EXPERIMENTAL AND NUMERICAL STUDIES ON THERMAL HYDRAULIC CHARACTERISTICS OF HE II THROUGH POROUS MEDIA

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ABSTRACT

To improve the stability of superconducting magnets cooled by He II, it is considered that the utilization of the porous media enhances the heat transport capability of He II. In the present studies, the thermal hydraulic characteristics of He II through porous media have been investigated experimentally and numerically. Experimental results are presented about 6 porous media with various porosity, thickness and average pore diameters. Numerical results are reported about 4 porous media among them. In the present numerical analysis, the two-dimensional calculations have been performed for the turbulent Gorter-Mellink regime to understand the heat and mass transfer of the He II through the porous media in the case of the large heat input. The present numerical model was based on the two-fluid model with the Gorter-Mellink mutual friction and dealt with the classical friction loss and the tortuosity in the porous media. This paper discusses the influence of these additional terms on the heat and mass flow of He II through the porous media at the turbulent Gorter-Mellink regime.

KEYWORDS: He II, heat and mass flow, porous media, turbulent two fluid model and tortuosity
PACS: 44.30.+v Heat flow in porous media

INTRODUCTION

For superconducting magnets cooled with He II, the conventional electrical insulation of the superconducting cables also works as the thermal insulation [1]. So, it is considered that porous media are applied as electrical insulation on superconducting cables, because the flow of the superfluid component through the porous media would make a good thermal connection between the He II and the superconducting cables. To use the porous media as the electrical insulation of superconducting magnets, it is important that the heat and mass flow of the He II through the porous media is investigated. In the present study, a numerical code, which can calculate the heat and mass transfer of He II both in micro channels of porous media and in He II bath and He II chamber simultaneously, have been developed [2]. The code is based on the Super-2D developed by Tatsumoto et al. [3], which is able to solve the two-dimensional heat transport in He II with FDM, and deals with the classical friction loss and the tortuosity in porous media. In the present calculations, the steady state problems of the heat and mass flow of the He II through the porous media were analyzed for various porous media. In this paper, the numerical results are compared with the experimental results and the validity of the present numerical model for the inside of the porous media is discussed.

EXPERIMENTAL APPARATUS AND NUMERICAL MODEL

Experimental Apparatus

FIGURE 1 shows a schematic view of an experimental apparatus for the porous media of #4 (See TABLE 1). The apparatus consists of 3 parts, which are an outer cylindrical vessel, an inner cylindrical vessel and a disk with a porous medium, made of SUS304. These parts are sealed with indium wire, the diameter of 1 mm. The space between the outer vessel and the inner vessel is evacuated to interrupt the heat flux from the He II bath to the He II chamber. The He II chamber is 40 mm high with the diameter of 60 mm. A heater and a germanium thermo sensor are fixed in the He II chamber. A pressure sensor, which is connected to the He II chamber by a narrow tube, is located out of the He II chamber. The He II chamber is only open to the He II bath through the porous medium at

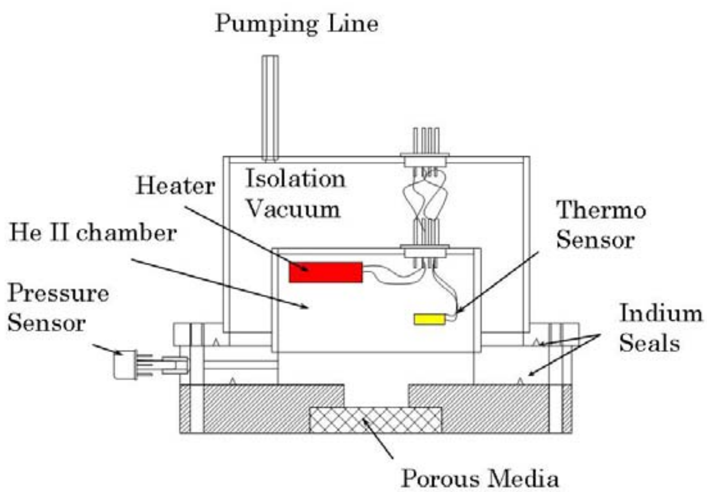

FIGURE 1. A schematic view of the experimental apparatus.

