Relativistically invariant analysis of Δ–isobar production in deuteron electrodisintegration: $e^- + d \to e^- + \Delta + N$: general analysis of polarization effects

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Abstract

The differential cross section and the polarization observables for Δ–isobar production in the deuteron electrodisintegration process, $e^- + d \to e^- + \Delta + N$, are calculated in a general formalism based on structure functions. The obtained expressions have a general nature, hold for one–photon–exchange, assuming P–invariance of the electromagnetic interaction and the conservation of the hadron electromagnetic current. The dependence of the differential cross section of the $e^- + d \to e^- + \Delta + N$ reaction on the vector and tensor polarizations of the deuteron target with unpolarized and longitudinally polarized electrons is considered. The general dependence of the asymmetries on two of five kinematic variables, the azimuthal angle φ and ϵ (linear polarization of the virtual photon) is calculated. A similar analysis is performed for the polarization of the nucleon produced in $\gamma^* d \to \Delta N$ reaction provided the electron beam is unpolarized or longitudinally polarized. Polarization effects, which are due to the strong $\Delta N-$ interaction in the final state are calculated. The photoproduction of the Δ–isobar on the deuteron target has been considered in detail, as a particular case. The differential cross section and various polarization observables have been derived in terms of the reaction amplitudes. The polarization observables due to the linear and circular polarizations of the photon, when the deuteron target is arbitrarily polarized have been derived in terms of the reaction amplitudes. The polarization of the final nucleon is also considered.

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1 Introduction

It is well known, that the Δ resonance dominates in the pion production processes and plays an important role in the physics driven by the strong interaction. Moreover, the mechanism of Δ resonance excitation has a dominant role in various nuclear phenomena at energies higher than the pion–production threshold (for the details see the reviews [1]).

High–precision measurements of the $N \to \Delta$ transition induced by photon (real or virtual) became possible with the availability of high–intensity GeV–energy electron beam facilities (such as Jefferson Laboratory, Bates, ELSA, MAMI) and of the high performance spectrometers, detectors and polarimeters (for recent review see [2]). It was predicted theoretically and proved experimentally that the electromagnetic $N \to \Delta$ transition is dominated by the magnetic ($M1$) dipole transition and that the two other quadrupole transitions (electric ($E2$) and Coulomb ($C2$)) are small. At moderate Q^2 (transfer momentum squared) the ratios $E2/M1$ and $C2/M1$ are at the level of a few percent.

The data on the $\pi^- p$ channel (both for the case of the photoproduction and electroproduction) from a neutron (deuterium) target are rather scarce, but of current interest in various accelerators. For example, recent results have been obtained to measure cross sections and various polarization observables for the $\pi^+ n$ [3] and $\pi^- p$ [4] channels and a program is ongoing at Jlab (CLAS collaboration).

The processes of the Δ–isobar excitation in the scattering of the electrons by nuclei, $e^- + A \to e^- + \Delta + A'$, where A and A' are nuclear states, as well as the processes $e^- + A \to e^- + N + A$", involve both hadron electrodynamics and nuclear dynamics. Thus, the simplest process of the Δ–isobar production in eA–collisions, $e^- + d \to e^- + \Delta + N$, brings information on the electromagnetic form factors of the $\Delta^0 \to n + \gamma^*$ transition (γ^* is the virtual photon). The investigation of the pion photo– and electroproduction off the neutron (deuteron) allows to determine the isotopic properties of the hadron electromagnetic current (namely, search for the isotensor contributions in the Δ–isobar excitation), provided the background contributions can be sufficiently controlled [5]. The amplitude for the excitation of the Δ–isobar on the free proton and on the free neutron is identical as long as isotensor components can be neglected. The comparison of the electromagnetic form factors for two transitions $\Delta^+ \to p + \gamma^*$ and $\Delta^0 \to n + \gamma^*$ was used earlier to estimate the possible admixture of the isotensor component in this current [6].

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For the purpose of nuclear physics itself, the process $e^- + A \to e^- + \Delta + A'$ is important, first of all, for clarifying the old standing problem of non–nucleonic degrees of freedom in the nuclei and their role in various processes involving nuclei [7]. The problem of the Δ–isobar behavior in the nuclear medium [8] is of great interest too. In particular, for the deuteron, despite the large precision and the new region of internal momentum explored, these questions (the relative role of different possible components in the deuteron wave function as the $\Delta\Delta$–configurations, $6q$–components, etc. are still under discussion [9]).

When addressing the problem of non–nucleonic degrees of freedom (in particular, the signals of the Δ–isobar excitation) in nuclear phenomena, it is natural to start these investigations from the most simple nuclear system, the two–nucleon system (deuteron). Using this approach, one may try to see whether the explicit inclusion of the resonance degrees of freedom (the most important Δ–isobar) in the nuclear wave functions improves our knowledge of the nuclear physics.

Due to the isospin conservation in the strong interactions, the Δ–isobar can contribute to the deuteron ground wave function in form of $\Delta\Delta$–admixture. One should note that problems connected with high spin of the Δ–isobar, as the non–physical components of the Δ–isobar wave function are discussed in the literature [10]. For on–mass–shell Δ–isobar these components can be removed by imposing additional constraints on the Rarita–Schwinger field describing the Δ–isobar [11]. But in nuclei the Δ–isobar must be off–mass–shell and, therefore, unphysical degrees of freedom are still possible. Therefore, to reduce the ambiguities inherent to any model–dependent analysis, it is natural to look for the Δ–isobar in few–nucleon systems, and/or in processes where this degree of freedom is excited through the simpler and well–known electromagnetic and weak interactions. The Δ–isobar can contribute both to the one–body current through direct coupling to the photon (real or virtual), and to the two–body currents, much in the same way as it comes into play in the intermediate nucleon–nucleon interaction. Exclusive reactions are expected to convey more information than simpler inclusive experiments, even if they are obviously more demanding for the theoretical analysis.

Pioneering calculations of the $\Delta\Delta$–admixture in the deuteron wave function have been done using static transition potentials with pion–exchange only [12] and it was found that the Δ–percentage P_Δ is of the order of 1%. Later on some improvements of this simple approach were made: inclusion of the ρ–meson exchange [13] and using a coupled–channel approach [14]. It was found that $P_\Delta \approx (0.4 - 0.8)\%$. The possible Δ–isobar configurations in nuclear wave functions have been searched for the three–nucleon systems, ^3He and ^3H.

As it was noted [7], resonances are a more efficient source of high momentum components in nuclear wave functions than the short–range repulsion in the nuclear force. As a result, a typical resonance Fermi momentum is about 0.3 to 0.6 GeV/c as compared to 0.1 GeV/c for a nucleon in the deuteron.

The experimental investigation of the polarization effects in various processes of the electron–deuteron scattering has been started some time ago. The experiments were done on the elastic scattering of unpolarized electrons by the tensor polarized deuteron target and also on the measurement of the tensor polarization of the recoil deuteron (for the purpose to separate the charge and quadrupole deuteron electromagnetic form factors) (for the details see the reviews [15]). To determine the neutron charge electromagnetic form factor G_{En}, several polarization measurements were done for the deuteron electrodisintegration process (see, for example, [16]).

The combination of 4π detectors with linearly and circularly polarized photon beams as well as polarized targets will provide access to new observables, very powerful for the extraction of specific resonance properties. The use of polarized proton and deuteron targets will allow measurement of double and triple polarization observables with polarized neutrons [17].

In Ref. [18] the non–nucleonic degrees of freedom in terms of the Δ–isobar were investigated and the contribution of the $\Delta\Delta$–component of the deuteron wave function was calculated in the framework of the Nambu–Jona–Lasinio model of light nuclei. It was found that $P_\Delta = 0.3\%$. This prediction agrees well with the experimental estimate $P_\Delta \leq 0.4\%$ at 90% of confidence level [19].

The experimental measurement of the $\Delta\Delta$–admixture in the deuteron wave function was done in a number of experiments which investigate pure hadronic reactions and processes induced by leptons as well. The analysis of the final state $\bar{N}NN\pi$ in the interaction of the antiproton beam with deuteron target lead to the result $P_\Delta = 16\%$ [20]. The $dp \to NNN\pi$ reactions were studied in [21] where an upper limit $P_\Delta \leq (1.1 \pm 0.3)\%$ was obtained. The processes of interaction of the positive (negative) pion with deuteron [22] ([23]) gave an upper limit of 0.8%(0.4%) for the $\Delta\Delta$–component of the deuteron wave function. In Ref. [24] the process of the inclusive Δ–isobar photoexcitation on the deuteron target, $\gamma d \to \Delta X$, was studied obtaining $P_\Delta \approx 3\%$. The latest estimate $P_\Delta \leq 0.3\%$ was made in the analysis of the neutrino–deuteron interaction [19].

Note also that many neutrino oscillation experiments were done in the kinematical region corresponding to a neutrino beam energy $\approx 1GeV$. But in this kinematical region the inelastic processes (mainly the quasi free Δ–isobar production) plays a significant role [25]. The theoretical predictions significantly underestimate the data in this region. On the other hand, the results of Ref. [26] lead to the suggestion that two–body currents may give sizable contribution in the dip region.

For high virtuality of the exchanged photon, the $^2H(e, e'p)n$ reaction is one of the simplest and best way to investigate the high–momentum components of the deuteron wave function, possible modifications to the internal structure of bound nucleons, and the nature of short–range nucleon correlations. It was found that the Δ–isobar production dominate over a large part of the phase space [27].

It is highly desirable to perform measurements on the deuteron in kinematics where the short–distance structure is emphasized. Such information can be only accessed in the context of reaction models which include a quantitative description of final–state interaction, meson–exchange currents, isobar configurations and so on. Experiments have already been done in such kinematics [27].

Theoretical studies of the inclusive electron–nucleus cross section at beam energies up to a few GeV show that, while the region of the quasielastic peak is quantitatively understood, the data in the Δ–isobar region are largely underestimated. In view of the rapid development of neutrino physics, the treatment of nuclear effects in data analysis is now regarded as one of the main sources of systematic uncertainty. Much of the information needed for this analysis can be extracted from the results of experimental and theoretical studies of electron–nucleus scattering. In this kinematical regime both quasielastic and inelastic processes, leading to the production of hadrons other than protons and neutrons, must be taken into account [28].

