coolant loop into production one and the coupling one, generating the electricity feeding the external neutron source. In this case the neutron production in the core and in the external source becomes *intrinsically coupled*. The ACS is taken as example in the present study, although this analysis may be expanded to other types of intrinsically coupled subcritical systems (for generalization of ACS concept see, for example, Ref. Slessarev and Bokov, 2004).

Operating in the critical mode with the subcritical core and coupled in a way discussed above, the hybrid system becomes similar to the critical reactor from the point of view of its dynamics (Gandini *et al.*, 2000; D'Angelo *et al.*, 2001). In this context the parameter f plays an essential role in the reactor kinetics: a proper choice of its nominal value f_0 guarantees the self-sustainability of the system with respect to the entire neutron balance (comprising fission neutrons and external neutrons). Any mismatch of this parameter to the value, necessary to maintain chain reaction in the core, would lead either to reactor runaway or to gradual attenuation of chain reaction in the absence of thermal feedbacks. That is why the parameter f may be considered as an analogue of reactivity $\rho = \left(k_{eff} - 1\right) / k_{eff}$ for the 'external' contribution to neutron balance in the core.

Gandini *et al.* (2000) assume that parameter f is "fixed at any time and slowly adjusted during burn-up to follow the subcriticality level evolution". One may anticipate that in practical situations f may vary from its nominal value. This can be due to uncertainties, technical failures, human errors, variation of energy production efficiency, etc. For this reason, and in view of the role played by f in the neutron balance, it becomes quite important to study kinetic or/and dynamic responses of a coupled hybrid system to fluctuations of f, and to compare it with response on reactivity fluctuations.

Nevertheless, one may expect that this response to an *equivalent* (see explanation below) perturbation of the parameter f would be *intrinsically different*. Preliminary considerations for such a statement are as follows. Indeed, the above analogy of f to reactivity is valid only in the case of the quasi-static variation of the reactor power. If the subcriticality level is chosen to be sufficiently large, the reactor core remains subcritical at any instant. This means that a self-sustaining nuclear reaction in the core remains impossible. On the other hand, any response of the external source on reactor power perturbation is delayed in time: the external neutron source "waits" for the fission energy arrival. As a result, any perturbation of the fraction f results in a prompt reactor response only in the very beginning of the transient. After that a slow transient takes place, limited by the inertia of heat transfer in the reactor.

Summarizing the above considerations, one may conclude that the fraction f would be *analogous to* ρ *from the point of view of a quasi-static neutron multiplication factor*, but its role for the kinetics of coupled system would be *quantitatively and qualitatively different*.

The goal of the present work is to elucidate and to qualify the above differences. We start with the introduction of a simple mathematical model, describing a coupled hybrid system. Within the framework of this model, we give an interpretation of external coupled source as a supplementary group of delayed neutrons. In order to inter-compare the influence of in-core reactivity variation and of f-parameter variation, an auxiliary quantity – '*source reactivity*' is utilized for convenience. This allows us to deduce the modified inhour equation for a coupled hybrid system. An analytical solution of the modified inhour equation is obtained in approximation of one group of delayed neutrons. The principal conclusion following from this analysis is that a response of the coupled system to the 'source reactivity' variation is intrinsically different from this one to the core reactivity variation. Namely, there is no equivalent of prompt criticality for 'source reactivity'.

2. External coupled source as a supplementary group of delayed neutrons

We introduce a simple mathematical model, describing a coupled hybrid system. This simplicity permits us to carry out a complete analytical study as well as to reveal and illustrate the most significant kinetic features of the system under consideration. Moreover, it permits us to carry out a direct comparison with conventional point kinetics of a critical reactor.

The equations of point kinetics for a coupled system can be presented in the 'classical' form (Waltar, Reynolds, 1981):

$$\frac{d}{dt}P(t) = \frac{\rho(t)-\beta}{\Lambda}P(t) + \sum_{g=1}^{G}\lambda_g \xi_g(t) + Q(t),$$
$$\frac{d}{dt}\xi_g(t) = \frac{\beta_g}{\Lambda}P(t) - \lambda_g \xi_g(t). \qquad (1)$$

where $\rho(t) = -r_0 + \delta\rho(t)$ is the core reactivity, $r_0 = \left(1-k_{eff}^0\right)/k_{eff}^0$ is the nominal subcriticality level and $\delta\rho(t)$ is the eventual reactivity variation; P is the reactor specific power; the term ξ_g describes the contribution of delayed neutron precursors of the g^{th}-group with the fraction β_g and the corresponding decay constant λ_g; $\beta = \sum_{g=1}^{G}\beta_g$ is the total fraction of delayed neutrons; Λ is the neutron generation time; the term $Q(t)$ describes an external source of neutrons.

It is generally assumed (see Refs. Slessarev *et al.*, 1999; Gandini *et al.*, 2000; D'Angelo *et al.*, 2001; Slessarev and Bokov, 2004) that the intensity of the external neutron source $Q(t)$ is *proportional* to the output power $P^{out}(t)$ of the reactor in coupled hybrid systems. Let us denote by f the fraction of produced power feeding the external source. Then one may express the intensity of the external source: $Q(t) = BfP^{out}(t)$, where B is the corresponding normalization coefficient. For neutron self-sustainability, in nominal conditions, the external source has to be equal to $Q_0 = r_0 P_0/\Lambda$ (where $P_0^{out} = P_0$, i.e. in the steady state the system has to evacuate all generated heat). Therefore, we find that $B = r_0/(f_0\Lambda)$ and in turn for the external neutron source:

$$Q(t) = \left(\frac{f(t)}{f_0}\right)\frac{r_0}{\Lambda}P^{out}(t). \qquad (2)$$

The nominal fraction f_0 of produced power devoted to feed the external neutron source depends on the peculiarities and the performance of the specific neutron production mechanism. In general it may be deduced from neutron economy:

$$f_0 = \frac{\nu r_0 c_n}{\eta_{el}\varphi^* \varepsilon_{fis}}. \qquad (3)$$

In this notation η_{el} is the reactor electric efficiency, ε_{fis} is the energy released per fission, ν is the mean number of fission neutrons, φ^* is the importance of the source neutrons, c_n is the

electric energy cost of neutron production, i.e. the electric energy consumed to generate one source neutron.

