Angular momentum and energy spread measurements by backscattering technique.

(Wire + ESA)

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1. Introduction

A main interest in the design of a high-intensity particle beam accelerator as the EURISOL driver is the control of the particle losses in the vacuum chamber. These losses, even concerning an extremely low fraction of the beam (10^{-4}-10^{-7}), can be sufficient to considerably complicate the maintenance of such an accelerator. Within this framework and in order to contribute to accelerator projects dedicated to rare isotope physics, the CEA is undertaking a research program on the theoretical and experimental study of the physical processes involved in halo formation around a high intensity beam in a particle accelerator. This research program is performed in collaboration with several French and international laboratories.

This note details the principle and the design of an innovative emittance measurement unit which aims to be "weakly" interceptive. "Weakly" means that the beam can continue to propagate in the pipe with similar properties compared to the case when the diagnostic is not inserted. It is planned to compare these emittance measurements to the predictions of the home-made codes which are used to simulate the EURISOL medium and high energy sections.

2. Principle of the device

In a nonrelativistic case, the energy of a backscattered proton on a target is given by the following equations (Fig.1):

$$E_p = \frac{m_p^2 + M_t^2 - 2m_p M_t \cos(2\psi)}{(m_p + M_t)^2} \times E_p^0;$$

and the backscattered angle θ is:

$$tg\theta = \frac{\sin(2\psi)}{{m_p}/{M_t} - \cos(2\psi)}.$$

where E_p^0 is the initial proton energy, E_p is the energy of scattered proton, m_p is the proton mass, M_t is the target nuclear mass, ψ is the backscattered angle of target nuclear.

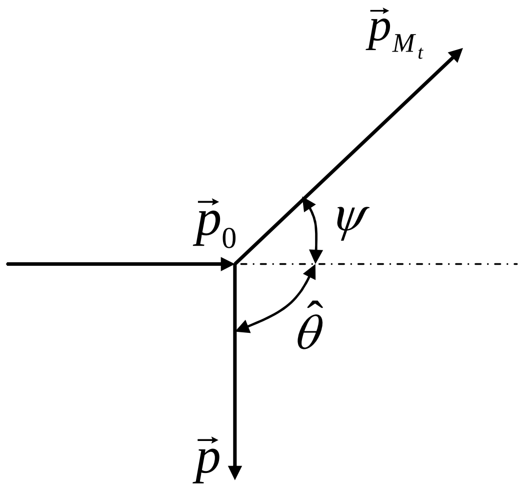

Fig.1: Scheme of a proton which is backscattered on a target.

And for a proton spectral measurement at angle $\hat{\theta} = 90°$, the equations become:

$$m_p / M_t = \cos(2\psi)\,; \qquad E_p = \frac{M_t^2 - m_p^2}{(m_p + M_t)^2} \times E_p^0 \quad \text{and} \quad d\theta = 2d\psi\,.$$

Typical proton spectrums with a nonzero energy and angular dispersions including energy losses are shown by the Fig.2 and the Fig.3.

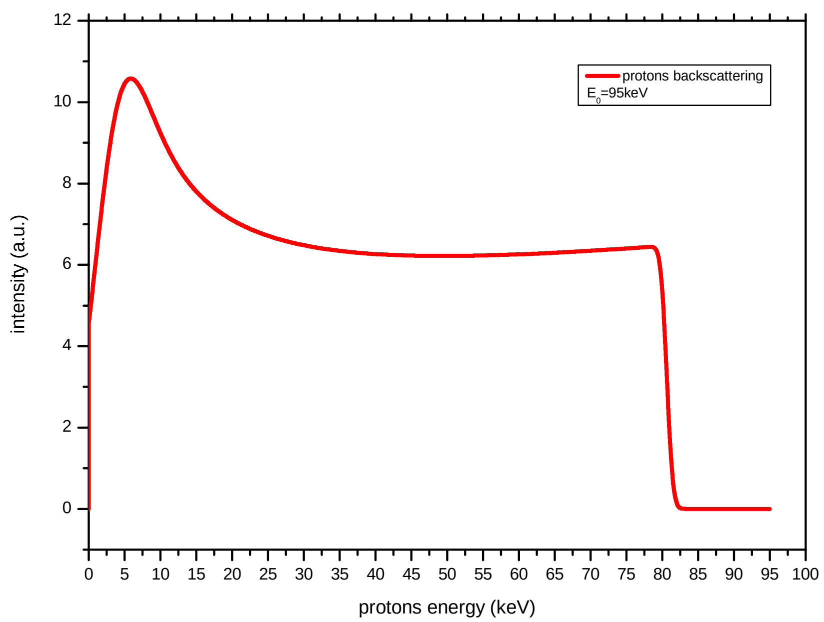

Fig.2 Backscattered protons spectrum (E_p^0=95keV, M_t = 12 (^{12}C)).

