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Stochastic wave excitation in rotating stars

S. Mathis[1,2], K. Belkacem[3], M. J. Goupil[3]

[1] CEA/DSM/IRFU/SAp, Laboratoire AIM CEA/DSM, CNRS, Université Paris Diderot, F-91191 Gif-sur-Yvette Cedex, France
[2] Observatoire de Paris, LUTH, CNRS, UMR 8102, F-92195 Meudon, France
[3] Observatoire de Paris, LESIA, CNRS, UMR 8109, F-92195 Meudon, France

Abstract

Wave propagation, excitation and associated transport are modified by the Coriolis and the centrifugal accelerations in rotating stars. In this work, we focus on the influence of the Coriolis acceleration on the volumetric stochastic excitation in convection zones of rotating stars. First, we present the complete formalism which has been derived and discuss the different terms which appear due to the Coriolis acceleration. Then, we use this formalism to compute the solar mode excitation rates and emphasize the peculiar behavior due to rotation. Consequences on wave transport in rotating stars are eventually discussed.

Session: STARS: convection

Introduction

The motivation of this work is to investigate the effect of uniform rotation on the stochastically excited modes amplitudes. Several issues can be addressed; is the amplitude of an non-axisymmetric mode ($m \neq 0$) the same as for an axisymmetric one ($m = 0$)? Are prograde and retrograde modes excited in the same manner and what are the consequences? This can have some important consequences from both an observational point of view as well as a theoretical one.

As a first step, we neglect the centrifugal acceleration that induces a deformation of the star. We then focus our attention on the effect of the Coriolis acceleration. We also restrict the study to uniform rotation. In the first section we present a formalism of stochastic excitation developed for a rotating star, and we apply it to the solar case by performing a perturbative development. Some consequences on the angular momentum transport by modes are briefly discussed in a third section. Conclusion and perspectives are provided in the last section.

Physical assumptions and formalism

Following Samadi & Goupil (2001) and Belkacem et al. (2008), we establish the inhomogeneous wave equation

$$\left(\partial_{t^2}-\vec{L}_\Omega\right)\vec{v}_{\rm osc}+\vec{\mathcal{C}}\left(\vec{v}_{\rm osc},\vec{u}_t\right)=\vec{\mathcal{S}}_t\left(\vec{u}_t\right).^1 \quad (1)$$

$\vec{v}_{\rm osc}$ is the velocity field of the waves and $\vec{u}_t$ is the turbulent one associated to the convective eddies. The $\vec{\mathcal{C}}\left(\vec{v}_{\rm osc},\vec{u}_t\right)$ vector field, which is not detailed here, is related to the wave-turbulence interaction that corresponds to the dynamical damping, η. $\vec{L}_\Omega$ is the operator that rules the wave dynamics in the case of the star free oscillations

$$\begin{aligned}\vec{L}_\Omega\left(\vec{v}_{\rm osc}\right)=&\ \vec{\nabla}\left[\alpha_s\vec{v}_{\rm osc}\cdot\vec{\nabla}s_0+{c_s}^2\vec{\nabla}\cdot\left(\rho_0\vec{v}_{\rm osc}\right)\right]-\vec{g}_{\rm eff}\vec{\nabla}\cdot\left(\rho_0\vec{v}_{\rm osc}\right)\\ &-\ \rho_0\Omega\partial_{t,\varphi}\vec{v}_{\rm osc}-2\,\rho_0\vec{\Omega}\times\partial_t\vec{v}_{\rm osc}-\rho_0 r\sin\theta\left(\partial_t\vec{v}_{\rm osc}\cdot\vec{\nabla}\Omega\right)\widehat{\mathbf{e}}_\varphi\,.\end{aligned} \quad (2)$$

ρ, $g_{\rm eff}$ and Ω are respectively the fluid density, effective gravity acceleration (including the centrifugal one) and angular velocity. c_s is the sound speed while $\alpha_s=\left(\partial p/\partial s\right)_\rho$, where p is the pressure and s the macroscopic entropy. X_0 and X_1 (where $X=\{\rho,p,s\}$) are respectively the hydrostatic value of X and its wave-associated fluctuation. Finally, (r,θ,φ) are the classical spherical coordinates.

