Measurements of the CKM angle γ using ADS, GLW and other methods

Marco Zito [a]

[a]Dapnia-SPP, CEA-Saclay,
91191 Gif-sur-Yvette Cedex, France

On behalf of the *BABAR* and Belle collaborations, we report on analyses sensitive to the angle γ and the sum of angles $2\beta+\gamma$ of the Unitarity Triangle.

1. Introduction

The angle γ (or ϕ_3) of the unitarity triangle is related to the phase of the CKM matrix element V_{ub} through $V_{ub} = |V_{ub}|e^{-i\gamma}$. We report on two classes of measurements: time independent measurements in decays $B^\pm \to D^0/\overline{D}^0 K^\pm$ sensitive to the angle γ; time dependent asymmetries in decays $B^0 \to D^{(*)\pm}\pi^\mp$ or $B^0 \to D^0 K^0$ sensitive to the sum of angles $2\beta+\gamma$.

2. Measurement of γ in $B^\pm \to D^0/\overline{D}^0 K^\pm$

The measurement of γ in charged B decays exploits the interference between $B^- \to D^{(*)0}K^{(*)-}$ and $B^- \to \overline{D}^{(*)0}K^{(*)-}$ (Fig. 1) that occurs when the $D^{(*)0}$ and the $\overline{D}^{(*)0}$ decay to common final states.

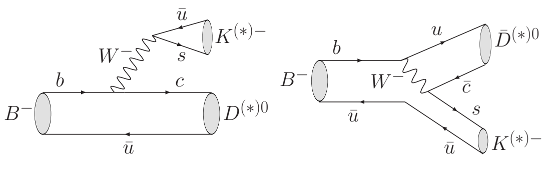

Figure 1. Feynman diagrams for $B^- \to D^{(*)0}K^{(*)-}$ and $\overline{D}^{(*)0}K^{(*)-}$.

Two methods are presented here. In the GLW method [1] the D^0 and the $\overline{D}^0$ decay to a CP eigenstate. In the ADS method [2] the D^0 from the favored $b \to c$ amplitude is reconstructed in the doubly-Cabbibo suppressed final state $K^+\pi^-$, while the $\overline{D}^0$ from the $b \to u$ suppressed amplitude is reconstructed in the favored final state $K^+\pi^-$. The experimental observables depend on two additional parameters: the magnitude r_B of the ratio of the amplitudes for the processes $B^- \to \overline{D}^0 K^-$ and $B^- \to D^0 K^-$ (Fig. 1) and the relative strong phase δ_B between these two amplitudes.

2.1. The GLW method and results

The results of the GLW analyses are expressed in terms of the ratios of partial rates $R_{CP\pm} = 2(\Gamma^-_{CP\pm} + \Gamma^+_{CP\pm})/(\Gamma^+ + \Gamma^-)$ and of the partial-rate charge asymmetries $A_{CP\pm} = (\Gamma^-_{CP\pm} - \Gamma^+_{CP\pm})/(\Gamma^-_{CP\pm} + \Gamma^+_{CP\pm})$, where $\Gamma^-_{CP\pm} = \Gamma(B^- \to D^0_{CP\pm}K^-)$, $\Gamma^+_{CP\pm} = \Gamma(B^+ \to \overline{D}^0_{CP\pm}K^+)$ and $\Gamma^\pm = \Gamma(B^\pm \to D^0 K^\pm)$

CP+ refers to the CP-even final states $\pi^+\pi^-$ and K^+K^- and *CP*– refers to the CP-odd final states $K^0_S\pi^0$, $K^0_S\phi$ and $K^0_S\omega$. $R_{CP\pm}$ and $A_{CP\pm}$ are related to the angle γ, the amplitude ratio r_B and the strong phase difference δ_B through the relations $R_{CP\pm} = 1 + r_B^2 \pm 2r_B\cos\delta_B\cos\gamma$ and $A_{CP\pm} = \pm 2r_B\sin\delta_B\sin\gamma/R_{CP\pm}$ [1], thus allowing a determination of the 3 unknowns (r_B, δ_B and γ) up to an 8 fold ambiguity in γ.

BABAR and Belle have published recently results, summarized in Table 1, using statistical samples larger than 200 million $B\overline{B}$ events [3–6]. The averages have been computed by the HFAG group [7]. Due to the limited statistics, the GLW method alone is not able to provide strong constraints on γ.

Table 1
Summary of *BABAR* and Belle measurements of the GLW observables R_{CP} and A_{CP}.

