High energy inelastic electron hadron scattering, in peripheral kinematics and sum rules for hadron form factors

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Abstract

Relations between differential cross section for inelastic scattering of electrons on hadrons and hadron form factors (sum rules) are derived on the basis of analytical properties of heavy photon forward Compton scattering on hadrons. Sum rules relating the slope of form-factors at zero momentum transfer and the anomalous magnetic moments of hadrons with integrated photo-production cross section on hadrons are obtained. The convergency of these integrals is provided by the difference of individual sum rules for different hadrons. Universal interaction of Pomeron with nucleons is assumed. Explicit formulae for the processes of electro-production on proton and light isobar nuclei are derived, in frame of the Sudakov's parametrization of momenta, wich is well adapted to peripheral kinematics. The light-cone form of the differential cross sections is also discussed. The accuracy of sum rules is estimated in frame of point-like hadrons and it is shown to be at the level of precision achievable by experiments. Suggestions and predictions for future experiments are also given.

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I. INTRODUCTION

Sum rules relating the form-factors of electron with the cross sections of electro-production processes at e^+e_- high energy collisions were firstly derived in 1974 [1]. In a series of papers the cross sections of processes such as

$$e_+e_- \to (2e_-e_+)e_+; (e_-\gamma)e_+; (e_-2\gamma)e_+; (e_-\mu_+\mu_-)e_+$$

were calculated in the so called peripheral kinematics, when the jet (consisting of the set of particles noted in parentheses) is moving closely to the initial electron direction in the center of mass reference frame (CMS). These cross sections do not decrease as a function of the CMS total beam energy $\sqrt{s}$. Moreover, they are enhanced by a logarithmical factor $\ln(s/m_e^2)$,which is characteristic for Weizsacker-Williams approximation. The contribution to the cross section for muon pair production due to the so called bremsstrahlung mechanism (corresponding to the virtual photon conversion into muon - anti-muon pair) was obtained in Ref. [1, 2]:

$$\sigma^{e^+e^-\to e^-\mu^+\mu^-e^+} = \frac{\alpha^4}{\pi\mu^2}\left[\left(ln\frac{s^2}{m_e^2\mu^2}\right)\left(\frac{77}{18}\xi_2 - \frac{1099}{162}\right) + const\right] \tag{1}$$

where $\xi_2 = \dfrac{\pi^2}{6}$.

In Ref. [3] Barbieri, Mignaco and Remiddi calculated the slope of the Dirac form factor of the electron for $q^2 \to 0$:

$$F_1'(0) = \frac{\alpha^2}{8\pi^2\mu^2}\left(\frac{77}{18}\xi_2 - \frac{1099}{162}\right). \tag{2}$$

The fact that the same coefficients appear in Eqs (1) and (2) suggests that a relation exists between inelastic cross section and elastic form factors. This relation was later on derived in Ref. [2].

Here we discuss an extension of these studies to strong interacting particles, considering QED interactions in the lowest order of perturbation theory. We derive sum rules which relate the nucleon and light nuclei form factors with the differential cross sections of electron scattering on the corresponding hadrons in peripheral kinematics. This kinematics corresponds to the region of very small values of Bjorken parameter x_B in deep inelastic scattering experiments.

This paper is organized as follows. After describing the relevant processes in peripheral kinematics, in terms of Sudakov variables (Section II), we remind the analytical properties of the advanced and the retarded parts of the virtual Compton scattering amplitudes (Section II). Then briefly the problem of restoring gauge invariance and formulation of the modified optical theorem are discussed.

Following the QED analysis of Ref. [2], we introduce the light cone projection of Compton scattering amplitude integrated on a contour in the s_2 plane, where s_2 is the invariant mass squared of the hadronic jet.

Sum rules (Section III) arise when the Feynman contour in the s_2 plane is closed to the left real axes singularities of the Compton amplitude and to the right ones. Sum rules obtained in such a way contain the left hand cut contribution which is difficult to interprete in terms of cross sections. Moreover, ultraviolet divergencies of contour integral arising from Pomeron Regge pole contribution are present. Therefore, the final sum rules consist of differences, constructed in such a way to compensate the Pomeron contributions and the left hand cuts, as well.

The applications to different kinds of targets, as proton and neutron, deuteron and light nuclei are explicitely given. Appendix A is devoted to an estimation of the cross section of proton photoproduction of a $p\bar{p}$ pair arising from effect of identity of protons in final state in the framework of a simple model. In Appendix B the details of kinematics of recoil target particle momentum is investigated in terms of Sudakov's approach.

A. Sudakov parametrization

Let us consider the process presented in Fig. 1, where the inelastic electron hadron interaction occurs through a virtual photon of momentum q. The particle momenta are indicated in the figure: $p^2 = M^2$, $p_1^2 = p_1'^2 = m^2$. The total energy is $s = (p + p_1)^2$ and the momentum transfer from the initial to the final electron is $t = (p_1 - p_1')^2$.

Let us introduce some useful notations, in order to calculate the differential cross section for the process of Fig. 1, where the the hadron is a proton in peripheral kinematics, i.e, $s \gg -t \simeq M^2$. Therefore $s = (p_1 + p)^2 = M^2 + m_e^2 + 2pp_1 \simeq 2pp_1 = 2ME \gg M^2 \gg m_e^2$.

The differential cross section can be written as:

$$d\sigma = \frac{1}{2 \cdot 2 \cdot 2s} \sum |\mathcal{M}|^2 d\Gamma. \tag{3}$$

Let us define two light-like vectors:

$$\tilde{p} = p - p_1 \frac{M^2}{s},\ \tilde{p}_1 = p_1 - p\frac{m^2}{s}$$

and a transversal vector, $q_\perp$, such that $\tilde{p} q_\perp = p_1 q_\perp = 0$.