The source terms that drive the eigenmodes are

$$\vec{\mathcal{S}}_t\left(\vec{u}_t\right)=\vec{\mathcal{S}}_{\rm SG}-\partial_t\left[\rho_1\left(\Omega\partial_\varphi\vec{u}_t+2\vec{\Omega}\times\vec{u}_t+r\sin\theta\left(\vec{u}_t\cdot\vec{\nabla}\Omega\right)\widehat{\mathbf{e}}_\varphi\right)\right]\,. \quad (3)$$

The term $\vec{\mathcal{S}}_{\rm SG}$ contains the source terms as derived by Samadi & Goupil (2001) and Belkacem et al. (2008), in which the dominant ones are the Reynolds and entropy contributions. The three last terms are those induced by rotation and can be re-expressed such as

$$\partial_t\left(2\,\rho_1\vec{\Omega}\times\vec{u}_t\right)=2\vec{\Omega}\times\partial_t\left(\rho_1\vec{u}_t\right)=-2\vec{\Omega}\times\left[\vec{\nabla}\cdot\left(\rho_0\vec{u}_t\right)\,\vec{u}_t\right] \quad (4)$$

$$\partial_t\left(\rho_1\Omega\partial_\varphi\vec{u}_t\right)=\Omega\left[\vec{\nabla}\cdot\left(\rho_0\vec{u}_t\right)\partial_\varphi\vec{u}_t\right] \quad (5)$$

$$\partial_t\left[\rho_1 r\,\sin\theta\left(\vec{u}_t\cdot\vec{\nabla}\Omega\right)\right]=-r\sin\theta\left[\vec{\nabla}\cdot\left(\rho_0\vec{u}_t\right)\vec{u}_t\right]\cdot\vec{\nabla}\Omega, \quad (6)$$

[1] Here the following notation for partial derivatives $\frac{\partial^n f}{\partial x^n}=\partial_{x^n}f$ is adopted.

where $\vec{\Omega}$ is supposed uniform and steady on a dynamical time scale. These last three terms scale as M_t^3 (M_t is the turbulent Mach number), while Samadi & Goupil (2001) have shown the Reynolds contribution scales as M_t^2. Thus the above rotational contribution can be ignored in front of the Reynolds one. Moreover, in the case where the turbulent convective motions are assumed to be anelastic ($\vec{\nabla}\cdot(\rho_0\vec{u}_t)=0$), they can be neglected. Therefore, the only source terms we must retain are the Reynolds and the entropy ones.

Following the procedure detailed by Samadi & Goupil (2001) and Belkacem et al. (2008), the power supplied into the modes (P) is derived

$$P=\left(C_R^2+C_S^2+C_c^2\right)/\left(8I\right) \tag{7}$$

where C_R^2, C_S^2, and C_c^2 are respectively the contributions of the Reynolds stresses, of the entropy fluctuation advection, and the crossed terms. The crossed terms are ignored in front of C_R^2 and C_S^2 (see Belkacem et al. 2008 for details). In addition, as shown above the source terms related to the rotation have been neglected.

The Reynolds stresses contribution is given by

$$C_R^2=4\pi^3\int \mathrm{d}m\;\,R(r)\;\,S_R(\omega_0), \tag{8}$$

where

$$S_R\left(\omega_0\right)=\int\frac{\mathrm{d}k}{k^2}\;E^2(k)\;\int\mathrm{d}\omega\;\chi_k(\omega+\omega_0)\;\chi_k(\omega)\,. \tag{9}$$

$E(k)$ and $\chi_k(\omega)$ are respectively the kinetic energy spectrum and the temporal correlation function which are also modified through the action of rotation on turbulence. The frequencies ω_0 and ω are associated with pulsation and convection, respectively. Furthermore

$$R(r) = R_{\rm spheroidal}+R_{\rm toroidal} \tag{10}$$

$$\text{with}\quad R_{\rm toroidal} = \frac{11}{15}L^2\left|\frac{\mathrm{d}\xi_T}{\mathrm{d}r}-\frac{\xi_T}{r}\right|^2 + \left|\frac{\xi_T}{r}\right|^2\left(\frac{11}{5}L^2(L^2-2)-\frac{8}{5}\mathcal{F}_{\ell,|m|}-\frac{2}{3}L^2\right) \tag{11}$$

where $R_{\rm spheroidal}$ is given by Eq. (23) of Belkacem et al. (2008), $\mathcal{F}_{\ell,|m|}=\frac{|m|(2\ell+1)}{2}\left[L^2-(m^2+1)\right]$, $L^2=\ell(\ell+1)$, and (ξ_r,ξ_H,ξ_T) are the radial, horizontal and toroidal components of the eigenfunction corresponding to a spherical harmonic (Y_ℓ^m). $R_{\rm spheroidal}$ corresponds to the non-rotating case.

It is modified by Coriolis acceleration through the modification of ξ_r and of ξ_H it induces.

