

SIMULATION OF THE LOSSES BY DECAY IN THE DECAY RING FOR THE BETA-BEAMS

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1. INTRODUCTION

The aim of the “beta-beams” is to accelerate and store high intensities of highly energetic β radioactive ions until they decay [1]. The problem is that the ions decay during all the acceleration process and the decay products will be lost on the walls of the different machines. The activation must be studied in the whole complex [2] (PS, SPS and the decay ring). However, since the stored intensity is the biggest in the decay ring, its design must take into account the power deposition problem very early. In this note, we will focus on the evaluation and the localization of this kind of losses in the decay ring. We will see that this determines the bend length.

When one of these ions decays, its rigidity changes slightly. Indeed, whereas its momentum change is negligible, its charge number has changed. If Z_0 , Z_1 , $(B\rho)_0$, $(B\rho)_1$ are respectively the charge number and the rigidity of the source ion and of the product ion, p their momentum,

we have : $(B\rho)_0 = \frac{p}{Z_0 e}$ and $(B\rho)_1 = \frac{p}{Z_1 e}$.

Thus, $\delta = \frac{\Delta(B\rho)}{(B\rho)_0} = \frac{Z_0}{Z_1} - 1$

The deviation due to δ is analogous to the one due to a momentum difference.

For instance, for ${}^6\text{He}^{2+}$ (which decays into ${}^6\text{Li}^{3+}$), $\delta = -1/3$ and ${}^{18}\text{Ne}^{10+}$ (which decays into ${}^{18}\text{F}^{9+}$), $\delta = +1/9$. This difference is so high that the product ions will quickly be lost when they enter a bend. Therefore, it is not possible to extract the decay products from the arc. We can only have a design with beam stoppers in the arc in order to limit the depositions in the magnetic elements. To evaluate our design, we added a module to the BETA code giving the amount of the losses on the ring walls. But it does not take into account the effects of the interaction of the decay products with the chamber: we can only predict the peaks of deposition. After hitting the wall, the decay products are supposed to be lost in the chamber. Before putting stoppers, it was necessary to have the right length for the bends in order to avoid the depositions there.

2. BEND LENGTH

The central trajectory of an ion stays the same after its disintegration in a straight section until it enters a dipole. It is strongly deviated besides. In [2], the calculation assumes a uniform repartition of the losses in the arc. But, in reality, the products of the decays in a drift will give a peak after crossing a bend magnet. As the bends are the most sensitive elements in the arc, we must avoid mostly the depositions there. The aim of this paragraph is to calculate the domain of variation of the bend angle as a function of the bend radius and of the chamber sizes at the first order. The dispersion function in a bend is given by: $\rho(1-\cos(\theta))$ with ρ the

bend radius and θ the bend angle. The length L of the bend is equal to $\rho\theta$. The dispersion in the dipole as a function of the path length is then $\rho\left(1 - \cos\left(\frac{L}{\rho}\right)\right)$. The deviation for the decay product from a radioactive ion is $\rho\delta\left(1 - \cos\left(\frac{L}{\rho}\right)\right)$ (Figure 1).

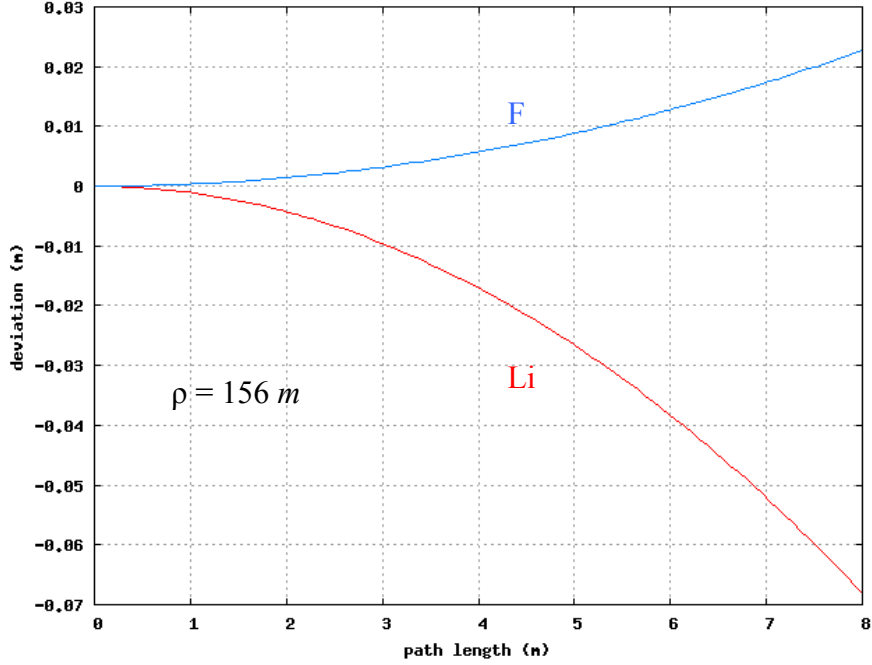


Figure 1 Deviation of the decay products vs bend length for the decay products $^6\text{Li}^{3+}$ and $^{18}\text{F}^{9+}$

The dipole has two degrees of freedom: its angle and its radius. We are going to fix the bend radius and calculate the bounds for the angle.