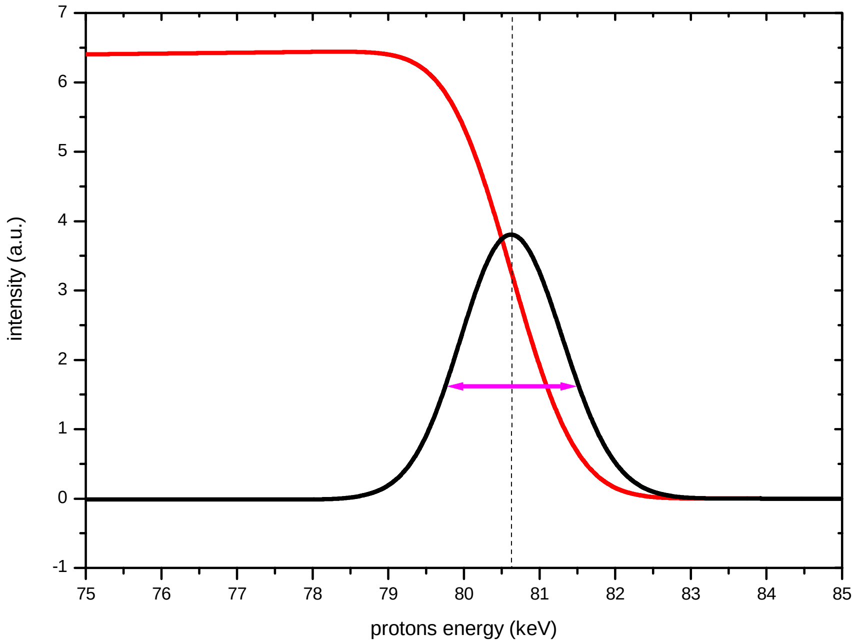

Fig.3 Spectral distribution of the initial proton beam.

The differential of spectrum front (Fig.3) gives a proton distribution which depends on the energy and the angular dispersions of the initial proton beam:

$$\sigma_{E_p}^2 = \left[\frac{2m_p \sqrt{M_t^2 - m_p^2}}{(m_p + M_t)^2} \times E_p^0 \right]^2 \times \sigma_\theta^2 + \left[\frac{M_t^2 - m_p^2}{(m_p + M_t)^2} \right]^2 \times \sigma_{E_p^0}^2 ;$$

with σ_θ^2 is the dispersion of momentum angular spread, $\sigma_{E_p^0}^2$ is the energy dispersion of initial proton beam. The measured distributions differ for different masses of the target nucleus (see Fig.4) due to the coefficient:

$$\boldsymbol{K} = \left[\frac{2m_p \sqrt{M_t^2 - m_p^2}}{(m_p + M_t)^2} \times E_p^0 \right]$$

For instance at $E_p^0 = 95 keV$, we have:

Target nature	K (eV/mrad)
^{9}Be	17.0
^{12}C	13.4
^{48}Ti	3.8
^{64}Cu	2.9
^{184}W	1.0

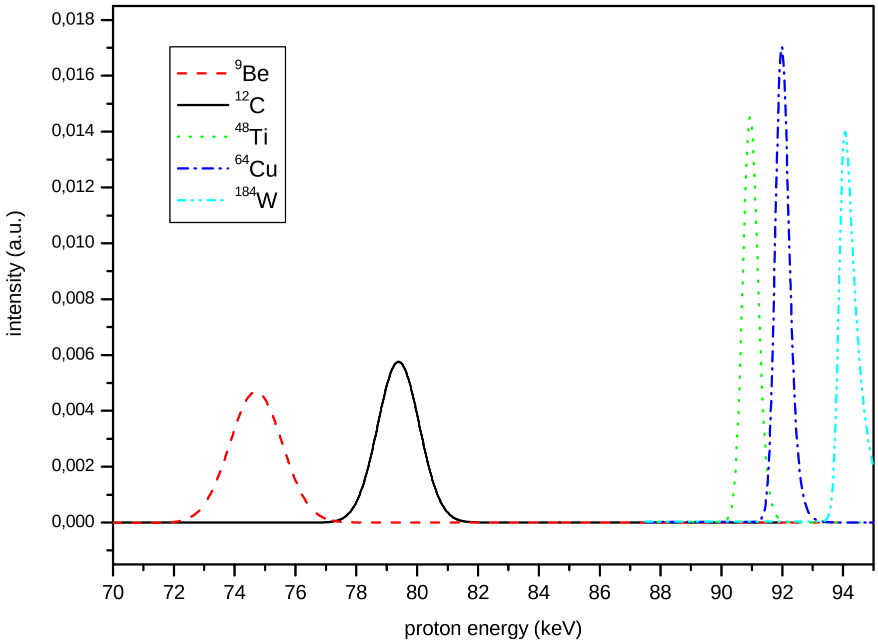

Fig.4 Distributions for various target nuclear masses.