Historically, the quasi–elastic cross section has been exploited in order to measure the neutron electric and magnetic form factors using mainly light ($A \leq 4$) nuclear targets. Today the emphasis has shifted to the search of possible in medium modifications of the nucleon form factors. At large momentum transfer it appears that only the low–ω (ω is the electron energy loss) side of the quasi–elastic peak can be exploited, the large–ω side is obscured by the overlap with Δ–isobar excitation [29].

In spite of a large complexity of the spin structure of the $e^- + d \to e^- + \Delta + N$ reaction amplitude in comparison with the $e^- + d \to e^- + n + p$ reaction amplitude, the mechanism of Δ–isobar production on the deuteron is more simpler. The reason is that in the impulse approximation the main mechanism for the $e^- + d \to e^- + \Delta + N$ reaction is described only by one Feynman diagram whereas for the $e^- + d \to e^- + n + p$ reaction one has to deal, at least, with four Feynman diagrams (due to the necessity to insure gauge invariance for the $\gamma^* + d \to n + p$ process [16]). Moreover, the single diagram in the impulse approximation for the $e^- + d \to e^- + \Delta + N$ reaction is determined, in good approximation, only by one form factor of the $\Delta \to N + \gamma^*$ transition - by the transition of the magnetic dipole type. All this makes the $e^- + d \to e^- + \Delta + N$ reaction more preferable for the determination of the spin structure of the deuteron wave function in comparison with the $e^- + d \to e^- + n + p$ reaction, from a theoretical point of view.

No wonder that the role of the polarization experiments in the $e^- + d \to e^- + \Delta + N$ and $e^- + d \to e^- + n + p$ reactions is also different. Although the form factors of the $\Delta \to N + \gamma^*$ transition are not presently better known than the nucleon form factors, such uncertainty has no effect on the calculations of different asymmetries for the $e^- + d \to e^- + \Delta + N$ reaction due to the polarizations of the colliding particles. One can expect that the various asymmetries in the $e^- + d \to e^- + \Delta + N$ reaction will be essentially constant, in the impulse approximation, over all the spectrum of the scattered electrons. This expectation derives from a factorization of polarization effects for the $e^- + N \to e^- + \Delta$ and $e^- + d \to e^- + \Delta + N$ processes, when the last one is considered in the impulse approximation. So, the search of deviations from this factorization deserves a special attention. Such deviations may be originated by dibaryon resonances [30], meson exchange currents and contribution of the $\Delta\Delta$–configuration in the deuteron ground state [7]. The other (small) form factors of the $\Delta \to N + \gamma^*$ transition may also lead to such deviations.

The $e^- + d \to e^- + \Delta + N$ reaction has been investigated earlier in inclusive set–up: the spectra of the scattered electrons show two peaks, one from quasi–elastic electron–nucleon scattering and another corresponding to the Δ–isobar excitation [31]. Experiments on the Δ–isobar excitation in the electron–nuclear scattering have been also carried out [32]. The detailed investigation of the Δ –isobar (and other nucleon resonances) production which is planned in a number of laboratories shows the importance of these processes in nuclear physics.

The theoretical analysis of the processes $\gamma + d \to N^* + N$ and $e^- + d \to e^- + N^* + N$ has been done in a few papers [33–36]. The cross sections of the reactions $e^- + d \to e^- + N^* + N$ ($N^* = \Delta(1232)$ and $N^*(1480)$) have been calculated in the framework of the non–relativistic impulse approximation [33]. The $\gamma + d \to \Delta + N$ reaction was considered in the relativistic impulse approximation with the help of the *dnp*–vertex formalism [34]. A similar approach was used in Ref. [35] for the analysis of the $e^- + d \to e^- + \Delta + N$ reaction. Estimates of polarization effects were done in Ref. [36] using the formalism of the spin–density matrix of the virtual nucleon [37].

For the study of polarization effects in the scattering of electrons by hadrons and nuclei it is necessary to distinguish the general analysis of the polarization phenomena on one side, and specific, model dependent, estimations of various asymmetries and polarizations, on the other side. The general analysis is based only on the most general properties of the hadron electrodynamics, such as the conservation of the hadron electromagnetic current, the invariance of the hadron electromagnetic interactions with respect to the space reflections and time reversal as well. The particular structure of the hadrons and nuclei participating in the reaction, is not essential in this case. The properties of the polarization phenomena, obtained in this way, are universal for all reactions of the same type.

In this paper we perform a general analysis of the structure of the differential cross section and various polarization observables for the $e^- + d \to e^- + \Delta + N$ reaction. The observables related to the cases of an arbitrary polarized deuteron target, longitudinally polarized electron beam, polarization of the outgoing nucleon, as well as the polarization transfer from electron to final nucleon, and the correlation of the electron and deuteron polarizations are considered in detail. The particular case of the process of the photoproduction of the Δ–isobar on the deuteron target has been considered in detail, separately. The differential cross section and various polarization observables have been derived in terms of the reaction amplitudes. The polarization observables due to the linear and circular polarizations of the photon, when the deuteron target is arbitrarily polarized, have been derived in terms of the reaction amplitudes. The polarization of the final nucleon is also considered. This analysis was done in frame of the structure function formalism.

The paper is organized as follows. In Section 2 the most general spin structure of the matrix element of the reaction $\gamma^* + d \to \Delta + N$ is given. The general structure of the differential cross section when the scattered electron and one of the hadrons are detected in coincidence, when the electron beam is longitudinally polarized (the polarization states of the deuteron target and of the final nucleon can be any) is also given here. In Section 3 the polarization observables due to the longitudinally polarized electron beam and unpolarized deuteron target (Section 3.1), or vector (tensor) polarized deuteron target (Section 3.2 (Section 3.3)) are derived. Section 4 gives the expressions for the nucleon polarization for the unpolarized and longitudinally polarized electron beam. Section 5 contains the helicity amplitudes in terms of the reaction scalar amplitudes. In Section 6 we consider $\gamma + d \to \Delta + N$ reaction and the polarization observables with unpolarized and linear or circular polarized photon beam and unpolarized deuteron target (Section 6.1), or vector (tensor) polarized deuteron target (Section 6.2 (Section 6.3)) are derived. Section 6.4 gives the expressions of the nucleon polarization for the unpolarized and linear or circular polarized photon beam. The main results are summarized in the Conclusion. Technical details are given in the Appendices.

2 The matrix element and the differential cross section

The general structure of the differential cross section for the $e^- + d \to e^- + \Delta + N$ reaction can be determined in the framework of the one–photon–exchange mechanism. The formalism in this section is based on the most general symmetry properties of the hadron electromagnetic interaction, such as gauge invariance (the conservation of the hadronic and leptonic electromagnetic currents) and P–invariance (invariance with respect to space reflections) and does not depend on the deuteron structure and on details of the reaction mechanism for $e^- + d \to e^- + \Delta + N$. In the one–photon–exchange approximation, the matrix element of the Δ –isobar production in the deuteron electrodisintegration process

$$e^-(k_1) + d(P) \to e^-(k_2) + \Delta(p_1) + N(p_2) \tag{1}$$

(the four–momenta of the corresponding particles are indicated in brackets) can be written as

$$M_{fi} = \frac{e^2}{k^2} j_\mu J_\mu, \ j_\mu = \bar{u}(k_2)\gamma_\mu u(k_1), \tag{2}$$

where $k_1(k_2)$ is the four–momentum of the initial (final) electron, $k = k_1 - k_2$, and J_μ is the electromagnetic current describing the transition $\gamma^* + d \to \Delta + N$ (γ^* is the virtual photon).

The electromagnetic structure of nuclei, as probed by elastic and inelastic electron scattering by nuclei, can be described by a set of response functions or structure functions [38]. Each of these structure functions is determined by different combinations of the longitudinal and transverse components of the electromagnetic current J_μ, thus providing different pieces of information about the nuclear structure or possible mechanisms of the reaction under consideration. The ones which are determined by the real parts of the bilinear combinations of the reaction amplitudes are nonzero in impulse approximation, those which originate from the imaginary part of the structure functions vanish in the absence of final state interaction.

The formalism of the structure functions is especially convenient for the investigation of polarization phenomena in the reaction (1). As a starting point, let us write the general structure of the differential cross section of the reaction (1), when the scattered electron and one of the hadrons are detected in coincidence, and the electron beam is longitudinally polarized (the polarization states of the deuteron target and of the final nucleon can be any):

$$\frac{d^3\sigma}{dE' d\Omega_e d\Omega_\Delta} = N \Big[H_{xx} + H_{yy} + \varepsilon \cos(2\varphi)(H_{xx} - H_{yy}) +$$
$$\varepsilon \sin(2\varphi)(H_{xy} + H_{yx}) - 2\varepsilon \frac{k^2}{k_0^2} H_{zz} -$$
$$\frac{\sqrt{-k^2}}{k_0} \sqrt{2\varepsilon(1+\varepsilon)} \cos\varphi (H_{xz} + H_{zx}) -$$
$$\frac{\sqrt{-k^2}}{k_0} \sqrt{2\varepsilon(1+\varepsilon)} \sin\varphi (H_{yz} + H_{zy}) \mp$$
$$i\lambda \sqrt{(1-\varepsilon^2)} (H_{xy} - H_{yx}) \mp$$
$$i\lambda \frac{\sqrt{-k^2}}{k_0} \sqrt{2\varepsilon(1-\varepsilon)} \cos\varphi (H_{yz} - H_{zy}) \pm$$
$$i\lambda \frac{\sqrt{-k^2}}{k_0} \sqrt{2\varepsilon(1-\varepsilon)} \sin\varphi (H_{xz} - H_{zx}) \Big], \quad (3)$$

with

$$N = \frac{\alpha^2}{64\pi^3} \frac{E'}{E} \frac{p}{MW} \frac{1}{1-\varepsilon} \frac{1}{(-k^2)},$$
$$|\vec{k}| = \frac{1}{2W} \sqrt{(W^2 + M^2 - k^2)^2 - 4M^2W^2}, \varepsilon^{-1} = 1 - 2\frac{\vec{k}^2_{Lab}}{k^2} \tan^2\left(\frac{\theta_e}{2}\right),$$
$$p = \frac{1}{2W} \sqrt{(W^2 + M_\Delta^2 - m^2)^2 - 4M_\Delta^2 W^2}, \; H_{\mu\nu} = J_\mu J_\nu^*. \quad (4)$$

