Target normal spin asymmetry and charge asymmetry for $e\mu$ elastic scattering and the crossed processes

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Abstract

Two kinds of asymmetry arise from the interference of the Born amplitude and the box-type amplitude corresponding to two virtual photons exchange, namely charge-odd and single spin asymmetries. In case of unpolarized particles the charge-odd correlation is calculated. It can be measured in a combination of electron muon and positron muon experiments. The forward-backward asymmetry is the corresponding quantity which can be measured for the crossed processes. In the case of a polarized muon the single spin asymmetry for annihilation and scattering channels has been calculated. The additional structure function arising from the interference is shown to suffer from infrared divergencies. The background due to electroweak interaction is discussed.

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I. INTRODUCTION

The motivation of this work is to give an accurate description of the process $e\bar{\mu} \to e\bar{\mu}(\gamma)$ $e\bar{e} \to \mu\bar{\mu}(\gamma)$ in frame of QED, in order to provide a basis for the comparison with experimental data. High precision experiments on the processes $e\bar{e} \to \tau\bar{\tau}$ and $e\bar{e} \to p\bar{p}$ are planned in future $c-\tau$ facilities. Moreover, the possibility of colliding $e\mu$ beam facilities has been discussed in framewok of programs on verification of Standard Model (SM) prediction.

Charge-odd and backward-forward asymmetries appear naturally from the interference of one and two photon exchange amplitudes in frame of QED and SM due to Z_0-boson exchange in Born approximation. But at the energy range reachable at $c-\tau$ factories, the relevant contribution of SM type is [1]:

$$\frac{\mathrm{d}\sigma_Z^{odd}}{\mathrm{d}\sigma_{QED}} \approx \frac{s}{M_Z^2} a_v a_a \approx 5 \cdot 10^{-5}, \quad 3 < \sqrt{s} < 5 \text{ GeV}. \tag{1}$$

which is quite small compared to QED effects.

The accuracy of results given below is determined by

$$\mathcal{O}\left(\frac{m_e^2}{m_\mu^2}, \frac{m_e^2}{m_\tau^2}\right) \sim 0.1\% \tag{2}$$

and the contribution of higher orders of QED $\alpha/\pi \approx 0.5\%$. Moreover we assume that all the velocities of the final heavy particles are finite in the annihilation as well as in the scattering channels. This is the reason why Coulomb factors are neglected.

Our paper is organized as follows. The annihilation channel and the scattering channel $e\bar{e} \to \mu\bar{\mu}$ are considered, In Secs. II and III respectively. In Sec. IV we take into account the soft photon emission and construct charge-odd and forward-backward asymmetries. Explicit form for additional structure G_3 for annihilation channel is given in Sec. V. In Sec. VI the one-spin asymmetries are investigated. The results are summarized in the Conclusions.

II. PROCESS $e^+ + e^- \to \mu^+ + \mu^-(\gamma)$

At first, we consider the process of creation of $\mu^+\mu^-$ pairs in electron-positron annihilation:

$$e^+(p_+) + e^-(p_-) \to \mu^+(q_+) + \mu^-(q_-). \tag{3}$$

The cross-section in the Born approximation, can be written as:

$$\frac{d\sigma_B}{dO_{\mu_-}} = \frac{\alpha^2}{4s}\beta(2-\beta^2+\beta^2c^2), \tag{4}$$

with $s=(p_++p_-)^2=4E^2$, $\beta^2=1-\frac{4m^2}{s}$, E is the electron beam energy in center of mass reference frame (implied for this process below), m, m_e are the masses of muon and electron, $c=\cos\theta$, and θ is the angle of μ_--meson emission to the electron beam direction.

The interference of the Born amplitude

$$M_B = \frac{i4\pi\alpha}{s}\bar{v}(p_+)\gamma_\mu u(p_-)\bar{u}(q_-)\gamma_\mu v(q_+),$$

with the box-type amplitude M_B, results in parity violating contributions to the differential cross section, i. e. the ones, changing the sign at $\theta\to\pi-\theta$,. As a consequence of charge-odd correlations we can construct:

$$A(\theta,\Delta E) = \frac{\mathrm{d}\sigma(\theta)-\mathrm{d}\sigma(\pi-\theta)}{\mathrm{d}\sigma_B(\theta)}. \tag{5}$$

Here we take into account as well the emission of an additional soft real photon with energy not exceeding some small value ΔE, so that $A(\theta,\Delta E)$ is free from the infrared singularities.

Part of the results presented here were previously derived in a paper by one of us (E. A. K.) in Ref. [2], and partially published in [3].

There are two box-type Feynman amplitudes (Fig. 1). We calculate only one of them, the uncrossed diagram (Fig. 1a) with matrix element

$$\begin{aligned} M_a &= i\alpha^2\int\frac{\mathrm{d}^4k}{i\pi^2}\frac{\bar{u}(q_-)Tv(q_+)\times\bar{v}(p_+)Zu(p_-)}{(\Delta)(Q)(P_+)(P_-)}, \\ (\Delta) &= (k-\Delta)^2-m_e^2, \quad (Q)=(k-Q)^2-m^2, \quad (P_\pm)=(k\mp P)^2-\lambda^2, \end{aligned} \tag{6}$$

with λ-"photon" mass and

$$\begin{aligned} T &= \gamma_\alpha(\hat{k}-\hat{Q}+m)\gamma_\beta, \quad Z=\gamma_\beta(\hat{k}-\hat{\Delta})\gamma_\alpha, \\ \Delta &= \frac{1}{2}(p_+-p_-), \quad Q=\frac{1}{2}(q_+-q_-), \quad P=\frac{1}{2}(p_++p_-). \end{aligned} \tag{7}$$

We will assume

$$m^2=\frac{s}{4}(1-\beta^2)\sim s\sim -t\sim -u. \tag{8}$$

The explicit form of kinematical variables used below is:

$$\begin{aligned} \Delta^2 = -P^2 = -\frac{s}{4}, \quad Q^2=-\frac{1}{4}s\beta^2, \quad \sigma=\Delta Q=\frac{1}{4}(u-t), \\ u=(p_--q_+)^2=-\frac{s}{4}(1+\beta^2+2\beta c), \quad t=(p_--q_-)^2=-\frac{s}{4}(1+\beta^2-2\beta c). \end{aligned} \tag{9}$$

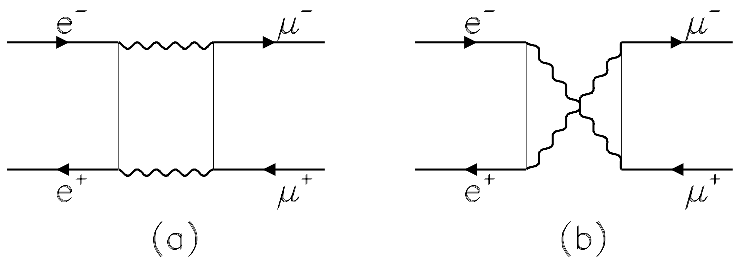

FIG. 1: Feynman diagrams for two-photon exchange in $e\bar{e} \to \mu\bar{\mu}$ process: box diagram (a) and crossed box diagram (b).

