Application of the structure function method to polarized and unpolarized electron-proton scattering

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Abstract

The cross section for polarized and unpolarized electron-proton scattering is calculated taking into account radiative corrections in leading and next-to leading logarithmical approximation. The expression of the cross section is formally similar to the cross section of the Drell-Yan process, where the structure functions of the electron play the role of Drell-Yan probability distributions. The main contribution to the K-factor arises from the interference of the Born amplitude with box-type terms, describing the exchange of two virtual photons between the electron and the proton. Proton form factors are assumed to decrease rapidly with the momentum transfer squared. The calculation of the box amplitude is done with the proton and the Δ-resonance in the intermediate state. Previous calculations are discussed and results of numerical estimations are given and discussed at the light of conflicting experimental results on proton electromagnetic form factors.

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I. INTRODUCTION

Radiative corrections (RC) to elastic (inelastic) electron-proton (ep) scattering cross sections can be classified in two types, according to the reaction mechanism which is assumed: one where a virtual photon is exchanged between electron and proton (class I) and a second one taking into account the two virtual photon exchange amplitude, arising from box-type Feynman diagrams in the lowest order of perturbation theory (PT)(class II). Both kinds of contributions to RC were considered in the literature, in detail, at the lowest order of PT for polarized and unpolarized cases.

The most elaborated consideration at the lowest order of PT was done in Ref. [1], where the approaches of previous papers (cf. the reference list in [1]) were considerably improved. The role of higher orders of PT was firstly considered for the unpolarized case in Ref. [2] and later for polarized case in Refs. [3] and [4].

The size of RC essentially depends on how the experiment was performed. In experiments where only the angle (Laboratory frame implied) of the scattered electron is measured, the initial electron emission can induce an enhancement of RC due to decreasing of the value of the momentum transfer squared, $Q^2 = -q^2$, between electron and proton.

This mechanism can be taken into account by writing the cross section in form of cross section of Drell-Yan process where the structure functions of the electron (SFs) play the role of probability distributions [2]. The set of SFs obey the renormalization group equations (Lipatov's equations). Their solutions are well known [5]. The formalism of SFs allows one to obtain the cross section in the so called "leading logarithmical approximation (LLA) i.e. taking correctly into account the terms of order $[(\alpha/\pi)\ln(Q^2/m_e^2)]^n$. It corresponds to collinear kinematics, where the photon is emitted in a direction close to the direction of the electron. Knowing the value of RC in the lowest order of PT, the non-leading contribution $(\alpha/\pi)[(\alpha/\pi)[\ln(Q^2/m_e^2)]^n$ can be calculated.

A different source of enhancement of cross section is related to the so called Weizsacker-Williams kinematics, where photons are emitted in non-collinear kinematics, and provide almost zero momentum $Q^2 < m_e^2$. This is not discussed in the present work.

A possible enhancement of the elastic cross section can be associated with box type Feynman diagram, due to the rapid decreasing of proton form factors (RDFF) in the case of proton intermediate state. A similar effect takes place when the $\Delta(33)$ resonance is present in the intermediate state of the box diagram, because the transition in the vertices $\gamma^* p\Delta$ shows also a rapid decreasing with Q^2.

The relative contribution of two photon exchange, from simple counting in α, would be of the order of the fine structure constant, $\alpha = \dfrac{e^2}{4\pi} \simeq \dfrac{1}{137}$: any contribution of two-photon exchange through its interference with the one-photon mechanism would not exceed 1%. On the other hand, more than 25 years ago it was observed [6] that the simple rule of α-counting for the estimation of the relative role of two-photon contribution to the amplitude of elastic electron hadron scattering does not hold at large momentum transfer. Using a Glauber approach for the calculation of multiple scattering contributions [7], it appeared that the relative role of two-photon exchange can increase significantly in the region of high momentum transfer, when the momentum squared is equally shared between the two photons.

Taking a simple model for nucleon form factors, based on the dipole parametrization:

$$G_E(q) = \frac{G_M(q)}{\mu} = \frac{M_0^4}{(Q^2+M_0^2)^2}, M_0^2 = 0.71 \text{ GeV}^2, \ \mu = 2.79 \tag{1}$$

an enhancement factor appears: $\mathcal{N}(z) = (z+1)^2/[(z/4)+1]^4$, where $z = Q^2/M_0^2$. The corresponding contribution arises in the loop calculation, when both exchanged photons have momenta close to $q/2$. This kinematical region differs from the "one soft photon" approach used in Ref. [1], when considering the box diagram.

Large interest in the 2γ contribution has arisen as a possible explanation of the discrepancy among electromagnetic proton form factors, when measured with two different methods: the polarization transfer method [8], which allows a precise measurement of the ratio of the electric to magnetic proton form factors [9] and the Rosenbluth separation, from unpolarized elastic ep cross section [10].