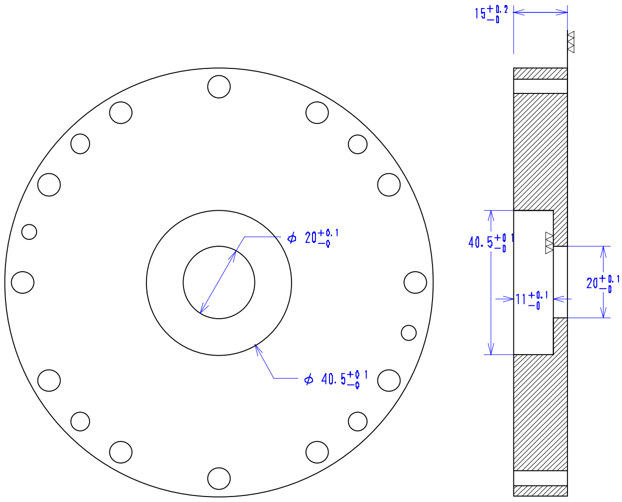

FIGURE 2. Bottom and side views of the bottom disk. The tested porous medium is attached into the center hole.

the bottom. In the present experiment, the apparatus is immersed in the pressurized He II bath.

FIGURE 2 shows schematic views of a disk with porous medium. The disk is 15 mm thick with the diameter of 120 mm and has a hole in the center of the disk. A porous medium, 10 mm thick with the diameter of 40 mm, is attached into the hole. The porous medium is made of Al_2O_3, the porosity is 32 % and the average pore diameter is 1 μm.

Numerical Model

The following basic equations are used in the present calculations.

Equation of continuity is

$$\frac{\partial}{\partial t}(\rho_s + \rho_n) + \nabla \cdot (\rho_s \vec{u}_s + \rho_n \vec{u}_n) = 0 \quad . \tag{1}$$

Equation of motion for total fluid is

$$\frac{\partial}{\partial t}(\rho_s \vec{u}_s + \rho_n \vec{u}_n) = -\nabla \cdot (\rho_s \vec{u}_s \vec{u}_s + \rho_n \vec{u}_n \vec{u}_n) - \nabla P + \eta\left(\nabla^2 \vec{u}_n + \frac{1}{3}\nabla(\nabla \cdot \vec{u}_n)\right) + \rho \vec{g} \quad . \tag{2}$$

Equation of motion for superfluid is

$$\frac{\partial \vec{u}_s}{\partial t} = -(\vec{u}_s \cdot \nabla)\vec{u}_s + s\nabla T - \frac{1}{\rho}\nabla P + \frac{\rho_n}{2\rho}\nabla|\vec{u}_n - \vec{u}_s|^2 + A_{GM}\rho_n|\vec{u}_n - \vec{u}_s|^2(\vec{u}_n - \vec{u}_s) + \vec{g} \quad . \tag{3}$$

Equation of entropy is

$$\frac{\partial}{\partial t}(\rho s) = -\nabla \cdot (\rho s \vec{u}_n) + \frac{\eta}{T}\left\{\frac{\partial u_{ni}}{\partial x_j}\frac{\partial u_{ni}}{\partial x_j} + \frac{\partial u_{nj}}{\partial x_i}\frac{\partial u_{ni}}{\partial x_j} - \frac{2}{3}(\nabla \cdot \vec{u}_n)^2\right\} + \frac{A_{GM}\rho_n\rho_s|\vec{u}_n - \vec{u}_s|^4}{T} \quad . \tag{4}$$

In the above equations, ρ is the density, η the viscosity, u the velocity, s the entropy, P the pressure, T the temperature, g the gravity and A_{GM} the constant of Gorter-Mellink. Subscripts of s and n express superfluid and normal fluid, respectively.

Assuming that both the superfluid and the normal fluid flow vertically through narrow straight tubes, the one-dimensional equations of motion for total fluid and entropy are changed into the following equations.