The contribution to the cross section of the amplitude arising from the crossed Feynman diagram (Fig. 1b), M_b, can be obtained from M_a by the crossing relation

$$\frac{\mathrm{d}\sigma_a(s,t)}{\mathrm{d}\Omega_\mu} = -\frac{\mathrm{d}\sigma_b(s,u)}{\mathrm{d}\Omega_\mu}, \tag{10}$$

which has the form

$$\frac{\mathrm{d}\sigma_a(s,t)}{\mathrm{d}\Omega_\mu} = \frac{\beta\alpha^3}{2\pi s^2}\mathrm{Re}[R(s,t)], \tag{11}$$

with

$$R(s,t) = \int \frac{\mathrm{d}^4 k}{i\pi^2}\frac{1}{(\Delta)(Q)(P_+)(P_-)}\frac{1}{4}Tr\left[(\hat{q}_- + m)T(\hat{q}_+ - m)\,\gamma_\mu\right] \times \frac{1}{4}Tr\left(\hat{p}_+ Z \hat{p}_- \gamma_\mu\right). \tag{12}$$

The scalar, vector and tensor loop momentum integrals are defined as:

$$J; J_\mu; J_{\mu\nu} = \int \frac{\mathrm{d}^4 k}{i\pi^2}\frac{1; k_\mu; k_\mu k_\nu}{(\Delta)(Q)(P_+)(P_-)} \tag{13}$$

Using symmetry properties, the vector and tensor integrals can be written as:

$$J_\mu = J_\Delta \cdot \Delta_\mu + J_Q \cdot Q_\mu, \tag{14}$$

$$J_{\mu\nu} = K_0 g_{\mu\nu} + K_P P_\mu P_\nu + K_Q Q^\mu Q^\nu + K_\Delta \Delta_\mu \Delta_\nu + K_x(Q_\mu \Delta_\nu + Q_\nu \Delta_\mu). \tag{15}$$

The quantity $R(s,t)$ can be expressed as a function of polynomials P_i as:

$$R = P_1 J + P_2 J_\Delta + P_3 J_Q + P_4 K_0 + P_5 K_\Delta + P_6 K_Q + P_7 K_P + P_8 K_x, \tag{16}$$

where the explicit form of polynomials is given in Appendix A. Using the explicit expression for the coefficients $J_\Delta, ..., K_x$ (See Appendix B) we obtain

$$R(s,t) \;=\; 4(\sigma - \Delta^2)(2\sigma - m^2)F + 16(\sigma - \Delta^2)(\sigma^2 + (\Delta^2)^2 - m^2\Delta^2)J$$

$$+4[(\Delta^2)^2 - 3\Delta^2\sigma + 2\sigma^2 - m^2\sigma]F_Q + 4[2(\Delta^2)^2 - 2\Delta^2\sigma + 2\sigma^2 - m^2\Delta^2]F_\Delta$$
$$+4[(\Delta^2)^2 + \Delta^2\sigma + m^2\Delta^2]G_Q + 4[-(\Delta^2)^2 + \sigma^2 - 2m^2\Delta^2]H_Q, \quad (17)$$

with the quantities $F \div H_Q$ given in Appendix B. Finally the charge-odd part of differential cross section has the form

$$\left(\frac{\mathrm{d}\sigma^{e\bar{e}}_{virt}(s,t)}{\mathrm{d}\Omega_\mu}\right)_{odd} = -\frac{\alpha^3\beta}{2\pi s}\mathcal{D}^{ann},$$
$$\mathcal{D}^{ann} = \frac{1}{s}[R(s,t) - R(s,u)] = (2 - \beta^2 + \beta^2c^2)\ln\left(\frac{1+\beta c}{1-\beta c}\right)\ln\frac{s}{\lambda^2} + \mathcal{D}^{ann}_V \quad (18)$$

$$\begin{aligned}
\mathcal{D}^{ann}_V &= \left(1 - 2\beta^2 + \beta^2c^2\right)\left[\frac{1}{1+\beta^2+2\beta c}\left(\ln\frac{1+\beta c}{2} + \ln\frac{s}{m^2}\right)\right.\\
&- \left.\frac{1}{1+\beta^2-2\beta c}\left(\ln\frac{1-\beta c}{2} + \ln\frac{s}{m^2}\right)\right]\\
&+ \beta c\left[\phi(\beta)\left(\frac{1}{2\beta^2} - 1 - \frac{\beta^2}{2}\right) - \frac{1}{\beta^2}\ln\frac{s}{m^2} - \frac{\pi^2}{6} + \frac{1}{2}\ln^2\frac{s}{m^2}\right.\\
&- \left.\frac{1}{2}\ln^2\frac{1-\beta c}{2} - \frac{1}{2}\ln^2\frac{1+\beta c}{2} + \mathrm{Li}_2\left(\frac{1+\beta^2+2\beta c}{2(1+\beta c)}\right) + \mathrm{Li}_2\left(\frac{1+\beta^2-2\beta c}{2(1-\beta c)}\right)\right]\\
&- \frac{m^2}{s}\left[\ln^2\frac{1-\beta c}{2} - \ln^2\frac{1+\beta c}{2} + 2\mathrm{Li}_2\left(\frac{1+\beta^2+2\beta c}{2(1+\beta c)}\right) - 2\mathrm{Li}_2\left(\frac{1+\beta^2-2\beta c}{2(1-\beta c)}\right)\right],
\end{aligned}$$

where $\phi(\beta) = sF_Q$, F_Q is given in Appendix B and

$$\mathrm{Li}_2(z) = -\int_0^z \frac{\mathrm{d}x}{x}\ln(1-x) \quad (19)$$

is the Spence function. The quantity $\mathcal{D}^{ann} - \mathcal{D}^{ann}_V$ suffers from infrared divergences, which will be compensated taking into account the soft photons contribution (see below).