The z axis is directed along the virtual photon momentum $\vec{k}$, the momentum of the detected Δ–isobar $\vec{p}$ lies in the xz plane (reaction plane); $E(E')$ is the energy of the initial (scattered) electron in the deuteron rest frame (Lab system); $d\Omega_e$ is the solid angle of the scattered electron in the Lab system, $d\Omega_\Delta(p)$ is the solid angle (value of the three-momentum) of the detected Δ–isobar in ΔN–pair center–of–mass system (CMS), M_Δ, M and m are the masses of the Δ–isobar, deuteron and nucleon, respectively; φ is the azimuthal angle between the electron scattering plane and the plane where the detected Δ–isobar lies (xz), $k_0 = (W^2 + k^2 - M^2)/2W$ is the virtual photon energy in the ΔN–pair CMS, W is the invariant mass of the final hadrons, $W^2 = M^2 + k^2 + 2M(E - E')$; λ is the degree of the electron longitudinal polarization, ε is the degree of the linear polarization of the virtual photon. The upper (bottom) sign in this formula corresponds to the electron (positron) scattering. This expression is valid for zero electron mass. Below we will neglect it wherever possible.

As it is seen from Eq. (3), the differential cross section and various polarization characteristics of the process under consideration are determined only by the space components of the hadronic tensor $H_{\mu\nu}$.

Assuming the conservation of the leptonic j_μ and hadronic J_μ electromagnetic currents the matrix element can be written as

$$M_{fi} = ee_\mu J_\mu = e\vec{l}\cdot\vec{J},\ e_\mu = \frac{e}{k^2}j_\mu,\ \vec{l} = \frac{\vec{e}\vec{k}}{k_0^2}\vec{k} - \vec{e}. \tag{5}$$

In CMS of the Δ–isobar and final nucleon we get

$$M_{fi} = e\vec{\chi}_2^+\vec{F}\chi_1^c = eF,$$

where $\vec{\chi}_2^+$ and χ_1^c are the Δ–isobar vector spinor and nucleon spinor, correspondingly.

Let us introduce, the orthonormal system of basic unit $\vec{m}, \vec{n}$, and $\hat{\vec{k}}$ vectors which are built from the momenta of the particles participating in the reaction under consideration

$$\hat{\vec{k}} = \frac{\vec{k}}{|\vec{k}|},\ \vec{n} = \frac{\vec{k}\times\vec{p}}{|\vec{k}\times\vec{p}|},\ \vec{m} = \vec{n}\times\hat{\vec{k}}.$$

The unit vectors $\hat{\vec{k}}$ and $\vec{m}$ define the $\gamma^* + d \to \Delta + N$ reaction xz–plane (the z axis is directed along the three–momentum of the virtual photon $\vec{k}$, the x axis is directed along the unit vector $\vec{m}$), and the unit vector $\vec{n}$ is perpendicular to the reaction plane.

In the analysis of polarization phenomena, it is convenient to use the amplitude F represented in the above orthonormal basis. The amplitude F can be chosen as

$$\vec{F} = \vec{m}F^{(m)} + \hat{\vec{k}}F^{(k)},$$

$$\begin{aligned}
F^{(m)} =& \vec{l}\cdot\vec{m}(if_1\vec{U}\cdot\vec{m} + if_2\vec{U}\cdot\hat{\vec{k}} + f_3\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\vec{m} + f_4\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\hat{\vec{k}} + \\
& f_5\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\vec{n} + f_6\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\vec{n}) + \vec{l}\cdot\vec{n}(if_7\vec{U}\cdot\vec{n} + f_8\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\vec{n} + \\
& f_9\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\vec{m} + f_{10}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\vec{m} + f_{11}\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\hat{\vec{k}} + f_{12}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\hat{\vec{k}}) + \\
& \vec{l}\cdot\hat{\vec{k}}(if_{13}\vec{U}\cdot\vec{m} + if_{14}\vec{U}\cdot\hat{\vec{k}} + f_{15}\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\vec{m} + f_{16}\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\hat{\vec{k}} + \\
& f_{17}\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\vec{n} + f_{18}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\vec{n}), \qquad (6)\\
F^{(k)} =& \vec{l}\cdot\vec{m}(if_{19}\vec{U}\cdot\vec{m} + if_{20}\vec{U}\cdot\hat{\vec{k}} + f_{21}\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\vec{m} + f_{22}\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\hat{\vec{k}} + \\
& f_{23}\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\vec{n} + f_{24}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\vec{n}) + \vec{l}\cdot\vec{n}(if_{25}\vec{U}\cdot\vec{n} + f_{26}\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\vec{n} + \\
& f_{27}\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\vec{m} + f_{28}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\vec{m} + f_{29}\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\hat{\vec{k}} + f_{30}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\hat{\vec{k}}) + \\
& \vec{l}\cdot\hat{\vec{k}}(if_{31}\vec{U}\cdot\vec{m} + if_{32}\vec{U}\cdot\hat{\vec{k}} + f_{33}\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\vec{m} + f_{34}\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\hat{\vec{k}} + \\
& f_{35}\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\vec{n} + f_{36}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\vec{n}), \qquad (7)
\end{aligned}$$

where $f_i (i = 1 - 36)$ are the scalar amplitudes, depending on three variables, which completely determine the reaction dynamics. If we single out the photon polarization vector $\vec{l}$, one can write the amplitude F as $F = F_i l_i$ and the hadronic tensor as $H_{ij} = F_i F_j^*$.

3 Polarization of the deuteron target

In the general case the deuteron polarization is described by the spin–density matrix. Let us start from the following general expression for the deuteron spin–density matrix in the coordinate representation [39]

$$\rho_{\mu\nu} = -\frac{1}{3}\left(g_{\mu\nu} - \frac{P_\mu P_\nu}{M^2}\right) - \frac{i}{2M}\varepsilon_{\mu\nu\alpha\beta} s_\alpha P_\beta + S_{\mu\nu}, \qquad (8)$$

where s_α is the four–vector describing the vector polarization of the target, with $s^2 = -1$, $s \cdot P = 0$. $S_{\mu\nu}$ is the tensor describing the tensor (quadrupole) polarization of the target, with $S_{\mu\nu} = S_{\nu\mu}$, $P_\mu S_{\mu\nu} = 0$, $S_{\mu\mu} = 0$. Due to these properties the tensor $S_{\mu\nu}$ has only five independent components. In Lab system all time components of the tensor $S_{\mu\nu}$ are zero and the tensor polarization of the target is described by five independent space components ($S_{ij} = S_{ji}$, $S_{ii} = 0$, $i, j = x, y, z$). The four–vector s_α is related to the unit vector $\vec{\xi}$ of the deuteron vector polarization in its rest system:

$$s_0 = -\vec{k}\vec{\xi}/M, \;\; \vec{s} = \vec{\xi} + \vec{k}(\vec{k}\vec{\xi})/M(M+\omega),$$

where ω is the deuteron energy in the $\gamma^* + d \to \Delta + N$ reaction CMS.

The hadronic tensor $H_{ij}(i, j = x, y, z)$ depends linearly on the target polarization and it can be represented as follows

$$H_{ij} = H_{ij}(0) + H_{ij}(\xi) + H_{ij}(S), \qquad (9)$$

where the term $H_{ij}(0)$ corresponds to the case of unpolarized deuteron target, and the term $H_{ij}(\xi)(H_{ij}(S))$ corresponds to the case of the vector(tensor-)-polarized target.

3.1 Unpolarized deuteron target

The general structure of the part of the hadronic tensor corresponding to unpolarized deuteron has the following form

$$H_{ij}(0) = \alpha_1 \hat{k}_i \hat{k}_j + \alpha_2 n_i n_j + \alpha_3 m_i m_j + \alpha_4 (\hat{k}_i m_j + \hat{k}_j m_i) + i\alpha_5 (\hat{k}_i m_j - \hat{k}_j m_i). \quad (10)$$

The structure functions α_i are real and depend on three invariant variables $s = W^2 = (k+P)^2$, k^2 and $t = (k-P)^2$. Let us emphasize that the structure function α_5 is determined by the strong interaction effects of the Δ–isobar and the nucleon in the final state and it vanishes for the pole diagram contribution in all kinematic range (independently on the particular parametrization of the $\gamma^* \Delta N-$ and $dnp-$vertexes). This is true for the non relativistic approach and for the relativistic one as well, when describing the $\gamma^* + d \to \Delta + N$ reaction. The scattering of longitudinally polarized electrons by unpolarized deuteron allows to determine the α_5 contribution. Then, the corresponding asymmetry is determined only by the strong interaction effects. More exactly, it is determined by the effects arising from non pole mechanisms of various nature (meson exchange currents can also induce nonzero asymmetry). Dibaryon resonances, if present, may also lead to nonzero asymmetry.