The equation of motion for total fluid in one dimension is

$$\frac{\partial}{\partial t}\left(\rho_s u_{sz}+\rho_n u_{nz}\right)+\frac{\partial}{\partial z}\left(\rho_s u_{sz}^2+\rho_n u_{nz}^2\right)=-\frac{\partial P}{\partial z}+\tau_W \quad , \tag{5}$$

The equation of entropy in one dimension is

$$\frac{\partial}{\partial t}(\rho s)+\frac{\partial}{\partial z}(\rho s u_{nz})=\frac{A_{GM}\rho_n\rho(u_{nz}-u_{sz})^4}{T}+\frac{\tau_W}{T}u_{nz} \quad , \tag{6}$$

where

$$\tau_W=-f\cdot\frac{\xi}{2A}\cdot\rho u_{nz}^2 \quad . \tag{7}$$

In the above equations, τ_w expresses the shear stress on the wall, ξ the wetted perimeter, f the pipe friction factor and A the cross-sectional area. In the present calculations, the pipe friction factor for each average pore diameter is used.

However, the flow passages in porous media are not straight but very narrow and winding actually. So, the Gorter-Mellink constant is modified in order to model the actual flow passages in the porous media. First, the value of the vortex line density is changed to deal with the influence of the average pore diameter. The equation of the vortex line density is described in detail as follows [4],

$$L_0^{1/2}=\frac{1}{2}\gamma\left|u_s-u_n\right|\left(1+\sqrt{1-\frac{4\alpha}{\gamma d\left|u_s-u_n\right|}}\right) \quad . \tag{8}$$

When the He II flows in a tube with the diameter over the order of mm,

$$L_0^{1/2}=\gamma\left|u_n-u_s\right| \quad . \tag{9}$$

When the He II flows in a narrow tube with the diameter of the order of μm,

$$L_0^{1/2}=\frac{1}{2}\gamma\left|u_n-u_s\right| \quad . \tag{10}$$

The mutual friction between superfluid and normal fluid is showed as follows,

$$\vec{F}_{sn}=L_0\vec{f}=A_{GM}\rho_n\rho_s(u_n-u_s)^3 \quad . \tag{11}$$

Thus the term of the mutual friction in the narrow tube with the diameter of the order of μm is a quarter of those in the tube with the diameter over the order of mm.

Second, the tortuosity of the porous media is imported into the present calculations

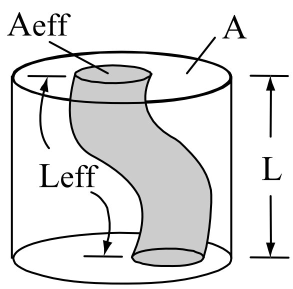

FIGURE 3. A schematic of the model of a flow passage in the porous media.

and the Gorter-Mellink constant is modified. The flow pass is not straight in the porous media since the superfluid helium can flow through only a space between particles. So, the effective channel length, called L_{eff}, and the effective channel cross sectional area, called A_{eff}, are defined as FIGURE 3 [5]. The porosity of the porous media is ε, while the tortuosity of the porous media is ω. The effective channel length is ω times the thickness of the porous media, L. $L_{eff} = \omega L$. From FIGURE 3, the effective channel length times the effective channel cross sectional area equals the total volume of the pore. $A_{eff} L_{eff} = \varepsilon A L$. Therefore, the effective channel cross sectional area is derived as $A_{eff} = \varepsilon A / \omega$ [8].