The following remark has to be made. As follows from Eqs. (2)-(3), $Q \propto \eta_{el} f / \left(c_n \varphi^*\right)$ and the intensity of the external source in the coupled hybrid system depends also on the reactor electric efficiency (η_{el}) and on the neutron production performance ($c_n \varphi^*$) of the external source. In general, these factors may vary and their perturbation acts in a similar way to a perturbation of the fraction of produced power feeding the external source ($\delta f = f - f_0$). In the present study we assume (for the sake of concreteness) that $\delta\eta_{el} = \delta c_n = \delta\varphi^* = 0$ but our approach may be easily generalized by the simple substitution: $f(t) \to \eta_{el}(t) f(t) / \left[c_n(t) \varphi^*(t)\right]$. For this reason, in our considerations below we utilize for the parameter f the term '*source efficiency*' assuming in such a way a broadened interpretation of this parameter as implicitly incorporating all the scope of above factors.

Let us introduce an auxiliary parameter – '*source reactivity*' as follows:

$$r \equiv r_0 \left(f / f_0\right), \tag{4}$$

i.e. a value proportional to the normalized fraction f and to the nominal subcriticality level r_0. Introduced in such a way, the parameter r determines the steady-state neutron multiplication of the system in the same manner as the core reactivity $\rho = \left(k_{eff} - 1\right) / k_{eff}$. Indeed, one can demonstrate that steady states ($\overline{P}, \overline{\xi}_g$) of the coupled system meet the conditions

$$(\rho + r)\overline{P} = 0, \quad \overline{\xi}_g = \overline{P}\beta_g / \left(\lambda_g \Lambda\right). \tag{5}$$

Introducing an entire neutron multiplication factor of the system m_{eff} by analogy with core neutron multiplication factor k_{eff}

$$\left(m_{eff} - 1\right) / m_{eff} = \rho + r, \tag{6}$$

we may express similarly the condition of neutron self-sustainability (analogue of criticality with regard to the entire neutron balance) for a coupled hybrid system:

$$m_{eff} = 1 \ . \tag{7}$$

Furthermore, as will be demonstrated below, parameter r determines the kinetic response of the coupled hybrid system to perturbation of the source efficiency like reactivity ρ determines the kinetic response of critical reactors on variation of the core neutron multiplication factor.

Now, let us return to Eqs. (1)-(2). To take into account the explicit dependence of P^{out} on t, one needs to describe the transfer of fission energy from the core to the energy producing mechanism. The simplest one-point thermo-hydraulic scheme of such a heat transfer can be presented by the Newton cooling model (Ref. Hetrick, 1971)

$$C_p \frac{dT_c(t)}{dt} = P(t) - H\theta(t), \tag{8}$$

where T_c is the core temperature and C_p is the heat capacity of the core. The first term on the right-hand side of Eq. (8) describes the rate of energy production (i.e. thermal power of the core) and the second describes the rate of thermal energy evacuation. The latter is assumed to be proportional to the core/environment (condenser) temperature difference, i.e.

$$P^{out}(t) \propto \theta(t) \equiv T_c(t) - T_{env}, \tag{9}$$

while H is the corresponding coefficient. Therefore, the source term in Eq. (1) can be rewritten as follows:

$$Q(t) = H\theta(t) r(t) / \Lambda. \tag{10}$$

Note, that the above simple model simulates the heat transfer inertia due to thermal resistance of the heat transfer system. This effect leads to the non-simultaneity of the realized power P and of the reactor output power P^{out}. Another physical mechanism leading to non-simultaneity, namely the time delay t_{SP}, arising physically from the transport of the coolant from the core to the turbine, is not included. Hence, our model is valid for the systems, where the 'thermal inertia' prevails. The issues of the 'delayed-argument' model (i.e. $P^{out} = P[t - t_{SP}]$) are discussed by Gandini *et al.* (2000) and D'Angelo *et al.* (2001).

The following remark has to be made with regard to Eq. (10). Within the framework of the above model there are, in principle, other ways to perturb the reactor equilibrium, related to the external neutron source. For example, *external* impact may manifest itself either by way of a perturbation of the heat-transfer coefficient H or via perturbation of the environment (condenser) temperature T_{env}. Both these events are of interest as they lead to corresponding transients in the hybrid system, but, in the absence of in-core feedback effects, they do not corrupt the entire neutron multiplication factor of the system. Therefore, in the present work it is assumed that $\delta H = 0$ and $\delta T_{env} = 0$.

Let us denote the variable component of the source reactivity for convenience

$$r(t) = r_0 + \delta r(t) = r_0 \left(1 + \varepsilon^{(r)}(t)\right), \tag{11}$$

where $\varepsilon^{(r)} = \delta r / r_0$ is the source reactivity variation expressed in values of initial subcriticality level. With the notations: $\xi^+ \equiv r_0 C_p \theta / \Lambda$, $\beta^+ \equiv r_0$, $\lambda^+ \equiv H / C_p$, one obtains from Eqs. (1), (6), (10) and (11) the following system of coupled equations:

$$\begin{cases} \dfrac{d}{dt} P = \dfrac{\delta\rho - \left(\beta + \beta^+\right)}{\Lambda} P + \sum\limits_{g=1}^{G} \lambda_g \xi_g + \left(1 + \varepsilon^{(r)}\right) \lambda^+ \xi^+; \\ \dfrac{d}{dt} \xi_g = \dfrac{\beta_g}{\Lambda} P - \lambda_g \xi_g, \quad g = \overline{1, G}; \\ \dfrac{d}{dt} \xi^+ = \dfrac{\beta^+}{\Lambda} P - \lambda^+ \xi^+. \end{cases} \tag{12}$$

One may note that in Eqs. (12) the external source imitates the evolution of precursors of delayed neutrons, i.e. the external source plays the role of a supplementary (artificial) group of delayed neutrons (Refs. Gandini et al., 2000, D'Angelo et al., 2001; Slessarev and Bokov, 2004). The parameters ξ^+ and λ^+ play the role of the effective concentration and of the effective decay constant, respectively. The subcriticality level plays the role of the fraction β^+ of the artificial delayed neutrons.