The experimental realization of the technique depends on the error bar. For our case, this value can be calculated with the following formula:

$$d^2E_p = \left[\frac{2m_p\sqrt{M_t^2 - m_p^2}}{(m_p + M_t)^2} \times E_p^0\right]^2 \times d^2\theta + \left[\frac{M_t^2 - m_p^2}{(m_p + M_t)^2}\right]^2 \times d^2E_p^0 \approx \underline{\underline{(2\frac{m_p}{M_t}E_p^0)^2 \times d^2\theta + d^2E_p^0}},$$

or:

$$\delta_{\exp}^2 \approx (2\frac{m_p}{M_t})^2 \times d^2\theta + \delta_0^2,$$

with $\delta_0 = \frac{dE_p^0}{E_p^0}$ is the resolution of the beam energy, $d\theta$ is the angular resolution of the transverse momentum spread.

The $\delta_{\exp} \approx 10^{-4}$ ($d\theta \approx 1$ mrad and $dE_p^0 \approx 10$ eV) is a good value for measuring the diagnostic system with an electrostatic analyzer (ESA). In this case, the ^{12}C wire can be used for a momentum spread measurement ($\delta_{\exp} \approx 2\frac{m_p}{M_t} \times d\theta$) and a ^{184}W wire can be used for a beam energy stability measurement ($\delta_{\exp} \approx \delta_0$) (see Fig.5).

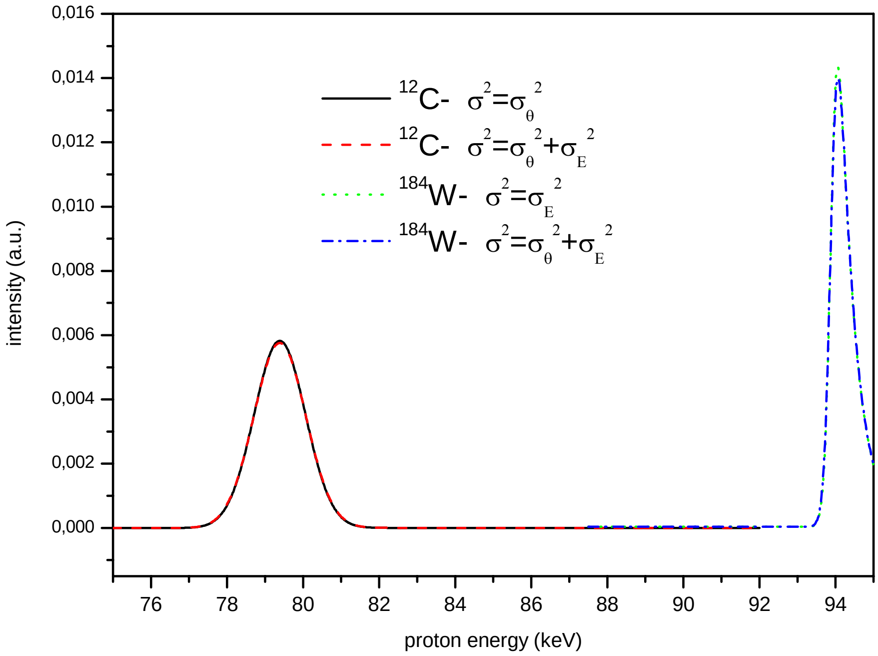

Fig.5 Distributions for ^{12}C and ^{148}W wires at $\boldsymbol{\sigma_\theta}$ = 50mrad and $\boldsymbol{\sigma_E}$ = 100eV.

3. Experimental set-up.

Technically, the experimental set up measurement would use the classical scheme of a wire-scanner including an ESA. The wire-target would move through the beam cross-section at $\hat{\theta} = 90°$ (Fig.6).
The parameter L_0 equal to 500 mm (it depends on the real construction of the beam line) is fixed for the calculations of the main ESA and the wire-target parameters for a hypothesis of an angular resolution of ≈1mrad and a maximum efficiency. The main ESA and wire target parameters are in the following tables.

Table 1: Main ESA parameters.

Main ESA parameters	Short names
Deflected angle	$\boldsymbol{\varphi_{ESA}}$
ESA radius	$\mathbf{r_{ESA}}$
Width of slits	$\boldsymbol{\Delta h_{in}}$ and $\boldsymbol{\Delta h_{out}}$
Height of slits	$\boldsymbol{\Delta l_{in}}$ and $\boldsymbol{\Delta l_{out}}$
Distances between electrode edges and slits	$\mathbf{l_{in}}$ and $\mathbf{l_{out}}$

Table 2: Main wire target parameters.