Next, the temperature gradient is shown as follows,

$$\nabla T = \frac{A_{GM}\rho_n}{s}\left|u_s - u_n\right|^3 . \tag{12}$$

Also, in the porous media, u_n is so small compared with u_s that u_n is able to be neglected.

$$u_s = \left(\frac{s}{A_{GM}\rho_n}\nabla T\right)^{\frac{1}{3}} . \tag{13}$$

Therefore, the mass flow rate of the superfluid component through the porous media is expressed as follows,

$$\dot{M} = \rho_s u_s A_{eff} = \rho_s \frac{A_{eff}}{L_{eff}^{1/3}}\left(\int \frac{s}{A_{GM}\rho_n} dT\right)^{1/3} = \frac{1}{\omega^{4/3}}\rho_s \frac{\varepsilon A}{L^{1/3}}\left(\int \frac{s}{A_{GM}\rho_n} dT\right)^{1/3}$$
$$= \rho_s \frac{\varepsilon A}{L^{1/3}}\left(\int \frac{s}{\left(\omega^4 A_{GM}\right)\rho_n} dT\right)^{1/3} = \rho_s \frac{\varepsilon A}{L^{1/3}}\left(\int \frac{s}{A_{GM}{}^{*}\rho_n} dT\right)^{1/3} . \tag{14}$$

The above equation is equivalent to that of the mass flow rate through the one-dimensional channel if a modified Gorter-Mellink constant, $A_{GM}{}^{*}$, is defined as $\omega^4 A_{GM}$. That is the reason why the present numerical code can solve the heat and mass transfer of He II both in micro channels of the porous media and in the He II bath and the He II chamber simultaneously if the modified Gorter-Mellink constant is used instead of the common Gorter-Mellink constant inside of the porous media.

The analytical area is divided into 50*40 rectangular meshes with the length of 0.4 mm. Time step is set to be 1*10-7 s and time integration is carried out explicitly. The adiabatic condition is used on the stainless wall of the test section. The slip condition is applied to the velocities of the superfluid there, while the non-slip condition is applied to the velocities of the total and normal fluid. The stepwise heat is applied uniformly to the top wall of the test section. The properties of the porous media, treated in the present calculations, are listed in TABLE 1. The tortuosity of the porous media, estimated from the experimental results, was applied to the present calculations.

TABLE 1. Properties of the porous media.

Number	#1	#2	#3	#4	#5	#6
Material	Al_2O_3	Al_2O_3	Al_2O_3	Al_2O_3	CSi	CSi
Thickness	2 mm	3 mm	4 mm	10 mm	1.2 mm	1.5 mm
Porosity	32 %	32 %	32 %	32 %	58 %	62 %
Average Pore Diameter	2 μm	2 μm	2 μm	1 μm	20 μm	10.8 μm
He II Bath Pressure	S. V. P.	S. V. P.	S. V. P.	1 atm	S. V. P.	S. V. P.

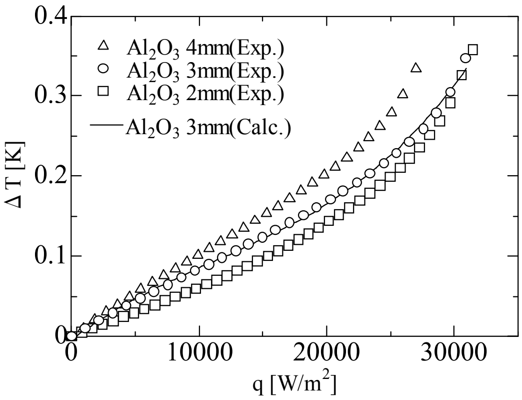

FIGURE 4. Temperature difference as a function of the thickness of the Al_2O_3 porous media in the saturated He II at 1.8 K. Open marks show the experimental results, while a solid line expresses the numerical results.

EXPERIMENTAL AND NUMERICAL RESULTS

Al_2O_3 Samples in Saturated He II

In this section, the results of the present numerical works for the previous experimental works by Maekawa et al. are reported [6]. In the previous experiments, the porous media of the Al_2O_3 with the thickness of 2, 3 and 4 mm (#1-#3 in TABLE 1), were tested in the saturated He II. The porosity of the porous media was 32 %, the average pore diameter 2 μm and the tortuosity estimated from the experiments 4.6. The calculation was performed about the porous media with thickness of 3 mm, using the properties of the porous media. FIGURE 4 shows the experimental and numerical results for the porous samples. The numerical result is in good agreement with the experimental data. The validity of the present numerical model was confirmed as to this porous media immersed in the saturated He II.