In Ref. [11] it was noted that the reason of the discrepancy lies in the slope of the reduced cross section as a function of ϵ, the virtual photon polarization. At the kinematics of the present experiments, radiative corrections can reach up to 40% on the cross section, and affect very strongly the slope, changing even its sign.

In Ref. [12] it was shown that the contribution of the sum of the nucleon and the Δ to the two photon exchange correction has an angular dependence compatible with both the polarization transfer and the Rosenbluth methods, for the measurement of the nucleon electromagnetic form factors. Unfortunately the kinematics of RDFF was not investigated in detail.

On the other hand, Ref. [3] is very detailed. The SFs method was applied to transferred polarization experiments. The size of this effect was an order of magnitude too small to bring the polarization data in agreement with the unpolarized ones. Therefore the conclusion of that paper was that one could not solve the discrepancy among the existing data. The SF method was also applied to polarization observables in Ref. [4], where it was shown that the corrections can become very large, if one takes into account the initial state photon emission. However the corresponding kinematical region is usually rejected in the experimental analysis, by appropriate selection on the scattered electron energy.

The motivation of the present paper is the compellent need for a precise expression of the radiative corrected cross section for ep elastic scattering, in both polarized and non-polarized cases, which is easy to handle for experimentalists and which has a sufficient accuracy.

Our paper is organized as follows. In section II we give the Drell-Yan formulae for cross sections in polarized and unpolarized cases. Section III is devoted to the calculation of the contribution to the K-factor for the unpolarized cross section and of the degree of transversal and longitudinal polarization of recoil proton. Numerical results are presented and discussed in Section IV. Conclusions summarize the main points of this work. Details of the methods used for the necessary integrations are presented in the Appendices.

II. DRELL-YAN EXPRESSION OF THE ep CROSS SECTIONS IN UNPOLARIZED AND POLARIZED CASES

In an experiment, the selection of elastic events requires a cut in the energy spectrum of the scattered electron, and one integrates over the events where the energy of the final electron, E', exceeds a threshold value $E' > Ey = Ec/\rho$, $\rho = 1 + (E/M)(1 - \cos\theta)$, $c < 1$ (E is the initial electron energy). The cross section in the case of unpolarized particles in frame of Drell-Yan approach is

$$\frac{d\sigma}{d\Omega} = \sigma_M \int_{z_1}^{1} dz D(z,\beta) \frac{\Phi(z)}{[1 - \Pi(Q_z^2)]^2} \left(1 + \frac{\alpha}{\pi} K_{unp}\right), \tag{2}$$

where where $\sigma_M = \alpha^2\cos^2(\theta/2)/\left(4E^2\sin^4(\theta/2)\right)$ is the Mott's cross section; $z_1 = c/[\rho - c(\rho-1)]$, and the non-singlet SF is:

$$D(z,\beta) = \frac{\beta}{2}\left[\left(1+\frac{3}{8}\beta\right)(1-z)^{\frac{\beta}{2}-1} - \frac{1}{2}(1+z)\right](1+O(\beta)), \tag{3}$$

$$\beta = \frac{2\alpha}{\pi}\left[\ln\frac{Q^2}{m_e^2} - 1\right], \qquad Q^2 = \frac{2E^2(1-\cos\theta)}{\rho}. \tag{4}$$

The other quantities entering in Eq. (2) are defined as

$$\Phi(z) = \frac{1}{z^2\epsilon_z\rho_z}\sigma_{red}(z), \qquad \sigma_{red}(z) = \tau_z G_M^2(Q_z^2) + \epsilon_z G_E^2(Q_z^2); \tag{5}$$

and

$$\tau = \frac{Q^2}{4M_p^2}, \qquad \frac{1}{\epsilon} = 1 + 2(1+\tau)\tan^2(\theta/2). \tag{6}$$

The quantities ϵ_z, Q_z^2, ρ_z, τ_z, can be obtained from ϵ, Q^2, ρ, τ by replacing the initial electron energy E by zE. The operator of vacuum polarization is taken as

$$\Pi(Q^2) = \frac{\alpha}{3\pi}\left(\ln\frac{Q^2}{m_e^2} - \frac{5}{3}\right). \tag{7}$$

The dependence of the differential cross section on the angle and the energy fraction of the scattered electron $y = 1/\rho$ can be written as:

$$\frac{d\sigma}{d\Omega dy} = \frac{\alpha^2\cos^2(\theta/2)}{4E^2\sin^4(\theta/2)}\int_{z_1}^{1}\frac{dz\rho_z}{z}D(z)D\left(\frac{y\rho_z}{z}\right)\frac{\phi(z)}{[1-\Pi(Q_z^2)]^2}, \tag{8}$$

The y dependence, at fixed momentum transfer and electron scattering angle, show a steep rise, at small y due to initial state emission, and a rise in the vicinity of the elastic value, $y = 1/\rho$. As an example, such dependence is shown in Fig. 1, for $\theta = 32.4^0$ and Q^2=3 GeV^2. The dashed lines show the kinematical cuts corresponding to c=0.95, 0.97 and 0.99, from left to right.