Despite the above analogy, the artificial group of delayed neutrons provides some unique properties from the point of view of reactor kinetics, namely:

(a) in contrast to the 'natural' groups of delayed neutrons, there is an ability (to some extent) to choose both the effective "decay constant" λ^+ and the fraction β^+;

(b) moreover, as discussed above, an external impact may perturb both λ^+ and ξ^+;

(c) a perturbation of the external source efficiency may lead to a mismatch between a decay of the 'fictitious' delayed neutron group [term $-\lambda^+\xi^+$ in Eqs. (12)] and of its contribution to the neutron balance [term $\left(1+\varepsilon^{(r)}\right)\lambda^+\xi^+$]. It is to be stressed, that some limited analogy to this phenomenon may be flow variation event in the circulating fuel systems.

In the next section we will use the mathematical model for the coupled hybrid system in the form of Eqs. (12) to explore its kinetic properties.

3. Inhour equation for the coupled hybrid system

As is well known, an asymptotic period T of the reactor kinetic response to reactivity perturbation is described by the characteristic equation, called the inhour equation or Nordheim equation (Refs. Hetrick, 1971; Ash, 1979). To obtain an analogue of the inhour equation for the coupled hybrid system, we follow the standard procedures, described for example in Refs. (Hetrick, 1971; Ash, 1979; Rozon, 1992). In the present section we use the simple but nonstrict approach, utilized in Ref. (Reuss, 2003), whereas in the Appendix the more rigorous Laplace transform method to one-group approximation of Eqs. (12) is applied.

Let us look for a solution of Eqs. (11) in the form:

$$P(t) = b_0 \exp(st); \quad \xi_g(t) = b_g \exp(st), \quad g = \overline{1,G}; \quad \xi^+(t) = b^+ \exp(st). \tag{13}$$

Substituting the above expressions into Eqs. (12), and after some rearrangement, we obtain a characteristic equation with respect to the variable ω:

$$\delta\rho + \frac{\varepsilon^{(r)}\lambda^+\beta^+}{\left(s+\lambda^+\right)} = s\left[\Lambda + \sum_{g=1}^{G}\frac{\beta_g}{\left(s+\lambda_g\right)} + \frac{\beta^+}{\left(s+\lambda^+\right)}\right]. \tag{14a}$$

It differs from the 'ordinary' inhour equation for a critical reactor by two extra terms (the last terms on the left- and on the right-hand sides of the equation). Moreover, the structure of the equation has changed. The last term on the right-hand side of this equation does not depend on $\varepsilon^{(r)}$. It represents, as expected, the supplementary group of delayed

neutrons. In other words, at $\varepsilon^{(r)} = 0$, Eq. (14a) collapses to an eventual inhour equation but with an extra group of delayed neutrons. On the contrary, in the 'ordinary' inhour equation there is no analogue for the term $\varepsilon^{(r)}\lambda^+\beta^+ / \left(s+\lambda^+\right)$. Hence, one can suppose, that its solution will differ not only quantitatively but also qualitatively.

An analytical study of this modified inhour equation would be relatively cumbersome. Nevertheless some preliminary remarks concerning its solution may be made. Rearrangement of Eq. (14a) in order to bring it into a form more appropriated for subsequent analysis gives:

$$s\left[\Lambda + \sum_{g=1}^{G} \frac{\beta_g}{\left(s+\lambda_g\right)} + \frac{\beta^+}{\left(s+\lambda^+\right)}\left(1+\varepsilon^{(r)}\right)\right] - \left(\delta\rho + \varepsilon^{(r)}\beta^+\right) = 0\,. \tag{14b}$$

The signs of the roots of the above characteristic equation give important information about the asymptotic behavior of the solution of Eq. (12). Thus, from the Descartes Rule of Signs (Meserve, 1982), it follows that if condition $\left(\delta\rho + \varepsilon^{(r)}\beta^+\right) < 0$ is fulfilled, then the characteristic equation (14a,b) has no *positive* real roots. This result is expected, as it follows from the physical meaning of the parameter $\varepsilon^{(r)}$, namely perturbation of the source reactivity; hence the condition $\left(\delta\rho + \varepsilon^{(r)}\beta^+\right) < 0$ signifies that in accordance with Eq. (6) $m_{eff} < 1$ and the chain reaction in the system has to decay with time.

A quantitative description of this decay, as well as a qualitative analysis for the case $\varepsilon^{(r)} > 0$ is given for the one-group approximation of Eqs. (12) in the next Section.

4. Approximation of one group of delayed neutrons

A simple analytical solution of Eq. (11) may be obtained in the one-group approximation for delayed neutrons. Moving to the effective concentration ξ of an "average" emitter of delayed neutrons with total fraction $\overline{\beta}$ and effective decay constant $\overline{\lambda}$, one obtains the coupled system of kinetic equations:

$$\begin{cases} \dfrac{d}{dt}P = \dfrac{\delta\rho - \overline{\beta}}{\Lambda}P + \left(1+\tilde{\varepsilon}^{(r)}\right)\overline{\lambda}\xi; \\ \dfrac{d}{dt}\xi = \dfrac{\overline{\beta}}{\Lambda}P - \overline{\lambda}\xi. \end{cases} \tag{15}$$

Note that in Eqs. (15) the source variation term is renormalized: $\tilde{\varepsilon}^{(r)} = \varepsilon^{(r)}\left(\beta^+ \big/ \overline{\beta}\right)$.