If we want to be sure that the accumulated beam does not hit the chamber, we must have $\rho|\delta|(1 - \cos(\theta)) + X_{beam} < X_{bend}$ with a half-aperture of X_{bend} for the dipole and a half beam size

of X_{beam} . The solution is $\theta_{max} = \arccos\left(1 - \frac{X_{bend} - X_{beam}}{\rho|\delta|}\right)$.

Moreover, the bend has to be long enough to avoid the depositions on the next magnetic elements. After a L_{SD} long drift, the deviation is $\rho|\delta|\left(1 - \cos(\theta) + \frac{L_{SD}}{\rho}\sin(\theta)\right)$. We must

have $\rho|\delta|\left(1 - \cos(\theta) + \frac{L_{SD}}{\rho}\sin(\theta)\right) - X_{beam} > X_{chamber}$ with a half-aperture of $X_{chamber}$ for the chamber.

The solution is

$$\theta_{min} = -\arccos\left(1/\sqrt{1 + \left(\frac{L_{SD}}{\rho}\right)^2}\right) + \arccos\left(\left(1 - \frac{X_{chamber} + X_{beam}}{\rho|\delta|}\right)/\sqrt{1 + \left(\frac{L_{SD}}{\rho}\right)^2}\right)$$

The beam size in the regular lattices is around ± 2 cm. We use half-apertures of 6 cm for the bends and 4 cm for the chamber. At $\gamma = 100$, the magnetic rigidity for $^6\text{He}^{2+}$ is 932 T.m. The bend radius is then 156 m for a 6 T field. We use 2 m long drifts. After calculation, if we consider $^6\text{He}^{2+}$, the maximum angle for the bend length is about $\text{Pi}/80$ (which corresponds to a

6.13 m length) and the minimum angle is around $\pi/89$ (which corresponds to a 5.5 m length). Therefore, the bend length must be between 5.5 m and 6.12 m to avoid the deposition in the bend and in the next magnet element (Figure 2).

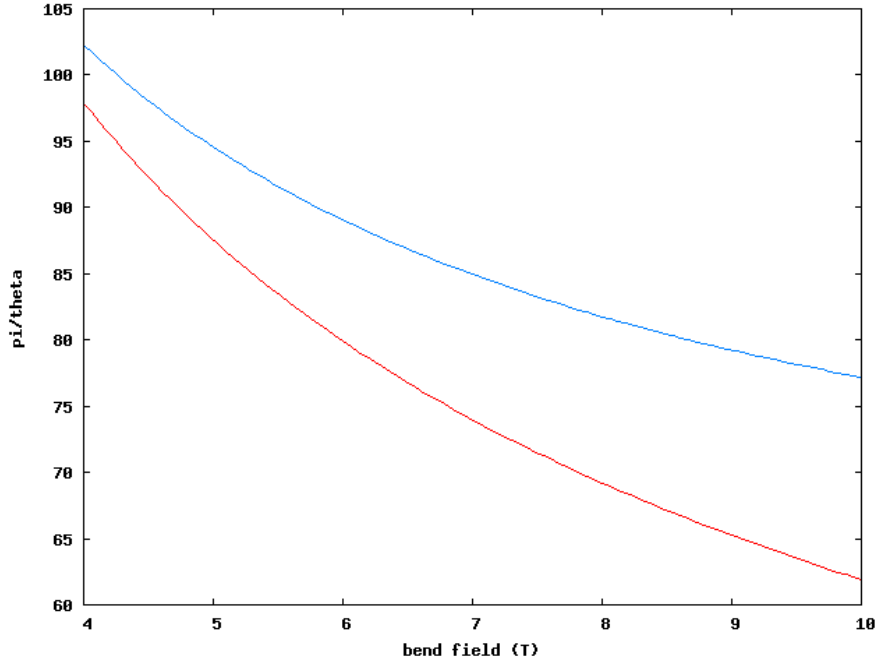


Figure 2 $\pi/L/\theta$ vs bend field to avoid peaks in the magnetic elements
In blue, maximum value. In red, minimum value.