In the chosen coordinate system, the different hadron tensor components, entering in the expression of the cross section (10), are related to the structure functions $\alpha_i (i = 1-5)$ by:

$$H_{xx} \pm H_{yy} = \alpha_3 \pm \alpha_2, \;\; H_{zz} = \alpha_1, \;\; H_{xz} + H_{zx} = 2\alpha_4, \\ H_{xz} - H_{zx} = -2i\alpha_5, \;\; H_{xy} \pm H_{yx} = 0, \;\; H_{yz} \pm H_{zy} = 0. \quad (11)$$

In the one–photon–exchange approximation, the general structure of the differential cross section for the reaction $d(\vec{e}, e'\Delta)N$ (in the case of longitudinally polarized electron beam and unpolarized deuteron target) can be written in terms of five independent contributions

$$\frac{d^3\sigma}{dE' d\Omega_e d\Omega_\Delta} = N\Big[\sigma_T + \varepsilon\sigma_L + \varepsilon\cos(2\varphi)\sigma_P + \sqrt{2\varepsilon(1+\varepsilon)}\cos\varphi\sigma_I + \lambda\sqrt{2\varepsilon(1-\varepsilon)}\sin\varphi\sigma_I'\Big], \quad (12)$$

where the individual contributions are related to the components of the spin–independent hadronic tensor, Eq. (10), by:

$$\sigma_T = H_{xx} + H_{yy}, \;\; \sigma_P = H_{xx} - H_{yy}, \;\; \sigma_L = -2\frac{k^2}{k_0^2} H_{zz}, \\ \sigma_I = -\frac{\sqrt{-k^2}}{k_0}(H_{xz} + H_{zx}), \;\; \sigma_I' = i\frac{\sqrt{-k^2}}{k_0}(H_{xz} - H_{zx}).$$

From the above equations, one can define a single–spin asymmetry which is due to the electron beam polarization:

$$\Sigma_e(\varphi) = \frac{d\sigma(\lambda=+1) - d\sigma(\lambda=-1)}{d\sigma(\lambda=+1) + d\sigma(\lambda=-1)} =$$
$$\frac{\sin\varphi\sqrt{2\varepsilon(1-\varepsilon)}\sigma'_I}{\sigma_T + \varepsilon\sigma_L + \varepsilon\cos(2\varphi)\sigma_P + \sqrt{2\varepsilon(1+\varepsilon)}\cos\varphi\sigma_I}, \qquad (13)$$

which contains a φ–dependence. Therefore, this asymmetry has to be measured in non coplanar geometry (out–of–plane kinematics).

We see that this asymmetry is determined by the structure function α_5 which is defined by the interference of the reaction amplitudes that characterize the absorption of virtual photons with nonzero longitudinal and transverse components of the electromagnetic current corresponding to the process $\gamma^* + d \to \Delta + N$. One finds that $\alpha_5 \sim \sin\vartheta$ independently from the reaction mechanism. It vanishes when the Δ–isobar emission angles are $\vartheta = 0°$ and $\vartheta = 180°$, due to the conservation of the total helicity of the interacting particles in the $\gamma^* + d \to \Delta + N$ reaction. The structure function α_5 is nonzero only if the complex amplitudes of the $\gamma^* + d \to \Delta + N$ reaction have nonzero relative phases. This is a very specific observable, which has no corresponding quantity in the Δ–isobar excitation in the deuteron photodisintegration process $\gamma + d \to \Delta + N$.

The study of the single–spin asymmetry Σ_e was firstly suggested for pion production in electron–nucleon scattering, $e + N \to e + N + \pi$ [40]. Afterward, this asymmetry has been discussed for the hadron production in the exclusive processes of the type $A(\vec{e}, eh)X$, where A is a nucleus and h is the detected hadron [41,42]. A number of experiments to measure the asymmetry Σ_e has already been done [43].

3.2 Vector–polarized deuteron target

The part of the hadronic tensor depending on the deuteron vector polarization has the following general structure:

$$\begin{aligned} H_{ij}(\xi) = & \vec{\xi}\vec{n}(\beta_1\hat{k}_i\hat{k}_j + \beta_2 m_i m_j + \beta_3 n_i n_j + \beta_4\{\hat{k}, m\}_{ij} + i\beta_5[\hat{k}, m]_{ij}) + \\ & + \vec{\xi}\vec{\hat{k}}(\beta_6\{\hat{k}, n\}_{ij} + \beta_7\{m, n\}_{ij} + i\beta_8[\hat{k}, n]_{ij} + i\beta_9[m, n]_{ij}) + \\ & + \vec{\xi}\vec{m}(\beta_{10}\{\hat{k}, n\}_{ij} + \beta_{11}\{m, n\}_{ij} + i\beta_{12}[\hat{k}, n]_{ij} + i\beta_{13}[m, n]_{ij}), \quad (14) \end{aligned}$$

where $\{a, b\}_{ij} = a_i b_j + a_j b_i, \ [a, b]_{ij} = a_i b_j - a_j b_i$.

Therefore, the dependence of the polarization observables on the deuteron vector polarization is determined by thirteen structure functions. On the basis of this formula one can make the following general conclusions:

- If the deuteron is vector–polarized and the polarization vector is perpendicular to the $\gamma^* + d \to \Delta + N$ reaction plane, then the dependence of the differential cross section on the ε and φ variables is the same as in the case of unpolarized target, and the non vanishing components of the $H_{ij}(\xi)$ tensor are:

$$H_{xx}(\xi) \pm H_{yy}(\xi) = (\beta_2 \pm \beta_3)\vec{\xi}\vec{n}, \; H_{zz}(\xi) = \beta_1\vec{\xi}\vec{n},$$
$$H_{xz}(\xi) + H_{zx}(\xi) = 2\beta_4\vec{\xi}\vec{n}, \; H_{xz}(\xi) - H_{zx}(\xi) = -2i\beta_5\vec{\xi}\vec{n}. \quad (15)$$

- If the deuteron target is polarized in the $\gamma^* + d \to \Delta + N$ reaction plane (in direction of the vector $\vec{k}$ or $\vec{m}$), then the dependence of the differential cross section of the $e^- + d \to e^- + \Delta + N$ reaction on the ε and φ variables is:
  · for unpolarized electron beam:

$$\varepsilon sin(2\varphi), \; \sqrt{2\varepsilon(1+\varepsilon)}sin\varphi,$$

  · for longitudinally polarized electron beam:

$$\pm i\lambda\sqrt{1-\varepsilon^2}, \;\; \mp i\lambda\sqrt{2\varepsilon(1-\varepsilon)}cos\varphi.$$

The differential cross section of the reaction $\vec{d}(\vec{e}, e'\Delta)N$, where the electron beam is longitudinally polarized and the deuteron target is vector–polarized, can be written as follows:

$$\frac{d^3\sigma}{dE'd\Omega_e d\Omega_\Delta} = \sigma_0\Big[1 + \lambda\Sigma_e + (A_x^d + \lambda A_x^{ed})\xi_x + (A_y^d + \lambda A_y^{ed})\xi_y + (A_z^d + \lambda A_z^{ed})\xi_z\Big], \quad (16)$$

where σ_0 is the unpolarized differential cross section, Σ_e is the beam asymmetry (the asymmetry induced by the electron–beam polarization), $A_i^d(i = x, y, z)$ are the analyzing powers due to the vector polarization of the deuteron target, and $A_i^{ed}(i = x, y, z)$ are the spin– correlation parameters. The direction of the deuteron polarization vector is defined by the angles ϑ^*, φ^* in the reference frame where the z axis is along the direction of the three–momentum transfer $\vec{k}$, and the y axis is defined by the vector product of the momenta of the detected Δ– isobar and the virtual photon (along the unit vector $\vec{n}$). The target analyzing powers and the spin–correlation parameters depend on the orientation of the deuteron polarization vector. The quantities Σ_e and A_i^d are T–odd observables and they are completely determined by the reaction mechanism beyond the impulse approximation, for example, by final–state interaction effects. On the contrary, the quantities A_i^{ed} are T–even observables and they do not vanish in absence of final–state interaction effects.

The expressions of the A_i^d and A_i^{ed} asymmetries can be explicitly written as functions of the azimuthal angle φ, of the virtual–photon linear polarization ε, and of the contributions of the longitudinal (L) and transverse (T) components (relative to the virtual–photon momentum $\vec{k}$) of the hadron electromagnetic current of the $\gamma^* + d \to \Delta + N$ reaction:

$$
\begin{aligned}
A_x^d\sigma_0 &= N\sin\varphi\left[\sqrt{2\varepsilon(1+\varepsilon)}A_x^{(LT)} + \varepsilon\cos\varphi A_x^{(TT)}\right],\\
A_z^d\sigma_0 &= N\sin\varphi\left[\sqrt{2\varepsilon(1+\varepsilon)}A_z^{(LT)} + \varepsilon\cos\varphi A_z^{(TT)}\right],\\
A_y^d\sigma_0 &= N\left[A_y^{(TT)} + \varepsilon A_y^{(LL)} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\varphi A_y^{(LT)} + \varepsilon\cos(2\varphi)\bar{A}_y^{(TT)}\right],\\
A_x^{ed}\sigma_0 &= N\left[\sqrt{1-\varepsilon^2}B_x^{(TT)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\varphi B_x^{(LT)}\right],\\
A_z^{ed}\sigma_0 &= N\left[\sqrt{1-\varepsilon^2}B_z^{(TT)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\varphi B_z^{(LT)}\right],\\
A_y^{ed}\sigma_0 &= N\sqrt{2\varepsilon(1-\varepsilon)}\sin\varphi B_y^{(LT)},
\end{aligned}
\tag{17}
$$

where the individual contributions to the considered asymmetries in terms of the structure functions β_i are given by

$$
\begin{aligned}
&A_x^{(TT)} = 4\beta_{11},\ A_y^{(TT)} = \beta_2 + \beta_3,\ \bar{A}_y^{(TT)} = \beta_2 - \beta_3,\ A_z^{(TT)} = 4\beta_7,\\
&A_x^{(LT)} = -2\frac{\sqrt{Q^2}}{k_0}\beta_{10},\ A_y^{(LT)} = -2\frac{\sqrt{Q^2}}{k_0}\beta_4,\ A_z^{(LT)} = -2\frac{\sqrt{Q^2}}{k_0}\beta_6,\\
&A_y^{(LL)} = 2\frac{Q^2}{k_0^2}\beta_1,\ B_x^{(TT)} = 2\beta_{13},\ B_z^{(TT)} = 2\beta_9,\\
&B_x^{(LT)} = -2\frac{\sqrt{Q^2}}{k_0}\beta_{12},\ B_y^{(LT)} = 2\frac{\sqrt{Q^2}}{k_0}\beta_5,\ B_z^{(LT)} = -2\frac{\sqrt{Q^2}}{k_0}\beta_8.
\end{aligned}
$$

At this stage, the general model–independent analysis of the polarization observables in the reactions $\vec{d}(e,e'\Delta)N$ and $\vec{d}(\vec{e},e'\Delta)N$ is completed. To proceed further in the calculation of the observables, one needs a model for the reaction mechanism and for the deuteron structure.