As noted above, β^+ and λ^+ are free parameters and their values may be optimized with the object of: (a) increasing the margin to prompt criticality; (b) slowing down eventual transients by increasing the mean neutron life time

$$l_{mean} = l\left(1 - \beta^+ - \sum_{g=1}^{G}\beta_g\right) + \frac{\beta^+}{\lambda^+} + \sum_{g=1}^{G}\frac{\beta_g}{\lambda_g}\,. \tag{16}$$

For these purposes β^+ and $\left(\lambda^+\right)^{-1}$ have to be increased as much as possible, and in most practical situations the artificial group (term β^+/λ^+) prevail over natural groups of delayed neutrons. Consequently, in this context one may neglect the contribution of the natural delayed neutrons and one may take:

$$\overline{\beta} = \beta^+ + \sum_{g=1}^{G} \beta_g \approx \beta^+, \quad \overline{\lambda} = \overline{\beta}\left[\beta^+\left(\lambda^+\right)^{-1} + \sum_{g=1}^{G} \beta_g \lambda_g^{-1}\right]^{-1} \approx \lambda^+, \quad \xi \approx \xi^+ \text{ and } \quad \tilde{\varepsilon}^{(r)} \approx \varepsilon^{(r)}. \quad (17)$$

Under these conditions the modified inhour formula Eq. (14) reduces after some rearrangement to the quadratic equation:

$$\Lambda s^2 + \left(\Lambda\overline{\lambda} + \overline{\beta} - \delta\rho\right)s - \overline{\lambda}\left(\delta\rho + \tilde{\varepsilon}^{(r)}\overline{\beta}\right) = 0. \quad (18)$$

Before writing the solution of Eq. (18), we analyze in detail the properties of its roots. Representing for convenience Eq. (18) in the form: $a_2 s^2 + a_1 s + a_0 = 0$ and taking into account that $a_2 = \Lambda > 0$ one obtains according to the Descartes sign rule:

i. If $\mathrm{sgn}(a_1) = 1$ and $\mathrm{sgn}(a_0) = 1$, the above equation has no *positive* roots. The physical meaning of these conditions is: the core is subcritical on prompt neutrons ($\delta\rho < \overline{\beta} + \Lambda\overline{\lambda}$) and the total neutron multiplication factor is less than "one" (i.e. $m_{eff} = 1/\left[1 - \left(\delta\rho + \tilde{\varepsilon}^{(r)}\overline{\beta}\right)\right] < 1$) correspondingly.

ii. If $\mathrm{sgn}(a_1) = 1$ and $\mathrm{sgn}(a_0) = -1$ then Eq. (18) has one *positive* root and one *negative* root. In this case the reactor core remains subcritical on prompt neutrons, but the overall neutron multiplication factor m_{eff} is greater than "one".

iii. If $\mathrm{sgn}(a_1) = -1$ then the condition $\mathrm{sgn}(a_0) = -1$ is fulfilled automatically, and Eq. (18) has one *positive* root and one *negative* root. Indeed, as follows from the definition of $\varepsilon^{(r)}$, its value area is: $\boldsymbol{E} = \left[-1, (1 - f_0)/f_0\right]$ and $\min_{\boldsymbol{E}}\left(\tilde{\varepsilon}^{(r)}\overline{\beta}\right) = -\beta^+$. Therefore condition $\mathrm{sgn}(a_1) = -1$ signifies, that $\delta\rho > \overline{\beta} + \Lambda\overline{\lambda}$, which leads to $\delta\rho + \tilde{\varepsilon}^{(r)}\overline{\beta} > \overline{\beta}\left(1 + \tilde{\varepsilon}^{(r)}\right) + \Lambda\overline{\lambda} > 0$, i.e. to condition $\mathrm{sgn}(a_0) = -1$.

A straightforward calculation of the determinant $D = a_1^2 - 4a_2 a_0$ of Eq. (18) proves that $D > 0$ if $\tilde{\varepsilon}^{(r)} \geq -1$, i.e. for the entire value area $\boldsymbol{E}$ of parameter $\varepsilon^{(r)}$. Consequently, Eq. (18) has neither complex nor double roots.

Case (i) corresponds to reactor power decreasing with time, whereas cases (ii) and (iii) correspond to solutions increasing with time.

5. Kinetic response to variation of source efficiency in the one-group approximation

The objective of the present study is to demonstrate the difference between kinetic responses of the coupled hybrid system to variation of core reactivity $(\delta\rho)$ and 'source' reactivity (δr), in particular, to inter-compare asymptotic reactor periods for these two cases.

The response to the reactivity insertion $(\delta\rho)$ is well known in the literature (e.g. Refs. Hetrick, 1971; Ash, 1979; Rozon, 1992; Reuss, 2003) and may be directly used for the inter-comparison. Therefore, in the following analysis we assume no reactivity variation ($\delta\rho = 0$)

and we focus our attention on the kinetic response of the coupled hybrid system to the source reactivity variation. In this case Eq. (18) reduces to

$$\Lambda s^2 + \left(\Lambda\overline{\lambda} + \beta^+\right)s - \varepsilon^{(r)}\overline{\lambda}\beta^+ = 0\,. \tag{19a}$$

Introducing parameter $u \equiv \Lambda\overline{\lambda}\,/\,\overline{\beta}$, Eq. (19a) can be rewritten in the form:

$$us^2 + \lambda^+(1+u)s - \varepsilon^{(r)}\left(\lambda^+\right)^2 = 0\,. \tag{19b}$$

The physical meaning of the parameter u is the ratio of the prompt neutron life-time Λ to the mean neutron life-time in the system Λ_{mean}. The latter is described by Eq. (16) and can be approximately taken as $\Lambda_{mean} \approx \overline{\beta}\,/\,\overline{\lambda}$. Indeed, estimates show that in all cases, interesting for eventual applications, the factor u may be considered as a small parameter. The prompt neutron life-time can vary by a few orders of magnitude: from $\Lambda \approx 10^{-7}$ s in fast-spectrum cores to $\Lambda \approx 10^{-3}$ s in thermal spectrum cores (Ref. Rozon 1992), i.e. one can suppose in further estimations that it does not exceed the value $\Lambda^{\max} \approx 10^{-3}$ s. The fraction of the supplementary group of delayed neutrons (subcriticality level) can vary from $\beta^+ = 350$ pcm in the case of 'beta-corrected' systems up to $\beta^+ = 5000$ pcm or, eventually, greater. Hence, it would be meaningful to assume that $\overline{\beta} \geq 700$ pcm. A value of λ^+ is, as the subcriticality level β^+, an object of optimization and so it can be, to some extent, chosen arbitrarily. A reasonable assessment for λ^+ would be $10^{-2} \div 10^{0}$ s^{-1}. Hence, we obtain the upper and the lower limits for parameter u: $10^{-8} < u < 10^{-1}$, i.e. $u \ll 1$.