III. SCATTERING CHANNEL

Let us consider now the elastic electron muon scattering

$$e(p_1) + \mu(p) \to e(p_1') + \mu(p')$$

which is the crossed process of (3). The Born cross section is the same for the scattering of electrons and positrons on the same target. Taking the experimental data from the scattering of electron and positron on the same target, one can measure the difference of the corresponding cross-sections which is sensitive to the interference of the one and two photon exchange amplitudes.

The Born cross sectin of electron scattering on muon at rest can be written in the form:

$$\begin{aligned} \frac{\mathrm{d}\sigma_B^{e\mu}}{\mathrm{d}\Omega} &= \frac{\alpha^2(s^2+u^2+2tm^2)}{2m^2\rho^2t^2}, \quad s = 2p_1p = 2mE, \\ t &= -2p_1p_1', \quad u = -2pp_1' = -\frac{s(\rho-1)}{\rho}, \; s+t+u=0, \\ u &= -2pp_1' - \frac{s}{\rho}, 2\rho = 1 + \frac{2E}{m}\sin^2\frac{\theta_e}{2}, \end{aligned} \tag{20}$$

where θ_e is the scattering angle. The charge-odd contribution to the cross section of $e\mu-$elastic scattering is:

$$\begin{aligned} \left(\frac{\mathrm{d}\sigma_{virt}^{e\mu}}{\mathrm{d}\Omega_e}\right)_{odd} &= -\frac{\alpha^3}{2\pi m^2\rho^2} Re\left(\mathcal{D}^{sc}\right), \\ \mathcal{D}^{sc} &= \frac{1}{t}[\mathcal{D}(s,t) - \mathcal{D}(u,t)] = \frac{2}{t^2}[s^2+u^2+2tm^2]\ln\frac{-u}{s}\ln\frac{-t}{\lambda^2} + \mathcal{D}_{virt}^{sc} \end{aligned} \tag{21}$$

with

$$\begin{aligned} \mathcal{D}_{virt}^{sc} &= \frac{s-u}{t}\left[\frac{1}{2}\ln^2\left(\frac{-t}{m^2}\right) - \frac{\tau}{1+\tau}\ln\left(\frac{-t}{m^2}\right) + m^2\bar{F}_Q\left(6\tau + 2 - \frac{2\tau^2}{1+\tau}\right)\right] \\ &+ \frac{s}{t}\left[-\ln^2\frac{s}{-t} + \pi^2 + 2\mathrm{Li}_2\left(1+\frac{m^2}{s}\right)\right] - \frac{u}{t}\left[-\ln^2\frac{u}{t} + 2\mathrm{Li}_2\left(1+\frac{m^2}{u}\right)\right] \\ &+ \frac{(1-2\tau)}{(-4\tau)}\left[2\ln\left(\frac{s}{-u}\right)\ln\left(\frac{-t}{m^2}\right) + \ln^2\left(\frac{-u}{m^2}\right) - \ln^2\left(\frac{s}{m^2}\right) + \pi^2\right. \\ &+ \left. 2\mathrm{Li}_2\left(1+\frac{m^2}{s}\right) - 2\mathrm{Li}_2\left(1+\frac{m^2}{u}\right)\right] + \left(2m^2 - \frac{su}{t}\right)\left[\frac{\ln\frac{s}{m^2}}{m^2+s} - \frac{\ln\frac{-u}{m^2}}{m^2+u}\right] \end{aligned} \tag{22}$$

with the help of the following relation:

$$m^2\bar{F}_Q = -\frac{1}{4\sqrt{\tau(1+\tau)}}\left[\pi^2 + \ln(4\tau)\ln x + \mathrm{Li}_2\left(-2\sqrt{\tau x}\right) - \mathrm{Li}_2\left(\frac{2\sqrt{\tau}}{\sqrt{x}}\right)\right], \tag{23}$$

where

$$x = \frac{\sqrt{1+\tau}+\sqrt{\tau}}{\sqrt{1+\tau}-\sqrt{\tau}} \text{ and } \tau = -t/(4m^2).$$

IV. SOFT PHOTON EMISSION

In this section the emission of soft real photons in the Lab reference frame for $e\mu$ scattering is calculated. Following Ref. [4], the odd part of cross section

$$\frac{\mathrm{d}\sigma^{soft}}{\mathrm{d}\sigma_0} = -\frac{\alpha}{4\pi^2}\cdot 2\int\frac{\mathrm{d}^3k}{\omega}\left(\frac{p_1'}{p_1'k} - \frac{p_1}{p_1k}\right)\left(\frac{p'}{p'k} - \frac{p}{pk}\right)_{S_0,\ \omega<\Delta\varepsilon} \tag{24}$$

must be calculated in the special reference frame S_0, where the sum of the three-momenta of the incident and of the recoil muon is zero $\vec{z} = \vec{k} + \vec{p}' = 0$. Really, in this frame, the on-mass shell condition of the scattered muon $\delta[(z-k)^2 - m^2]$, $z = p_1 + p - p_1'$ does not depend on the direction of the emitted photon. The photon energy can be determined as the difference of the energy of the scattered electron and the corresponding value for the elastic case: the maximum value of the photon energy $\Delta\varepsilon$ in the S_0 frame is related with the energy of the scattered electron, detected in the Lab frame ΔE as (see [4], Appendix C),

$$\Delta\varepsilon = \rho\Delta E. \tag{25}$$

The calculation of the soft photon integral with $\omega < \Delta\varepsilon$ can be performed using t'Hooft and M. Veltman approach (see [5], Section 7). We find

$$\begin{aligned} \frac{\mathrm{d}\sigma_{e\mu}^{soft}}{\mathrm{d}\Omega} &= -\frac{\alpha}{\pi}\left\{2\ln\rho\ln\left[\frac{(2\rho\Delta E)^2}{\lambda^2 x}\right] + \mathcal{D}_{soft}^{sc}\right\}, \\ \mathcal{D}_{soft}^{sc} &= -2\mathrm{Li}_2\left(1-\frac{1}{\rho x}\right) + 2\mathrm{Li}_2\left(1-\frac{\rho}{x}\right), \end{aligned} \tag{26}$$

which is in agreement with Ref. [4].