To summarize, the chamber dimensions strongly determine the bounds for the bend parameters. Since the stored intensities for $^{18}\text{Ne}^{10+}$ are more than one order of magnitude lower than for $^6\text{He}^{2+}$, the deposition of its decay products is negligible. The bend length is then optimized for $^6\text{He}^{2+}$.

3. SIMULATION OF THE LOSSES IN THE RING

We have quantified the losses in the arcs at the first order to locate the deposition peaks and to know where we need to insert beam stoppers. The principle is described below.

We divide each element of the structure in small pieces, which gives NB elements. We initialize an array of which each entry corresponds to an element. For each element i (i going from 1 to NB), we make the structure begin there. Besides, we cut the envelope in N_{sigma} envelopes centered on the closed orbit (Figure 3). Each envelope j corresponds to a number n_j of standard deviations σ for the beam distribution f_σ . We transport the beam and we locate the elements Max_i^j and Min_i^j where the extremities of the envelope j (corresponding to $n_j\sigma$) hit the chamber. Since the power of the particles comprised between the beam envelopes at $n_1\sigma$

and $n_2\sigma$ is given by the formula $P'L_i \int_{n_1\sigma}^{n_2\sigma} f_\sigma(x)dx$ where P' is the power lost per meter, L_i the

length of the element i , we are able to calculate the power comprised between the extremities of two following envelopes. We assume that the deposition is uniform between them. Thus, to have a power lost per meter, we have to divide by the length of the deposition area. Besides, we add each one of these contributions in the array to have the total deposited power in each element.

We have to precise that our simulation does not take into account the contributions due to multipolar effects (sextupolar , octupolar effects...). For example, the sextupolar kick is not visible in our simulation. It is necessary to use another code to study the blow-up. Recently, we are in collaboration with Dr Jones from TRIUMF to better locate the losses in the ring thanks to its code ACCSIM [3] which realizes quite good tracking simulations. It takes into account the interactions with the matter (possibilities to use FLUKA or GEANT4) and the sextupolar kicks.

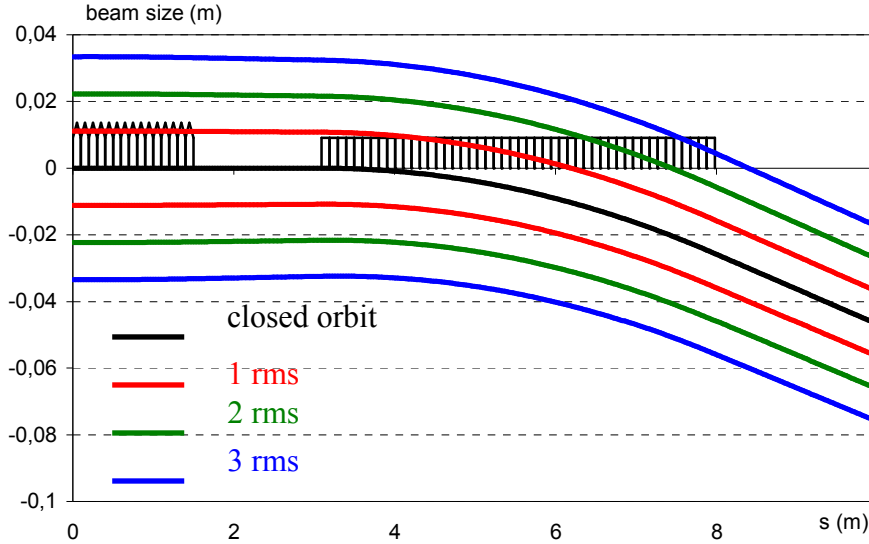


Figure 3. Beam size of the ions after decaying in the first element for different numbers of rms

4. RESULTS

The number of ions stored in the decay ring as a function of time is:

$$N_{stored}(t) = N_0 2^{\frac{-t}{\gamma\tau_{1/2}}}$$

with N_0 the stored number of ions just after the merging
 γ the relativistic factor
 $\tau_{1/2}$ the half-time

The kinetic energy of the stored beam is: $T_{stored}(t) = (\gamma - 1)E_0 I_0 2^{\frac{-t}{\gamma\tau_{1/2}}}$ with E_0 the rest energy of an ion

The lost power by decay is then:
$$P_{stored}(t) = \frac{\gamma - 1}{\gamma} \frac{E_0 N_0 \text{Ln}(2)}{\tau_{1/2}} 2^{\frac{-t}{\gamma\tau_{1/2}}}$$

The maximum lost power by decay is:
$$P_{max} = \frac{\gamma - 1}{\gamma} \frac{E_0 N_0 \text{Ln}(2)}{\tau_{1/2}}$$