Therefore $\tilde{p}^2 = \mathcal{O}\left(\frac{m^2 M^4}{s^2}\right)$ and similarly $\tilde{p}^2 = \mathcal{O}\left(\frac{m^4 M^2}{s^2}\right)$. Terms of order $\mathcal{O}\left(\frac{M^2}{s}, \frac{m^2}{M^2}\right)$ compared those of order 1 will be systematically neglected. In the Laboratory system, with an appropriate choice of the axis, the four vectors are written, in explicit form, as: $\tilde{p}_1 \approx p_1 = E(1,1,0,0)$, and $\tilde{p} = \frac{M}{2}(1,-1,0,0)$, and $q_\perp = (0,0,q_x,q_y)$, $q_\perp^2 = -\vec{q}^2 < 0$, which is essentially a two-dimensional vector. Let us express the four momentum of the exchanged photon in the Sudakov parametrization, in infinite momentum frame, as function of two (small) parameters α and β (Sudakov parameters):

$$q = \alpha \tilde{p} + \beta \tilde{p}_1 + q_\perp. \tag{4}$$

The on-mass shell condition for the scattered electron can be written as:

$$p_1'^2 - m^2 = (p_1 - q)^2 - m^2 = -\vec{q}^2 + s\alpha\beta - \alpha s - \beta m^2 = 0 \tag{5}$$

where we use the relation:

$$2p_1\tilde{p}_1 = 2p_1\left(p_1 - p\frac{m^2}{s}\right) = m^2, \tag{6}$$

similarly one can find $2p\tilde{p} = M^2$.

From the definition, Eq. (4), and using Eq. (5), the momentum squared of the virtual photon is:

$$q^2 = s\alpha\beta - \vec{q}^2 = -\frac{\vec{q}^2 + m^2\beta^2}{1-\beta} < 0.$$

The variable β is related to the invariant mass of the proton jet, the set of particles moving close to the direction of the initial proton:

$$s_2 = (q+p)^2 - M^2 + \vec{q}^2 \simeq s\beta$$

neglecting small terms as $s\alpha\beta$ and αM^2 (Weizsacker-Williams approximation [4]). We discuss later the consequences of such approximation on the s dependence of the cross section. In these notation the phase space of the final particle

$$d\Gamma = (2\pi)^4 \frac{d^3 p_1'}{2\epsilon_1'(2\pi)^3} \Pi_1^n \frac{d^3 q_i}{2\epsilon_i (2\pi)^3} \delta^4(p_1 + p - p_1' - \sum_i^n q_i), \tag{7}$$

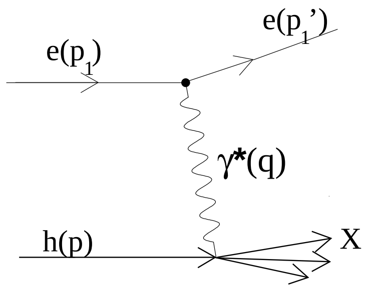

FIG. 1: Feynman diagram for inelastic electron hadron scattering .

introducing an auxiliary integration on the photon transferred momentum $\int d^4q\delta^4(p_1 - q - p_1') = 1$ can be written as

$$d\Gamma = (2\pi)^{-3}\delta^4((p_1-q)^2 - m^2)d^4q d\Gamma_H, \tag{8}$$

where $d\Gamma_H$ is the hadron phase space:

$$d\Gamma_H = (2\pi)^4\delta^4(p + q - \sum_i^n q_i)\Pi_1^n \frac{d^3q_i}{2\epsilon_i(2\pi)^3}. \tag{9}$$

In the Sudakov parametrization:

$$d^4q = \frac{s}{2}d\alpha d\beta d^2q_\perp \simeq \frac{ds_2}{2s}d(s\alpha)d^2\vec{q}, \tag{10}$$

one obtains:

$$d\Gamma = \frac{ds_2}{2s}d^2\vec{q}(2\pi)^{-3}d\Gamma_H. \tag{11}$$

The matrix elements, expressed in terms of the Sudakov parameters can be rewritten in the form:

$$\mathcal{M} = \frac{4\pi\alpha}{q^2}\overline{u}(p_1')\gamma^\mu u(p_1)\mathcal{J}_H^\nu g_{\mu\nu}. \tag{12}$$

It is convenient to use here the Gribov representation of the numerator of the (exact) Green function in the Feynman gauge for the exchanged photon:

$$g_{\mu\nu} = (g_\perp)_{\mu\nu} + \frac{2}{s}(\tilde{p}_\mu\tilde{p}_{1\nu} + \tilde{p}_\nu\tilde{p}_{1\mu}). \tag{13}$$

All three terms, in the right hand side of the previous equation, give contributions to the matrix element proportional to

$$1 : \frac{s}{M^2} : \frac{M}{s}.$$

So, the main contribution (with power accuracy) is

$$\mathcal{M} = \frac{4\pi\alpha}{q^2}\frac{2}{s}\overline{u}(p_1')\hat{p}u(p_1)\mathcal{J}_H^\nu p_{1\nu} = \frac{8\pi\alpha s}{q^2}N\frac{\mathcal{J}_H^\nu p_{1\nu}}{s}, \tag{14}$$

with $N = \frac{1}{s}\overline{u}(p_1')\hat{p}u(p_1)$. One can see explicitly the proportionality of the matrix element of peripheral processes to s, in the high energy limit, $s \gg -t$. It follows from the relation $\sum_{pol}|N|^2 = \frac{1}{s^2}Tr\hat{p}_1'\hat{p}\hat{p}_1\hat{p} = 2$ and from the fact that the quantity $\frac{1}{s}\mathcal{J}_H^\nu p_{1\nu}$ is finite in this limit. Such term can be further transformed using the conservation of the hadron current $\mathcal{J}_H^\nu q_\nu \simeq (\beta p_1 + q_\perp)_\nu \mathcal{J}_H^\nu = 0$, which leads to

$$\frac{1}{s}\mathcal{J}_H^\nu p_{1\nu} = \frac{1}{s\beta}\vec{q}\cdot\vec{\mathcal{J}}_H = \frac{|\vec{q}|}{s_2}(\vec{e}\cdot\vec{\mathcal{J}}_H), \tag{15}$$

where $\vec{e} = \vec{q}/|\vec{q}|$ is the polarization vector of the virtual photon. As a result, one finds:

$$\sum|\mathcal{M}|^2 = \frac{(8\pi\alpha s)^2|\vec{q}|^2}{\left[(\vec{q})^2 + m^2\left(\frac{s_2}{s}\right)^2\right]^2}\frac{2}{s_2^2}\left(\vec{\mathcal{J}}_H\cdot\vec{e}\right)^2. \tag{16}$$