The charge asymmetry for $e^{\pm}\mu$ scattering has the form:

$$\begin{aligned} &\frac{\mathrm{d}\sigma^{e^-\mu\to e^-\mu(\gamma)} - \mathrm{d}\sigma^{e^+\mu\to e^+\mu(\gamma)}}{\mathrm{d}\sigma^{e^-\mu\to e^-\mu(\gamma)} + \mathrm{d}\sigma^{e^+\mu\to e^+\mu(\gamma)}}\frac{\mathrm{d}\sigma_{e\mu}^{virt} + \mathrm{d}\sigma_{e\mu}^{soft})}{\mathrm{d}\Omega_B^{e\mu}} = \frac{\alpha}{\pi}\left[\Xi - 2\ln\rho\ln\frac{(2\rho\Delta E)^2}{-tx}\right], \\ &\Xi = Re\left[-\frac{t^2\mathcal{D}_{virt}^{sc}}{s^2+u^2+2tm^2} - \mathcal{D}_{soft}^{sc}\right]. \end{aligned} \tag{27}$$

The function Ξ is shown in Fig. 2 as a function of $\cos\theta_e$ for given E/m .

The odd contributions to the differential cross section for the process $e^+ + e^- \to \mu^+ + \mu^-$, due to soft photon emission, has the form:

$$(\mathrm{d}\sigma_{soft}^{e^+e^-\to\mu^+\mu^-(\gamma)})_{odd} = \mathrm{d}\sigma_0\left(-\frac{\alpha}{4\pi^2}\right)2\int\frac{\mathrm{d}^3k}{\omega}\left(-\frac{p_-}{p_-k} + \frac{p_+}{p_+k}\right)\left(\frac{q_+}{q_+k} - \frac{q_-}{q_-k}\right)_{S_0,\omega<\Delta\varepsilon}. \tag{28}$$

Again, the integration must be performed in the special frame S^0, where $\bar{p}_+ + \bar{p}_- - \bar{q}_+ = \bar{q}_- + \bar{k} = 0$. In this frame we have

$$\begin{aligned} (q_- + k)^2 - m^2 &= 2(E_- + \omega)\omega \approx 2m\omega = (p_+ + p_- - q_+)^2 - m^2 = 4E(E - \varepsilon_+), \\ E - \varepsilon_+ &= \frac{m}{2E}\Delta\varepsilon. \end{aligned} \tag{29}$$

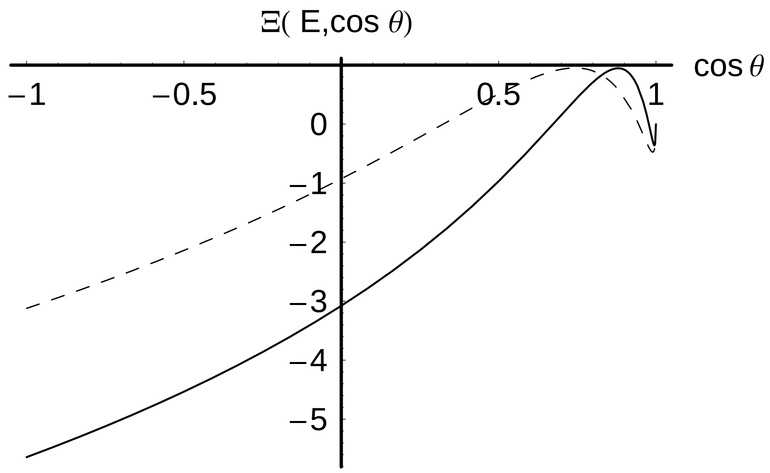

FIG. 2: $\Xi(s, \cos\theta)$ for $E = 5m$ (dashed line) and $E = 10m$, m is muon mass.

In the elastic case $E - \varepsilon_+^{el} = 0$ and the photon energy in the Lab system is

$$\Delta E = \varepsilon_+^{el} - \varepsilon_+ = \frac{m}{2E}\Delta\varepsilon. \tag{30}$$

The t'Hooft-Veltman procedure for soft photon emission contribution leads to:

$$\frac{\mathrm{d}\sigma_{ann}^{soft}}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_0}{\mathrm{d}\Omega}.\frac{2\alpha}{\pi}\left[\ln\left(\frac{4E\Delta E}{m\lambda}\right)^2 \ln\frac{1+\beta c}{1-\beta c} + \mathcal{D}_S^{ann}\right] \tag{31}$$

with

$$\begin{aligned}\mathcal{D}_s^{ann} \; = \; & \frac{1}{2}\mathrm{Li}_2\left(\frac{-2\beta(1+c)}{(1-\beta)(1-\beta c)}\right) + \frac{1}{2}\mathrm{Li}_2\left(\frac{2\beta(1-c)}{(1+\beta)(1-\beta c)}\right) \\ & -\frac{1}{2}\mathrm{Li}_2\left(\frac{-2\beta(1-c)}{(1-\beta)(1+\beta c)}\right) - \frac{1}{2}\mathrm{Li}_2\left(\frac{2\beta(1+c)}{(1+\beta)(1+\beta c)}\right).\end{aligned} \tag{32}$$

The total contribution (virtual and soft) is free from infrared singularities and has the form

$$\begin{aligned}\frac{\mathrm{d}\sigma_{ann}}{\mathrm{d}\Omega} \; &= \frac{\alpha^3\beta}{2\pi s}(2-\beta^2+\beta^2c^2)\Upsilon, \quad \Upsilon = 2\ln\frac{1+\beta c}{1-\beta c}\ln\left(\frac{2\Delta E}{m}\right) + \Phi(s,\cos\theta), \\ \Phi(s,\cos\theta) &= \mathcal{D}_S^{ann} - \frac{\mathcal{D}_V^{ann}}{2-\beta^2+\beta^2c^2}.\end{aligned} \tag{33}$$

The quantity $\Phi(s, \cos\theta)$ is presented in Fig. 3.