3.3 Tensor–polarized deuteron target

The part of the hadronic tensor $H_{ij}(S)$, which depends on the deuteron tensor polarization, has the following general structure:

$$
\begin{aligned}
H_{ij}(S) = &S_{ab}\hat{k}_a\hat{k}_b(\gamma_1\hat{k}_i\hat{k}_j + \gamma_2 m_i m_j + \gamma_3 n_i n_j + \gamma_4\{\hat{k},m\}_{ij} + i\gamma_5[\hat{k},m]_{ij}) +\\
&S_{ab}m_a m_b(\gamma_6\hat{k}_i\hat{k}_j + \gamma_7 m_i m_j + \gamma_8 n_i n_j + \gamma_9\{\hat{k},m\}_{ij} + i\gamma_{10}[\hat{k},m]_{ij}) +
\end{aligned}
$$

$$S_{ab}\{\hat{k},m\}_{ab}(\gamma_{11}\hat{k}_i\hat{k}_j+\gamma_{12}m_im_j+\gamma_{13}n_in_j+\gamma_{14}\{\hat{k},m\}_{ij}+i\gamma_{15}[\hat{k},m]_{ij})+$$
$$S_{ab}\{\hat{k},n\}_{ab}(\gamma_{16}\{\hat{k},n\}_{ij}+\gamma_{17}\{m,n\}_{ij}+i\gamma_{18}[\hat{k},n]_{ij}+i\gamma_{19}[m,n]_{ij})+$$
$$S_{ab}\{m,n\}_{ab}(\gamma_{20}\{\hat{k},n\}_{ij}+\gamma_{21}\{m,n\}_{ij}+i\gamma_{22}[\hat{k},n]_{ij}+i\gamma_{23}[m,n]_{ij}). \quad (18)$$

In this case, the dependence of the polarization observables on the deuteron tensor polarization is determined by 23 structure functions.

From this equation one can conclude that:

- If the deuteron is tensor polarized so that only S_{zz}, S_{yy} and $(S_{xz}+S_{zx})$ components of the quadrupole polarization tensor are nonzero, then the dependence of the differential cross section of the $e^-+d\to e^-+\Delta+N$ reaction on the parameter ε and on the azimuthal angle φ must be the same as in the case of the unpolarized target (more exactly, with similar $\varepsilon-$ and $\varphi-$ dependent terms).
- If the deuteron is polarized so that only $(S_{xy}+S_{yx})$ and $(S_{yz}+S_{zy})$ components of the quadrupole polarization tensor are nonzero, then the typical terms follow $\sin\varphi$ and $\sin(2\varphi)$ dependencies - for deuteron disintegration by unpolarized electron beam, and terms which do not depend on ε, φ, and $\cos\varphi$ - for deuteron disintegration by longitudinally polarized electron beam.

In polarization experiments it is possible to prepare the deuteron target with definite spin projection on some quantization axis. The corresponding asymmetry is usually defined as

$$A=\frac{d\sigma(\lambda_d=+1)-d\sigma(\lambda_d=-1)}{d\sigma(\lambda_d=+1)+d\sigma(\lambda_d=-1)},$$

where $d\sigma(\lambda_d)$ is the differential cross section of the $e^-+d\to e^-+\Delta+N$ reaction when the quantization axis for the deuteron spin (in the ΔN–pair CMS) coincides with its momentum, i.e., the deuteron has helicity λ_d. From an experimental point of view, the measurement of an asymmetry is more convenient than a measurement of a cross section, as most of systematic experimental errors and other multiplicative factors cancel in the ratio.

The general form of the hadron tensor $H_{ij}(\lambda_d)$, which determines the differential cross section of the process under consideration for the case of the deuteron with helicity λ_d, can be written as

$$H_{ij}^{(\lambda_d=\pm1)}=\delta_1\hat{k}_i\hat{k}_j+\delta_2m_im_j+\delta_3n_in_j+\delta_4\{\hat{k},m\}_{ij}+i\delta_5[\hat{k},m]_{ij}\pm$$
$$\pm\delta_6\{\hat{k},n\}_{ij}\pm i\delta_7[\hat{k},n]_{ij}\pm\delta_8\{m,n\}_{ij}\pm i\delta_9[m,n]_{ij}. \quad (19)$$

The reaction amplitude is real in the Born (impulse) approximation. So, assuming the T-invariance of the hadron electromagnetic interactions, we can do the following statements, according to the deuteron polarization state:

- The deuteron is unpolarized. Since the hadronic tensor $H_{ij}(0)$ has to be symmetric (over i,j indexes) in this case, the asymmetry in the scattering of longitudinally polarized electrons vanishes.
- The deuteron is vector polarized. Since the hadronic tensor $H_{ij}(\xi)$ has to be antisymmetric in this case, then the deuteron vector polarization can manifest itself in the scattering of longitudinally polarized electrons. The perpendicular target polarization (normal to the $\gamma^* + d \to \Delta + N$ reaction plane) leads to a correlation of the following type: $\pm i\lambda\sqrt{2\varepsilon(1-\varepsilon)}\sin\varphi$. The longitudinal and transverse (along or perpendicular to the virtual–photon momentum) target polarization (lying in the $\gamma^* + d \to \Delta + N$ reaction plane) leads to two correlations of the following type : $\mp i\lambda\sqrt{1-\varepsilon^2}$ and $\mp i\lambda\sqrt{2\varepsilon(1-\varepsilon)}\cos\varphi$.
- The deuteron is tensor polarized. The hadronic tensor $H_{ij}(S)$ is symmetric in this case. In the scattering of longitudinally polarized electrons the contribution proportional to λS_{ab} vanishes. If the target is polarized so that only the $(S_{xy} + S_{yx})$ or $(S_{yz} + S_{zy})$ components of the quadrupole polarization tensor are nonzero, then in the differential cross section only the following two terms are present: $\varepsilon\sin(2\varphi)$ and $\sqrt{2\varepsilon(1+\varepsilon)}\sin\varphi$. For all other target polarizations the following structures are present: a term which does not depend on ε and φ variables as well as terms with the following dependencies: 2ε, $\varepsilon\cos(2\varphi)$, and $\sqrt{2\varepsilon(1+\varepsilon)}\cos\varphi$.

The differential cross section of the Δ–isobar excitation in the scattering of longitudinally polarized electrons by tensor polarized deuteron target (in the coincidence experimental setup) has the following general structure

$$
\begin{aligned}
\frac{d^3\sigma}{dE' d\Omega_e d\Omega_\Delta} = N\Big\{ & \sigma_T + A^T_{xz}Q_{xz} + A^T_{xx}(Q_{xx} - Q_{yy}) + A^T_{zz}Q_{zz} + \\
& \varepsilon\Big[\sigma_L + A^L_{xz}Q_{xz} + A^L_{xx}(Q_{xx} - Q_{yy}) + A^L_{zz}Q_{zz}\Big] + \\
& \sqrt{2\varepsilon(1+\varepsilon)}\cos\varphi\Big[\sigma_I + A^I_{xz}Q_{xz} + A^I_{xx}(Q_{xx} - Q_{yy}) + A^I_{zz}Q_{zz}\Big] + \\
& \sqrt{2\varepsilon(1+\varepsilon)}\sin\varphi(A^I_{xy}Q_{xy} + A^I_{yz}Q_{yz}) + \varepsilon\sin(2\varphi)(A^P_{xy}Q_{xy} + A^P_{yz}Q_{yz}) + \\
& \varepsilon\cos(2\varphi)\Big[\sigma_P + A^P_{xz}Q_{xz} + A^P_{xx}(Q_{xx} - Q_{yy}) + A^P_{zz}Q_{zz}\Big] + \\
& \lambda\sqrt{2\varepsilon(1-\varepsilon)}\sin\varphi\Big[\sigma'_I + \bar{A}^I_{xz}Q_{xz} + \bar{A}^I_{xx}(Q_{xx} - Q_{yy}) + \bar{A}^I_{zz}Q_{zz}\Big] + \\
& \lambda\sqrt{2\varepsilon(1-\varepsilon)}\cos\varphi\Big[\bar{A}^I_{xy}Q_{xy} + \bar{A}^I_{yz}Q_{yz}\Big] + \\
& \lambda\sqrt{1-\varepsilon^2}\cos\varphi\Big[A^T_{xy}Q_{xy} + A^T_{yz}Q_{yz}\Big]\Big\},
\end{aligned}
$$

where the quantities $Q_{ij}(i,j=x,y,z)$ are the components of the quadrupole polarization tensor of the deuteron in its rest system (the coordinate system is specified similarly to the case of the ΔN–pair CMS). These components satisfy the following conditions: $Q_{ij}=Q_{ji}$, $Q_{ii}=0$. By writing this formula we took into account the following relation: $Q_{xx}+Q_{yy}+Q_{zz}=0$.

A general property of these tensor asymmetries is that they vanish in the region of the quasi–elastic scattering. This can be explained as follows. All the asymmetries are determined by the convolution $X_{\mu\nu}S_{\mu\nu}$, where the tensor $X_{\mu\nu}$ is built with the four–momenta describing the $d\to np$ transition. Due to the condition $P_\mu S_{\mu\nu}=0$, the most general form of this tensor is

$$X_{\mu\nu}=a_1g_{\mu\nu}+ia_2[\gamma_\mu,\gamma_\nu]+a_3\gamma_\mu p_\nu+a_4\gamma_\nu p_\mu+a_5p_\mu p_\nu,$$

where p_μ is the four–momentum of the nucleon–spectator. However, if we take into account that $S_{\mu\nu}g_{\mu\nu}=0$, $S_{\mu\nu}=S_{\nu\mu}$, then the convolution $X_{\mu\nu}S_{\mu\nu}$ is determined by a_3, a_4 and a_5. From the condition $P_\mu S_{\mu\nu}=0$, it follows that the time components of the $S_{\mu\nu}$ tensor vanish in the Lab system. Therefore, the convolution $X_{\mu\nu}S_{\mu\nu}$ turns out to be proportional to the nucleon–spectator three–momentum which is zero at the peak of the quasi–elastic scattering.