With the help of Eqs. (7,14,16) the cross section for the considered process can be written as:

$$d\sigma^{(e+p\to e+jet)} = \frac{\alpha^2 d^2q\vec{q}^2 ds_2}{\pi\left[(\vec{q})^2 + m^2\left(\frac{s_2}{s}\right)^2\right]^2 s_2^2}(\vec{e}\cdot\vec{\mathcal{J}}_H)^2 d\Gamma_H. \tag{17}$$

Let us note that the differential cross section at $\vec{q}^2 \neq 0$ does not depend on the CMS energy $\sqrt{s}$. In the logarithmic (Weizsacker-Williams) approximation, the integral over the transverse momentum $\vec{q}$, at small $\vec{q}^2$, gives rise even to large logarithm:

$$\sigma_{tot}^{\ell} = \pi\alpha\, ln\left(\frac{s^2Q^2}{M^4m^2}\right)\int_{s_{th}}^{\infty}\frac{ds_2}{s_2}\sigma_{tot}^{(\gamma+p\to X)}(s_2), \tag{18}$$

where $s_{th} = (M+m_\pi)^2 - M^2$, Q^2 is the characteristic momentum transfer squared, $Q^2 \simeq M^2$, and we introduced the total cross section for real polarized photons interacting with protons:

$$\sigma_{tot}^{(\gamma+p\to X)}(s_2, q^2 = 0) = \frac{\alpha\pi}{s_2}\int(\vec{\mathcal{J}}_H\cdot\vec{e})^2 d\Gamma_H. \tag{19}$$

The differential cross section (18) is closely related (due to the optical theorem) with the s-channel discontinuity of the forward amplitude for electron-proton scattering with the same intermediate state: a single electron and a jet, moving in opposite directions (see Figs. 2a,

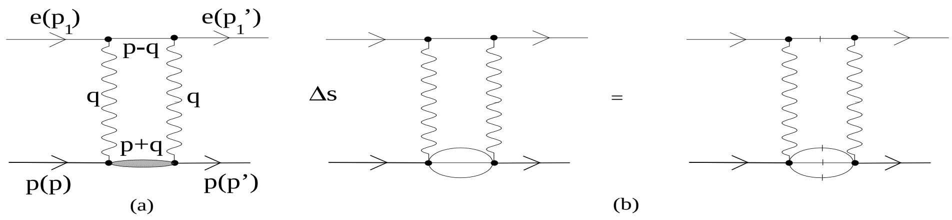

FIG. 2: Feynman diagram for e^+e^- scattering at the order α^3.

2b) where, by Cutkovsky rule, the denominators of the "cutted" lines in the Feynman graph of Fig. 2b must be replaced by:

$$\frac{1}{q^2 - M^2 + i0} \to -2\pi i \delta(q^2 - m^2). \tag{20}$$

For the spin averaged forward scattering amplitude one has:

$$\Delta_s A(s) = \frac{4s\alpha}{\pi^2} \int \frac{d^2 q_\perp \vec{q}^2}{(q^2)^2} \int \frac{ds_2}{s_2} \sigma^{(\gamma^* p \to X)}(s_2, q) \tag{21}$$

with

$$\sigma^{(\gamma^* p \to X)}(s_2, q) = \int \frac{4\pi\alpha}{2 \cdot 2 \cdot 2s_2} (\vec{\mathcal{J}}_H \cdot \vec{e})^2 d\Gamma_H. \tag{22}$$

From the formulae given above one obtains

$$\frac{d\Delta_s \bar{A}^{eY \to eY}}{d^2 q} = 2s \frac{d\sigma^{eY \to eY}}{d^2 q}, \tag{23}$$

where $\bar{A}^{eY \to eY}$ is the forward scattering amplitude, averaged over the spin states. This relation can be considered the differential form of the optical theorem.

Let us now consider the discontinuity of forward scattering amplitude with the initial hadron intermediate state, we call it a "pole contribution". For the case of elastic electron-proton scattering we have

$$\frac{d\Delta_s \bar{A}^{ep \to ep}}{d^2 q} = \frac{(4\pi\alpha)^2}{(q^2)^2 s (2\pi)^2} S, \tag{24}$$

with

$$S = Tr(\hat{P} + M_p)\Gamma(q)(\hat{P}' + M_p)\Gamma(-q)^*,$$

and

$$\Gamma(q) = F_1 \hat{p}_1 - \frac{1}{2M} F_2 \hat{q} \hat{p}_1.$$

A simple calculation gives $Sp = 2s^2[(F_1)^2 + \tau(F_2)^2]$, with $\tau = \vec{q}^2/(4M_p^2)$.