Wire-target parameters	Short names
Diameter of wire	$\mathbf{d_0}$
Effective size of wire	$\boldsymbol{\Delta l_{eff}}$

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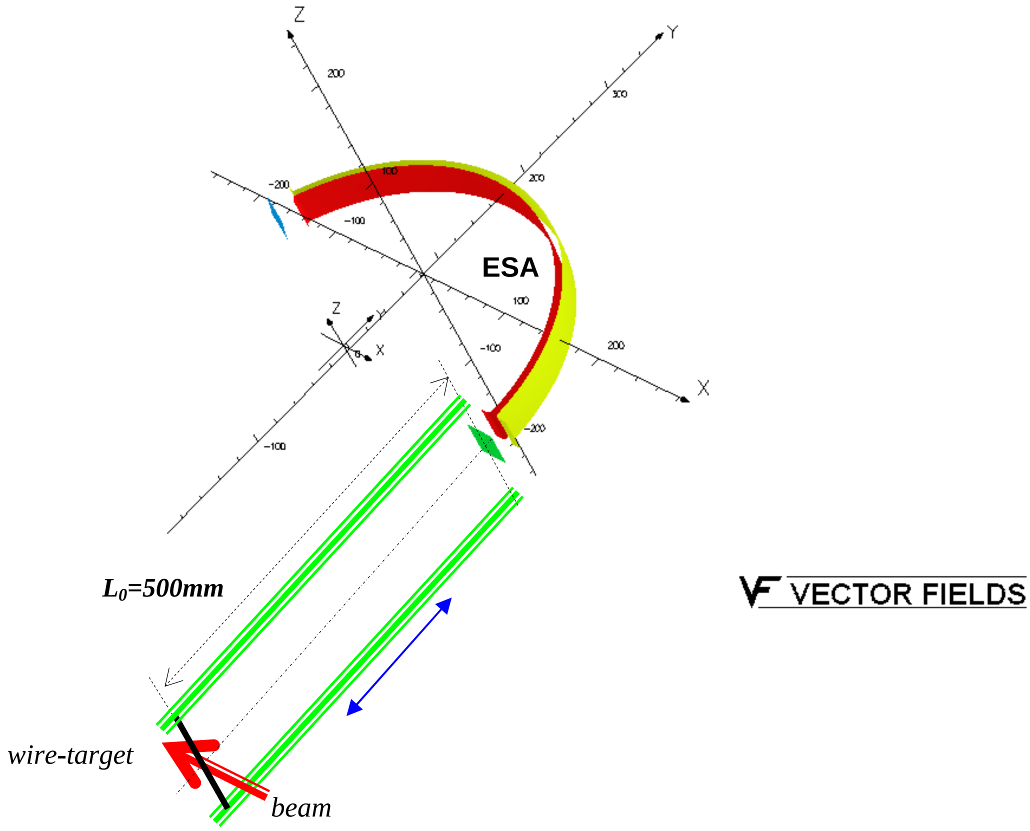

Fig.6 Experimental set-up

4. Calculation of ESA parameters.

In this section, the calculations of the ESA and wire-target main parameters based on the trajectory calculations of charge particles which move in a central electrostatic field $U= -\alpha/r$ will be performed. The electrical potential in this system is inversely proportional to the trajectory radius [L.D. Landau, E.M. Lifshitz, Vol.**1,** Mechanics]. For a starting point with coordinates (r_0,φ_0) in respect to the "potential center", we found:

$$\arccos\frac{M/r + m\alpha/M}{\sqrt{2mE + m^2\alpha^2/M}} = \varphi + const\,,$$

where $E = \frac{mv^2}{2} - \frac{\alpha}{r_0}$ is the total energy of the charged particle, $M = mr_0 v\cos(d\theta)$ is the initial angular momentum of the particle and $d\theta$ is the initial angle deviation of velocity vector relative to the tangent of the particle trajectory. It can be reduced to a more simple view:

$$\cos(\varphi+\varphi_0) = \frac{\cos(d\theta)r_0/r - a(v)/\cos(d\theta)}{\sqrt{[1-a(v)]^2 + a(v)^2 tg^2(d\theta)}}\,;$$