Thus, in the general case, the number of independent asymmetries $A^m_{ij}(W,k^2,\vartheta)$, $i,j=x,y,z; m=T,P,L,I$, contributing to the exclusive cross section of the Δ–isobar excitation is 23 for the scattering of longitudinally polarized electrons by a tensor polarized deuteron target, 16(7) for the scattering of unpolarized (longitudinally polarized) electrons $A^m_{ij}(W,k^2,\vartheta)$, where $i,j=x,y,z;m=T,P,L,I$. These asymmetries can be related to the structure functions γ_i which are the bilinear combinations of the 36 independent scalar amplitudes describing the $\gamma^*+d\to\Delta+N$ reaction, by the following relations:

$$\begin{aligned}
&A^T_{xz}=2\frac{\omega}{M}(\gamma_{12}+\gamma_{13}),\ A^T_{xx}=\frac{1}{2}(\gamma_7+\gamma_8),\\
&A^T_{zz}=\frac{\omega^2}{M^2}(\gamma_2+\gamma_3)-\frac{1}{2}(\gamma_7+\gamma_8), A^L_{xz}=-4\frac{\omega}{M}\frac{k^2}{k_0^2}\gamma_{11},\ A^L_{xx}=-\frac{k^2}{k_0^2}\gamma_6,\\
&A^L_{zz}=-\frac{k^2}{k_0^2}(2\frac{\omega^2}{M^2}\gamma_1-\gamma_6), A^I_{xz}=-4\frac{\omega}{M}\frac{\sqrt{-k^2}}{k_0}\gamma_{14},\ A^I_{xx}=-\frac{\sqrt{-k^2}}{k_0}\gamma_9,\\
&A^I_{zz}=-\frac{\sqrt{-k^2}}{k_0}(2\frac{\omega^2}{M^2}\gamma_4-\gamma_9), A^I_{xy}=-4\frac{\sqrt{-k^2}}{k_0}\gamma_{20},\\
&A^I_{yz}=-4\frac{\omega}{M}\frac{\sqrt{-k^2}}{k_0}\gamma_{16},\ A^P_{xy}=4\gamma_{21},\ A^P_{yz}=4\frac{\omega}{M}\gamma_{17}, A^P_{xz}=2\frac{\omega}{M}(\gamma_{12}-\gamma_{13}),\\
&A^P_{xx}=\frac{1}{2}(\gamma_7-\gamma_8),\ A^P_{zz}=\frac{\omega^2}{M^2}(\gamma_2-\gamma_3)-\frac{1}{2}(\gamma_7-\gamma_8),\\
&\bar{A}^I_{xz}=4\frac{\omega}{M}\frac{\sqrt{-k^2}}{k_0}\gamma_{15},\ \bar{A}^I_{xx}=\frac{\sqrt{-k^2}}{k_0}\gamma_{10},\ \bar{A}^I_{zz}=\frac{\sqrt{-k^2}}{k_0}(2\frac{\omega^2}{M^2}\gamma_5-\gamma_{10}),
\end{aligned}$$

$$\bar{A}^I_{xy}=-4\frac{\sqrt{-k^2}}{k_0}\gamma_{22},\ \bar{A}^I_{yz}=-4\frac{\omega}{M}\frac{\sqrt{-k^2}}{k_0}\gamma_{18},\ A^T_{xy}=4\gamma_{23},\ A^T_{yz}=4\frac{\omega}{M}\gamma_{19}.$$

One can see from this formula that the scattering of unpolarized electrons by a tensor polarized deuteron target with components $Q_{xy}=Q_{yz}=0$, is characterized by the same $\varphi-$ and ε–dependences as in the case of the scattering of unpolarized electrons by unpolarized deuteron target. If $Q_{xy}\neq 0$, $Q_{yz}\neq 0$, then new terms of the type $\sqrt{2\varepsilon(1+\varepsilon)}\sin\varphi$ and $\varepsilon\sin(2\varphi)$ are present in the cross section. The asymmetries with upper indexes $T, P(L)$ are determined only by the transverse (longitudinal) components of the electromagnetic current for the $\gamma^*+d\to\Delta+N$ reaction, while the asymmetries with upper index I are determined by the interference of the longitudinal and transverse components of the electromagnetic current.

Using the explicit form for the amplitude of the reaction under consideration it is easy to obtain the expression for the hadronic tensor H_{ij} in terms of the scalar amplitudes f_i $(i=1,...,36)$ .Appendix A contains the formulas for the structure functions $\alpha_i,\beta_i,\gamma_i$ in terms of the scalar amplitudes, which describe the polarization effects in the $e^-+d\to e^-+\Delta+N$ reaction due the deuteron polarization.

Let us stress again that the results listed above have a general nature and are not related to a particular reaction mechanism. They have been derived assuming the one–photon–exchange mechanism, the spin one nature of the photon, the P-invariance of the hadron electromagnetic interaction, and the hadron electromagnetic current conservation. Other possible, model dependent contributions to the deuteron structure such as meson–exchange current, the $D-$ wave admixture in the deuteron ground state, a $\Delta\Delta-$ component, six–quark configuration etc., do not affect the general results of this section.

4 Nucleon polarization

Let us write the matrix element of the reaction under consideration in the following form (the vector indexes of the virtual photon and Δ–isobar are singled out)

$$M_{fi}=el_i\chi^*_{2k}F_{ki}\chi^c_1.$$

The polarization properties of the nucleon, produced in the $\gamma^*+d\to\Delta+N$ reaction, are determined by the $\vec{P}_{ij}$ tensor

$$\vec{P}_{ij}=Tr\rho_{lk}F_{ki}\vec{\sigma}F^+_{lj},\ i,j=x,y,z, \tag{20}$$

where ρ_{lk} is the Δ–isobar spin–density matrix. Let us consider the case when the deuteron target and produced Δ–isobar are unpolarized. The nucleon polarization vector $\vec{P}$ (multiplied by the unpolarized differential cross section $d^3\sigma/dE' d\Omega_e d\Omega_\Delta$) is given by an expression obtained from Eq. (3), replacing the components of the hadronic tensor H_{ij} by the corresponding $\vec{P}_{ij}$ tensor components. The tensor $\vec{P}_{ij}$ can be represented in the following general form:

$$\vec{P}_{ij} = \hat{\vec{k}} P_{ij}^{(k)} + \vec{m} P_{ij}^{(m)} + \vec{n} P_{ij}^{(n)}.$$

Assuming the P-invariance of the hadron electromagnetic interaction, we can write the tensor structure of the quantities $P_{ij}^{(k)}$, $P_{ij}^{(m)}$, and $P_{ij}^{(n)}$, in terms of the structure functions P_i, $i = 1 - 13$, which depend on three independent kinematical variables: k^2, W, and t:

$$\begin{aligned}
P_{ij}^{(k)} &= P_1\{\hat{k}, n\}_{ij} + P_2\{m, n\}_{ij} + iP_3[\hat{k}, n]_{ij} + iP_4[m, n]_{ij}, \\
P_{ij}^{(m)} &= P_5\{\hat{k}, n\}_{ij} + P_6\{m, n\}_{ij} + iP_7[\hat{k}, n]_{ij} + iP_8[m, n]_{ij}, \\
P_{ij}^{(n)} &= P_9\hat{k}_i\hat{k}_j + P_{10}m_im_j + P_{11}n_in_j + P_{12}\{\hat{k}, m\}_{ij} + iP_{13}[\hat{k}, m]_{ij}.
\end{aligned} \tag{21}$$

The expressions for the structure functions P_i, in terms of the scalar amplitudes f_i, $i = 1 - 36$, are given in Appendix B. We can see that the symmetric parts (with respect to the i, j indexes) of the tensors in this equation (which correspond to eight structure functions P_i, $i = 1, 2, 5, 6, 9, 10, 11, 12$) determine the components of the polarization vector of the nucleon produced in collisions of unpolarized electrons with an unpolarized deuteron target, for the reaction $d(e, e'\vec{N})\Delta$. The antisymmetric parts of the tensors in Eq. (21), (that is, the five structure functions P_i, $i = 3, 4, 7, 8, 13$) determine the components of the polarization vector of the nucleon produced in collisions of longitudinally polarized electrons with an unpolarized deuteron target, for the reaction $d(\vec{e}, e'\vec{N})\Delta$.

Moreover, it can be shown that eight structure functions P_1, P_2, P_5, P_6, P_{9-12} (in the symmetric parts of the corresponding tensors) determine the T–odd contributions to the nucleon polarization vector $\vec{P}$ (for the scattering of unpolarized electrons), whereas the five structure functions P_3, P_4, P_7, P_8, P_{13} (in the antisymmetric parts of the corresponding tensors) determine the T–even contributions to the nucleon polarization vector $\vec{P}$ (for the scattering of longitudinally polarized electrons).

These five T–even structure functions are nonzero even when the $\gamma^* + d \to \Delta + N$ reaction amplitudes are real functions, which is true in the framework of impulse approximation. In the scattering of the longitudinally polarized electrons, they determine the nucleon polarization induced by the absorption of circularly polarized virtual photons (by unpolarized deuteron target) in the $\gamma^* + d \to \Delta + N$ reaction: the polarization is transferred from the electron to the produced nucleon by the virtual photon. The eight T-odd structure functions, defined above, are nonzero only for complex $\gamma^* + d \to \Delta + N$ reaction amplitudes (with different relative phases).