Al_2O_3 Samples in Pressurized He II

The calculations were carried out as to the porous media of Al_2O_3 immersed in the

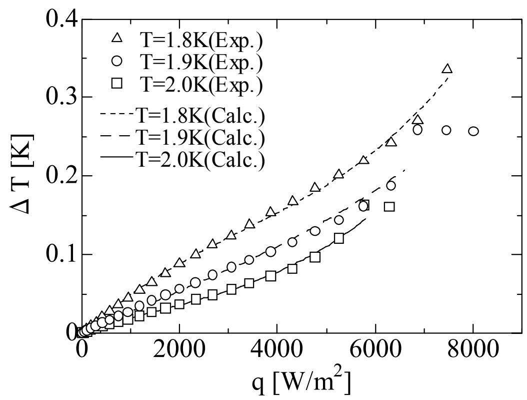

FIGURE 5. Temperature difference as a function of the bath temperature for the Al_2O_3 porous media with 10 mm thick in the pressurized He II. Open marks show the experimental results, while solid and dashed lines express the numerical results.

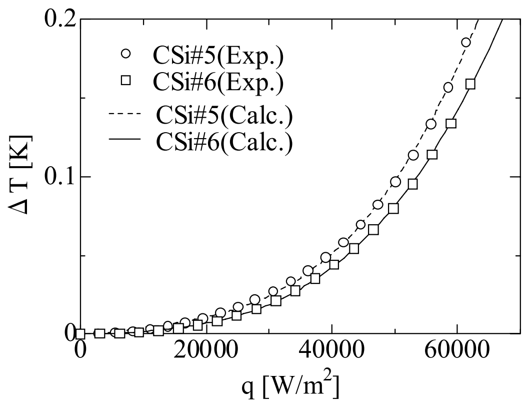

FIGURE 6. Temperature difference through the CSi porous media in the saturated He II at 1.9 K. Open marks show the experimental results, while solid and dashed lines express the numerical results.

pressurized He II before the experiments were conducted. The thickness of the porous media was 10 mm, the porosity 32 %, the average pore diameter 1 μm (#4 in TABLE 1). The tortuosity was assumed to be the same value in the case of the experiments above. The numerical results are shown with the results of the following experiment in FIGURE 5. The calculations were consistent with the experimental results, but a little discrepancy between the numerical outputs and the experimental data occurred in the regime of the high heat flux. Subsequently, it was found that the accurate tortuosity of the porous media with thickness of 10 mm, obtained from the results of the following experiments, was 4.8. It is guessed that the difference of the tortuosity depends on the influence of the variation of the average pore diameter. It is expected that the numerical results will be closer to the experimental results if the corrected tortuosity is applied to the calculations.

Influence of Properties of Porous Media

To verify the validity of the numerical model for various porous media, the numerical analyses were performed on two porous media of CSi tested by Baudouy et al. (#5 and #6 in TABLE 1) [7]. The properties of the porous media differed far from the porous samples of Al_2O_3 (See TABLE 1). The thickness of the porous media are 1.2 and 1.5 mm, the porosity 58 and 62 %, the average pore diameter 20 and 10.8, and the tortuosity estimated from the experiments 1.7 and 1.8, respectively. FIGURE 6 shows the experimental and numerical results for the porous samples of CSi. The numerical results agree with the experimental data. It was confirmed that the present numerical model could be extended for various porous media.

CONCLUSION

In the present study, a numerical code, which dealt with the classical friction loss and the tortuosity in the porous media, have been developed and the steady state problems of the heat and mass flow of the He II through the porous media were analyzed about various porous media. As the result of the comparison between the calculations and the experiments, the numerical outputs agreed with the experimental data for 4 porous samples. The calculations were also consistent with the experiments both in the saturated He II and in the pressurized He II. Consequently, the present numerical code could calculate the steady state heat and mass transfer of the He II through various porous media if the properties of porous media such as the porosity, the thickness, the average pore diameter and the tortuosity were applied to the numerical code.

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