For a starting point ($r_0, \varphi_0 = 0$) and $d\theta = 0$ we find:

$$r = \frac{r_0}{a(v) + [1 - a(v)]\cos(\varphi)}$$

The parameter $a(v) = \alpha / mr_0 v^2$ is the trajectory criteria (circular, elliptical and hyperbolic). Then, with the condition $a(v) = \alpha / mr_0 v_0^2 = 1$ and $r = r_0$ =constant for circular trajectory, it can be found that $\alpha = mr_0 v_0^2$. For $a(v) = \alpha / mr_0 v^2 = v_0^2 / v^2 = E_0 / E$, the equations to calculate the ESA parameters are:

$$\cos(\varphi + \varphi_0) = \frac{\cos(d\theta) r_0 / r - E_0 / E\cos(d\theta)}{\sqrt{\left[1 - E_0 / E\right]^2 + (E_0 / E)^2 tg^2(d\theta)}};$$

$$r = \frac{r_0}{E_0 / E + \left[1 - E_0 / E\right]\cos(\varphi)} \text{ at } d\theta = 0$$

$$dr \approx r_0 [1 - \cos(\varphi)] dE / E_0$$

The results for realistic parameters of backscattered protons are in the Fig.7. This picture shows two variants of the ESA orientation, relative to the initial beam direction, due to scattered proton energy depending on $d\theta$:

$$E^{sc} = E_0^{sc} \pm K d\theta,$$

where $d\theta = \pm 0.7$ mrad, K=13.4 eV and $E_0^{sc} = 80$ keV is the energy of backscattered protons. Depending on the sign, we have two possible angles for the ESA:

$$1) E^{sc} = E_0^{sc} + K d\theta \Rightarrow \varphi_0 \approx 80.5^0 \Rightarrow \varphi_{ESA} = 2 \cdot (180 - \varphi_0) \approx 199^0$$
$$2) E^{sc} = E_0^{sc} - K d\theta \Rightarrow \varphi_0 \approx -80.5^0 \Rightarrow \varphi_{ESA} = 2 \cdot \varphi_0 \approx 161^0$$

The first variant corresponds to the left side of the picture and has a deflection angle 199°. The starting point is at 80.5° and the backscattered protons are deflected to the opposite direction compared to the beam one.
The second variant corresponds to the right side of the picture and has a deflection angle 161°. The starting point is at -80.5° and the backscattered protons are deflected to the same direction compared to the beam one.