Due to the tensor structure of the quantities $\vec{P}_{ij}$, in the scattering of unpolarized electrons by unpolarized deuterons, the polarization component of the nucleon which is orthogonal to the $\gamma^* + d \to \Delta + N$ reaction plane is characterized by the same ε and φ dependences as in the unpolarized case. The polarization vector of the nucleons polarized in the $\gamma^* + d \to \Delta + N$ reaction plane (components P_x and P_z) is characterized by two dependences: $\varepsilon \sin(2\varphi)$ and $\sqrt{2\varepsilon(1+\varepsilon)} \sin\varphi$.

To prove these statements, we explicitly single out the dependence of the nucleon polarization on the kinematic variables φ and ε. In the general case, the vector of the nucleon polarization can be represented as the sum of two terms: $\vec{P}^{(0)}$ and $\vec{P}^{(\lambda)}$, where the polarization $\vec{P}^{(0)}$ corresponds to the unpolarized electron beam (the so-called induced polarization) and the polarization $\vec{P}^{(\lambda)}$ corresponds to the longitudinally polarized electron beam (polarization transfer). So, the components of the nucleon polarization vector $\vec{P}$ in the reactions $d(e, e'\vec{N})\Delta$, $d(\vec{e}, e'\vec{N})\Delta$ are given by:

$$\vec{P} = \vec{P}^{(0)} + \lambda \vec{P}^{(\lambda)},$$

$$\begin{aligned}
P_x^{(0)}\sigma_0 &= N \sin\varphi[\sqrt{2\varepsilon(1+\varepsilon)}P_x^{(LT)} + \varepsilon\cos\varphi P_x^{(TT)}], \\
P_z^{(0)}\sigma_0 &= N \sin\varphi[\sqrt{2\varepsilon(1+\varepsilon)}P_z^{(LT)} + \varepsilon\cos\varphi P_z^{(TT)}], \\
P_y^{(0)}\sigma_0 &= N[P_y^{(TT)} + \varepsilon P_y^{(LL)} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\varphi P_y^{(LT)} + \varepsilon\cos(2\varphi)\bar{P}_y^{(TT)}], \\
P_x^{(\lambda)}\sigma_0 &= N[\sqrt{1-\varepsilon^2}R_x^{(TT)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\varphi R_x^{(LT)}], \\
P_z^{(\lambda)}\sigma_0 &= N[\sqrt{1-\varepsilon^2}R_z^{(TT)} + \sqrt{2\varepsilon(1-\varepsilon)}\cos\varphi R_z^{(LT)}], \\
P_y^{(\lambda)}\sigma_0 &= N\sqrt{2\varepsilon(1-\varepsilon)}\sin\varphi R_y^{(LT)}],
\end{aligned} \tag{22}$$

where $\sigma_0 = d^3\sigma/dE'd\Omega_e d\Omega_\Delta$ is the unpolarized differential cross section of the reaction under consideration, and the individual contributions to the polarization vector in terms of the structure functions P_i are:

$$\begin{aligned}
&P_x^{(TT)} = 4P_6,\ P_y^{(TT)} = P_{10} + P_{11},\ \bar{P}_y^{(TT)} = P_{10} - P_{11},\ P_z^{(TT)} = 4P_2, \\
&P_x^{(LT)} = -2\frac{\sqrt{Q^2}}{k_0}P_5,\ P_y^{(LT)} = -2\frac{\sqrt{Q^2}}{k_0}P_{12},\ P_z^{(LT)} = -2\frac{\sqrt{Q^2}}{k_0}P_1, \\
&P_y^{(LL)} = 2\frac{Q^2}{k_0^2}P_9,\ R_x^{(TT)} = 4P_8,\ R_z^{(TT)} = 4P_4,
\end{aligned}$$

$$R_x^{(LT)} = -2\frac{\sqrt{Q^2}}{k_0}P_7,\ R_y^{(LT)} = 2\frac{\sqrt{Q^2}}{k_0}P_{13},\ R_z^{(LT)} = -2\frac{\sqrt{Q^2}}{k_0}P_3.$$

The expressions for the structure functions P_i in terms of the reaction amplitudes are general and do not depend on the details of the reaction mechanism. As explicitly shown in Appendix B, each of the 13 structure functions $P_i(W, k^2, t)$, $i = 1-13$, carries independent information about the scalar amplitudes. Therefore, measurement of all these structure functions is, in principle, necessary to perform the complete $\gamma^* + d \to \Delta + N$ experiment.

5 Helicity amplitudes

Since the spin structure of the matrix element of the reaction under consideration is quite complicated, it is convenient to perform the unitarization procedure (taking into account the final state interaction, i.e., the $N\Delta \to N\Delta$ scattering effects) with the help of the helicity amplitudes formalism. As it was shown above, the reaction $\gamma^* + d \to \Delta + N$ is described by 36 independent amplitudes.

Let us introduce the set of the helicity amplitudes $h_{\lambda\lambda'}(k^2, W, \vartheta)$ (where λ and λ' are the helicities of the initial (γ^*+d) and final $(\Delta+N)$ states) and consider the amplitudes

$$h_{\lambda\lambda'} =< \lambda_\Delta, \lambda_N|T|\lambda_\gamma, \lambda_d >= \vec{\chi}_2^{*}(\lambda_\Delta)\vec{F}(\lambda_\gamma, \lambda_d)\chi_1^c(\lambda_N),$$

where $\lambda_\gamma, \lambda_d, \lambda_N$ and λ_Δ are the helicities of the virtual photon, deuteron, nucleon and Δ– isobar, respectively, with $\lambda = \lambda_\gamma - \lambda_d$ and $\lambda' = \lambda_\Delta - \lambda_N$. We choose the following convention:

$$\begin{aligned}
&h_1 =< \frac{1}{2}\frac{1}{2}|T|11 >,\ h_2 =< -\frac{1}{2} - \frac{1}{2}|T|11 >,\ h_3 =< \frac{1}{2}\frac{1}{2}|T|10 >, \qquad (23)\\
&h_4 =< -\frac{1}{2} - \frac{1}{2}|T|10 >,\ h_5 =< \frac{1}{2}\frac{1}{2}|T|1-1 >,\\
&h_6 =< -\frac{1}{2} - \frac{1}{2}|T|1-1 >,\ h_7 =< \frac{1}{2} - \frac{1}{2}|T|11 >,\ h_8 =< -\frac{1}{2}\frac{1}{2}|T|11 >,\\
&h_9 =< \frac{1}{2} - \frac{1}{2}|T|10 >, h_{10} =< -\frac{1}{2}\frac{1}{2}|T|10 >,\ h_{11} =< \frac{1}{2} - \frac{1}{2}|T|1-1 >,\\
&h_{12} =< -\frac{1}{2}\frac{1}{2}|T|1-1 >,\ h_{13} =< \frac{1}{2}\frac{1}{2}|T|01 >,\ h_{14} =< \frac{1}{2}\frac{1}{2}|T|00 >,\\
&h_{15} =< \frac{1}{2}\frac{1}{2}|T|0-1 >,\ h_{16} =< -\frac{1}{2}\frac{1}{2}|T|01 >,\ h_{17} =< -\frac{1}{2}\frac{1}{2}|T|00 >,\\
&h_{18} =< -\frac{1}{2}\frac{1}{2}|T|0-1 >, h_{19} =< \frac{3}{2}\frac{1}{2}|T|11 >,\ h_{20} =< -\frac{3}{2} - \frac{1}{2}|T|11 >,
\end{aligned}$$

$$h_{21} =< \frac{3}{2}\frac{1}{2}|T|10 >,\ h_{22} =< -\frac{3}{2} - \frac{1}{2}|T|10 >,\ h_{23} =< \frac{3}{2}\frac{1}{2}|T|1-1 >,$$
$$h_{24} =< -\frac{3}{2} - \frac{1}{2}|T|1-1 >,\ h_{25} =< \frac{3}{2} - \frac{1}{2}|T|11 >,$$
$$h_{26} =< -\frac{3}{2}\frac{1}{2}|T|11 >,\ h_{27} =< \frac{3}{2} - \frac{1}{2}|T|10 >,\ h_{28} =< -\frac{3}{2}\frac{1}{2}|T|10 >,$$
$$h_{29} =< \frac{3}{2} - \frac{1}{2}|T|1-1 >,\ h_{30} =< -\frac{3}{2}\frac{1}{2}|T|1-1 >,$$
$$h_{31} =< \frac{3}{2}\frac{1}{2}|T|01 >,\ h_{32} =< \frac{3}{2}\frac{1}{2}|T|00 >,\ h_{33} =< \frac{3}{2}\frac{1}{2}|T|0-1 >,$$
$$h_{34} =< -\frac{3}{2}\frac{1}{2}|T|01 >,\ h_{35} =< -\frac{3}{2}\frac{1}{2}|T|00 >,\ h_{36} =< -\frac{3}{2}\frac{1}{2}|T|0-1 > .$$

We choose the helicity amplitudes in such a way that the first 18 helicity amplitudes (corresponding to the Δ–isobar helicities $\pm 1/2$) coincide with the helicity amplitudes for the deuteron electrodisintegration reaction $\gamma^* + d \to n + p$ [44]. As it was shown above, the matrix element of the process under consideration can be described in terms of the scalar amplitudes. The formulas relating the two sets of independent amplitudes f_i and h_i are given in Appendix C.

6 Δ–isobar production in deuteron photodisintegration process

Let us consider the particular case of the Δ–isobar production, in the deuteron photodisintegration reaction

$$\gamma(k) + d(P) \to \Delta(p_1) + N(p_2), \tag{24}$$

where the four–momenta of the particles are given in the brackets. Of course, all observables for this reaction can be obtained using the formulas presented above for the case of the virtual photon, but it is rather tedious procedure. So, it is worth to have the expressions for the differential cross section and various polarization observables which are suitable for the analysis of the future data on this reaction.

The matrix element of this reaction can be written as

$$M = eA_\mu J_\mu = -eA_i J_i, \tag{25}$$

where A_μ is the photon polarization four–vector and we use the transverse gauge: $\vec{k}\cdot\vec{A} = 0$ ($\vec{k}$ is the photon momentum).