In Ref. [4] the components of the recoil proton polarization (transversal $\mathcal{P}_t$ and longitudinal $\mathcal{P}_\ell$) were calculated in frame of the Drell-Yan approach:

$$\left(\mathcal{P}_t\frac{d\sigma}{d\Omega}\right)_{corr} = -\lambda\int_{z_1}^{1} dz D(z,\beta)\frac{\alpha^2}{Q_z^2}\left(\frac{1}{\rho_z}\right)^2\sqrt{\frac{\tau_z}{\tan^2(\theta/2)(1+\tau_z)}}G_E(Q_z^2)G_M(Q_z^2)\left(1+\frac{\alpha}{\pi}K_t\right); \tag{9}$$

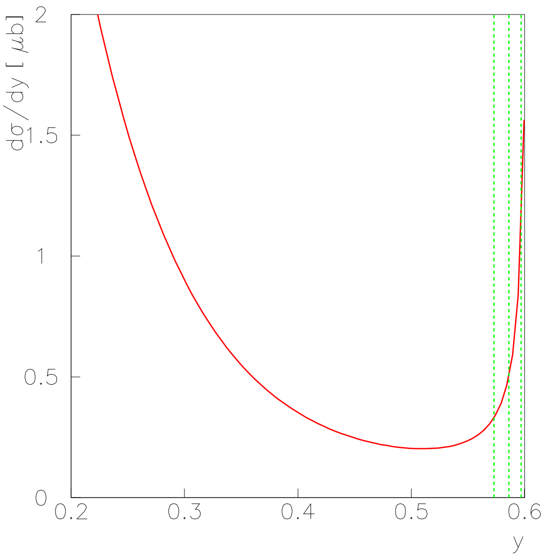

FIG. 1: The y dependence of the elastic differential cross section, at $\theta = 32.4^0$ and Q^2=3 GeV^2.

$$\left(\mathcal{P}_\ell \frac{d\sigma}{d\Omega}\right)_{corr} = -\lambda \int\limits_{z_1}^{1} dz D(z,\beta) \frac{\alpha^2}{2M^2} \left(\frac{1}{\rho_z}\right)^2 \sqrt{1 + \frac{1}{\tan^2(\theta/2)(1+\tau_z)}} G_M^2(Q_z^2) \left(1 + \frac{\alpha}{\pi} K_\ell\right). \tag{10}$$

where $\lambda = \pm 1$ is the chirality of the initial electron;

The factors K_{unp} and $K_{t,\ell}$ contain the contribution of the 2γ exchange diagrams, and they are calculated in next section.

III. CALCULATION OF THE K FACTORS CONTRIBUTION FROM THE 2γ EXCHANGE

A. Proton intermediate state

We parameterize the loop momentum of the box-type Feynman amplitude in such a way, that the denominators of Green function are $(\pm\kappa + q/2)^2 + \lambda^2$ for the photon, whereas for the electron (e) and the for the proton (p) they have a form $(e) = (\pm\kappa + \Delta)^2 - m_e^2$, $\Delta = \frac{1}{2}(p_1 + p_1')$, $(p) = \left(\kappa + \frac{1}{2}(p+p')\right)^2 - M^2$, where the sign '$-$' for the electron corresponds to the Feynman diagram for the two photon box (Fig.2a) and the sign '$+$' corresponds to the crossed box diagram (Fig. 2b). The assumption of a rapid decreasing of form factors

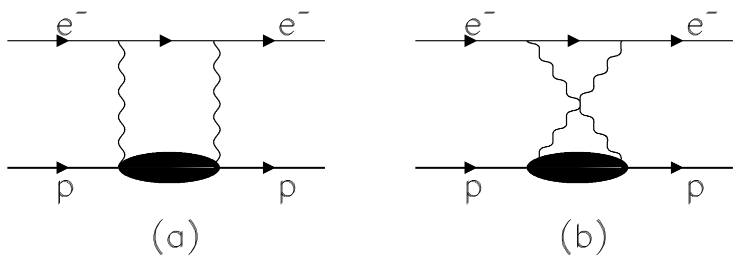

FIG. 2: Feynman diagrams for two-photon exchange in elastic ep scattering: box diagram (a) and crossed box diagram (b).

implies that we can neglect the dependence on the loop momentum κ in the denominators of the photon Green function as well as in the arguments of the form factors, which results in ultraviolet divergences of the loop momentum integrals. Therefore they should be understood as convergent integrals with the cut-off restriction $|\kappa^2| < M^2\tau$:

$$\int \frac{d^4\kappa}{i\pi^2} \frac{N_\pm(\Delta, Q)}{\left((\pm\kappa+\Delta)^2 - m_e^2\right)\left((\kappa+Q)^2 - M^2\right)} \theta\left(M^2\tau - |\kappa^2|\right) = I_\pm \cdot N_\pm(\Delta, Q). \tag{11}$$

where $\Delta = \frac{1}{2}(p_1 + p_1'),\ Q = \frac{1}{2}(p + p')$. The explicit form of $I_\pm$ is given in Appendix A. $N_\pm(\Delta, Q)$ is the Feynman diagram numerator defined below.