The roots of the characteristic equation (19) are;

$$\omega_{1,2}^{(r)} = \frac{\overline{\lambda}}{2u}\left[-(1+u) \pm \sqrt{(1+u)^2 + 4\tilde{\varepsilon}^{(r)}u}\right] = \frac{\overline{\lambda}}{2u}\left[-(1+u) \pm R\left(u,\tilde{\varepsilon}^{(r)}\right)\right], \tag{20}$$

where the function $R\left(u,\tilde{\varepsilon}^{(r)}\right) = \sqrt{(1+u)^2 + 4\tilde{\varepsilon}^{(r)}u}$ is introduced for convenience and the superscript (r) over $\omega_{1,2}$ denotes the solution for source reactivity variation. Another way to deduce the characteristic equation (19) is demonstrated in Appendix, where an exact solution of Eq. (15) is obtained, applying the Laplace Transforms method. It is shown that a perturbation of the source reactivity $\tilde{\varepsilon}^{(r)}$ leads to the transient

$$P(t) = P_0\sum_{i=1}^{2} A_i\left(u,\tilde{\varepsilon}^{(r)}\right)\exp\left(\omega_i^{(r)}t\right), \tag{21}$$

Where the $\omega_i^{(r)}$ are given by Eq. (20) and the coefficients $A_{1,2}\left(u,\tilde{\varepsilon}^{(r)}\right)$ are assigned by the expressions:

$$A_1\left(u,\tilde{\varepsilon}^{(r)}\right) = \frac{1}{2}\left(1 + \frac{1+u+2\tilde{\varepsilon}^{(r)}}{R\left(u,\tilde{\varepsilon}^{(r)}\right)}\right), \qquad A_2\left(u,\tilde{\varepsilon}^{(r)}\right) = \frac{1}{2}\left(1 - \frac{1+u+2\tilde{\varepsilon}^{(r)}}{R\left(u,\tilde{\varepsilon}^{(r)}\right)}\right). \tag{22}$$

One may consider in accordance with above estimates, that conditions $u \ll 1$ and $2\tilde{\varepsilon}^{(r)}u \ll 1$ are fulfilled in any practical circumstance. In this situation the above solution for reactor power [Eqs. (21)-(22)] can be simplified. Thus, expanding $R\left(u,\tilde{\varepsilon}^{(r)}\right)$ in a Taylor series up to 1st order in u:

$$R\left(u,\tilde{\varepsilon}^{(r)}\right) = 1 + \left(1 + 2\tilde{\varepsilon}^{(r)}\right)u + O\left(u^2\right) \tag{23}$$

one obtains for the roots of the characteristic equation

$$\begin{aligned} \omega_1^{(r)} &= \tilde{\varepsilon}^{(r)}\overline{\lambda} + O\left(u^2\right) \approx \delta r\left(\overline{\lambda}/\overline{\beta}\right), \\ \omega_2^{(r)} &= -\overline{\lambda}\left[u^{-1} + \left(1 + \tilde{\varepsilon}^{(r)}\right)\right] + O\left(u^2\right) \approx -\left[\overline{\beta}/\Lambda + \overline{\lambda} + \delta r\left(\overline{\lambda}/\overline{\beta}\right)\right]. \end{aligned} \tag{24}$$

Similar simplification for the coefficients $A_{1,2}\left(u,\tilde{\varepsilon}^{(r)}\right)$ yields:

$$A_1\left(u,\tilde{\varepsilon}^{(r)}\right) \approx \left(1 + \tilde{\varepsilon}^{(r)}\right)\left(1 - 2\tilde{\varepsilon}^{(r)}u\right), \qquad A_2\left(u,\tilde{\varepsilon}^{(r)}\right) \approx -\tilde{\varepsilon}^{(r)}\left(1 - 2u\left(1 + \tilde{\varepsilon}^{(r)}\right)\right). \tag{25}$$

Hence, we obtain the following approximate (at $u \to 0$) expression:

$$P(t) = P_0\left[\left(1 + \tilde{\varepsilon}^{(r)}\right)\exp\left(\tilde{\varepsilon}^{(r)}\overline{\lambda}t\right) - \tilde{\varepsilon}^{(r)}\exp\left(\overline{\beta}t/\Lambda\right)\right]. \tag{26}$$

We complete these asymptotic analytical results with numerical illustrations calculated in accordance with Eqs. (20)-(22). Dependences of the dimensionless roots $\Omega_{1,2} \equiv \omega_{1,2}^{(r)}/\overline{\lambda}$ and of the coefficients $A_{1,2}$ on $\tilde{\varepsilon}^{(r)}$ for different values of parameter u are presented in Figs. 1, 2. These calculations were performed for $-1 \le \tilde{\varepsilon}^{(r)} \le 10$ and cover the range of all possible values of the parameter u (see above estimates).

It follows from Figs. 1a,b and from Eq. (24), that the first root $\omega_1^{(r)} > 0$ corresponds to an increasing with time solution (an asymptotic gradual growth), whereas the second root $\omega_2^{(r)} < 0$ describes a prompt jump of reactor power (a term, rapidly decreasing with time). As long as quantitative results for the dependences $\Omega_{1,2}\left(u,\tilde{\varepsilon}^{(r)}\right)$ are concerned, one may establish, that at $u \lesssim 10^{-3}$, all curves of Ω_1 collapse to only one, i.e. dependence on this parameter disappears when $u \to 0$; in this case (i.e. at $u \lesssim 10^{-3}$) the second root Ω_2 does not depend on $\tilde{\varepsilon}^{(r)}$ and may be approximately assumed to be $\Omega_2 = 1/u$. A particularity of the curves for $\Omega_1\left(u,\tilde{\varepsilon}^{(r)}\right)$, compared with similar calculation for the reactivity insertion in a critical reactor, is their monotony. Thus, for $\tilde{\varepsilon}^{(r)} = 1$, i.e. for the value which would lead to criticality on prompt neutrons if k_{eff} were modified, one obtains $\Omega_1 \approx 1$ (i.e. $\omega_1^{(r)} \approx \overline{\lambda}$). Even if $\tilde{\varepsilon}^{(r)} = 10$ (!), then Ω_1 increases only by a factor of $6 \div 10$. Summarizing the above considerations, we conclude that if the source reactivity varies, *the asymptotic reactor period $T^{(r)}$ will be of the order of the inverse effective decay constant $\overline{\lambda}$* in any circumstances.