The second source term, the entropy fluctuation contribution is obtained as

$$C_S^2 = \frac{4\pi^3\,\mathcal{H}}{\omega_0^2} \int \frac{\mathrm{d}r}{r^2}\,\alpha_s^2 \left(\left| D_\ell\, \frac{\mathrm{d}\,(\ln \mid \alpha_s \mid)}{\mathrm{d}\ln r} - \frac{\mathrm{d}D_\ell}{\mathrm{d}\ln r} \right|^2 + L^2\ |D_\ell|^2 \right) \mathcal{S}_S(\omega_0) \quad (12)$$

with $\mathcal{H}$ an anisotropy factor (defined in Samadi & Goupil 2001), $D_\ell = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\xi_r\right) - \frac{L^2}{r}\,\xi_H$ and

$$\mathcal{S}_S(\omega_0) = \int \frac{\mathrm{d}k}{k^4}\,E(k)\,E_s(k) \int \mathrm{d}\omega\,\chi_k(\omega_0+\omega)\,\chi_k(\omega), \quad (13)$$

E_s being the spectrum associated to the entropy turbulent fluctuations. As for $R_{\mathrm{spheroidal}}$ no direct change are due to uniform rotation.

Application to the excitation of solar oscillation modes

In this section, we apply the formalism to spheroidal solar oscillation modes for which $2\Omega/\omega_0$ (where ω_0 is (hereafter) the mode frequency in the non-rotating case) is such that they are only slightly perturbed by the Coriolis acceleration. In this case, we get respectively for each displacement eigenmode component (cf. Unno et al. 1989)

$$\xi_\alpha = \xi_\alpha^{(0)} + m\left(\frac{2\Omega}{\omega_0}\right)\xi_\alpha^{(1)} \quad \text{and} \quad \xi_T = \left(\frac{2\Omega}{\omega_0}\right)\xi_T^{(1)} \quad (14)$$

where $\alpha = \{r, H\}$, $\xi_\alpha^{(0)}$ being the component in the non-rotating case for which $\xi_T^{(0)} = 0$, and $\xi_\alpha^{(1)}$ and $\xi_T^{(1)}$ are given by Unno et al (1989).

Using these expansions in Eq. (7), we get

$$P_{n,l,m} = P_{n,l,m}^{(0)} + m\left(\frac{2\Omega}{\omega_0}\right)P_{n,l,m}^{(1)} \quad (15)$$

so that the excitation rate is different for prograde $(m < 0)$ and retrograde $(m > 0)$[2] modes since it depends explicitly on m. To better quantify this bias introduced by the Coriolis acceleration, we define

$$\delta P_m/P_{-m} = (P_m - P_{-m})/P_{-m} \quad (16)$$

[2] The mode phase is expanded as $\exp\left[i\,(m\varphi + \omega_0 t)\right]$.

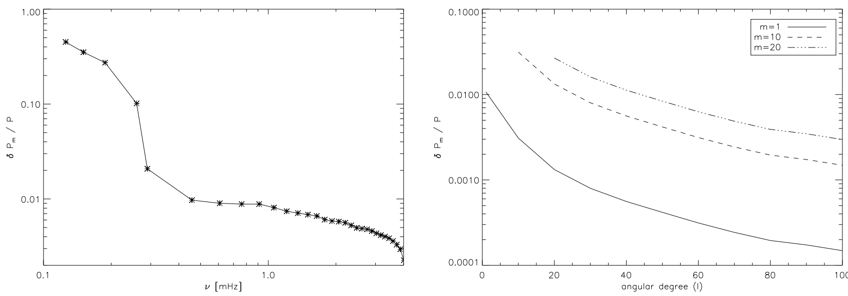

Figure 1: **Left panel:** Bias between prograde and retrograde modes (see Eq.(16)) for $\ell = 1$ modes, computed with a standard solar model. **Right panel:** Bias between prograde and retrograde modes as a function of the mode angular degree (ℓ) and for a radial order $n = 5$.

which is plotted in Fig. (1). In this first application, the simplest turbulent spectrum of Kolmogorov is assumed in evaluating Eqs (9) and (13). For low-frequency g modes that are excited in the bottom of the convection zone, where $2\Omega \approx \omega_c$ (ω_c being the convective frequency) the effect of rotation on the convective velocity field has to be taken into account (work in progress).

First, $\delta P_m/P_{-m}$ scales as $2\Omega/\omega_0$. Therefore, in the solar case, we find that acoustic mode excitation rates are only weakly affected by the Coriolis acceleration while gravity modes are affected up to $50\,\%$ for the most low-frequency modes. On the other hand, for a given m, $\delta P_m/P_{-m}$ increases for decreasing ℓ (it becomes maximum for $l = |m|$); in other words the bias is stronger for low-ℓ degrees.