For the case of electron-deuteron scattering we use the electromagnetic vertex of deuteron in the form [5]

$$< \xi^{\lambda'}(P')|J_\mu^{EM}(q)|\xi^\lambda(P) >= d_\mu[F_1(\xi^{\lambda'*}\xi^\lambda) - \frac{F_3}{2M_d^2}(\xi^{\lambda'*}q)(\xi^\lambda q)] + F_2[\xi^\lambda_\mu(q\xi^{\lambda'*}) - \xi^{\lambda'*}_\mu(\xi^\lambda q)], \tag{25}$$

where $d_\mu = (P'+P)_\mu, q_\mu = (P'-P)_\mu$ and $\xi^\lambda(P)$ is the polarization vector of deuteron in chiral state λ, with the following properties:

$$\xi^2 = -1, (\xi(P)P) = 0; \ \sum_\lambda \xi^\lambda(P)_\mu \xi^{\lambda*}(P)_\nu = g_{\mu\nu} - \frac{P_\mu P_\nu}{M_d^2}. \tag{26}$$

For the forward scattering amplitude, averaged over the spin states one finds:

$$\frac{d\Delta_s \bar{A}^{ed\to ed}}{d^2q} = \frac{2s(4\pi\alpha)^2}{3(q^2)^2(2\pi)^2}T, \tag{27}$$

with

$$T = 2(F_1)^2 + (F_1 + 2\tau_d(1+\tau_d)F_3)^2 + 2\tau_d(F_2)^2, \tau_d = \frac{\vec{q}^2}{4M_d^2}. \tag{28}$$

We note that the amplitude corresponding to crossed box-type Feynman diagram has a zero s-channel discontinuity.

II. VIRTUAL COMPTON SCATTERING ON PROTON

Let us examine the different contributions to the total amplitude for virtual photon Compton scattering on a proton (hadron). Keeping in mind the baryon number conservation low we can separate all possible Feynman diagram to four classes. In one, which will be named as class of retarded diagram (the corresponding amplitude is denoted as A_1) the initial state photon is first absorbed by nucleon line and after emitted the scattered photon. Another class (advanced, A_2), corresponds to the diagrams, in which scattered photon is first emitted along the nucleon line and the point of absorption is located after. The third class corresponds to the case when both photons do not interact with initial nucleon line. The corresponding amplitude is denoted as A_P. The fourth class contains diagrams in which only one of external photons interacts with the nucleon line. The corresponding notation is A_{odd} (see Fig. 3):

$$A^{\mu\nu}(s,q) = A_1^{\mu\nu}(s,q) + A_2^{\mu\nu}(s,q) + A_P^{\mu\nu}(s,q) + A_{odd}. \tag{29}$$

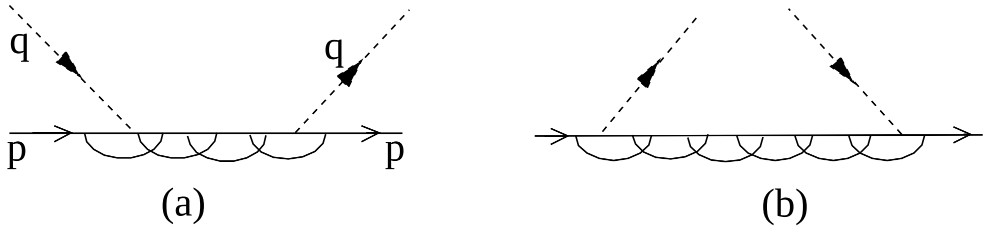

FIG. 3: Illustration of retarded (a) and advanced (b) virtual photon emission and absorbtion diagrams. The diagram containing Pomeron is not considered here.

The amplitude $A_P^{\mu\nu}(s,q)$ corresponds to the Pomeron type Feynman diagram (Fig. 4e) and gives the non-vanishing contribution to the total cross sections in the limit of a large invariant mass squared of initial particles $s_2 \to \infty$. The fourth class amplitude can be relevant in experiments measuring charge-odd effects and it will not be considered here. One can show explicitly that the amplitudes for each of the four classes are gauge invariant. The arguments in favor of this, are essentially the same as for the QED case [6].

Let us discuss now the analytical properties of the retarded part of the forward Compton scattering of a virtual photon on a proton, $A_1(s_2,q)$ (see Fig. 4) in the s_2 plane. Due to general principles, the singularities - poles and branch points - are situated on the real axis.

These singularities are illustrated in Fig. 5. On the right side the pole at $s_2 = 0$ correspond to one nucleon exchange in the s_2 channel (Fig. 5a), the right hand cut starts at the pion-nucleon threshold, $s_2 = (M+m_\pi)^2 - M^2$. The left cut, related with the u channel 3-nucleon state of the Feynman amplitude is illustrated in Fig. 4f. It is situated rather far from the origin at $s_2 = -8M^2$. It can be shown that it is the nearest singularity of the right hand cut. Really, the u-channel cut corresponding to $2\pi N$ state can not be realized without exotic quantum number states (see Fig. 4h).

III. SUM RULES

Following Ref. [6] let us introduce the quantity

$$\int_C ds_2 \frac{p_1^\mu p_1^\nu A_{1\mu\nu}^{\gamma^* p\to\gamma^* p}}{s^2(\vec{q}^2)^2} = \frac{dI}{d\vec{q}^2}, \tag{30}$$

with the Feynman contour C in the s_2 plane as shown in Fig. 5a.

Sum rules appear when one considers the equality of the path integrals along the contours obtained by deformating C in such a way to be closed to the left and to the right side (Fig. 5). As a result one finds:

$$F(\vec{q}^2) = \frac{d\sigma_{left}}{d\vec{q}^2} - \frac{d\sigma_{el} - d\sigma_{el}^B}{d\vec{q}^2} = \frac{d\sigma_{inel}}{d\vec{q}^2}, \tag{31}$$

where $\frac{d\sigma_{left}}{d\vec{q}^2}$ indicates the contribution of left cut [1];

$$\frac{d\sigma_{el}^B}{d\vec{q}^2} = \frac{4\pi\alpha^2 Z^2}{(\vec{q}^2)^2}.$$

The latter is generally the Born cross section of the scattering of an electron on any hadron with charge Z, when the strong interaction is switched off, and $\frac{d\sigma_{el}}{d\vec{q}^2}$ is the elastic electron hadron cross section, when the strong interaction is switched on, in the lowest order of QED coupling constant. This quantity can be expressed in terms of electromagnetic form factors of corresponding hadrons.

