Meridional circulation in the radiation zones of rotating stars: origins, behaviors and consequences on stellar evolution

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Stellar radiation zones are the seat of meridional currents. This circulation has a strong impact on the transport of angular momentum and the mixing of chemicals that modify the evolution of stars. First, we recall in details the dynamical processes that are taking place in differentially rotating stellar radiation zones and the assumptions which are adopted for their modelling in stellar evolution. Then, we present our new results of numerical simulations which allow us to follow in 2D the secular hydrodynamics of rotating stars, assuming that anisotropic turbulence enforces a shellular rotation law and taking into account the transport of angular momentum by internal gravity waves. The different behaviors of the meridional circulation in function of the type of stars which is studied is discussed with their physical origin and their consequences on the transport of angular momentum and of chemicals. Finally, we show how this work is leading to a dynamical vision of the evolution of rotating stars from their birth to their death.

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1 Dynamics of stellar radiation zones

Rotation, and more precisely differential rotation, has a major impact on the internal dynamics of stars. First, as it is known from the theory of rotating stars, rotation induces some large-scale meridional circulation in radiation zones, which act to transport simultaneously the angular momentum, the chemicals and the magnetic field by advection. This circulation is due to the differential rotation, the transport of angular momentum and the action of the perturbing forces on the thermal imbalance, namely the centrifugal and the Lorentz forces (cf. Busse 1982, Zahn 1992, Maeder & Zahn 1998, Garaud 2002, Mathis & Zahn 2004-2005, Rieutord 2006). Next, differential rotation induces hydrodynamical turbulence in radiative regions through various instabilities: the secular and the dynamical shear instabilities, the baroclinic and the multidiffusive instabilities. This hydrodynamical turbulence acts to reduce the gradients of angular velocity and of chemical composition; this is the reason why it is modelled as a diffusive process (cf. Talon & Zahn 1997, Maeder 2003, Mathis et al. 2004). On the other hand, rotation has a strong impact on the stellar magnetism. In radiation regions, it interacts with fossil magnetic fields, the associated secular torque of the Lorentz force and the magnetohydrodynamical instabilities such as the Tayler-Spruit and the multidiffusive magnetic instabilities having a strong impact on the transport of angular momentum and of chemicals (cf. Charbonneau & Mac Gregor 1993; Garaud 2002; Mathis & Zahn 2005; Spruit 1999; Spruit 2002; Menou et al. 2004; Maeder & Meynet 2004; Eggenberger et al. 2005; Braithwaite & Spruit 2005; Braithwaite 2006; Brun & Zahn 2006; Kitchatinov & Rüdiger 2007; Zahn, Brun & Mathis 2007). Finally, internal gravity waves, which are excited at the borders of convective zones, propagate inside radiation zones where they extract or deposit angular momentum where they are damped leading to a modification of the angular velocity profile and thus of the chemicals distribution due to the shear modification (cf. Goldreich & Nicholson 1989, Talon et al. 2002, Talon & Charbonnel 2005). Note also that rotation modifies stellar winds and mass loss (cf. Maeder 1999). Therefore, differential rotation has impera-



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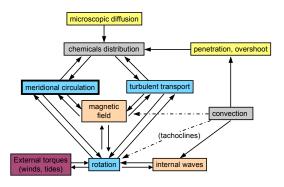


Fig. 1 Dynamical transport processes in stellar radiation zones: those regions are the seat of highly non-linear interactions between the differential rotation, the meridional circulation, the turbulence, the magnetic field and the internal waves.

tively to be taken into account to get a coherent picture of the internal dynamics and of the evolution of the stars.

2 Modelling

To obtain a dynamical vision of stellar radiation zones, the complete equations of (magneto)hydrodynamics have thus to be solved. To achieve this goal, assumptions are made. First, secular effects and their consequences on stellar evolution are studied. Therefore, secular time-scales associated to the nuclear evolution of stars are chosen. On the other hand, stellar radiation regions are stably stratified regions. Then, the buoyancy force, which is the main restoring force, acts to inhibit turbulent motions in the vertical direction. This leads to a strongly anisotropic turbulent transport which is more efficient in the horizontal direction (on an isobar) than in the vertical one. Therefore, horizontal eddytransport coefficients are larger than those in the vertical direction so that the horizontal gradients of scalar fields such as rotation, temperature and chemical concentration smaller than their vertical gradients (cf. Zahn 1992, Maeder & Zahn 1998 and Mathis & Zahn 2004). Finally, physical processes which have dynamical time-scales and need a high angular resolution description, such as hydro- or magnetohydrodynamical instabilities and turbulence, are treated using prescriptions issued from laboratory experiments or numerical simulations. From the technical point of view, the dynamical equations are expanded in spherical functions (cf. Mathis & Zahn 2005 and references therein), the modal lagrangian equation in time and radius which are obtained being directly implemented in stellar evolution codes which are mostly one-dimensional. This is the first step to achieve the highly multi-scales problem in time and in space of the dynamical stellar evolution that could not be yet studied with Direct Numerical Simulations.