Then the expressions for K-factors can be written as:

$$K_i = -2\mathcal{N}\left(Q^2/M_0^2\right)\frac{U^i(\Delta, Q)}{Z_i}, \qquad i = unp, x, z, \tag{12}$$

where $\mathcal{N}$ is the enhancement factor defined above. Z_i, $i = unp, x, z$ are the modulo squared of the Born amplitude which are singled out in the definition of the K-factor :

$$Z_{unp} = \left(1 - \frac{Q^2\left(M^2 + s\right)}{s^2}\right)\left[\frac{g_e^2 + \tau g_m^2}{1+\tau} + 2\tau g_m^2 \tan^2(\theta/2)\right], \tag{13}$$

$$Z_t = -\frac{1}{\rho} g_e g_m \sqrt{\frac{\tau}{1+\tau}} \sin\theta, \tag{14}$$

$$Z_\ell = \frac{Q^2}{2E^2} g_m^2 \sqrt{\frac{\tau}{1+\tau}} \left(\frac{E}{M} - \tau\right). \tag{15}$$

with $g_e = 1$, $g_m = \mu$ (the form factors dipole dependence is extracted as the enhancement factor $\mathcal{N}(z)$ in (12)). In the unpolarized case the expression for $U^{unp}(\Delta,Q)$ is:

$$\begin{aligned} U^{unp}(\Delta,Q) &= \frac{1}{s^2M^2\tau}\cdot\frac{1}{4}Tr\left[\left(\hat{p}'+M\right)\Gamma_\lambda\hat{Q}\Gamma_\eta\left(\hat{p}+M\right)\bar{\Gamma}_\mu\right]\times \\ &\times\left\{I_+\cdot\frac{1}{4}Tr\left[\hat{p}_1'\gamma_\lambda\hat{\Delta}\gamma_\eta\hat{p}_1\gamma_\mu\right]+I_-\cdot\frac{1}{4}Tr\left[\hat{p}_1'\gamma_\eta\hat{\Delta}\gamma_\lambda\hat{p}_1\gamma_\mu\right]\right\} \end{aligned} \tag{16}$$

where $\Gamma_\alpha=\gamma_\alpha-\frac{\mu}{4M}\gamma_\alpha\hat{q}$, $\bar{\Gamma}_\alpha=\gamma_\alpha+\frac{\mu}{2M}\gamma_\alpha\hat{q}$. The quantities $U^{t,\ell}(\Delta,Q)$ for polarized case can be obtained from (16) by the following replacements:

$$\gamma_\mu\to\gamma_\mu\gamma_5 \tag{17}$$

in the lepton traces and

$$\left(\hat{p}'+M\right)\to\left(\hat{p}'+M\right)\hat{a}_{t,\ell}\gamma_5 \tag{18}$$

in the proton traces. Here $a_{t,\ell}$ is the final proton polarization vector (i.e. $(a_{t,\ell}p')=0$) and corresponds to different orientations of the proton polarization. If the final proton is polarized along the x-axis, one finds:

$$(a_tp)=0,\qquad (a_tp_1)=-\frac{E^2}{2M\rho}\frac{\sin\theta}{\sqrt{\tau(1+\tau)}}, \tag{19}$$

whereas in case of polarization along the z-axis:

$$(a_\ell p)=2M\sqrt{\tau(1+\tau)},\qquad (a_\ell p_1)=M\sqrt{\frac{\tau}{1+\tau}}\left(\frac{E}{M}-1-2\tau\right). \tag{20}$$

B. The Δ resonance contribution

Let us write the structure of the vertex for the transition $\Delta(p)\to\gamma(q)+P(p')$, following the formalism of Ref. [13] (and references therein):

$$M(\Delta\to\gamma P) \;=\; eg_{\Delta N}\,\sqrt{3/2}\,\bar{u}(p',\eta)\left(\gamma_\mu-\frac{1}{M_\Delta}p'_\mu\right)u_\nu(p,\lambda)F_{\mu\nu}(q), \tag{21}$$

where $F_{\mu\nu}(q)=e_\mu(q)q_\nu-e_\nu(q)q_\mu$ is the Maxwell tensor, $e(q)$ is the polarization vector of virtual photon, η and λ are the chiral states of the nucleon and of the Δ-resonance and $\sqrt{3/2}g_{\Delta p}\approx 1.56\mu$ (μ is the anomalous magnetic moment of the proton).