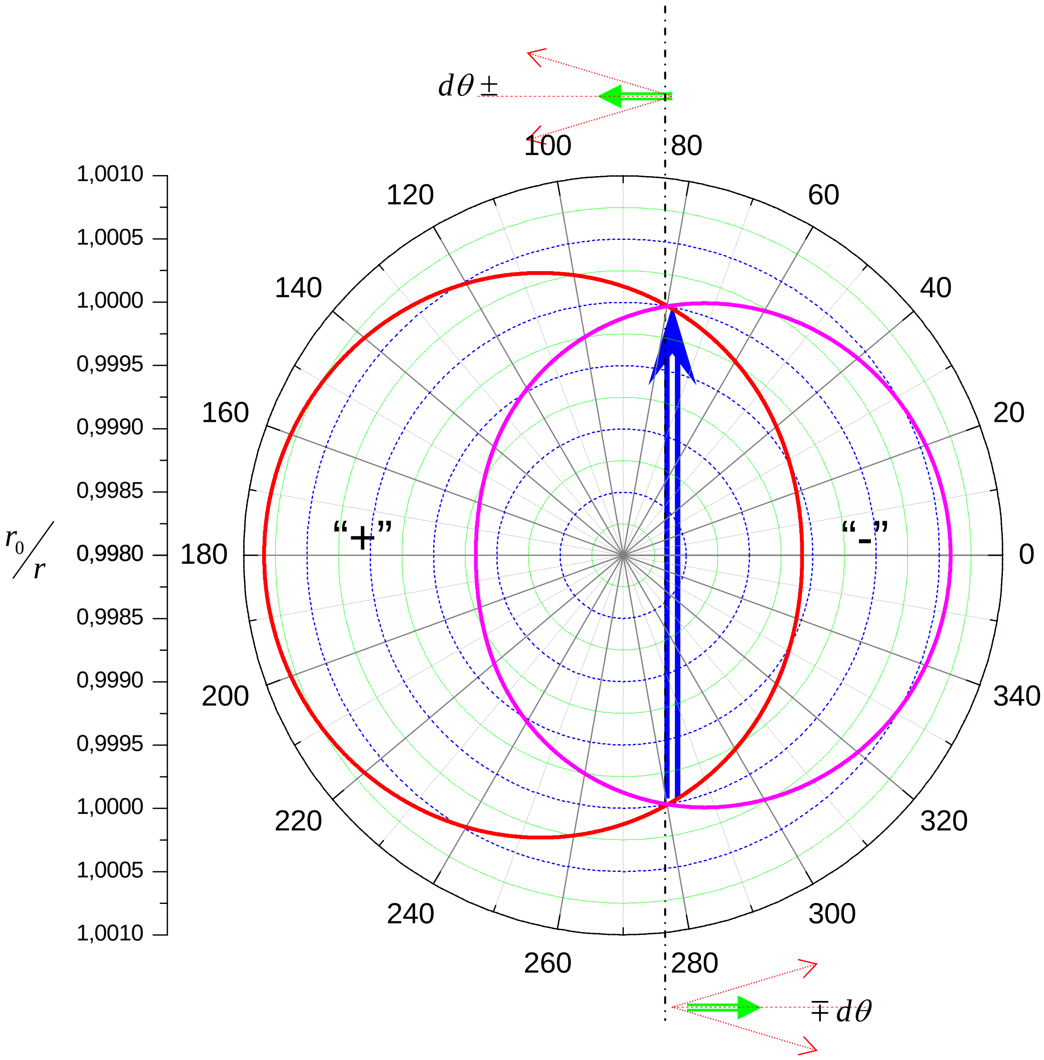

Fig.7

From a magnification M_{ESA}=1, it can be calculated a radius of the ESA and the width of the entrance and exit slits for the second variant ($\varphi_{ESA} \approx 161°$). For such configuration, we find:

$$M_{ESA} = \Delta h_{in} / \Delta h_{out} = 1$$

$$L_{ESA} = L_0 = 500 \text{ mm} \Rightarrow r_{ESA} = r_0 = 178 \text{ mm}$$

and:

$$\Delta h_{in} = \Delta h_{out} = 2 \cdot dr \approx 90 \text{ µm}$$

4.2 Bandpass and resolution.

There are three sources which contribute to the error bar measurements or the bandpass (BP):

$$BP = 2.36 \cdot d\sigma_{\varepsilon} = 2.36\sqrt{(d\sigma_{ESA})^2 + (d\sigma_U)^2 + (d\sigma_{\theta_v})^2}$$

The first source ($d\sigma_{ESA}$) limits the wire-target diameter. It can be deduced geometrically from the scheme in the Fig.8 with:

$$(d_0/2)^2 + (\Delta h_{in})^2 = (2d\theta \cdot L_0)^2$$

$$d_0 = 1.4mm$$

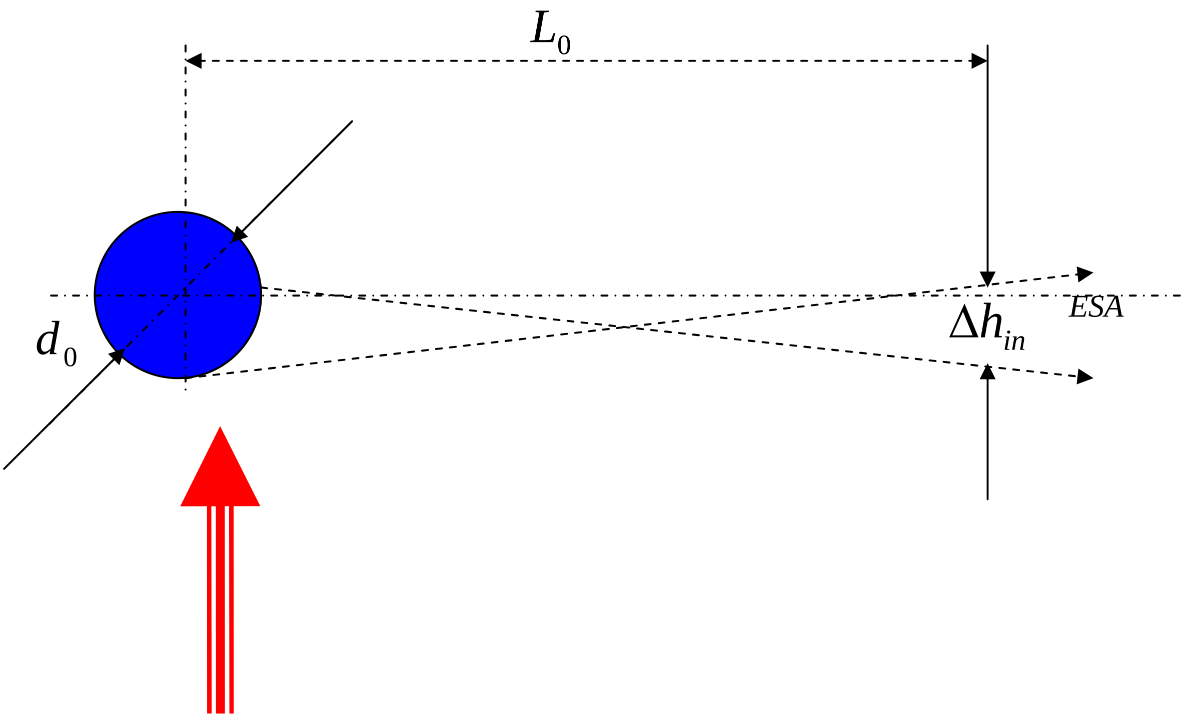

Fig.8

The second source, $d\sigma_U$, is the stability of the ESA electrodes voltage and this can be calculated from the classical formula for ESA:

$$\frac{\Delta U}{d} = \frac{2}{r_{ESA}} \cdot E_0^{sc}$$

$$d = 6mm$$

$$^{+}U = 2651.9V$$

$$^{-}U = -2742.9V$$

$$dU = \frac{2}{r_{ESA}} \cdot K \cdot d\theta \approx 0.3V$$

The third source, $d\sigma_{\theta_v}$, is the effective size of the wire-target and it will be chosen taking into account the maximum emittance range.

Since the technique aims to measure the horizontal component of the angular spread (depending on the wire-target nature), the influence of the vertical component on the experimental value must be less than the error bar on $d\theta$. The diagram in the Fig.9 shows that the maximum of vertical emittance component value α_ε is confined by a change of the average scattering angle on value $d\theta$.

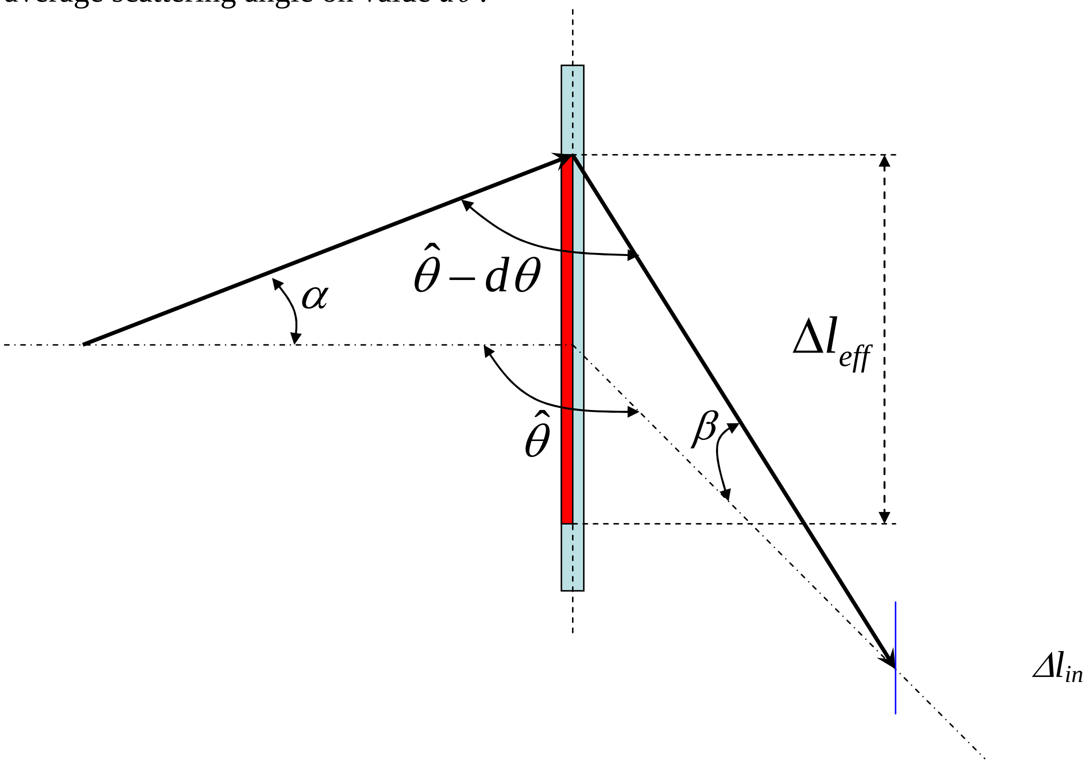

Fig.9

This dependency can be written as:

$$\cos(\theta) = \sin(\alpha_\varepsilon) \cdot \sin(\beta)$$

where α_ε is the vertical component of emittance; β is the angular size of wire-target. Taking into account the smallness of angles (α_ε and β), it can be written as:

$$2(d\theta_v) \approx \sin(\Delta\alpha_\varepsilon) \times d\beta_{target}$$

where $d\theta_v$[rad] is the error bar, $\Delta\alpha_\varepsilon$ [rad] is the full range of emittance, from "-" to "+", and $d\beta_{target}$[rad] is the angular effective size of wire-target. For values:

$$d\theta_v = 0.7mrad$$
$$\Delta\alpha_\varepsilon \approx \Delta\theta_\varepsilon = 100mrad \Rightarrow d\beta_{target} = 14mrad;$$

with $\Delta\theta_\varepsilon$ the horizontal angular spread of the initial proton beam.