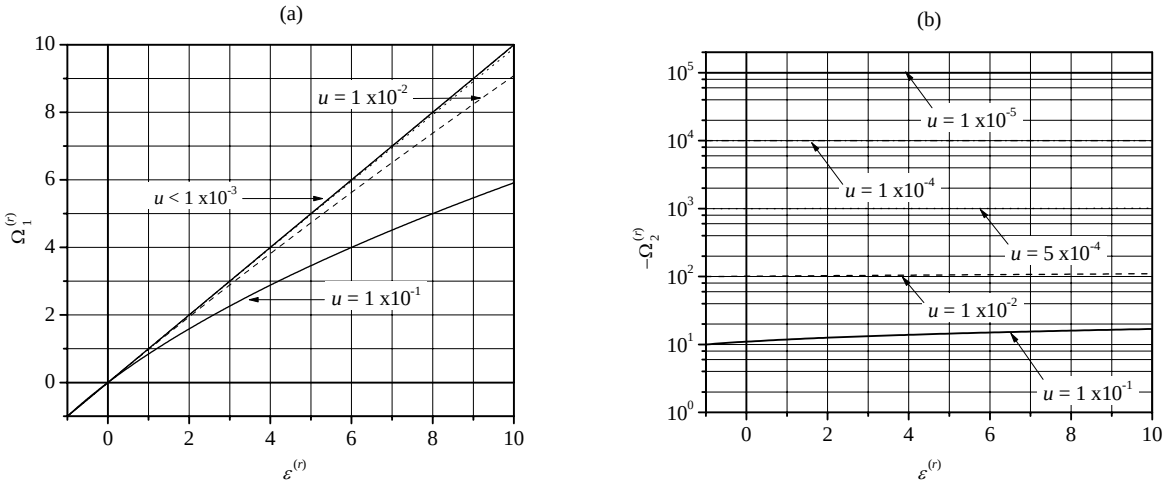

Figure 1. Dependence of the dimensionless roots Ω_1 (a) and Ω_2 (b) of the modified inhour equation on the source efficiency variation $\tilde{\varepsilon}^{(r)}$ at different values of parameter u.

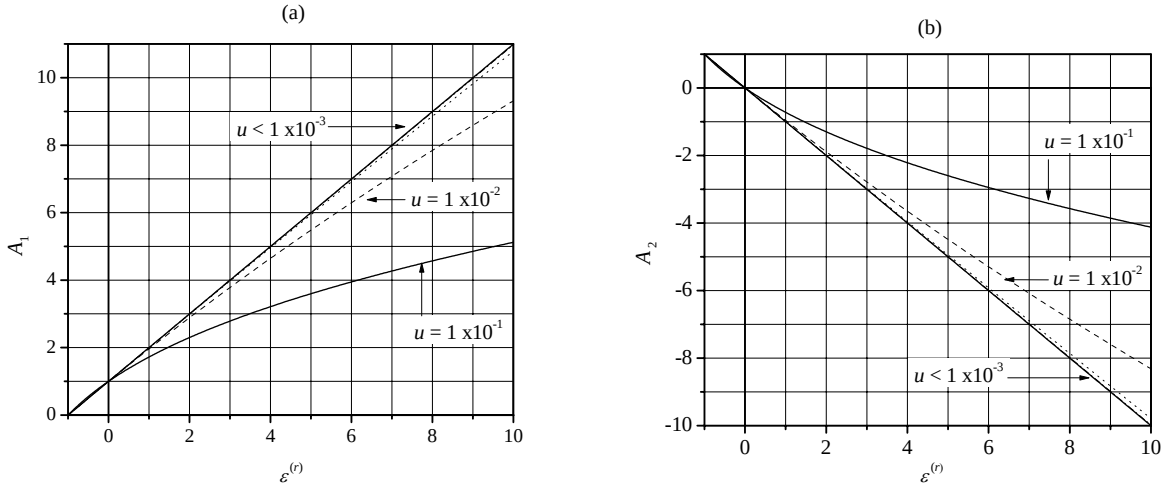

Figure 2. Dependence of the coefficients A_1 (a) and A_2 (b) on source efficiency variation $\tilde{\varepsilon}^{(r)}$ for different values of parameter u.

Hence, if the effective neutron multiplication factor of the system m_{eff} is modified by means of the source reactivity r, *there is no analogue to prompt criticality* (with consequent drastic decrease of the reactor period), existing in the conventional critical reactor when reactivity $\rho > \beta$ is inserted. In the next Section we discus this particularity in detail.

Figs. 2a,b demonstrate, that coefficients $A_{1,2}$ have nearly linear dependence on $\tilde{\varepsilon}^{(r)}$ when $u \to 0$. One can note that if $u \lesssim 10^{-3}$ they collapse, in accordance with Eq. (25), to asymptotes $\lim\limits_{u\to 0} A_1\left(u, \tilde{\varepsilon}^{(r)}\right) = 1 + \tilde{\varepsilon}^{(r)}$ and $\lim\limits_{u\to 0} A_2\left(u, \tilde{\varepsilon}^{(r)}\right) = -\tilde{\varepsilon}^{(r)}$, correspondingly. This permits us to estimate the magnitude of the prompt power jump (after the second term in Eq. (26) has disappeared):

$$\Delta P^{(r)}_{prompt} \approx P_0\left(A_1 - 1\right) \approx \tilde{\varepsilon}^{(r)} P_0\,, \tag{27}$$

i.e. it is proportional to perturbation of the source efficiency.

