Properties of Electromagnetic Hadron Form Factors from (polarized) Proton-Antiproton Annihilation

Egle Tomasi-Gustafsson
DAPNIA/SPhN, CEA/Saclay, 91191 Gif-sur-Yvette Cedex, France
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A unique possibility of studying electromagnetic proton form factors is offered by (polarized) high energy antiprotons beams. The measurement of the differential cross section of $\bar{p}+p \to \ell + +\ell^-$, $\ell=\mu$ or e allows the individual determination of the moduli of the electric G_E and the magnetic G_M form factors. Symmetry properties based on C-invariance allow to test the reaction mechanism (one or two photon exchange) through the even property of the cross section with respect to specific kinematical variables. Model independent properties of the observables can be derived and a method to measure form factors (which are complex) and their relative phase is proposed.

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I. INTRODUCTION

The experimental determination of the elastic proton electromagnetic form factors (FFs) at large momentum transfer is presently of large interest, due to the availability of electron beams in the GeV range with high intensity and high polarization, large acceptance spectrometers, hadron polarized targets, and hadron polarimeters. The possibility of extending the measurements of these fundamental quantities, which contain dynamical information on the nucleon structure, has inspired experimental programs at JLab, Frascati, Novosibirsk, Bejing and at future machines, such as FAIR, both in the space-like (SL) and in the time-like (TL) regions.

This short contribution contains mainly statements, their proof can be found in the quoted references.

The traditional way to measure proton electromagnetic FFs in consists in the Rosenbluth separation [1], i.e. the determination of the ϵ dependence of the reduced elastic differential cross section, at fixed momentum transfer squared, $Q^2=-q^2$, assuming that the interaction occurs through the exchange of one-photon (Q^2 is, then, the invariant mass of the virtual photon) [1].

High precision data on the ratio of the electric to magnetic proton FFs at large Q^2 have been recently obtained [2] through the polarization transfer method suggested by A.I. Akhiezer and M.P. Rekalo [3]. Such data showed that the ratio of electric to magnetic FF $R(Q^2)=\mu G_E(Q^2)/G_M(Q^2))$ ($\mu=2.79$ is the magnetic moment of the proton) deviates from unity as Q^2 increases, reaching a value of $\simeq 0.35$ at $Q^2=5$ GeV^2, contrary to what is obtained from the Rosenbluth separation [4, 5]: $G_M(Q^2)\simeq \mu G_E(Q^2)\simeq G_D(Q^2)=(1+Q^2/0.71\ \mathrm{GeV}^2)^{-2}$. Assuming such scaling behavior $R(Q^2)\simeq 1$, $G_M(Q^2)$ has been extracted up to $Q^2\simeq 31$ GeV^2 [6].

No experimental bias has been found in both types of measurements, the experimental observables being the differential cross section on the one hand, and the polarization of the outgoing proton in the scattering plane (more precisely the ratio between the longitudinal and the transverse polarization), on the other hand. The discrepancy is not at the level of these observables, as shown by the extraction of G_M in Ref. [7], but, instead, of the slope of the ϵ dependence of the reduced cross section, which is directly related to $G_E(Q^2)$, i.e. the derivative of the differential cross section, with respect to ϵ [8]. This puzzle has given rise to many speculations and different interpretations, suggesting in particular, the presence of 2γ exchange [10]. In previous papers [11] it was shown that the present data do not give any evidence of the presence of the 2γ mechanism, in the limit of the experimental errors. C-invariance and crossing symmetry require a very specific non linear ϵ dependence of the reduced cross section [12–14], whereas the data do not show any deviation from linearity.

Radiative corrections to the unpolarized cross section can reach 30-40% at large Q^2, and, as usually applied, induce a large correlation in the parameteres of the Rosenbluth fit [8]. In the original papers [15, 16], the authors stated already that when $\Delta E\to 0$, the measured cross section becomes negatively infinite, whereas physical arguments require that it should vanish and that this problem would be overcome taking into account higher order radiative corrections. It was shown in Ref. [17] that higher order corrections can be taken into account with very high precision following the structure function method [18] solving the discrepancy between polarized and unpolarized measurements of FFs.