Mode/Exp	A_{CP+}	A_{CP-}	R_{CP+}	R_{CP-}
$B \to D^0 K$/*BABAR*	$+0.35 \pm 0.13 \pm 0.04$	$-0.06 \pm 0.13 \pm 0.04$	$0.90 \pm 0.12 \pm 0.04$	$0.86 \pm 0.10 \pm 0.05$
$B \to D^0 K$/Belle	$+0.06 \pm 0.14 \pm 0.05$	$-0.12 \pm 0.14 \pm 0.05$	$1.13 \pm 0.16 \pm 0.08$	$1.17 \pm 0.14 \pm 0.14$
Average	$+0.22 \pm 0.10$	-0.09 ± 0.10	0.90 ± 0.10	0.94 ± 0.10
$B \to D^{*0} K$/*BABAR*	$-0.10 \pm 0.23^{+0.03}_{-0.04}$	-	$1.06 \pm 0.26^{+0.10}_{-0.09}$	-
$B \to D^{*0} K$/Belle	$-0.20 \pm 0.22 \pm 0.04$	$+0.13 \pm 0.30 \pm 0.08$	$1.41 \pm 0.25 \pm 0.06$	$1.15 \pm 0.31 \pm 0.12$
average	-0.15 ± 0.16	$+0.13 \pm 0.31$	1.25 ± 0.19	1.15 ± 0.33
$B \to D^0 K^*$/*BABAR*	$-0.08 \pm 0.19 \pm 0.08$	$-0.26 \pm 0.40 \pm 0.12$	$1.96 \pm 0.40 \pm 0.11$	$0.65 \pm 0.26 \pm 0.08$

Table 2
Summary of *BABAR* and Belle ADS measurements.

Mode/Exp	R_{ADS}	R_{ADS} 90% C.L.limit	r_B	r_B 90% C.L.limit
$B \to D^0 K$/*BABAR*	$0.013^{+0.011}_{-0.009}$	< 0.029		$r_B < 0.23$
$B \to D^0 K$/Belle	$0.000 \pm 0.008 \pm 0.001$	< 0.0139		$r_B < 0.18$
$B \to D^{*0}_{(D^0\pi^0)} K$/*BABAR*	$-0.002^{+0.010}_{-0.006}$	< 0.023		$r_B^{*2} < (0.16)^2$
$B \to D^{*0}_{(D^0\gamma)} K$/*BABAR*	$0.011^{+0.018}_{-0.013}$	< 0.045		
$B \to D^0 K^*$/*BABAR*	$0.046 \pm 0.031 \pm 0.08$		$r_B = 0.28^{+0.006}_{-0.010}$	

2.2. ADS Results

In the ADS method the decays $D^0 \to K^+ \pi^-$ and $\overline{D}^0 \to K^+ \pi^-$ are used. The overall effective branching ratio for the final state $B^- \to [K^+\pi^-]_{D^0} K^-$ is expected to be small ($\sim 10^{-7}$), but the two interfering diagrams are of the same order of magnitude and large asymmetries are therefore expected. The favored decay mode $B^- \to [K^-\pi^+]_{D^0} K^-$ is used to normalize the measurement and cancel many experimental systematics. The main experimental observables are the ratio R_{ADS} of the suppressed to favored modes and the B^-/ B^+ asymmetry:

$$\begin{aligned} R_{ADS} &= \frac{\mathcal{B}([K^+\pi^-]_{D^0}K^-) + \mathcal{B}([K^-\pi^+]_{D^0}K^+)}{\mathcal{B}([K^-\pi^+]_{D^0}K^-) + \mathcal{B}([K^+\pi^-]_{D^0}K^+)} \\ &= r_D^2 + 2 r_D r_B \cos\gamma \cos(\delta_B + \delta_D) + r_B^2 \quad (1) \\ A_{ADS} &= \frac{\mathcal{B}([K^+\pi^-]_{D^0}K^-) - \mathcal{B}([K^-\pi^+]_{D^0}K^+)}{\mathcal{B}([K^+\pi^-]_{D^0}K^-) + \mathcal{B}([K^-\pi^+]_{D^0}K^+)} \\ &= 2 r_D r_B \sin\gamma \sin(\delta_B + \delta_D)/R_{ADS}, \quad (2) \end{aligned}$$

where $r_B = |A(B^- \to \overline{D}^0 K^-)/A(B^- \to D^0 K^-)|$ and $r_D = |A(D^0 \to K^+\pi^-)/A(D^0 \to K^-\pi^+)| = 0.060 \pm 0.003$[8] are the suppressed to favored B and D amplitude ratios, and δ_B and δ_D are the strong phase differences between the two B and D decay amplitudes, respectively. R_{ADS} is highly sensitive to r_B^2. A summary of the *BABAR* and Belle ADS results can be found in Table 2, and more details on the analysis in Ref.[9–11]. For the D^0K and $D^{*0}K$ channels limits on R_{ADS} and r_B are extracted. *BABAR* has recently presented an ADS analysis based on the the three-body decay mode $D^0 \to K^+\pi^-\pi^0$. Some complications arise due to the variations of the amplitude and phase over the Dalitz phase space. No statistically significant signal is observed in the suppressed mode and the 95% CL upper limits $R_{ADS} < 0.039$ and $r_B < 0.185$ are set [12].