Using the notation Φ^2, for the generalized form factors squared, one can write the following expression for the scattering of electrons on a hadron Y with charge Z:

$$Z^2 - \Phi^2(-\vec{q}^2) = \frac{2\vec{q}^2}{\pi\alpha^2}\frac{d\sigma^{eY\to eX}}{d\vec{q}^2}. \tag{32}$$

For a spin-zero target the quantity Φ^2 coincide with its squared charge form factor.

For the case of electron scattering on a spin one-half particle (proton, ^3He,^3H), which is described by two form factors, F_1 (Dirac) and F_2 (Pauli) one can write:

$$Z_i^2 - F_{1i}^2(-\vec{q}^2) - \tau_i F_{2i}^2(-\vec{q}^2) = \frac{2\vec{q}^2}{\pi\alpha^2}\frac{d\sigma^{eY_i\to eX_i}}{d\vec{q}^2}, \tag{33}$$

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[1] The equality of the numbers in (1,2) derives from the absence of left cut contribution, which is known for planar Feynman diagram amplitudes.

with

$$\tau_i = \frac{\vec{q}^2}{4M_i^2}; Z_p = Z_{^3H} = 1; Z_{^3He} = 2.$$

For the scattering of electron on deuteron we have:

$$1 - \frac{1}{3}[2F_1^2(-\vec{q}^2) + [F_1(-\vec{q}^2) + 2\tau_d(1+\tau_d)F_3(-\vec{q}^2)]^2 + 2\tau_d F_2^2(-\vec{q}^2)] = \frac{2\vec{q}^2}{\pi\alpha^2}\frac{d\sigma^{ed\to eX}}{d\vec{q}^2}. \quad (34)$$

These equations can be directly tested in experiments with electron-hadron colliders.

Let us consider the differential form of these sum rules: $\left.\frac{d}{d\vec{q}^2}(\vec{q}^2)^2 F(\vec{q}^2)\right|_{\vec{q}^2=0}$, which can be expressed in terms of the static properties of hadrons, such as charge radii, anomalous magnetic moments, etc. and of the photo-production total cross section.

Considering formally this derivative at $\vec{q}^2 = 0$ we obtain

$$\frac{d}{d\vec{q}^2}\Phi_Y^2|_{\vec{q}^2=0} = \frac{2}{\pi^2\alpha}\int\limits_{s_{th}}^{\infty}\frac{ds_2}{s_2}\sigma_{tot}^{\gamma Y\to X}(s_2). \quad (35)$$

Unfortunately the sum rule in this form can not be experimentally verified, due to the divergence of the integral in the right hand side of this equation. This is due to the increasing of the photoproduction cross section at large values of the initial CM energy squared s_2, as a consequence of the Pomeron Regge pole contribution. The universal character of Pomeron interaction with the nucleons can be confirmed by the data collected in the Particle Data Group [7]:

$$[\sigma^{\gamma p}(s_2) - \sigma^{\gamma n}(s_2)]|_{s_2\to\infty} = [2\sigma^{\gamma p}(s_2) - \sigma^{\gamma d}(s_2)]|_{s_2\to\infty} \to 0. \quad (36)$$

In Ref. [8] the difference of proton and neutron sum rule was derived

$$\frac{1}{3} < r_p^2 > + \frac{1}{4M^2}\left[\kappa_n^2 - \kappa_p^2\right] = \frac{2}{\pi^2\alpha}\int_{\omega_n}^{\infty}\frac{d\omega}{\omega}\left[\sigma^{\gamma p\to X}(\omega) - \sigma^{\gamma n\to X}(\omega)\right]. \quad (37)$$

We use here the known relations

$$F_1(-\vec{q}^2) = 1 - \frac{1}{6}\vec{q}^2 < r^2 > + O((\vec{q}^2)^2); F_2(0) = \kappa,$$

with $< r^2 >$, κ-are the charge radius squared and the anomalous magnetic moment of nucleon (in units $\hbar/Mc$).

In Ref. [8], it was verified that this sum rule is fulfilled within the experimental errors: both sides of the equation equal 1.925 mb. Here the Pomeron contribution is compensated in the difference of proton and neutron total cross photo-production cross sections.

In Ref. [9] a similar combination of cross sections was considered for A=3 nuclei:

$$\frac{2}{3} < r^2_{^3He} > -\frac{1}{3} < r^2_{^3H} > -\frac{1}{4M^2}\left[\kappa^2_{^3He} - \kappa^2_{^3H}\right] = \frac{2}{\pi^2\alpha}\int_{\omega_{th}}^{\infty}\frac{d\omega}{\omega}\left[\sigma^{\gamma^3He\to X}(\omega) - \sigma^{\gamma^3H\to X}(\omega)\right]. \tag{38}$$

In a similar way, the combination of cross sections of electron scattering on proton and deuteron leads to the relation

$$\frac{1}{3} < r_d^2 > -\frac{F_3(0)}{3M_d^2} - \frac{1}{6M_d^2}F_2(0)^2 - 2[\frac{1}{3} < r_p^2 > -\frac{1}{4M_p^2}\kappa_p^2] = \frac{2}{\pi^2\alpha}\int\limits_{\omega_{th}}^{\infty}\frac{d\omega}{\omega}[\sigma_{tot}^{\gamma d\to X}(\omega) - 2\sigma_{tot}^{\gamma p\to X}(\omega)], \tag{39}$$

where ω_{th} for deuteron and proton are different: $(\omega_{th})_d = 2,2MeV, (\omega_{th})_p = m_\pi + \frac{m_\pi^2}{2M_p} \approx 140$ MeV.