Mode-induced transport

Let us now examine the mode-induced transport of angular momentum. The eulerian flux of angular momentum introduced by the Reynolds stresses, for each azimuthal order m is given by (see Lee & Saio 1993)

$$\mathcal{F}_{\rm AM;m} = \int_{4\pi} \rho_0 r \sin\theta \, u_{r;m} u^*_{\varphi;m} {\rm d}\widetilde{\Omega} \quad \text{where} \quad \vec{u} = i\,\omega_0\,\vec{\xi}, \tag{17}$$

${\rm d}\widetilde{\Omega} = \sin\theta {\rm d}\theta {\rm d}\varphi$ being the solid angle. In the non-rotating adiabatic case, we get

$$\mathcal{F}_{\rm AM;m} + \mathcal{F}_{\rm AM;-m} = 0; \tag{18}$$

therefore, modes do not transport any net flux of angular momentum. In the rotating dissipative case, introducing Eq.(14) into Eq.(17) , we get

$$\begin{aligned}&\mathcal{F}_{\rm AM;m}+\mathcal{F}_{\rm AM;-m}=\\&-2\mathcal{D}\left(\eta\right)\rho_0 r\sum_{n,l}\omega_0^2\left(2\Omega/\omega_0\right)\mathcal{I}_m\left\{\left[A^2_{l,m}\right]^{(0)}\ \mathcal{G}_1+\left[A^2_{l,m}\right]^{(1)}\ \mathcal{G}_2\right\}\neq 0\end{aligned}\tag{19}$$

where $\mathcal{I}_m$ denotes the imaginary part, and

$$\mathcal{G}_1=\left[m^2\left(\xi^{(1)}_{r;l}\xi^{(0)*}_{H;l}+\xi^{(0)}_{r;l}\xi^{(1)*}_{H;l}\right)+\xi^{(0)}_{r;l}\left(\alpha^m_{l-1}\xi^{(1)}_{T;l-1}-\beta^m_{l+1}\xi^{(1)}_{T;l+1}\right)^*\right]\tag{20}$$

$$\mathcal{G}_2=-m^2\,\xi^{(0)}_{r;l}\xi^{(0)*}_{H;l}\tag{21}$$

with

$$\alpha_{\ell,m}=\ell\sqrt{\frac{\left(\ell+1\right)^2-m^2}{\left(2\ell+1\right)\left(2\ell+3\right)}}\quad\text{and}\quad\beta_{\ell,m}=\left(\ell+1\right)\sqrt{\frac{\ell^2-m^2}{\left(2\ell+1\right)\left(2\ell-1\right)}}.\tag{22}$$

$\mathcal{D}$ corresponds to the phase shift between $u_{r;m}$ and $u_{\varphi;m}$ due to dissipative processes (e.g., the thermal diffusion or the viscous friction) that causes a net transport of angular momentum (Goldreich & Nicholson 1989). Here, we assume that the damping is quasi-independent of m since $\omega_0\gg m\,\Omega$ for the considered modes. Then, the amplitude is developed as for the power, *i.e.*

$$A^2_{\ell,m}=\left[A^2_{\ell,m}\right]^{(0)}+m\left(\frac{2\Omega}{\omega_0}\right)\left[A^2_{\ell,m}\right]^{(1)}\tag{23}$$

$$=\frac{P^{(0)}}{2\eta I\omega_0^2}+m\left(\frac{2\Omega}{\omega_0}\right)\frac{P^{(1)}}{2\eta I\omega_0^2}\,.\tag{24}$$

Therefore, the Coriolis acceleration introduces extra biases between prograde and retrograde waves through the modifications of the eigenfunctions ($\mathcal{G}_1$) and of the excitation rate ($[A^2_{l,m}]^{(1)}$).

Conclusion

In this work, we derive the formalism that allows to treat the stochastic excitation of modes by convective regions in presence of rotation. Then, we applied it, as a first application, to the solar spheroidal oscillations. We show that a bias between pro- and retrograde waves is introduced in the excitation by the Coriolis acceleration. It can be relatively important for low-frequency g-modes while it is quite negligible for acoustic ones. We showed that the azimuthal asymmetries both in eigenfunctions and their excitation rates introduce an extra contribution. The associated mode-induced transport of angular momentum remains to be quantified as in Talon & Charbonnel (2005).

Future works must apply the formalism to the case of rapid rotators for both inertial and gravito-inertial modes (Dintrans & Rieutord 2000, Rieutord et al. 2001) and include the effect of differential rotation.

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DISCUSSION

Guzik: Do you see the asymmetry in excitation introduced by the Coriolis acceleration in any stellar g-mode data? Where we should look for it?

Mathis: I have not seen it yet in any data, but we have to look at this to eventually get an additional observational constraint on gravity mode behaviour.