The investigation of this question in TL region is expected to shed new light on the reaction mechanisms involved and on the comprehension of the nucleon structure. If the momentum transfer is shared between the two virtual photons, the two–photon–exchange (TPE) contribution can become important with increasing q^2 in case of elastic electron–proton scattering, as the fast decrease of FFs can compensate the additional factor of $\alpha=e^2/(4\pi)\simeq 1/137$ (the electromagnetic fine constant). This was already indicated more than thirty years ago [19], and recently discussed for elastic electron deuteron data in [20].

This problem becomes very actual as such mechanism could in principle be detected in very precise measurements and, if present, would prevent the simple extraction of FFs and of hadron properties in electron scattering measurements. The TPE contribution in the $\bar{p}+p \to e^+ + e^-$ reaction results in a nonlocal spin structure of the matrix element and in an additional amplitude. This makes the study of the hadron structure much more involved with respect to the case of the one–photon–exchange mechanism.

At our knowledge, the annihilation reaction $\bar{p}+p \to \ell^+ + \ell^-$, $\ell = e$ or μ was firstly considered in Ref. [21] in the case of unpolarized particles, the general case of polarized initial particles (antiproton beam or/and proton target) in $\bar{p}+p \to e^+ + e^-$ has been firstly discussed in Ref. [22], with particular attention to the determination of the phases of FFs, and more recently in Ref. [23], assuming the one photon exchange mechanism. In case of two photon exchange, a general, model independent analysis of polarized and unpolarized observables can be found in Ref. [24].

II. OBSERVABLES AND FORM FACTORS FOR $p+\overline{p} \to \ell^+ + \ell^-$

The calculation of the cross section and of the polarization observables for the process $\bar{p}+p \to \ell^+ + \ell^-$, $\ell = e$ or μ, in the annihilation channel are more conveniently performed in the center of mass system (CMS). Let us choose the z axis along the direction of the incoming antiproton, the y axis normal to the scattering plane, and the x axis to form a left-handed coordinate system.

The starting point of the analysis of the reaction $p+\overline{p} \to e^+ + e-$ is the standard expression of the matrix element in framework of one-photon exchange mechanism:

$$\begin{aligned}\mathcal{M} &= \frac{e^2}{q^2}\overline{u}(-k_2)\gamma_\mu u(k_1)\overline{u}(p_2)\\ &\left[F_{1N}(q^2)\gamma_\mu - \frac{\sigma_{\mu\nu}q_\nu}{2m}F_{2N}(q^2)\right]u(-p_1), \quad (1)\end{aligned}$$

where p_1, p_2, k_1 and k_2 are the four-momenta of initial antiproton and proton and the final electron and positron respectively, $q^2 > 4m^2$, $q = k_1 + k_2 = p_1 + p_2$. F_{1N} and F_{2N} are the Dirac and Pauli nucleon electromagnetic FFs, which are complex functions of the variable q^2 - in the TL region of momentum transfer.

The particular relation between the nucleon electromagnetic FFs at threshold: $G_E(q^2) = G_M(q^2)$, $q^2 = 4m^2$ is related, from a physical point of view, to the dominance of S-state.

The complete derivation of the formulas for the unpolarized cross section, the angular asymmetry and all the polarization observables can be found in Ref. [24]. Here we only stress the main properties of the interesting observables.

A. The cross section

In order to calculate the cross section when all particles are unpolarized, one has to sum the matrix element squared over the polarization of the final particles and to average over the polarization of initial particles. The cross section in CMS is given by:

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \mathcal{N}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right], \quad (2)$$

where $\tau = q^2/(4m^2)$ and $\mathcal{N} = \alpha^2/(4\sqrt{q^2(q^2-4m^2)})$ is a kinematical factor. This formula was firstly obtained in Ref. [21].

The angular dependence of the cross section, Eq. (2), results directly from the assumption of one-photon exchange, where the photon has spin one and the electromagnetic hadron interaction satisfies the $P-$invariance. Therefore, the measurement of the differential cross section at three angles (or more) would also allow to test the presence of 2γ exchange.

The electric and the magnetic FFs are weighted by different angular terms in the cross section, Eq. (2). One can define an angular asymmetry, $\mathcal{R}$, with respect to σ_0, the differential cross section measured at $\theta = \pi/2$, σ_0 [25]:

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \sigma_0\left[1+\mathcal{R}\cos^2\theta\right], \quad (3)$$

where $\mathcal{R}$ can be expressed as a function of FFs:

$$\mathcal{R} = \frac{\tau|G_M|^2 - |G_E|^2}{\tau|G_M|^2 + |G_E|^2}. \quad (4)$$

This observable should be very sensitive to the different underlying assumptions on FFs, therefore, a precise measurement of this quantity, which does not require polarized particles, would be very interesting.