We use here a similar expansion for the deuteron form factor F_1, normalized to $F_1(0) = 1$, and introduce its square charge radius. The numerical values for the other quantities can be found in Ref. [10]:

$$F_2(0) = -\frac{M_d}{M_p}\mu_d;\ \mu_d = 0.857;\ 2F_3(0) = 1 + F_2(0) - M_d^2 Q_d;\ Q_d = 0.2859 \text{ fm}^2.$$

We find for the left side of formula (39) using the data from Ref. [7] and the parametrization from Ref. [11]:

$$\frac{2}{\pi^2\alpha}\left\{\int\limits_{0.020}^{0.260}\frac{d\omega}{\omega}\sigma_{tot}^{\gamma d\to X}(\omega) + \int\limits_{0.260}^{16}\frac{d\omega}{\omega}\left[\sigma_{tot}^{\gamma d\to X}(\omega) - 2\sigma_{tot}^{\gamma p\to X}(\omega)\right]\right\} = 0.86fm^2 \tag{40}$$

IV. CONCLUSIONS

The left cut contribution has no direct interpretation in terms of cross section. In analogy with QED case, it can be associated with the contribution to the cross section of the process $ep \to e2p\bar{p}$, electro-production of a proton antiproton pair on a proton target, arising the identity of the two protons in the final state.

Fortunately the threshold of this process is located quite far. Using this fact we can estimate its contribution in the framework of a QED-like model with nucleons and pions (ρ-mesons), omitting form factor effects (so we put them equal to the coupling constants of nucleons with pions and vector mesons).

In this paper we applied the optical theorem, which connects the s-channel discontinuity of the forward scattering amplitude with the total cross section. This statement holds for the complete scattering amplitude, whereas we consider only part of it, A_1. We can explicitly point out on the Feynman diagram (see Fig. 4 g), contributing to A_2, which has three nucleon s-channel state. The relevant contribution can be interpreted as the identity effect of proton photo-production of a $p\bar{p}$ pair.

The explicit calculation in the framework of our approach is given in Appendix A. The order of magnitude I of the contribution to the derivative with respect to $\vec{q}^2$ at $\vec{q}^2 = 0$, of the scattering amplitudes entering in the sum rules is

$$I = \frac{g^4 M^2}{\pi^3 s_{2min}^2}.$$

In order to estimate the strong coupling constant, the PDG value for the total cross section of scattering of pion on proton, was taken: $\sigma_{tot}^{\pi p} = 20$ mb [7]. Keeping in mind the ρ-meson t-channel contribution $\sigma^{\pi p} \sim g^4/(4\pi m_\rho^2)$ and the minimal value of the three nucleon invariant mass squared $s_{2min} = 8M_p^2$ one finds $I \approx (1/15)$ mb. Comparing this value with the typical values of the right and left hand sides of the sum rules, which is of order of 2 mb, we estimate the error arising by omitting the left cut contribution and by replacing our incomplete cross section, by a measurable one, at the level of 3%.

Using the data [7] for γp and γd cross sections and the parametrization from Ref. [11] we find

$$< r_d > \approx \; 1.94 \text{ fm}.$$

This quantity in a satisfactory agreement with prediction of models based dispersion relations [12] where is $< r_d > \sim 2$ fm. The reason for a discrepancy which does not exceed the error of 3% inherent to our approach) can be attributed to the lack of data for γp and γd cross sections near the threshold.

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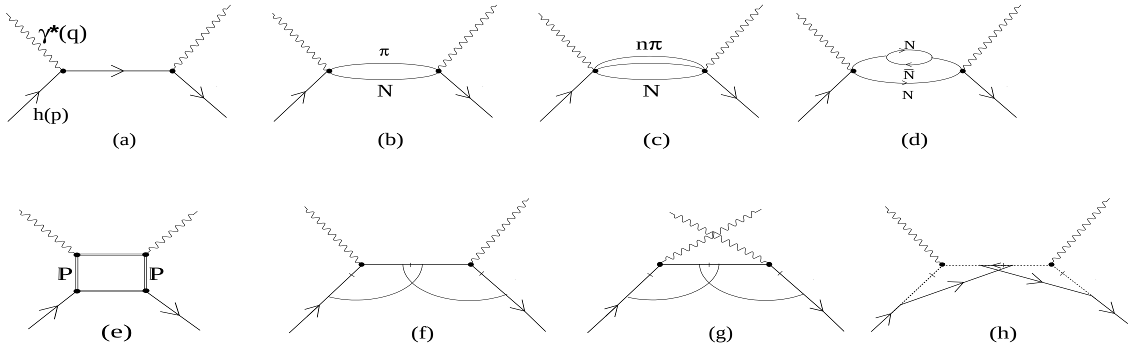

FIG. 4: Feynman diagrams for forward virtual Compton scattering on the proton, contributing to the retarded part of the amplitude: intermediate state in s_2 channel for single proton (a), $N\pi$ (b), $Nn\pi$ (c), $\sum N\overline{N}$ (d), two jets s_2 channel state, with two Pomeron t channel state (d), $3N$ intermediate state in u channel (e), $3N$ intermediate state in u_2 channel (f). The Feynman diagram of the A_2 set which has s channel discontinuity is illustrated in (g) and an example of exotic u channel state in (h).