The relevant asymmetry can be constructed from (5)

$$A = \frac{4\alpha}{\pi}\Upsilon. \tag{34}$$

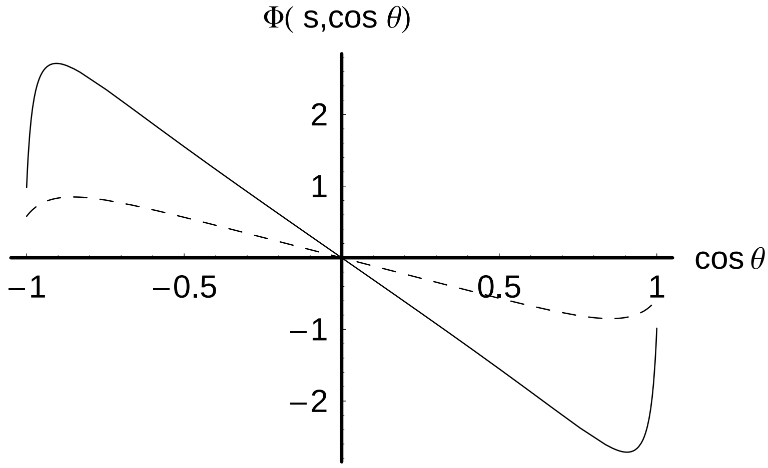

FIG. 3: $\Phi(s, \cos\theta)$, for $s = 10m^2$ (dashed line) and $s = 20m^2$, m is muon mass.

V. DERIVATION OF THE ADDITIONAL STRUCTURE: ANNIHILATION CHANNEL

Let us start from the following form of the matrix element for the process $e^+(p_+) + e^-(p_-) \to \mu^+(q_+) + \mu^-(q_-)$ in presence of 2γ exchange:

$$M_2 = \frac{i\alpha^2}{s}\bar{v}(p_+)\gamma_\mu u(p_-) \times \bar{u}(q_-)\left(G_1\gamma_\mu - \frac{G_2}{m}\gamma_\mu \hat{P} + 4\frac{1}{s}G_3\hat{\Delta}Q_\mu\right)v(q_+), \tag{35}$$

where the amplitudes G_i are complex functions of the two kinematical variables s, and t.

To calculate the structure G_3 from the 2γ amplitude (see Eq. (6)), both Feynman diagrams (Figs. 1a and 1b) must be taken into account. Similarly to Sec. II, only one of them can be calculated explicitly (the uncrossed one), whereas the other can be obtained from this one by appropriate replacements.

To extract the structure G_3 we multiply Eq. (35) subsequently by

$$\begin{aligned} &\bar{u}(p_-)\gamma_\lambda v(p_+) \times \bar{v}(q_+)\gamma_\lambda u(q_-), \\ &\bar{u}(p_-)\hat{Q}v(p_+) \times \bar{v}(q_+)u(q_-), \\ &\bar{u}(p_-)\hat{Q}v(p_+) \times \bar{v}(q_+)\hat{\Delta}u(q_-), \end{aligned} \tag{36}$$

and perform the summation on fermions spin states.

Solving the algebraical set of equations we find

$$G_1^a = \frac{1}{\beta^4\sin^4\theta}\left\{(8B^a + A^a\beta^2\sin^2\theta)(1-\beta^2\cos^2\theta) - 4C^a\beta\cos\theta\left[2-\beta^2(1+\cos^2\theta)\right]\right\},$$

$$G_2^a = \frac{1}{\beta^4\sin^4\theta}\left\{\beta(1-\beta^2)(A^a\beta\sin^2\theta-8C^a\cos\theta)+4B^a\left[2-\beta^2(1+\cos^2\theta)\right]\right\} \qquad (37)$$

$$G_3^a = \frac{1}{\beta^3\sin^4\theta}\left[-A^a\beta^2\sin^2\theta\cos\theta-8B^a\cos\theta+4\beta C^a(1+\cos^2\theta)\right],$$

with

$$A^a = \int\frac{\mathrm{d}^4k}{i\pi^2}\frac{1}{(\Delta)(Q)(P_+)(P_-)}\frac{1}{s}Tr(\hat{p}_+Z\hat{p}_-\gamma_\lambda)\times\frac{1}{4}Tr\left[(q_-+m)T(\hat{q}_+-m)\gamma_\lambda\right],$$

$$B^a = \int\frac{\mathrm{d}^4k}{i\pi^2}\frac{1}{(\Delta)(Q)(P_+)(P_-)}\frac{m}{s^2}Tr(\hat{p}_+Z\hat{p}_-\hat{Q})\times\frac{1}{4}Tr\left[(\hat{q}_-+m)T(\hat{q}_+-m)\right], \qquad (38)$$

$$C^a = \int\frac{\mathrm{d}^4k}{i\pi^2}\frac{1}{(\Delta)(Q)(P_+)(P_-)}\frac{1}{s^2}Tr(\hat{p}_+Z\hat{p}_-\hat{Q})\times\frac{1}{4}Tr\left[(\hat{q}_-+m)T(\hat{q}_+-m)\hat{\Delta}\right].$$

The explicit value for G_3^a is:

$$\begin{aligned}
G_3^a &= \frac{2s}{\beta^3(1-c^2)^2}\left\{\frac{1}{2}G_Q(1-c^2)\beta^3(1-\beta c)\right.\\
&+ \frac{1}{2}H_Q\beta^2(1-c^2)\left[c(-3+5\beta^2)-\beta-\beta c^2\right]\\
&+ F_\Delta c\left[1-4\beta^2+2\beta^4+c^2\beta^2(3-4\beta^2)-2\beta c(1-2\beta^2)\right]\\
&+ F_Q\beta\left[-c^2+\beta c\left(-\frac{1}{2}-4\beta^2c^2+\frac{5}{2}c^2\right)+\beta^2\left(-\frac{1}{2}+2\beta^2c^2+\frac{3}{2}c^2\right)\right]\\
&- 2Js\beta^2c(1-c^2)(1-\beta^2)(1-\beta c)\\
&+ \left. Fc\left[1+\beta^2c^2-2\beta^4-4\beta^4c^2+\beta c(-3+4\beta^2+2\beta^4+\beta^2c^2)\right]\right\}.
\end{aligned} \qquad (39)$$

The contributions from the crossed Feynman diagram can be obtained from Eqs. 39 by:

$$(A^b,B^b,C^b)_{crossed} = -[A^a,B^a,C^a(\cos\theta\to-\cos\theta)]_{uncrossed}. \qquad (40)$$