Similar to the GLW analysis, more statistics is needed to constraint γ from the ADS method.

3. $\sin(2\beta + \gamma)$ measurements

Time-dependent asymmetries in $B^0 \to D^{(*)}\pi$, $D^{(*)}\rho$ and $B^0 \to D^{(*)0}K^0$ can be used to constrain $\sin(2\beta + \gamma)$[13]. As β is well known from $b \to c\bar{c}s$, a constraint on the angle γ follows. The $B^0 \to D^{(*)}\pi$ method uses an interference between the usual Cabibbo-favored $b \to c$ channel and the doubly-Cabibbo suppressed $b \to u$ channel (Fig.2). These two amplitudes have a relative weak phase of γ, and a weak phase of 2β is provided by the $B^0\overline{B}^0$ mixing. These modes have the advantage of a "large" ($\sim 0.5\%$) branching fraction but the price to pay is the small ratio r of the suppressed to favored amplitudes,

$$r = \left| \frac{A(B^0 \to D^{(*)+}h^-}{A(\overline{B}^0 \to D^{(*)+}h^-} \right| \propto \lambda^2 (\sim 0.02).$$

This results in small CP-asymmetries. The ratio r cannot be measured directly, but can be estimated from the recent *BABAR* measurement [14] of the branching fractions

$$\begin{aligned} \mathcal{B}(B^0 \to D_s^+ \pi^-) &= (1.3 \pm 0.3 \pm 0.2) \times 10^{-5} \\ \mathcal{B}(B^0 \to D_s^{*+} \pi^-) &= (2.8 \pm 0.6 \pm 0.5) \times 10^{-5} \end{aligned}$$

using theoretical assumptions to obtain:

$$\begin{aligned} r_{D\pi} &= (1.3 \pm 0.2(\text{stat}) \pm 0.1(\text{syst})) \times 10^{-2} \\ r_{D^*\pi} &= (1.9 \pm 0.2(\text{stat}) \pm 0.2(\text{syst})) \times 10^{-2}. \end{aligned}$$

For $B^0 \to D\rho$, r is estimated in a similar way from a measurement of $B^0 \to D_s^+ \rho^-$ [15], yielding $r_{D\rho} = (0.3 \pm 0.6(\text{stat}) \pm 0.1(\text{syst})) \times 10^{-2}$.

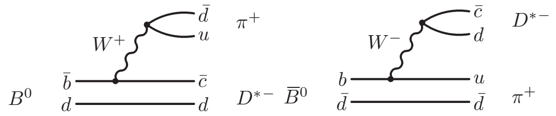

Figure 2. Feynman diagrams for the Cabibbo-favored decay $B^0 \to D^{*-}\pi^+$ (left) and the Cabibbo-suppressed decay $\overline{B}^0 \to D^{*-}\pi^+$ (right).

The experimental observables are the coefficients $S^\pm$ and C of the $\sin(\Delta m \Delta t)$ and $\cos(\Delta m \Delta t)$ terms in the time dependent asymmetries of $B^0(\bar{B}^0) \to D^{(*)\pm}\pi^\mp$ (or $D^{(*)\pm}\rho^\mp$). For small values of r, the parameter $S^\pm$ is given by $S^\pm \simeq 2r\sin(2\beta + \gamma \pm \delta)$, where δ is the strong phase difference between the $b \to u$ and $b \to c$ decay amplitudes. The parameters $S^\pm$ can be rewritten in terms of $a = 2r\sin(2\beta+\gamma)\cos(\delta)$ and $c^{lept} = \cos(2\beta + \gamma)[2r\sin(\delta)]$ for lepton-tagged events. In the case of kaon tags, CP violation effects on the tag side pollute the measurement of the c parameters.