6. Discussion. Comparison with the case of reactivity insertion

In contrast to conventional critical reactors, in coupled hybrid systems there are two ways to affect the total neutron multiplication factor: either by means of reactivity insertion $(\delta\rho)$ or by means of modification of the source efficiency. Introduction of the source reactivity r gives us an easy-to-use tool making it possible to inter-compare these two modes, since this parameter has exactly the same meaning from the point of view of the steady neutron multiplication factor as the core reactivity $\rho = \left(k_{eff} - 1\right) / k_{eff}$. However, as was mentioned above, transients, as a response to an equivalent multiplication factor perturbation $\delta r = \delta\rho$ can differ. To quantify this eventual difference, let us inter-compare the important kinetic characteristics: roots of the inhour equation in both cases. Particular attention is paid to the inter-comparison of the asymptotic periods $T^{(r)}$ and $T^{(\rho)}$ for these two cases. A solution for the case of r-variation was obtained in the previous Sections [Eqs. (20)-(26)].

As for case of step-wise reactivity insertion, there exists a vast literature from which a solution could be taken (e.g. Refs. Hetrick, 1971; Ash, 1979; Rozon, 1992; Reuss, 2003). Thus, supposing $\Lambda \ll \overline{\beta} / \overline{\lambda}$ and having introduced parameter $\varepsilon^{(\rho)} \equiv \delta\rho / \overline{\beta}$ we can write the solution for this case in the following way:

$$P(t) = P_0 \left[\frac{1}{1-\varepsilon^{(\rho)}} \exp\left(\frac{\varepsilon^{(\rho)}}{1-\varepsilon^{(\rho)}} \overline{\lambda} t \right) - \frac{\varepsilon^{(\rho)}}{1-\varepsilon^{(\rho)}} \exp\left(-\frac{\overline{\beta}}{\Lambda} \left(1-\varepsilon^{(\rho)}\right) t \right) \right]. \tag{28}$$

Two ultimate limits are considered in our analysis:

1. Small reactivities: $\delta\rho \ll \overline{\beta}$.

In this case a reactivity insertion leads to the following roots of the characteristic equation:

$$\omega_1^{(\rho)} = \frac{\overline{\lambda}\delta\rho}{\overline{\beta} - \delta\rho} \approx \overline{\lambda} \frac{\delta\rho}{\overline{\beta}} = \overline{\lambda}\varepsilon^{(\rho)}, \quad \omega_2^{(\rho)} = \frac{\delta\rho - \overline{\beta}}{\Lambda} \approx -\frac{\overline{\beta}}{\Lambda}, \tag{29}$$

where $\varepsilon^{(\rho)} \equiv \delta\rho / \overline{\beta} \ll 1$. A comparison with the result for source variations [Eq. (24)] demonstrates that for small perturbations of neutron multiplication factor $\varepsilon = \varepsilon^{(\rho)} = \varepsilon^{(r)} \ll 1$ one finds: $\omega_1^{(\rho)} = \omega_1^{(r)}$ and $\omega_2^{(\rho)} = \omega_2^{(r)}$. Consequently, the asymptotic reactor period in both cases would be identical $T^{(r)} = T^{(\rho)} = \left(\overline{\lambda}\varepsilon^{(\rho)}\right)^{-1}$ and large when compared with the effective generation time of precursors of delayed neutrons $\overline{\tau} = \ln(2) / \overline{\lambda}$.

We can also inter-compare the prompt power jumps in these two cases. In the case of reactivity insertion we obtain from Eq. (28) : $\Delta P_{prompt}^{(\rho)} = P_0 \varepsilon^{(\rho)} / \left(1 - \varepsilon^{(\rho)}\right)$. Hence, the ratio of prompt jumps in equivalent circumstances $\left(\varepsilon = \varepsilon^{(\rho)} = \varepsilon^{(r)}\right)$ is:

$$\Delta P_{prompt}^{(r)} / \Delta P_{prompt}^{(\rho)} = 1 - \varepsilon. \tag{30}$$

i.e. the case of the source reactivity variation is more advantageous when compared with core reactivity variation, as in the case of *positive* reactivity insertion $(\varepsilon > 0)$, it results in lesser prompt power jump.

2. Great reactivities: $\delta\rho > 1.5\overline{\beta}$ $\left(\varepsilon^{(\rho)} > 1.5\right)$

In this case the reactor becomes super critical on prompt neutrons and an analysis of the inhour equation yields the well known result:

$$\omega_1^{(\rho)} = \frac{\delta\rho - \overline{\beta}}{\Lambda} \approx \frac{\overline{\beta}}{\Lambda}\left(\varepsilon^{(\rho)} - 1\right) > 0\,, \quad \omega_2^{(\rho)} = \frac{\overline{\lambda}\delta\rho}{\overline{\beta} - \delta\rho} \approx -\frac{\overline{\lambda}}{\overline{\beta}}\left(\frac{\varepsilon^{(\rho)}}{\varepsilon^{(\rho)} - 1}\right) < 0\,. \tag{31}$$

As one may remark, the positive root $\omega_1^{(\rho)}$ of the characteristic equation, governing the rate of power growth, increases drastically when compared with Eq. (29) while Eqs. (24) remain valid for variation of source reactivity. Let us assess the ratio $\omega_1^{(\rho)} / \omega_1^{(r)}$ for the equivalent perturbation of the neutron multiplication factor $\varepsilon = \varepsilon^{(\rho)} = \varepsilon^{(r)} > 1$:

$$\frac{\omega_1^{(\rho)}}{\omega_1^{(r)}} \approx \frac{\overline{\beta}}{\Lambda\overline{\lambda}} \frac{\left(\varepsilon^{(\rho)} - 1\right)}{\tilde{\varepsilon}^{(r)}} \approx \frac{\overline{\beta}}{\Lambda\overline{\lambda}} = \frac{1}{u} \gg 1\,. \tag{32}$$

The asymptotic period for the case of variation of the source efficiency becomes much greater than the asymptotic period in the case when reactivity is inserted $T^{(r)} / T^{(\rho)} = u^{-1} \gg 1$. In addition, its value remains comparable with the effective generation time of precursors of delayed neutrons $T^{(r)} \sim 1 / \overline{\lambda}$.