As one can see, in the quantities G_1, G_2, and G_3 infrared divergencies are present. For clearness, let us show explicitly the cancellation of infrared divergencies of the box (as in form of Eq. 35) - Born interference with soft photon contribution. The infrared divergent parts of G_i^a can be written in the form:

$$G_1^a|_{ir} = \frac{2s^2}{\beta^2(1-c^2)}J(1-\beta c)\left[1-2\beta^2+\beta^4+\frac{5}{4}(1+c^2)\beta^2-(1-c^2)\beta^4\right]$$

$$G_2^a|_{ir} = \frac{4s^2}{\beta^2(1-c^2)}J(1-\beta c)\left[2-4\beta^2+2\beta^4+(1-c^2)\beta^2-(1-c^2)\beta^4\right]$$

$$G_3^a|_{ir} = \frac{4s^2}{\beta^2(1-c^2)}J(1-\beta c)(-2)(1-\beta^2)\beta c \qquad (41)$$

The expression for J is given in Eq. (B1). The interference of the uncrossed photon box amplitude with the Born one can be written in the form:

$$-G_2^a+(2-\beta^2+\beta^2c^2)G_1^a+G_1^a\beta^3c(1-c^2) = -4(2-\beta^2+\beta^2c^2)\ln\frac{s}{\lambda^2}\ln\frac{m^2-t}{m_e^2} \qquad (42)$$

Here we can see that the infrared divergent contribution is proportional to the Born elemnt square. this fact can be used to eliminate the infrared dipendence of G_1, G_2, and G_3 structures by taking into account soft photon emission.

VI. ONE SPIN ASYMMETRY

Let us consider now the process of electron interaction with a heavy lepton. The target spin asymmetry for heavy fermions production process $e^+(p_+) + e^-(p_-) \to p(q_+) + \bar{p}(q_-)$ (in CMS frame) is defined as

$$\frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} = (\vec{a}\vec{n})R_n, \tag{43}$$

where $\vec{a}$ is the proton polarization vector, $\vec{n} = (\vec{q}_- \times \vec{p}_-) \,/\, |\vec{q}_- \times \vec{p}_-|$ is the unit vector normal to the scattering plane, $\mathrm{d}\sigma^{\uparrow}$ is the cross section of processes with proton polarization vector $\vec{a}$, $\mathrm{d}\sigma^{\downarrow}$ is the cross section of processes with proton polarization vector $-\vec{a}$. Thus the denominator of the left hand side in Eq. (43) is the unpolarized cross section of process $e^+ + e^- \to \ell^+ + \bar{\ell}^-$.

The difference of cross sections in (43) is originated by the s-channel discontinuity of interference of the Born-amplitude with TPE amplitude

$$\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow} \sim Re \sum \left(A^{+}_{elastic} \cdot A_{TPE} + A_{elastic} \cdot A^{+}_{TPE} \right). \tag{44}$$

Using the density matrix of final proton $u(p)\bar{u}(p) = (\hat{p} + M)(1 - \gamma_5 \hat{a})$ one gets

$$\begin{aligned} &Re \sum \left(A^{+}_{elastic} \cdot A_{TPE} + A_{elastic} \cdot A^{+}_{TPE} \right) = 32 \frac{(4\pi\alpha)^3 \, (2\pi i)^2}{s\pi^2} Re(Y), \\ &Y = \int \frac{dk}{i\pi^2} \frac{1}{(\Delta)(Q)(+)(-)} \times \frac{1}{4} Tr \left[\hat{p}_1 \gamma^{\alpha} \hat{p}'_1 \gamma^{\mu} \left(\hat{k} - \hat{\Delta} \right) \gamma^{\nu} \right] \times \\ &\times \frac{1}{4} Tr \left[(\hat{p} - M) \left(-\gamma_5 \hat{a} \right) \gamma_{\alpha} \left(\hat{p}' + M \right) \gamma_{\nu} \left(\hat{k} - \hat{Q} + M \right) \gamma_{\mu} \right]. \end{aligned} \tag{45}$$

Performing the loop-momenta integration the right hand side of Eq. (45) can be expressed in terms of basic integrals (see Appendix B)

$$Re(Y) = 4M(a, \Delta, Q, P) Im \left(F_Q - G_Q + H_Q \right), \tag{46}$$

where $(a, \Delta, Q, P) \equiv \varepsilon^{\mu\nu\rho\sigma} a_{\mu} \Delta_{\nu} Q_{\rho} P_{\sigma} = (\sqrt{s}/2)^3 (\vec{a}\vec{n}) \beta \sin\theta$. Using the expressions listed in Appendix B we have:

$$Im \left(F_Q - G_Q + H_Q \right) = \frac{\pi}{s} \psi(\beta) = \frac{\pi}{s\beta^2} \left(\frac{1 - \beta^2}{\beta} \ln \frac{1 + \beta}{1 - \beta} - 2 \right). \tag{47}$$

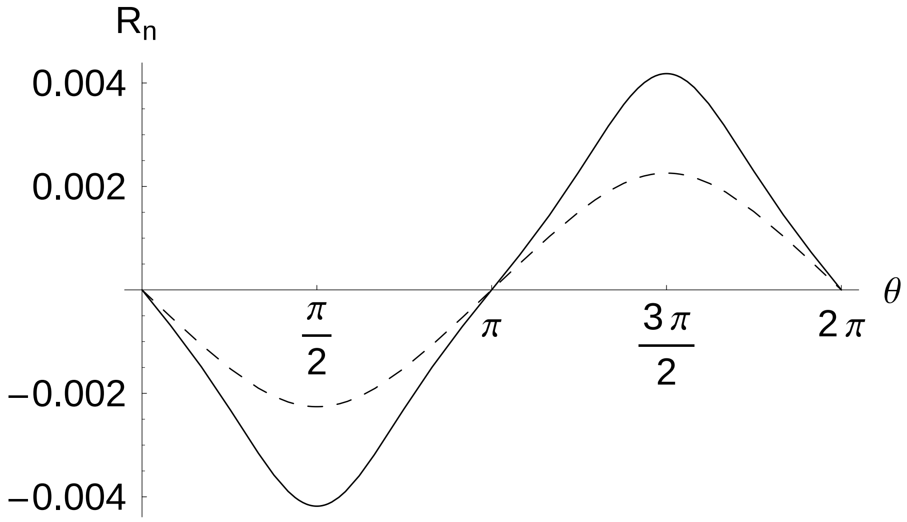

FIG. 4: Asymmetry R_n for the case of lepton pair creation for energies $s = 15$ GeV^2 (dashed line) and $s = 20$ GeV^2 (solid line).