The Green function of the Δ resonance, neglecting its width, is

$$\frac{D_{\mu\nu}(p)}{p^2-M_\Delta^2+i0}, \tag{22}$$

with

$$D_{\mu\nu}(p) = \sum_{\lambda} u_\mu(p,\lambda)\bar{u}_\nu(p,\lambda) =$$
$$= (\hat{p}+M)\left[-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{1}{3M}\left[\gamma_\mu p_\nu - \gamma_\nu p_\mu\right] + \frac{2}{3M^2}p_\mu p_\nu\right]. \qquad (23)$$

The transition vertices associated with form factors are of the same form as the dipole ones for the nucleons. The part of the virtual Compton scattering of the proton amplitude which enters in the box amplitude is:

$$\bar{u}(p')[p']_\mu D_{\rho\sigma}(p_2)[p]_\nu u(p) F_{\mu\rho}(k_1) F^*_{\sigma\nu}(k_2),$$

$$k_{1,2} = \pm\kappa + \frac{q}{2}, \quad p_2 = \kappa + Q, \quad [p]_\mu = \gamma_\mu - \frac{1}{M}p_\mu.$$

Thus, in unpolarized case, the contribution of the Δ-resonance to the K-factor can be written in the form of simple box-type diagram (12) with [1]

$$U_\Delta^{unp} = \frac{1}{s^2M^2\tau}\cdot\frac{1}{4}Tr\left[(\hat{p}'+M)\,[p']_\mu D_{\rho\sigma}(Q)[p]_\nu\,(\hat{p}+M)\,\bar{\Gamma}_\eta\right]\times$$
$$\times\left\{I_+\cdot\frac{1}{4}Tr\left[\hat{p}'_1 P^{\mu\nu\rho\sigma}\hat{p}_1\gamma^\eta\right] + I_-\cdot\frac{1}{4}Tr\left[\hat{p}'_1 R^{\mu\nu\rho\sigma}\hat{p}_1\gamma^\eta\right]\right\}, \qquad (24)$$

where

$$P_{\mu\nu\rho\sigma} = \frac{1}{4}\left[\gamma_\rho q_\nu - \gamma_\nu q_\rho\right]\hat{\Delta}\left[\gamma_\sigma q_\mu - \gamma_\mu q_\sigma\right],$$
$$R_{\mu\nu\rho\sigma} = \frac{1}{4}\left[\gamma_\sigma q_\mu - \gamma_\mu q_\sigma\right]\hat{\Delta}\left[\gamma_\rho q_\nu - \gamma_\nu q_\rho\right].$$

The contributions in the polarized cases can be obtained from (24) via the same replacement rules (17), (18).

IV. RESULTS AND DISCUSSION

The numerical results strongly depend on the inelasticity cut, in the scattered electron energy spectrum. The results shown here correspond to $c = 0.97$. This value has been chosen because it corresponds to the energy resolution of modern experiments. The unpolarized

[1] Here we use the approximation $(M_\Delta - M)/M \ll 1$ therefore we can use the same $I_\pm$ as in case of proton intermediate state.

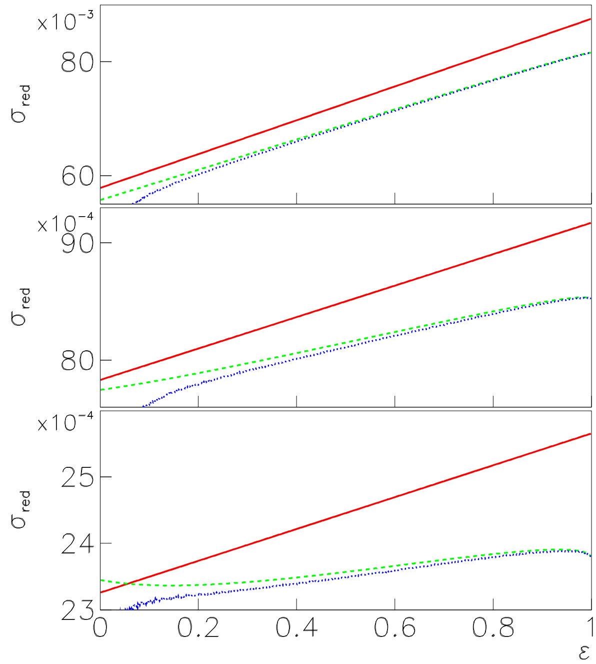

FIG. 3: The ϵ-dependence of the elastic differential cross section, for $Q^2 = 1$, 3, and 5 GeV^2, from top to bottom: Born cross section (solid line), Drell-Yan cross section (dashed line), full calculation (dash-dotted line).

cross section has been calculated assuming the dependence of form factors on Q^2 given by Eq. 1. In Fig. 3 the results are shown as a function of ϵ, for $Q^2 = 1$, 3, and 5 GeV^2, from top to bottom. The calculation based on the structure function method, from Eq. (2) is shown as dashed lines. The full calculation, including the two-photon exchange contribution is shown as dash-dotted lines. For comparison the results corresponding to the Born reduced cross section are shown as solid lines. One can see that the main effect of the present calculation is to modify and lower the slope of the reduced cross section. This effect gets larger with Q^2. Non-linearity effects are small. Including two-photon exchange modifies very little the results, in the kinematical range presented here.