We can determine the height of the wire-target and the ESA entrance slit by using the classical formula for dispersions:

$$2d\theta \Big/ \sin(\Delta\alpha_\varepsilon) \approx d\beta_{target} = \sqrt{\Delta l_{eff}^2 + \Delta l_{in}^2} \Big/ L_0$$
$$\Delta l_{eff} = \Delta l_{in} = 5mm$$

To finish, for a maximum efficiency, the stability of ESA voltage must be less than 0.3V:

$$d\sigma_\varepsilon = \sqrt{(d\sigma_{ESA})^2 + (d\sigma_U)^2 + (d\sigma_{\theta_v})^2} \approx 1mrad$$
$$d\sigma_{ESA} = d\sigma_{\theta_v} = 0.7mrad \Rightarrow d\sigma_U \le 0.3V$$

5. Modelling calculation of scattered proton spectrum.

This section shows estimates for the number of collected backscattered protons for realistic parameters of the experimental set-up. This number of collected backscattered protons is plot in the figure Fig.10 for a exposition time of one second. The following tables show the parameters which have been used for the calculations:

Initial beam parameters	Values
Proton energy	$E_0 = 95$ keV
Beam current	$I_0 = 80$ mA
Beam diameter	$D_0 = 50$ mm
Angular spread	$K \cdot \sigma_\theta = 500$ eV
Energy spread	σ_E=100 eV

Wire-target sizes (^{12}C)	Values
Diameter of wire	$\mathbf{d_0}$=1.4mm
Effective size of wire	-$\mathbf{\Delta l_{eff}}$ =5mm

ESA parameters	Values
Deflected angle	$\varphi_{\mathbf{ESA}}$=161° (electrodes at 150°)
ESA radius	$\mathbf{r_{ESA}}$=178mm
Width of slits	$\Delta\mathbf{h_{in}}$=$\Delta\mathbf{h_{out}}$=90μm
Height of slits	$\Delta\mathbf{l_{in}}$ =5mm
Distances between	$\mathbf{l}_{in}$=15mm, $\mathbf{l_{out}}$=20mm

The distances between electrode edges and slits were computed with the help of OPERA calculations. The detector could be a MCP “F4655-12 Hamamatsu” like.

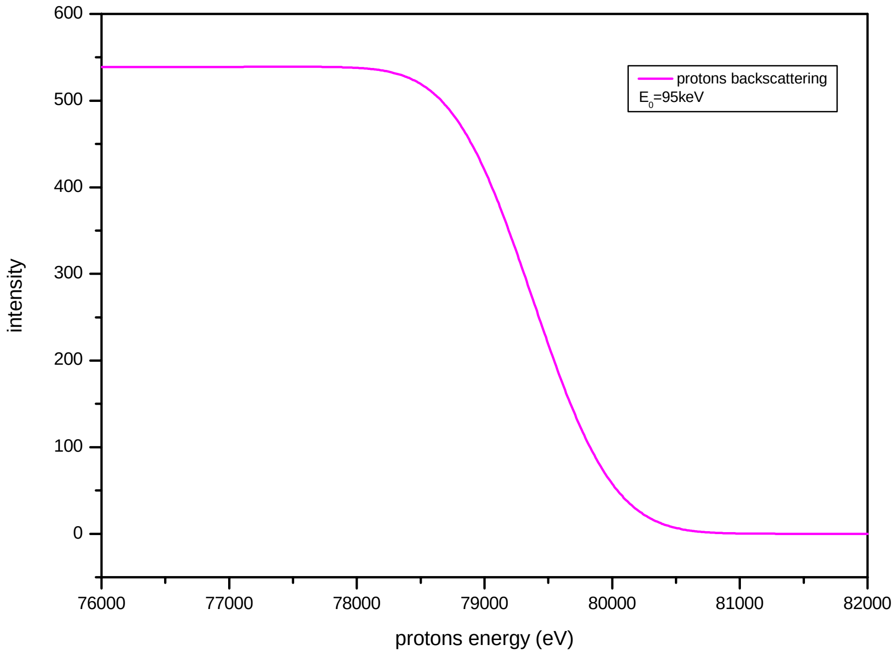

Fig.10: Number of collected proton spectrum with an exposition time of 1s.

6. Conclusion and prospects

This note details the principle and the design of an innovative transverse emittance measurement unit which aims to be non interceptive. The interaction between the wire and the beam is assumed to be weak enough to be considered negligible. One major advantage of this device is that it allows, depending on the wire material, to measure either the transverse momentum or the energy spread.
A proposal for the main parameters is detailed and could be the starting point to a construction phase.

7. Acknowledgements

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