The total cross section contains the moduli squared of the FFs:

$$\sigma(q^2) = \mathcal{N}\frac{8}{3}\pi\left[2|G_M|^2 + \frac{1}{\tau}|G_E|^2\right]. \quad (5)$$

FFs are complex in TL region and polarization phenomena allow to determine their relative phase.

B. Polarization observables

In case of polarized antiproton beam with polarization $\vec{P}$, the only non zero analyzing power is related to P_y and the terms related to $|G_E|^2$ and $|G_M|^2$ vanish:

$$\left(\frac{d\sigma}{d\Omega}\right)_0 A_y = \frac{\mathcal{N}}{\sqrt{\tau}}\sin 2\theta Im(G_M G_E^*). \quad (6)$$

It differs only by sign from the corresponding analyzing power when the target is polarized. One can see that this analyzing power, being T-odd, does not vanish in $p+\overline{p} \to \ell^+ + \ell^-$, even in one-photon approximation, due to the fact that FFs are complex in time-like region. This is a principal difference with elastic ep scattering. Let us note also that the assumption $G_E = G_M$ implies $A_y = 0$, independently from any model taken for the calculation of FFs.

In the case when both the proton and the antiproton are polarized, among the nine possible terms, the correlation coefficients A_{xy}, A_{yx}, A_{zy}, and A_{yz}, vanish, while A_{xx}, A_{yy}, and A_{zz} contain some combinations of the FFs moduli squared. The most interesting nonzero components are:

$$\left(\frac{d\sigma}{d\Omega}\right)_0 A_{xz} = \left(\frac{d\sigma}{d\Omega}\right)_0 A_{zx} = \frac{1}{\sqrt{\tau}} \sin 2\theta Re G_E G_M^* \mathcal{N}, \tag{7}$$

which contain the real part of the product $G_E G_M^*$.

Therefore, in order to determine the relative phase of FFs, in TL region, the interesting observables are A_y, and A_{xz} which contain, respectively, the imaginary and the real part of the product $G_E G_M^*$.

III. TWO PHOTON EXCHANGE

The matrix element, in presence of the TPE mechanism, contains three complexe amplitudes, functions of two kinematical variables (s and q^2), instead that two FFs, functions only of q^2. Note that, unlike the case of elastic electron–nucleon scattering in the Born approximation, the hadronic tensor in the time–like region contains a symmetric part even in the Born approximation due to the fact that nucleon FFs are complex. Taking into account the TPE contribution leads to additional terms in the symmetric part of this tensor. If the charges of the final leptons are not detected, then the interference term between the one- and two–photon–exchange channels will not contribute to the differential cross section. This symmetry between the positron and the electron can then be used either to eliminate or extract the contribution of the TPE mechanism.

Tn case of two photon exchange, the differential cross section of the reaction $p+\overline{p} \to e^+ + e-$ for the case of unpolarized particles has the form:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4q^2}\sqrt{\frac{\tau}{\tau-1}} D, \tag{8}$$

$$\begin{aligned} D &= (1+\cos^2\theta)(|G_M|^2 + 2Re G_M \Delta G_M^*) + \\ & \frac{1}{\tau}\sin^2\theta(|G_E|^2 + 2Re G_E \Delta G_E^*) + \\ & 2\sqrt{\tau(\tau-1)}\cos\theta \sin^2\theta Re(\frac{1}{\tau}G_E - G_M)F_3^*. \end{aligned}$$

In the Born approximation, at the reaction threshold where $q^2 = 4m^2$, one has $G_M = G_E$ and the differential cross section becomes $\theta-$independent . This is not anymore true in presence of TPE terms. The TPE contribution is an odd function of the variable $\cos\theta$ and does not contribute to the differential cross section for $\theta = 90^0$.