BABAR and Belle use two experimental methods for reconstructing $B^0(\overline{B}^0) \to D^{(*)}\pi$ and $D^{(*)}\rho$ decays. They perform either exclusive reconstruction, where the hadronic decay modes with $D^0 \to K^-\pi^+$, $K^-\pi^+\pi^0$ and $K^-\pi^+\pi^-\pi^+$ are fully reconstructed, or partial reconstruction of $D^{*\pm}\pi^\mp$, where only the slow π from $D^* \to D^0\pi$ is reconstructed. *BABAR* has published results based on a statistics of 232×10^6 $B\overline{B}$ events [16,17] while Belle has released a new result based on an integrated luminosity of $357 fb^{-1}$, corresponding to approximately 386×10^6 $B\overline{B}$ events [18]. These results are reported in Fig.3.

BABAR sets the lower limits $|\sin(2\beta + \gamma)| > 0.64$ (0.40) at 68% (90%) C.L. The constraints [19] in the ρ-η plane are shown in Fig. 4.

4. Search for $\mathbf{B}^0 \to D_s^{(*)+} a_{0(2)}^-$

It was recently suggested to use the decays $B^0 \to D^{(*)+} a_{0(2)}^-$ for measuring $\sin(2\beta + \gamma)$ [20]. These decay can proceed through two diagrams similar to those of Fig.2 and it is expected that the V_{cb} amplitude is significantly suppressed respective to the V_{ub} amplitude, giving significant CP-asymmetries.

The V_{ub}-mediated part of the $B^0 \to D^{(*)+} a_{0(2)}^-$ decay amplitude can be related to $B^0 \to D_s^{(*)+} a_{0(2)}^-$ using $\tan(\theta_{\text{Cabibbo}}) = |V_{cd}/V_{cs}|$ and the ratio of the decay constants $f_{D_s^{(*)}}/f_{D^{(*)}}$.

BABAR finds no evidence for these decays and set upper limits at 90% C.L. on the branching fractions [21]: $\mathcal{B}(B^0 \to D_s^+ a_0^-) < 1.9 \times 10^{-5}$, $\mathcal{B}(B^0 \to D_s^{*+} a_0^-) < 3.6 \times 10^{-5}$, $\mathcal{B}(B^0 \to D_s^+ a_2^-) < 1.9 \times 10^{-4}$, and $\mathcal{B}(B^0 \to D_s^{*+} a_2^-) < 2.0 \times 10^{-4}$. These upper limits suggest that the branching ratios of $B^0 \to D^{(*)+} a_{0(2)}^-$ are too small for CP-asymmetry measurements given the present statistics of the B-factories.

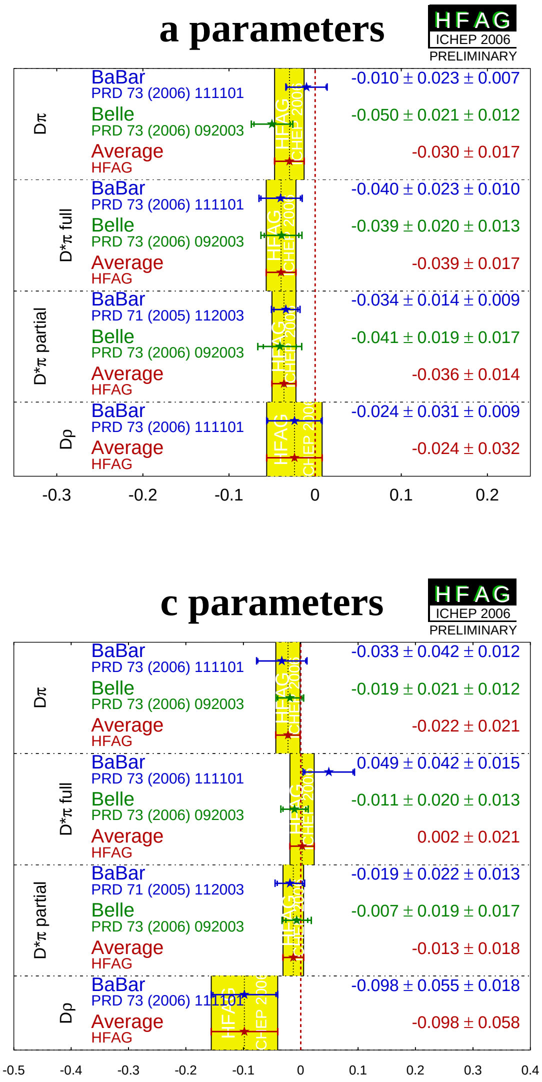

Figure 3. *BABAR* and Belle $\sin(2\beta+\gamma)$ results and HFAG averages [7].