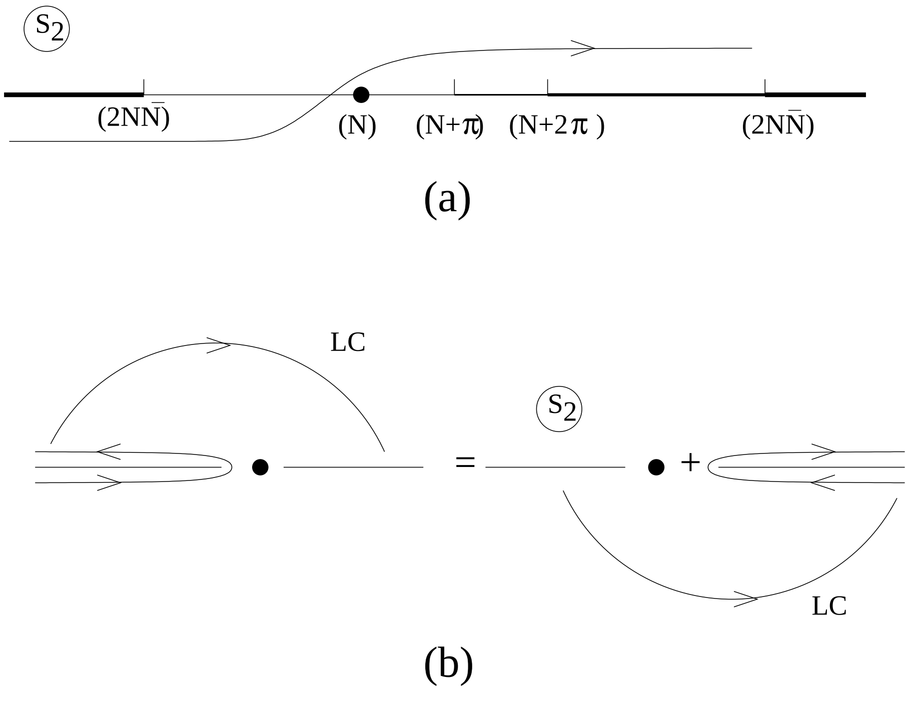

FIG. 5: Illustration of singularities along the s_2 real axis with the open contour C (a), and with the contour C closed (b), corresponding o Fig. 4. LC stays for large circle contribution.

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VI. APPENDIX A: EFFECT OF IDENTITY OF PROTONS TO THE CROSS SECTION OF $2p\bar{p}$ PHOTO-PRODUCTION CROSS SECTION

The contribution to the s-channel discontinuity of the part of the scattering amplitude $A_2^{\gamma^* p\to\gamma^* p}$, arising from the interference of the amplitudes for the creation of a proton-antiproton pair of bremsstrahlung type, due to identity of protons in the final state has the form:

$$\Delta_s A_2(s,q) = -\frac{16g^4}{s^2}\int ds_2 d\Gamma_3 \frac{S}{(q_1^2-m_\pi^2)(q_2^2-m_\pi^2)}, \tag{41}$$

with $q_1 = P_1 - P, q_2 = P_2 - P$ and

$$S = \frac{1}{4}Sp(\hat{P}+M)\gamma_5(\hat{P}_1+M)V_1(\hat{P}_3-M)V_2(\hat{P}_2+M)\gamma_5,$$

where $V_1 = \gamma_5\frac{\hat{q}-\hat{P}_3+M}{d_3}\hat{p}_1 + \hat{p}_1\frac{\hat{P}_1-\hat{q}+M}{d_1}\gamma_5$;
$V_2 = \gamma_5\frac{-\hat{q}+\hat{P}_2+M}{d_2}\hat{p}_1 + \hat{p}_1\frac{-\hat{P}_3+\hat{q}+M}{d_3}\gamma_5$;
$d_{1,2,3} = (q-P_{1,2,3})^2 - M^2, s_2 = (P+q)^2 - M - q^2$.

The element of phase volume can be written as:

$$d\Gamma_3 = (2\pi)^4\delta^4(P+q-P_1-P_2-P_3)\Pi_1^3\frac{d^3P_i}{2E_i(2\pi)^3}.$$

Here we consider pions interacting with nucleons with coupling constant g. A similar expression can be written for the case when one or both pions are replaced by ρ-meson. It can be shown that the corresponding contributions are approximately one order of magnitude smaller than those from pions.

We use Sudakov's parametrization of momenta:

$$q = \alpha\tilde{P} + \beta p_1 + q_\perp;\ P = \tilde{P} + \frac{M^2}{s}p_1;\ P_i = \alpha_i\tilde{P} + \beta_i p_1 + P_{i\perp}. \tag{42}$$

Using the formulae given above all the relevant quantities can be written as:

$$\int ds_2 d\Gamma_3 = \frac{1}{4(2\pi)^3}\frac{d^2P_1 d^2P_2 d\alpha_1 d\alpha_2}{\alpha_1\alpha_2\alpha_3}; \alpha_1+\alpha_2+\alpha_3 = 1; \vec{P}_1+\vec{P}_2+\vec{P}_3 = \vec{q} \tag{43}$$

and

$$
\begin{aligned}
s_2 &= s\beta = -M^2 + \sum_1^3 \frac{\vec{P}_i^2 + M^2}{\alpha_i}; \\
q_i^2 &= -\frac{\vec{P}_i^2 + (1-\alpha_i)^2}{\alpha_i}, \ i = 1,2; \ d_i = -s_2\alpha_i + 2\vec{q}\vec{P}_i - \vec{q}^2, \ i = 1,2,3.
\end{aligned} \tag{44}
$$

Note that the quantities V_i can be written as

$$
V_1 = s\gamma_5 A_{13} + \frac{\gamma_5 \hat{q}\hat{p}_1}{d_3} - \frac{\hat{p}_1\hat{q}\gamma_5}{d_1}; \ V_2 = s\gamma_5 A_{23} - \frac{\gamma_5\hat{q}\hat{p}_1}{d_2} + \frac{\hat{p}_1\hat{q}\gamma_5}{d_3}, \tag{45}
$$

with

$$
A_{13} = \frac{\alpha_1}{d_1} - \frac{\alpha_3}{d_3}; \ A_{23} = \frac{\alpha_2}{d_2} - \frac{\alpha_3}{d_3}.
$$

In this form, the gauge invariance of the contribution to the forward scattering amplitude is explicitly seen, namely this quantity vanishes at $\vec{q} \to 0$, (we see that replacements $\hat{q}\hat{p}_1 = \hat{q}_\perp \hat{p}_1, \hat{p}_1\hat{q} = \hat{p}_1\hat{q}_\perp$ in $V_{1,2}$ can be done).