For the current values,

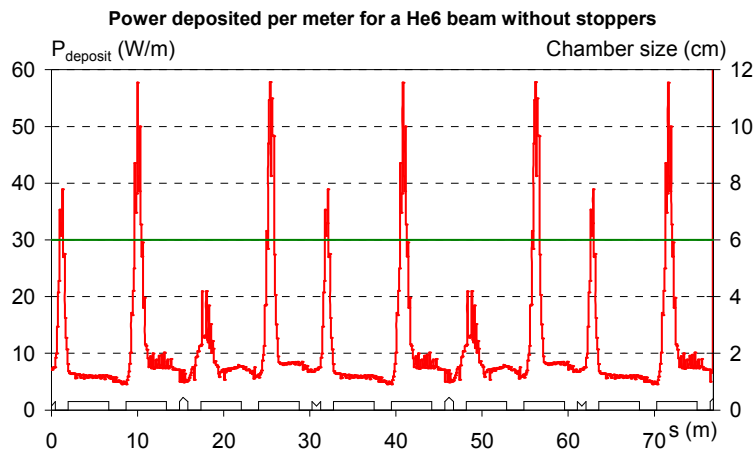
	⁶ He	¹⁸ Ne
gamma	100	100
rigidity (T.m)	932	559
E ₀ of an ion at rest (GeV)	5.67	17
Stored intensity	9.71 10 ¹³	3.11 10 ¹²
Stored beam energy (kJ)	8800	846
Lost power (stored beam) (kW)	75.5	3.5
Lost power (stored beam) (W/m)	10.8	0.5

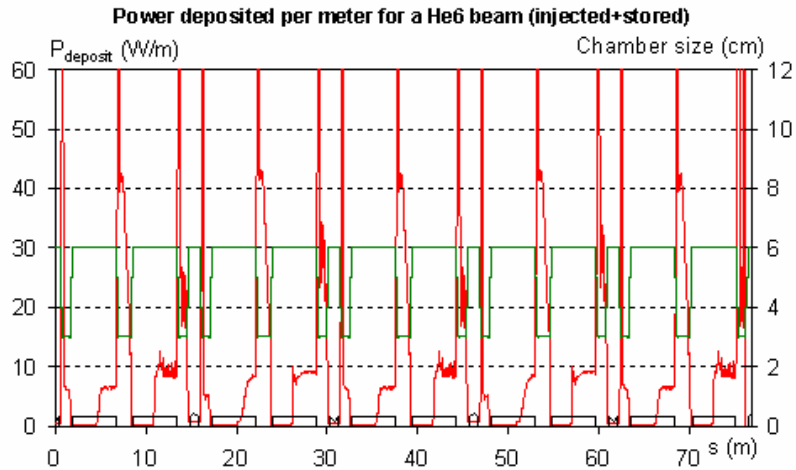
The lost power by ¹⁸Ne¹⁰⁺ is one order of magnitude lower than by ⁶He²⁺, so we can neglect it. If ¹⁹Ne¹⁰⁺ is chosen later instead of ¹⁸Ne¹⁰⁺, the design will have to take into account the two species. We assume that the transverse distribution is Gaussian. If we do the simulation for the regular FODO lattice in the ring of which the parameters are in the Table 1, we obtain the Figure 4:

Table 1 Parameters of the magnetic elements for the regular FODO lattice in the ring

QUADRUPOLES		
function	length (m)	strength (m ⁻²)
focusing quadripole	1	0.0765
defocusing quadripole	1	-0.068
BEND		
bend angle (rad)	bend radius (m)	
Pi/100	156	

The emittance value that we used was $9 \pi \text{ mm.mrad}$ for 3σ [4].





**Figure 4 Losses in the regular FODO lattice for 6He^{2+}
In red, losses in the regular FODO lattice. In green chamber size**

The Figure 4 proves that it is possible to limit the losses in the magnetic elements by putting beam stoppers in the structure. It is important to notify that the interactions with the matter are not calculated. The decay products are supposed to be absorbed by the stopper.

5. CONCLUSION

The amount of losses is so high that we have to optimize the design to enable the insertion of beam stoppers in the ring. For the moment, we consider only ${}^6\text{He}^{2+}$ because the ${}^{18}\text{Ne}^{10+}$ intensity is one order of magnitude lower. It is not the case if the ${}^{19}\text{Ne}^{10+}$ is chosen instead. In this case, the design will have to be optimized for the deposition of the both species. In order to evaluate our structure, we have written a code which quantifies the depositions in the ring. At the moment, it does not take into account the sextupolar kicks and the interaction with the matter. However, it is quite fast and locates the deposition peaks. We are looking forward to working with ACCSIM [3] to compare with our results.

6. REFERENCES

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- [3]. ACCSIM code, <http://www.triumf.ca/compserv/accsim.html>
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