Thus, after standard algebra, the following expression for spin asymmetry can be obtained for the processes $e^+ + e^- \to \vec{\ell}^+ + \ell^-$:

$$R_n = 2\alpha \frac{M}{\sqrt{s}} \frac{\beta\psi(\beta)\sin\theta}{2 - \beta^2 \sin^2\theta}. \tag{48}$$

and it is shown in Fig. 4, as a function of θ at several values of s.

Such considerations apply to the scattering channel when the initial lepton is polarized. Similarly to (47) one finds

$$Im_s\left(\bar{F}_Q - \bar{G}_Q + \bar{H}_Q\right) = -\frac{\pi}{s + M^2}. \tag{49}$$

(note that the s-channel imaginary part vanishes for the crossed photon diagram amplitude). The contribution of the polarization vector appears in the same combination

$$(a, \Delta, Q, P) = \frac{1}{2}(a, p, p_1, q) = \frac{ME^2}{2\rho}\sin\theta(\vec{a}\vec{n}). \tag{50}$$

The single spin asymmetry for the process $e^- + \vec{\ell} \to e^- + \ell$ (the initial lepton is polarized) has the form:

$$\frac{\mathrm{d}\sigma^\uparrow - \mathrm{d}\sigma^\downarrow}{\mathrm{d}\sigma^\uparrow + \mathrm{d}\sigma^\downarrow} = (\vec{a}\vec{n})T_n \tag{51}$$

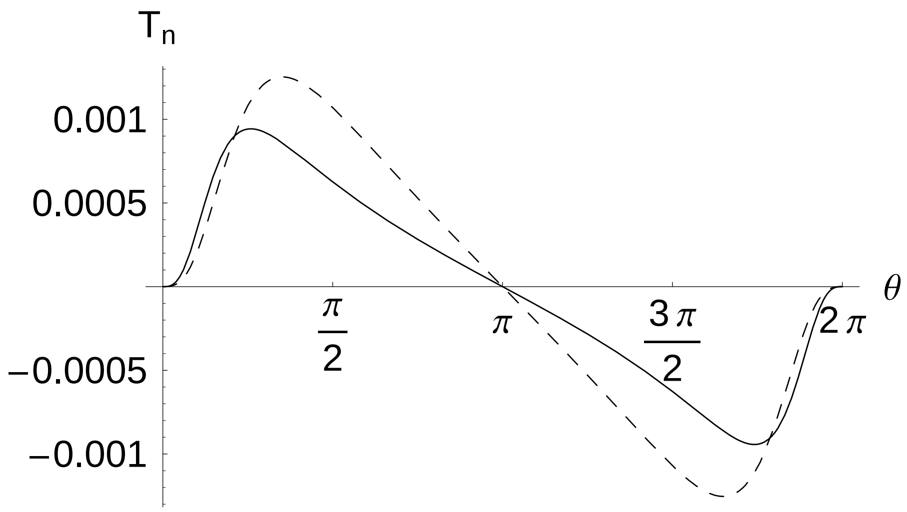

FIG. 5: Asymmetry T_n in case of scattering on τ lepton for $E = 3$ GeV (dashed line) and $E = 10$ GeV (solid line), E is the energy of the electron.

with

$$T_n = \frac{\alpha}{2M^2}\frac{s^2}{s+M^2}(1+\tau)\frac{\epsilon}{\rho(1+2\tau\tan^2\frac{\theta_e}{2})}\sin\theta\tan^2\frac{\theta}{2}. \quad (52)$$

This quantity is shown in Fig. 5 for the case of structureless proton as a function of θ, for two values of s. The asymmetry decreases when the c.m.s. energy growth, so on experiment it is useful to measure the asymmetry near the treshold of heavy lepton production.

VII. CONCLUSIONS

We calculated QED radiative corrections to the differential cross-section of the processes $e^+ + e^- \to \mu^+ + \mu^-(\gamma)$, $e^\pm + \mu \to e^\pm + \mu(\gamma)$, arising from the interference between the Born and the box-type amplitudes. The relevant part of soft photon emission contribution, which eliminates the infrared singularities, was also considered.

Angular asymmetry, charge asymmetry as well as target spin asymmetry were calculated. These quantities are free from infrared and electron mass singularities. Numerical applications show that these observables are large enough to be measured (see Figs. 2-5).

The parametrization (35) for the contribution to the matrix element arising from box-type diagrams in terms of three additional functions $G_i(s,t)$, $i = 1, 2, 3$ suffers from infrared divergencies, which can be eliminated by taking into account soft photon emission expressed in terms of structures G_1, G_2, G_3. This procedure results in replacing $\ln(m/\lambda)$ with $\ln(\Delta E)/m$.

VIII. APPENDIX A: TRACE CALCULATION.

The explicit expressions for the polynomials P_i are:

$$\begin{aligned}
P_1 &= 8\{-(\Delta^2)^3 - \Delta^2\sigma^2 + 2\sigma^3 + [(\Delta^2)^2 - 2\Delta^2\sigma]m^2\}, \\
P_2 &= 16[(\Delta^2)^2\sigma - \sigma^3 + \Delta^2\sigma m^2], \\
P_3 &= 8(\{2(\Delta^2)^2\sigma - 2\sigma^3 + m^2[(\Delta^2)^2 + 2\Delta^2\sigma - \sigma^2] + \Delta^2 m^4\}, \\
P_4 &= 8[5(\Delta^2)^2 - 6\Delta^2\sigma + 5\sigma^2 - 5\Delta^2 m^2], \\
P_5 &= 8[(\Delta^2)^3 - 2(\Delta^2)^2\sigma + \Delta^2(\sigma^2 - m^2\Delta^2)], \\
P_6 &= 8\{(\Delta^2)^3 - 2(\Delta^2)^2\sigma + \Delta^2\sigma^2 + m^2[-(\Delta^2)^2 - 2\Delta^2\sigma + 2\sigma^2] - 2\Delta^2 m^4\}, \\
P_7 &= 8[-(\Delta^2)^3 + 2(\Delta^2)^2\sigma - \Delta^2\sigma^2 + (\Delta^2)^2 m^2], \\
P_8 &= 8\{-(\Delta^2)^3 + (\Delta^2)^2\sigma - 3\Delta^2\sigma^2 + 3\sigma^3 - m^2[(\Delta^2)^2 + 3\Delta^2\sigma]\}.
\end{aligned} \tag{53}$$

IX. APPENDIX B: USEFUL INTEGRALS.

In the calculation of $e\mu$ scattering we use the following set of scalar integrals with three and four denominators [2].