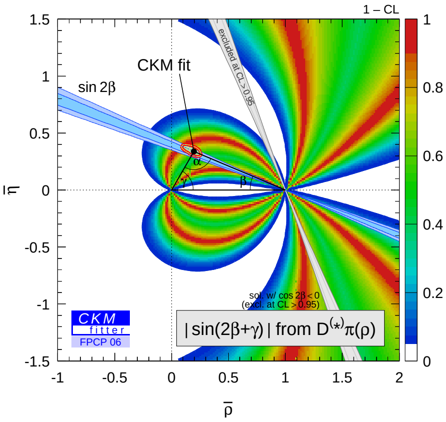

Figure 4. Constraints in the ρ-η plane from $\sin(2\beta+\gamma)$ measurements [19]. The white area is excluded at the 95% CL.

5. Study of $\mathbf{B}^0 \to D^{(*)0}K^{(*)0}$

In the decay modes $\overline{B}^0 \to D^{(*)0}\overline{K}^0$ the *CP* asymmetry appears as a result of the interference between two diagrams leading to the same final state $D^{(*)0}K^0_S$.

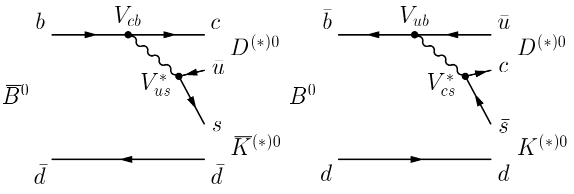

Figure 5. The decay diagrams for the $b \to c$ transition $\overline{B}^0 \to D^{(*)0}\overline{K}^0$ and the $\bar{b} \to \bar{u}$ transition $B^0 \to D^{(*)0}K^0$.

A direct determination of the relevant ratio of decay amplitudes $r_B^{(*)}$ is not possible but insight into the B decay dynamics affecting $r_B^{(*)}$ can be gained by measuring a similar amplitude ratio $\tilde{r}_B \equiv |\mathcal{A}(\overline{B}^0 \to \overline{D}^0\overline{K}^{*0})/\mathcal{A}(\overline{B}^0 \to D^0\overline{K}^{*0})|$ using the self-tagging decay $\overline{K}^{*0} \to K^-\pi^+$.

BABAR has recently published a new measurement of the $B^0 \to D^{(*)0}K^0_S$, $\overline{B}^0 \to D^0\bar{K}^{*0}$ and $\overline{B}^0 \to \bar{D}^0\bar{K}^{*0}$ branching fractions, based on a data sample of 226×10^6 $B\overline{B}$ events [22]. Defining $\mathcal{B}(\tilde{B}^0 \to D^{*0}\tilde{K}^0) \equiv (\mathcal{B}(\overline{B}^0 \to D^{*0}\overline{K}^0) + \mathcal{B}(B^0 \to D^{*0}K^0))/2$ and $\mathcal{B}(\tilde{B}^0 \to D^0\tilde{K}^0) \equiv (\mathcal{B}(\overline{B}^0 \to D^0\overline{K}^0) + \mathcal{B}(B^0 \to D^0K^0))/2$, the results of this measurement are:

$$\begin{aligned}
\mathcal{B}(\tilde{B}^0 \to D^0\tilde{K}^0) &= (5.3 \pm 0.7 \pm 0.3) \times 10^{-5} \\
\mathcal{B}(\tilde{B}^0 \to D^{*0}\tilde{K}^0) &= (3.6 \pm 1.2 \pm 0.3) \times 10^{-5} \\
\mathcal{B}(\overline{B}^0 \to D^0\overline{K}^{*0}) &= (4.0 \pm 0.7 \pm 0.3) \times 10^{-5} \\
\mathcal{B}(\overline{B}^0 \to \overline{D}^0\overline{K}^{*0}) &< 1.1 \times 10^{-5} \text{ at 90\% C.L.}
\end{aligned}$$

From the absence of signal for the V_{ub} mediated mode $\overline{B}^0 \to \overline{D}^0\overline{K}^{*0}$, the limit $\tilde{r}_B < 0.40$ at 90% C.L. is obtained suggesting that a substantially larger data sample is needed for a competitive measurement of $\sin(2\beta+\gamma)$ in these decays.

6. Conclusion and prospects

The angle γ/ϕ_3 is the most difficult to measure of the Unitarity Triangle angles at the B-factories. Very promising progress has been made in constraining it over the past few years. All the measurements presented here are statistically limited and will improve with the increase of statistics. Unfortunately, perspectival studies at the 1 ab^{-1} horizon indicate that the GLW and ADS analyses will not be competitive with Dalitz (GGSZ) methods. On the other hand $\sin(2\beta+\gamma)$ measurements using $B^0 \to D^{(*)}\pi$ provide already interesting constraints in the ρ-η plane and deserve to be pursued.

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