One can note that behavior of the coupled system is essentially different for the case of source reactivity variation. The explanation is rather simple: in this case the core remains subcritical and works in the mode of 'energy amplifier'. If some perturbation of the external source effectiveness leads to a prompt change in the core power, development of the consequent transient will be limited by the rate of energy transfer from the core to the neutron production mechanism (e.g. proton accelerator).

The above result can be reformulated in the following way: the change in the neutron multiplication factor of the coupled hybrid system through the effectiveness of the external source does not affect essentially its asymptotic period. This findings leads to important conclusion concerning the operation of these systems: from the point of view of reactor kinetics, it is preferable to regulate the neutron multiplication factor by means of the source reactivity. These results allow us to give another practical recommendation: it is preferable to envisage reactivity reserves (e.g. devoted to compensate eventual reactivity swing) in the form of the source reactivity. In this case an erroneous insertion of these reserves will not lead to drastic decrease of the reactor period.

7. Conclusions

In the present study we have proposed an approach allowing to elucidate the role of the source efficiency in kinetics of the coupled system. The total neutron multiplication factor m_{eff} and the source reactivity r are introduced by analogy with core neutron multiplication factor k_{eff} and core reactivity ρ. The source reactivity r becomes a valuable mean to compare variation of the source effectiveness with reactivity insertion.

With the support of a simple mathematical model describing the coupling of the subcritical core and of the external neutron source we demonstrate that the latter may be treated as a supplementary group of delayed neutrons. As was shown, this resemblance between 'natural' and 'artificial' delayed neutrons is not absolute: some new opportunities arise and they have to be taken into account when the kinetics of the coupled hybrid system is considered.

The modified inhour equation, which takes into account the ability to modify source reactivity, is deduced and an analysis of its roots is performed. An asymptotic reactor period, in the case of source reactivity variation, is obtained as a solution of this modified inhour equation.

The main conclusion that ensues from the above analysis is: the kinetic response of the coupled hybrid system to 'source reactivity' variation is intrinsically different from that to core reactivity, in particular, when great (when compared with the effective fraction of delayed neutrons) reactivity is introduced. Namely, there is no equivalent of prompt criticality (accompanied by drastic decrease of the reactor period) for 'source reactivity'. These results allow us to give a practical recommendation: it is preferable to have reactivity reserves in the form of the source reactivity from the point of view of reactor kinetics. In this case an erroneous insertion of these reserves will not lead to drastic decrease of the reactor period.

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Appendix. Solution of the one-group kinetic equation for coupled hybrid system

In the present section we will utilize the following notation for convenience:

$$q \equiv \overline{\beta} / \Lambda , \quad \lambda \equiv \overline{\lambda} , \; \varepsilon \equiv \delta r / \overline{\beta} . \tag{A1}$$

With this notation and supposing that there is no in-core reactivity variation (i.e. $\delta\rho = 0$) Eqs. (15) may be rewritten in the following way:

$$\begin{cases} \dfrac{d}{dt} P(t) = -qP(t) + (1+\varepsilon)\lambda\xi(t), \\ \dfrac{d}{dt} \xi(t) = qP - \lambda\xi(t). \end{cases} \tag{A2}$$

To solve the above system of equations it is useful to apply the Laplace Transformation method. We will denote the Laplace Transform for some arbitrary function of time $F(t)$ as follows: $\hat{F}(s) = \mathcal{L}[F(t)]$. Then, the Laplace transform of Eq.(A2) yields

$$\begin{cases} s\hat{P}(s) - P_0 = -q\hat{P}(s) + (1+\varepsilon)\lambda\hat{\xi}(s), \\ s\hat{\xi}(s) - \xi_0 = q\hat{P}(s) - \lambda\hat{\xi}(s), \end{cases} \tag{A3}$$

where $P_0 = P(t=0)$ and $\xi_0 = \xi(t=0)$. These equations may be solved algebraically for $\hat{P}(s)$ with some rearrangement

$$\hat{P}(s) = \frac{(s+\lambda)P_0 + (1+\varepsilon)\lambda\xi_0}{(s+\lambda)(s+q) - (1+\varepsilon)q\lambda} = \frac{(s+\lambda)P_0 + (1+\varepsilon)\lambda\xi_0}{(s-\omega_1)(s-\omega_2)}, \tag{A4}$$

where

$$\omega_{1,2} = \frac{-(q+\lambda) \pm \sqrt{(q+\lambda)^2 + 4\varepsilon\lambda q}}{2} \tag{A5}$$

are the roots of the characteristic equation

$$(s+\lambda)(s+q)-(1+\varepsilon)\lambda q = 0\,. \tag{A6}$$

Making use of the notation $u = \lambda / q$ and $R(u,\varepsilon) = \sqrt{(1+u)^2 + 4\varepsilon u}$, one can rewrite above expression in the following way

$$\omega_{1,2} = \frac{\lambda}{2u}\left[-(1+u) \pm R(u,\varepsilon)\right]. \tag{A7}$$

Applying the inverse Laplace Transformation $P(t) = \mathcal{L}^{-1}\left[\hat{P}(s)\right]$ to Eq. (A4) one obtains the solution of Eqs. (A3)

$$P(t) = \frac{P_0}{(\omega_1 - \omega_2)}\sum_{i=1}^{2}(-1)^{i-1}\left[(\omega_i + \lambda) + (1+\varepsilon)\lambda\frac{\xi_0}{P_0}\right]\exp(\omega_i t)\,, \tag{A8}$$

Where the ω_i are given by expression Eq. (A7). If at $t = 0$ the system was in equilibrium, then the initial condition yields $\xi_0 = P_0 / u$. After some rearrangement, the solution of Eqs. (A3) can be written in the form:

$$P(t) = P_0\sum_{i=1}^{2} A_i\left(u, \varepsilon^{(r)}\right)\exp(\omega_i t)\,, \tag{A9}$$

Where the coefficients A_i are given by

$$A_i\left(u, \varepsilon^{(r)}\right) = \frac{1}{2}\sum_{i=1}^{2}(-1)^{i-1}\left[(-1)^{i-1} + \frac{1+u+2\varepsilon^{(r)}}{R\left(u, \varepsilon^{(r)}\right)}\right]. \tag{A10}$$