$$\begin{aligned}
F_\Delta &= \frac{-i}{\pi^2}\int \frac{\mathrm{d}^4k}{(\Delta)(P_+)(P_-)} = \frac{1}{s}\left[\frac{\pi^2}{6} + \frac{1}{2}\ln^2\frac{s}{m_e^2}\right], \\
F_Q &= \frac{-i}{\pi^2}\int \frac{\mathrm{d}^4k}{(Q)(P_+)(P_-)} \\
&= \frac{1}{s\beta}\left[\frac{1}{2}\ln^2\frac{1-\beta}{2} - \frac{1}{2}\ln^2\frac{1+\beta}{2} + \mathrm{Li}_2\left(\frac{1+\beta}{2}\right) - \mathrm{Li}_2\left(\frac{1-\beta}{2}\right)\right], \\
H &= \frac{-i}{\pi^2}\int \frac{\mathrm{d}^4k}{(\Delta)(Q)(P_+)} = G = \frac{-i}{\pi^2}\int \frac{\mathrm{d}^4k}{(\Delta)(Q)(P_-)} = -\frac{1}{2(m^2-t)}\left[\ln^2\frac{m^2-t}{m^2}\right. \\
&+ \left.\left(2\ln\frac{m^2-t}{m^2} + \ln\frac{m^2}{m_e^2}\right)\ln\frac{m^2}{\lambda^2} - \frac{1}{2}\ln^2\frac{m^2}{m_e^2} - 2\mathrm{Li}_2\left(-\frac{t}{m^2-t}\right)\right], \\
F &= \frac{1}{2}sJ - G = -\frac{1}{2(m^2-t)}\left[\left(2\ln\frac{m^2-t}{m^2} + \ln\frac{m^2}{m_e^2}\right)\ln\frac{s}{m^2}\right. \\
&- \left.\ln^2\frac{m^2-t}{m^2} + \frac{1}{2}\ln^2\frac{m^2}{m_e^2} + 2\mathrm{Li}_2\left(-\frac{t}{m^2-t}\right)\right],
\end{aligned}$$

$$J = \frac{-i}{\pi^2}\int \frac{\mathrm{d}^4k}{(\Delta)(Q)(P_+)(P_-)} = -\frac{1}{s(m^2-t)}\left[\left(2\ln\frac{m^2-t}{m^2}+\ln\frac{m^2}{m_e^2}\right)\ln\frac{s}{\lambda^2}\right]. \qquad (54)$$

The terms proportional to m_e^2/s, m_e^2/m_μ^2 were neglected. Notations follow Ref. (7).

The vector integrals with three denominators are:

$$\begin{aligned}
&\frac{1}{i\pi}\int \frac{k^\mu \mathrm{d}^4k}{(\Delta)(Q)(P_+)} = H_P P^\mu + H_\Delta \Delta^\mu + H_Q Q^\mu, \quad H_Q = \frac{1}{t}\ln\frac{m^2-t}{m^2},\\
&H_\Delta = \frac{1}{m^2-t}\left(-\ln\frac{m^2}{m_e^2} - \frac{m^2+t}{t}\ln\frac{m^2-t}{m^2}\right),\\
&H_P = H + \frac{1}{m^2-t}\left(\ln\frac{m^2}{m_e^2} + 2\ln\frac{m^2-t}{m^2}\right),\\
&\frac{1}{i\pi}\int \frac{k^\mu \mathrm{d}^4k}{(\Delta)(P_+)(P_-)} = G_\Delta \Delta^\mu, \quad G_\Delta = \frac{1}{s}\left(-2\ln\frac{s}{m_e^2} + \frac{1}{2}\ln^2\frac{s}{m_e^2} + \frac{\pi^2}{6}\right),\\
&\frac{1}{i\pi}\int \frac{k^\mu \mathrm{d}^4k}{(Q)(P_+)(P_-)} = G_Q Q^\mu, \quad G_Q = \frac{1}{s-4m^2}\left(-2\ln\frac{s}{m^2} + sF_Q\right).
\end{aligned} \qquad (55)$$

Four denominator vector and tensor integrals were defined in (13). The relevant coefficients are:

$$\begin{aligned}
J_\Delta &= \frac{1}{2d}\Big[(F+F_\Delta)\sigma - Q^2(F+F_Q)\Big],\\
J_Q &= \frac{1}{2d}\Big[(F+F_Q)\sigma - \Delta^2(F+F_\Delta)\Big], \quad F = \frac{1}{2}sJ - G, \quad d = \Delta^2 Q^2 - \sigma^2,\\
K_0 &= -\frac{1}{2\sigma}\Big[\sigma(F - G + H_P + H_\Delta + H_Q) + H_\Delta(\sigma - \Delta^2) - H_Q(\sigma - Q^2)\\
&\quad +2P^2 J_\Delta(\Delta^2 - 2\sigma) + \Delta^2 G_\Delta - Q^2 G_Q - 2p^2 Q^2 J_Q\Big],\\
K_\Delta &= -\frac{1}{2\sigma d}\Big[Q^2\sigma(G - F - H_P - 3H_\Delta + 6P^2 J_\Delta)\\
&\quad +(\Delta^2 Q^2 + \sigma^2)(H_\Delta - 2P^2 J_\Delta - G_\Delta) - Q^4(H_Q - 2P^2 J_Q - G_Q)\Big],\\
K_P &= \frac{1}{2P^2\sigma}\Big[2\sigma(H_\Delta - 2P^2 J_\Delta + H_p + \frac{1}{2}F - \frac{1}{2}G) + Q^2(H_Q - 2P^2 J_Q - G_Q)\\
&\quad -\Delta^2(H_\Delta - 2P^2 J_\Delta - G_\Delta)\Big],\\
K_Q &= -\frac{1}{2\sigma d}\Big[-\Delta^2\sigma A_P + 2\Delta^4 A_\Delta + (\sigma^2 - 2\Delta^2 Q^2)A_Q\Big],\\
K_x &= -\frac{1}{2d}\left(\sigma A_P + Q^2 A_Q - 2\Delta^2 A_\Delta\right),
\end{aligned} \qquad (56)$$

where we used

$$A_\Delta = H_\Delta + 2\Delta^2 J_\Delta - G_\Delta,\ A_Q = H_Q + 2\Delta^2 J_Q - G_Q,\ A_P = F - G + H_P + 3H_\Delta + 6\Delta^2 J_\Delta. \quad (57)$$

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