The Q^2-dependence of the unpolarized cross section is shown in Fig. 4, for electron scattering angles equal to $\theta = 85°$, $60°$, and $20°$, from top to bottom. The $G_D^2(Q^2)$-dependence has been removed, in order to enhance the differences among the calculations.

The results for the polarized case are shown in Figs. 5 and 6 respectively for the longi-

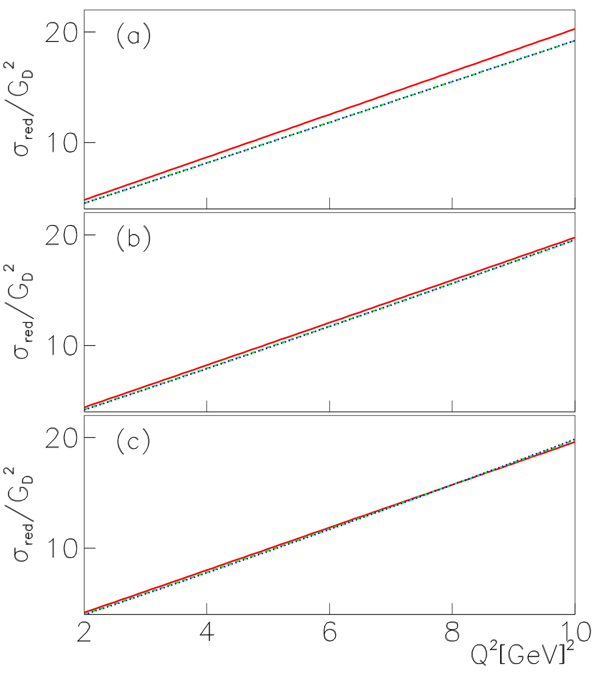

FIG. 4: The Q^2-dependence of the elastic differential cross section, at $\theta = 85^0$, $60°$, and $20°$. Notations as in Fig. 3.

tudinal and the transversal components of the proton polarization. The relative effect on the polarization is much smaller than on the unpolarized cross section but the ϵ dependence is different for the longitudinal and for the transversal components. Again the effect of the two photon contribution is negligible, in both cases.

It is particularly interesting to look at the ratio of the longitudinal to transverse components of the proton polarization, which is the object of experimental measurements and which is directly related to the form factor ratio (Fig. 7). The calculation based on Born approximation, would give a constant value equal to one, due to the ansatz used for the form factors from (1). The results from the present calculation differ very little, within 1%. However the two photon contribution depends on Q^2 and becomes larger as Q^2 increases. The present results suggest that an appropriate treatment of radiative corrections constitutes the solution of the discrepancy between form factors extracted by the Rosenbluth or by the recoil polarization method.

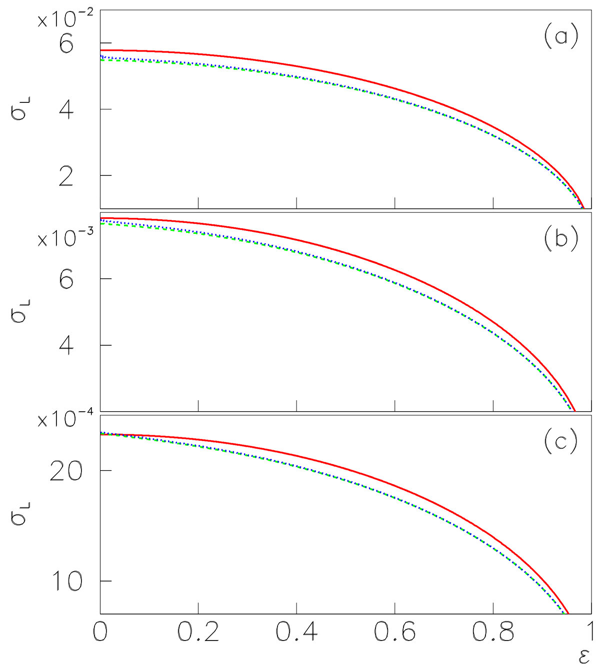

FIG. 5: The ϵ-dependence of the longitudinal proton polarization, $Q^2 = 1$, 3, and 5 GeV^2, from top to bottom. Notations as in Fig. 3.

V. CONCLUSION

We have considered radiative corrections in case of quasielastic kinematics, when the scattered electron has energy close to the elastic value. We considered two types of corrections: the real photon emission related to the electron vertex, that we calculated in frame of the structure function approach, and the two-photon exchange box diagram. We did not consider the photon emission from the proton, which is expected to be small. The enhancement due to RC has been explicitly calculated in QED, taking into account the fast decreasing of nucleon FFs.