The calculation of the trace leads to the result:

$$
\frac{S}{s^2} = A_{13}A_{23}S_1 + A_{13}\left[\frac{1}{d_3} + \frac{1}{d_2}\right] S_2 + A_{23}\left[\frac{1}{d_3} + \frac{1}{d_1}\right] S_3 + \left[\frac{1}{d_3} + \frac{1}{d_1}\right]\left[\frac{1}{d_3} + \frac{1}{d_2}\right] S_4, \tag{46}
$$

with

$$
\begin{aligned}
S_1 &= M^4 + \frac{1}{2}M^2\vec{q}^2 + (PP_1)(P_2P_3) + (P_3P_1)(P_2P) - (PP_3)(P_2P_1); \\
S_2 &= \frac{M^2}{2}[\alpha_2(\vec{q}\vec{P}_2) - \alpha_3(\vec{q}\vec{P}_3) - (\alpha_1 + 2\alpha_3)(\vec{q}\vec{P}_1)] + \frac{1}{2}[(\vec{q}\vec{P}_3)[(P_1P_2) - \\
&\quad \alpha_2(PP_1) - \alpha_1(PP_2)] + (\vec{q}\vec{P}_2)[(P_1P_3) + \alpha_3(PP_1) - \alpha_1(PP_3)] - \\
&\quad (\vec{q}\vec{P}_1)[(P_3P_2) - \alpha_2(PP_3) - \alpha_3(PP_2)]]; \\
S_3 &= \frac{M^2}{2}[\alpha_3(\vec{q}\vec{P}_3) - \alpha_1(\vec{q}\vec{P}_1) + \\
&\quad (\alpha_2 + 2\alpha_3)(\vec{q}\vec{P}_2)] + \frac{1}{2}[(\vec{q}\vec{P}_1)[-(P_3P_2) - \\
&\quad \alpha_3(PP_2) + \alpha_2(PP_3)] + (\vec{q}\vec{P}_2)[(P_1P_3) - \alpha_3(PP_1) - \alpha_1(PP_3)] - \\
&\quad (\vec{q}\vec{P}_3)[(P_1P_2) - \alpha_1(PP_2) - \alpha_2(PP_1)]]. \\
S_4 &= -\frac{\alpha_3\vec{q}^2}{2}[M^2\alpha_3 + \alpha_1(PP_2) + \alpha_2(PP_1) - (P_1P_2)];
\end{aligned} \tag{47}
$$

The invariants entering in this expression have the form

$$
2PP_i = \frac{\vec{P}_i^2 + M^2(1+\alpha_i^2)}{\alpha_i}; \ 2P_iP_j = \frac{(\alpha_i\vec{P}_j - \alpha_j\vec{P}_i)^2 + M^2(\alpha_i^2 + \alpha_j^2)}{\alpha_i\alpha_j}.
$$

The numerical integration of this expression confirms the estimation given above within 10%.

VII. APPENDIX B: CORRELATION BETWEEN THE MOMENTUM AND THE SCATTERING ANGLE OF RECOIL PARTICLE IN LAB. FRAME

The idea of expanding four-vectors of some relativistic problem using as a basis two of them (Sudakov's parametrization) becomes useful in many regions of quantum field theory. It was crucial for studying the double logarithmical asymptotic of the amplitudes of the processes with large transversal momenta. Being applied in condition of peripheral kinematics, it essentially coincides with the infinite momentum frame approach.

Let us consider here an application to the study of the kinematic of peripheral process of jet formation on a particle at rest. An experimental approach is the measurement of the recoil particle momentum distribution. For instance this method was used in the process of electron-positron pair production by linearly polarized photon on electron in solid target (atomic electrons). Here the correlation between the recoil momentum in the initial photon plane and the plane of photon polarization is used to determine the degree of photon polarization [14].

The Sudakov's parametrization allows to give a transparent explanation of the correlation between the angle of emission of a recoil particle of mass M, with the recoil momentum in the laboratory reference frame [14]:

$$\frac{\vec{P}'}{M} = \frac{2\cos\theta_p}{\sin^2\theta_p}; \frac{E'}{M} = \frac{1+\cos^2\theta_p}{\sin^2\theta_p}, \tag{48}$$

where $\vec{P}'$, E' are the three-momentum and the energy of the recoil particle ($(E')^2 - (P')^2 = M^2$); θ_p is the angle between the beam axes $\vec{k}$ in the rest frame of target particle

$$\gamma(k) + P(P) \to jet + P(P'),\ s = 2kP = 2M\omega, P - P' = q,\ P^2 = (P')^2 = M^2.$$

The kinematics corresponds to the main contribution to the cross section, for the case when the jet moves close to projectile direction. Using the Sudakov representation for the transfer momentum $q = \alpha\tilde{P} + \beta k + q_\perp$, and the recoil particle on mass shell condition $(P-q)^2 - M^2 \approx -s\beta - \vec{q}^2 = 0$, we obtain for the ratio of the squares of the transversal and longitudinal components of the three-momentum of the recoil particle:

$$\tan^2\theta_p = \frac{\vec{q}^2}{(\beta\omega)^2} = \frac{4M^2}{\vec{q}^2}, \vec{q}^2 = (P')^2\sin^2\theta_p. \tag{49}$$

Eq. (48) follows immediately from (49).

This correlation was firstly mentioned in Ref. [15] where the production of electrons from the matter was investigated. It was proven in Ref. [14].

Eq. (48) can be applied in experiments with collisions of high energy protons scattered on protons in the matter.

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