3 Numerical simulation of secular transport: the hydrodynamical case with a 'shellular' rotation

Here, we focus our study on the highly non-linear interactions between the differential rotation, the meridional circulation, the turbulence and the internal waves (the magnetic field B is not taken into account). The numerical simulations of this work have been computed with stellar evolution codes STAREVOL and the Geneva code (the reader is referred respectively to Palacios et al. 2006 and Meynet & Maeder 2000 for their detailed description). The hydrodynamical case, where the differential rotation is assumed to be shellular, $\overline{\Omega}(r)$, due to the strong horizontal turbulence which enforces the angular velocity to be constant on an isobar, is considered. The transport equations, namely the Navier-Stockes, the continuity and the heat transport equations, and their associated boundary conditions are implemented following Mathis & Zahn (2004) in STAREVOL and Maeder & Zahn (1998) in the Geneva code. On the other hand, the vertical and the horizontal shear-induced turbulent transports are treated using the prescriptions derived in Zahn (1992), Talon & Zahn (1997), Maeder (2003) and Mathis et al. (2004).

The results presented here are issued from the numerical simulation of the evolution of a $1.5~M_{\odot}$ star with a solar metallicity (Z = 0.02) and an initial equatorial rotation velocity $V_{\rm ini} = 100 \ {\rm km \cdot s^{-1}}$ computed with STAREVOL. The age of the presented model is 1.7×10^9 yrs with a central hydrogen mass fraction $X_{\rm c}=0.32.$ We follow for each time-step the internal hydrodynamics of the radiation zone(s) of the star: the differential rotation profile, the associated temperature and mean molecular weight fluctuations and the meridional circulation pattern (see Fig. 2). Moreover, diagnosis tools have been developed to identify what are the dominant physical processes which are driving the angular momentum transport, the meridional circulation and the chemicals mixing. Looking at Fig. (3), one can identify that meridional circulation is mainly driven by the barotropic and the thermal diffusion terms. The former one is associated to the perturbation of the thermal imbalance in the stellar radiation region by the centrifugal force. In the case of a uniform rotation, it is the only term which remains. The term due to the nuclear energy production has to be taken into account only in the region where nuclear reactions occur, here in the centre, while the non-stationary term is completely negligible for this main-sequence star where structural adjustments are weak. Besides, if we study the value of the meridional circulation using the angular momentum equation, the circulation is mainly driven in the external part of the star by the extraction of angular momentum associated to the surface braking. The circulation then allows the advection of the angular momentum to the surface and the concept of the "wind-driven" meridional circulation which has been proposed by Zahn (1992) is veriAstron. Nachr. / AN (2006) 791

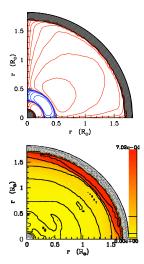


Fig. 2 Meridional circulation in a $1.5M_{\odot}$, Z_{\odot} rotating star when around half of the central hydrogen is burnt. **Top:** meridional currents where red and blue loops turn anticlockwise and clockwise respectively. **Bottom:** norm of the meridional circulation (scale in cm·s⁻¹). Hatched areas indicate the convective zones.

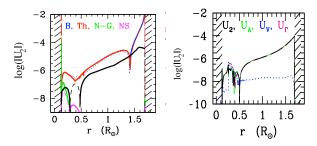


Fig. 3 Decompositions of the radial function U_2 (where the radial component U_r is: $U_r = U_2 \, (r) \, P_2 \, (\cos \theta)$) of the meridional circulation presented in Fig. (2). Left: the barocline, $U_{\rm B}$, and thermal diffusion, $U_{\rm Th}$, terms are dominant while the nuclear and gravitational heating, $U_{\rm N-G}$, and the non-stationary, $U_{\rm NS}$, terms are only perturbations. Right: in the external part $(r>0.5 \, R_{\odot})$ the meridional circulation is mainly driven by the angular momentum extraction, U_{Γ} , the shear contribution, $U_{\rm V}$, becoming of the same order of magnitude only in the inner part.

fied. Then, in the case of a star which does not loose angular momentum through winds or mass-loss, we achieved a quasi-asymptotic state where the differential rotation and the circulation are adjusting themselves to the lagrangian structural adjustments. Finally, if internal gravity waves are taken into account (cf. Talon & Charbonnel 2005), a complex multi-cellular pattern appears which is due to the successive extraction fronts of angular momentum which are driven by the surface braking. Thus, once again we get a "wind-driven" circulation.

Moreover, the angular momentum transport is here dominated by the advection by the meridional circulation while its flux transported by the shear-induced turbulence is smaller at least by an order of magnitude except in the region near the centre. Finally, concerning the transport coefficients, the meridional circulation is the dominating process in the transport of chemicals while the fundamental hypothesis concerning the anisotropic turbulent transport is verified.

4 Conclusion

In this work, a coherent description of the dynamical transport processes which take place in stellar radiation zones has been undertaken. Each of them and their respective effects on angular momentum and chemical transport are identified and modelled in a consistent way. A two-dimensional picture of the internal dynamics of stellar radiation zones is obtained and the first step of the numerical implementation of the theoretical results in stellar evolution codes, namely the purely hydrodynamical case where it is assumed that the strong horizontal turbulent transport enforces a shellular rotation law, has been achieved. Work is now in progress to implement the differential rotation in latitude, the magnetic field and the gravito-inertial waves while theoretical work is engaged to derive prescriptions for MHD instabilities and waves excitation. Thus, we are now entering in a new exciting period of the story of stellar evolution where we hope to be able to obtain a dynamical vision of the Hertzsprung-Russel diagram in support to space and ground based instrumentations.

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