The two photon contribution is parameterized in terms of a K-factor in the structure function approach. The K-factor can contribute for less than few percent to the unpolarized cross section. Its contribution is different for the polarized cross section, and very small on the ratio of the longitudinal to transversal components.

The main effect of the present calculation of RC is visible on the unpolarized cross

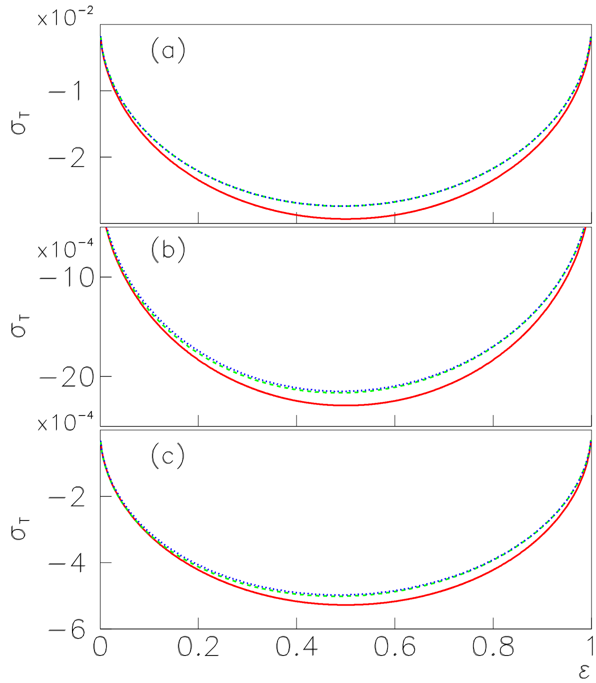

FIG. 6: Same as Fig. 5 for the transversal proton polarization.

section: it changes noticeably the slope of the ϵ dependence of the reduced cross section, in comparison with the Born approximation. This slope is directly related to the electric form factor, therefore applying RC as suggested here to the unpolarized cross section, would solve the discrepancy between form factors extracted from the Rosenbluth method and from the recoil polarization method.

We considered both Δ and nucleon intermediate states. Their contributions, of opposite sign, partially compensate, but the nucleon contribution is larger. This result is consistent with Ref. [12].

In [4] it was shown that the corrections on the polarization observables can be very large, if the cut parameter is small, see Fig. 1. This is due to the initial state photon emission, which is normally excluded in the experimental analysis.

In conclusion, the SF method is a very powerful tool to calculate RC to elastic *ep* scattering. In particular, it takes precisely into account collinear photon emission. The two photon contribution is negligible in the considered kinematical range. The correction to the ratio of longitudinal to transverse proton polarization is small. But the correction on the

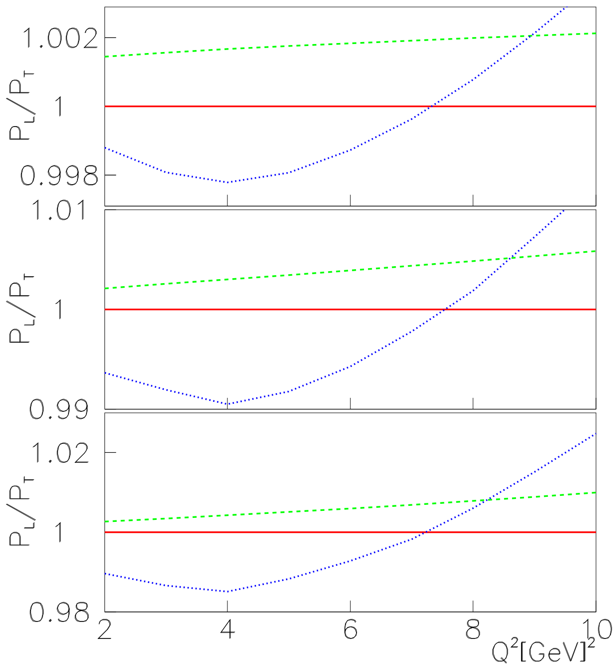

FIG. 7: The Q^2-dependence of the longitudinal to transversal components of the proton polarization, at $\theta = 85^0$, $60°$, and $20°$. Notations as in Fig.3.

unpolarized cross section has the effect and the size required to solve the discrepancy among proton form factors.