A. Single spin Polarization observables

The presence of a symmetrical part in the hadronic tensor (5) leads to a non–zero single–spin asymmetry which can be written as

$$\begin{aligned} A_y(\theta) &= \frac{2\sin\theta}{\sqrt{\tau}D}\Big[\\ & \cos\theta\ Im(G_M G_E^* + G_M \Delta G_E^* - G_E \Delta G_M^*) \\ & + \sqrt{\tau(\tau-1)} Im(\cos^2\theta\ G_M + \sin^2\theta\ G_E)F_3^*\Big]. \end{aligned} \tag{9}$$

Again, in the Born approximation this expression reduces to the result of Ref. [22, 23]. One can see that $A_y(\theta)$ is determined by the spin vector component which is perpendicular to the reaction plane. $A_y(\theta)$, being a T–odd quantity, does not vanish even in the one–photon–exchange approximation due to the complex nature of the nucleon FFs in the time–like region. This is a principal difference with the elastic electron–nucleon scattering. Let us consider two particular kinematical cases. For $\theta = 90^0$, $A_y(\theta)$ vanishes in the Born approximation. The presence of the TPE contributions leads to a non–zero value of this quantity:

$$A_y(90^0) = 2\frac{\sqrt{\tau-1}}{\overline{D}} Im G_E F_3^*, \quad \overline{D} = D(\theta = 90^0),$$

which is expected to be small due to the fact that it is determined by the interference of the one–photon and two–photon exchange amplitudes and is of the order of α. This asymmetry is an increasing function of the variable q^2, due, on one side, to the presence of the kinematical factor containing τ and on the other side, to the steep decreasing of the nucleon FFs with q^2 while the TPE mechanism becomes more important when q^2 increases. So, the measurement of this asymmetry at $\theta = 90^0$ as a function of q^2 can give information about the size of the TPE contribution.

At threshold, in the Born approximation, $A_y^{th}(\theta)$ has to vanish, due to the relation $G_E = G_M$. Including the TPE contributions, at threshold, this asymmetry can still be equal to zero, if $\Delta G_E = \Delta G_M$. In this case the differential cross section does not contain any explicit dependence on the angular variable θ, but only through the amplitudes $\Delta G_{E,M}$ which, in the general case, depend on the variable θ. In the expression of the cross section (8), the contribution of the one–photon–exchange diagram leads to an even function of $\cos\theta$, whereas the TPE contribution leads to four new terms which are smaller by a factor of α.

This property can by used to suggest a method to determine the FFs, even in presence of TPE. The TPE contribution can be canceled (extracted) by considering the sum (difference) of the differential cross section at two complementary angles:

$$\frac{d\sigma_+}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma_+}{d\Omega}(\pi - \theta) = 2\frac{d\sigma^{Born}}{d\Omega}(\theta). \quad (10)$$

Taking this sum at two different angles, one can determine the ratio of the moduli of the FFs, $R = |G_E|/|G_M|$:

$$\frac{d\sigma_+}{d\Omega}(\theta_1) : \frac{d\sigma_+}{d\Omega}(\theta_2) = \frac{\tau(1+x_1^2) + (1-x_1^2)R^2}{\tau(1+x_2^2) + (1-x_2^2)R^2}, \quad (11)$$

with $x_i = \cos\theta_i$. Similar symmetry properties apply to the asymmetry:

$$\begin{aligned} \Delta A_y(\theta) &= A_y(\theta) - A_y(\pi - \theta) \\ &= 2\sin(2\theta)\frac{\sqrt{\tau}R\sin\delta}{\tau(1+x_2^2) + (1-x_2^2)R^2}, \quad (12) \end{aligned}$$

where δ is the relative phase between G_E and G_M.

IV. CONCLUSIONS

Model independent relations among TL form factors and experimental observables for the reaction $\bar{p} + p \to \ell + +\ell^-$, $\ell = \mu$ or e are derived. C-invariance requires specific symmetry properties of the differential cross section and of single polarization observables which allow to select the TPE mechanism, if present. In this case, it is still possible to extract nucleon FFs, but with a more complicated procedure. In SL region, it would require the measurement of three T-odd or five T-even polarization observables, or the measurement of the differential cross section for electron and positron scattering in the same kinematical conditions. In TL region, precise measurements of the differential cross section and of one spin observables at different angles allow the determination of the moduli of the FFs and of their relative phase.

No experimental evidence has been found up to now of the presence of TPE in the existing data at moderate Q^2. Moreover, an exact calculation, possible in QED, shows that TPE amplitude is very small [26]. The measurement of such mechanism would invalidate most of the results obtained in electroproduction experiments on hadrons. A more realistic and effective way to reconcile polarized and unpolarized measurements for *ep* elastic scattering has been suggested in Ref. [17], where the necessity to take into account higher order radiative corrections particularly in the electron vertex, has been pointed out.

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