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Appendix A: Calculation of $I_\pm$

In this appendix we perform the following integration:

$$I_\pm = Re \int \frac{d^4\kappa}{i\pi^2} \frac{\theta\left(M^2\tau - |\kappa^2|\right)}{(\Delta_\pm)\,(Q)}, \tag{A1}$$

$$(\Delta_\pm) = (\pm\kappa + \Delta)^2 - m_e^2,$$
$$(Q) = (\kappa + Q)^2 - M^2,$$

where $\Delta = \frac{1}{2}(p_1 + p_1'),\ Q = \frac{1}{2}(p + p')$. First we perform Wick-rotation ($\kappa_0 \to i\kappa_0$) and imply the cut-off provided by θ-function using that parameterizing

$$Re\int \frac{d^4\kappa}{i\pi^2}\frac{\theta\left(M^2\tau - |\kappa^2|\right)}{(\Delta_\pm)(Q)} = \frac{2}{\pi}\int\limits_{-M\sqrt{\tau}}^{M\sqrt{\tau}} d\kappa_0 \int\limits_0^{M\sqrt{\tau - k_0^2/M^2}} dk\ k^2 \int\limits_{-1}^{1} d(\cos\theta_\kappa) Re\frac{1}{(\Delta_\pm)(Q)}, \quad \text{(A2)}$$

where $k = |\vec{\kappa}|$. We also performed the integration over the azimuthal-angle ϕ_κ. Now let us consider the integral in the Breit-system where $q_0 = 0$ and $\vec{p}_1 = -\vec{p}_1'$. Thus $\vec{\Delta} = 0$, $p_0 = p_0' = E'$, $|\vec{p}_1| = M\sqrt{\tau}$, $\vec{Q}^2 = M^2\cot^2(\theta_e/2)$, $E' = M\sqrt{\tau + 1/\sin^2(\theta_e/2)}$, where θ_e is the electron scattering angle in laboratory frame.

Before integrating over angle θ_κ let us write the explicit expression for real part of integrand:

$$Re\frac{1}{(\Delta_\pm)(Q)} = \frac{a\left(a + b\cos\theta_\kappa\right) \mp \delta_1\delta_2}{(a^2 + \delta_1^2)\left((a + b\cos\theta_\kappa)^2 + \delta_2^2\right)},$$

where $a = -\kappa_0^2 - k^2 + M^2\tau$, $b = -2k|\vec{Q}|$, $\delta_1 = 2\kappa_0 M\sqrt{\tau}$. $\delta_2 = 2\kappa_0 E'$. The integration over θ_κ is straightforward and results in:

$$I_\pm = -\frac{1}{\pi|\vec{Q}|}\int\limits_{-M\sqrt{\tau}}^{M\sqrt{\tau}} d\kappa_0 \int\limits_0^{M\sqrt{\tau - k_0^2/M^2}} dk\ k\frac{1}{a^2 + \delta_1^2} \times$$
$$\times \left\{\frac{a}{2}\ln\left(\frac{(a+b)^2 + \delta_2^2}{(a-b)^2 + \delta_2^2}\right) \mp \delta_1 \arctan\left(\frac{2b\delta_2}{a^2 - b^2 + \delta_2^2}\right)\right\}. \quad \text{(A3)}$$

Appendix A: Method for the integration of the D function

Let us consider the integral

$$\mathcal{I} = \int_{x_0}^{1} D(x)\phi(x)dx. \quad \text{(A1)}$$

The partition function $D(x)$ has a discontinuity for $x = 1$ and has the following properties:

$$\mathcal{I} = \int_0^1 D(x)dx = 1;\ D(x)|_{x\neq 0} = \frac{\beta}{4}\frac{1 + x^2}{1 - x}. \quad \text{(A2)}$$

Using (A2) one can write $\int_0^1 D(x)dx = \int_0^{1-\epsilon} D(x)dx + \int_{1-\epsilon}^1 D(x)dx = 1$. Therefore Eq. (A2) becomes:

$$\begin{aligned} \mathcal{I} &= \int_{x_0}^{1-\epsilon} dx D(x)\phi(x) + \int_{1-\epsilon}^{1} dx D(x)\phi(1) \\ &= \frac{\beta}{4}\int_{x_0}^{1-\epsilon} dx \frac{1+x^2}{1-x}\phi(x) + \left(1 - \int_0^{1-\epsilon} dx \frac{\beta}{4}\frac{1+x^2}{1-x}\right)\phi(1). \end{aligned} \tag{A3}$$

After elementary integration, Eq. (A3) becomes:

$$\mathcal{I} = \frac{\beta}{4}\int_{x_0}^{1-\epsilon} dx \frac{1+x^2}{1-x}\left[\phi(x) - \phi(1) + \phi(1)\right] + \phi(1)\left[1 - \frac{\beta}{4}\int_0^{1-\epsilon} dx \frac{1+x^2}{1-x}\right] \tag{A4}$$

$$= \phi(1)\left[1 - \frac{\beta}{4}\left(2\ln\frac{1}{1-x_0} - x_0 - \frac{x_0^2}{2}\right)\right] + \frac{\beta}{4}\int_{x_0}^{1} dx \frac{1+x^2}{1-x}\left[\phi(x) - \phi(1)\right] + \mathcal{O}(\beta^2), \tag{A5}$$

removing therefore the singularity.

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