The differential cross section in CMS (not averaged over the spins of the initial particles) can be written as

$$\frac{d\sigma}{d\Omega}=\frac{\alpha}{8\pi}\frac{p}{W}\frac{1}{W^2-M^2}\rho_{ij}H_{ij}, \tag{26}$$

where $\rho_{ij}=A_iA_j^*$ and hadronic tensor is determined as $H_{ij}=J_iJ_j^*$. The notation of the other quantities have been defined in previous sections.

In the reaction CMS, the quantity J_i can be written as

$$J_i=\vec{\chi}_2^+\vec{G}_i\chi_1^c, \tag{27}$$

where $\vec{\chi}_2^+$ and χ_1^c are the Δ–isobar vector spinor and nucleon spinor, respectively. The quantity $\vec{G}_i$ can be chosen as

$$\vec{G}_i=\vec{m}G_i^{(m)}+\hat{\vec{k}}G_i^{(k)}, \tag{28}$$

with

$$\begin{aligned}G_i^{(m)}=&m_i(ig_1\vec{U}\cdot\vec{m}+ig_2\vec{U}\cdot\hat{\vec{k}}+g_3\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\vec{m}+g_4\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\hat{\vec{k}}+\\&g_5\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\vec{n}+g_6\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\vec{n})+n_i(ig_7\vec{U}\cdot\vec{n}+g_8\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\vec{n}+\\&g_9\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\vec{m}+g_{10}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\vec{m}+g_{11}\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\hat{\vec{k}}+g_{12}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\hat{\vec{k}}),\\G_i^{(k)}=&m_i(ig_{13}\vec{U}\cdot\vec{m}+ig_{14}\vec{U}\cdot\hat{\vec{k}}+g_{15}\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\vec{m}+g_{16}\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\hat{\vec{k}}+\\&g_{17}\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\vec{n}+g_{18}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\vec{n})+n_i(ig_{19}\vec{U}\cdot\vec{n}+g_{20}\vec{\sigma}\cdot\vec{n}\vec{U}\cdot\vec{n}+\\&g_{21}\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\vec{m}+g_{22}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\vec{m}+g_{23}\vec{\sigma}\cdot\vec{m}\vec{U}\cdot\hat{\vec{k}}+g_{24}\vec{\sigma}\cdot\hat{\vec{k}}\vec{U}\cdot\hat{\vec{k}}),\end{aligned} \tag{29}$$

where $g_i(i=1-24)$ are the scalar amplitudes, depending on two variables (energy and scattering angle), which completely determine the reaction dynamics, and $\vec{U}$ is the deuteron polarization vector.

The hadronic tensor $H_{ij}(i,j=x,y,z)$ depends linearly on the target polarization and it can be represented as follows

$$H_{ij}=H_{ij}(0)+H_{ij}(\xi)+H_{ij}(S), \tag{30}$$

where the term $H_{ij}(0)$ corresponds to the case of the unpolarized deuteron target, and the term $H_{ij}(\xi)(H_{ij}(S))$ corresponds to the case of the vector (tensor-)-polarized target. Let us consider the polarization observables of the $\gamma+d\to\Delta+N$ reaction for each contribution to the hadronic tensor H_{ij}.

The general structure of the hadronic tensor for unpolarized deuteron target has following form

$$H_{ij}(0) = a_1 m_i m_j + a_2 n_i n_j, \tag{31}$$

where a_1 and a_2 are the structure functions which can be expressed in terms of the reaction scalar amplitudes. The expressions of these structure functions can be found in Appendix D.

The differential cross section of this reaction for the case of unpolarized particles can be written as

$$\frac{d\sigma_{un}}{d\Omega} = N(a_1 + a_2),\ \ N = \frac{\alpha}{16\pi}\frac{p}{W}\frac{1}{W^2 - M^2}. \tag{32}$$

Let us consider the case when photon is polarized. The general expression of the photon polarization vector is determined by two real parameters β and δ and it can be written as [45]

$$\vec{A} = \cos\beta\vec{m} + \sin\beta exp(i\delta)\vec{n}. \tag{33}$$

If the parameter $\delta = 0$ then the photon polarization vector describe the linear polarization state of the photon, directed at an angle β with respect to the x axis. The parameters $\beta = \pi/4$ and $\delta = \pm\pi/2$ denote the circular polarization of the photon. Arbitrary β and δ correspond to elliptic photon polarization.

The differential cross section in the case of polarized photon has the following form

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{un}}{d\Omega}(1 + A_\perp \cos 2\beta), \tag{34}$$

where $A_\perp$ is the asymmetry due to the linear polarization of the photon and it can be written as

$$A_\perp = \frac{d\sigma/d\Omega(\beta = 0^0) - d\sigma/d\Omega(\beta = 90^0)}{d\sigma/d\Omega(\beta = 0^0) + d\sigma/d\Omega(\beta = 90^0)}. \tag{35}$$

This asymmetry has following form in terms of the structure functions

$$\frac{d\sigma_{un}}{d\Omega}A_\perp = N(a_1 - a_2)\ or\ A_\perp = \frac{a_1 - a_2}{a_1 + a_2}. \tag{36}$$

Note that circular polarization of the photon does not contribute to the differential cross section due to the P–invariance of the hadron electromagnetic interaction.

6.2 *Vector polarized deuteron target*

For $\gamma + d \to \Delta + N$, the dependence of the polarization observables on the deuteron vector polarization is determined by six structure functions. The part of the hadronic tensor which depends on the deuteron vector polarization has the following general structure:

$$H_{ij}(\xi) = \vec{\xi}\vec{n}(b_1 m_i m_j + b_2 n_i n_j) + \vec{\xi}\hat{\vec{k}}(b_3\{m,n\}_{ij} + ib_4[m,n]_{ij}) + \\ + \vec{\xi}\vec{m}(b_5\{m,n\}_{ij} + ib_6[m,n]_{ij}), \tag{37}$$

where b_i, $(i = 1 - 6)$ are the structure functions, depending on two variables, which can be expressed in terms of the reaction scalar amplitudes. The expressions of these structure functions are given in Appendix D.

The part of the differential cross section of the $\gamma + d \to \Delta + N$ reaction which depends on the deuteron vector polarization, for the case of arbitrarily polarized photon, can be written as

$$\frac{d\sigma_v}{d\Omega} = \frac{d\sigma_{un}}{d\Omega}\Big[A_y\xi_y + C_y^l \cos 2\beta \xi_y + \sin 2\beta \cos\delta(C_x^l\xi_x + C_z^l\xi_z) + \\ + \sin 2\beta \sin\delta(C_x^c\xi_x + C_z^c\xi_z)\Big], \tag{38}$$

where A_y is the asymmetry due to the vector polarization of the deuteron target when the photon is unpolarized (the so–called single target asymmetry). This asymmetry is due to the component of the polarization vector $\vec{\xi}$ describing the vector polarization of the target, which is normal to the reaction plane. If the reaction amplitudes are real functions (as, for example, in the impulse approximation) then this asymmetry is equal to zero. The quantities $C^l_{x,y,z}(C^c_{x,z})$ are the correlation coefficients due to the vector polarization of the deuteron target when the photon is linearly (circularly) polarized. The correlation coefficients $C^l_{x,y,z}$ are zero when the amplitudes are real. The correlation coefficients $C^c_{x,z}$ are determined by the components of the polarization vector lying in the reaction plane and they are non–zero, in general, for the real amplitudes. All these polarization observables can be expressed in terms of the structure functions $b_i(i = 1 - 6)$ as:

$$\frac{d\sigma_{un}}{d\Omega}A_y = N(b_1+b_2),\ \frac{d\sigma_{un}}{d\Omega}C_y^l = N(b_1-b_2),$$
$$\frac{d\sigma_{un}}{d\Omega}C_x^l = 2Nb_5,\ \frac{d\sigma_{un}}{d\Omega}C_z^l = 2Nb_3,$$
$$\frac{d\sigma_{un}}{d\Omega}C_x^c = 2Nb_6,\ \frac{d\sigma_{un}}{d\Omega}C_z^c = 2Nb_4.$$

6.3 Tensor polarized deuteron target

For $\gamma + d \to \Delta + N$, the dependence of the polarization observables, on the deuteron tensor (quadrupole) polarization is completely determined by ten structure functions. The part of the hadronic tensor which depends on the tensor (quadrupole) polarization of the deuteron target has the following general structure:

$$H_{ij}(S) = S_{ab}\hat{k}_a\hat{k}_b(c_1m_im_j + c_2n_in_j) + S_{ab}m_am_b(c_3m_im_j + c_4n_in_j) + S_{ab}\{\hat{k},m\}_{ab}(c_5m_im_j + c_6n_in_j) + S_{ab}\{\hat{k},n\}_{ab}(c_7\{m,n\}_{ij} + ic_8[m,n]_{ij}) + S_{ab}\{m,n\}_{ab}(c_9\{m,n\}_{ij} + ic_{10}[m,n]_{ij}), \qquad (39)$$

where $c_i (i = 1 = 10)$ are structure functions, which depend on two variables. Their expressions in terms of the reaction scalar amplitudes are given in Appendix D.

The part of the differential cross section of the $\gamma + d \to \Delta + N$ reaction which depends on the deuteron tensor polarization, for the case of arbitrarily polarized photon, can be written as

$$\frac{d\sigma_t}{d\Omega} = \frac{d\sigma_{un}}{d\Omega}\Big\{A_{zz}Q_{zz} + A_{xx}(Q_{xx}-Q_{yy}) + A_{xz}Q_{xz} + \cos 2\beta\Big[C_{zz}^lQ_{zz} + C_{xx}^l(Q_{xx}-Q_{yy}) + C_{xz}^lQ_{xz}\Big] + \sin 2\beta\cos\delta(C_{xy}^lQ_{xy} + C_{yz}^lQ_{yz}) + \sin 2\beta\sin\delta(C_{xy}^cQ_{xy} + C_{yz}^cQ_{yz})\Big\}, \qquad (40)$$

where A_{zz}, A_{xx} and A_{xz} are the asymmetries due to the tensor polarization of the deuteron target when the photon is unpolarized. These asymmetries are non–zero, in the general case, if the reaction amplitudes are real, in contrast to the A_y asymmetry. The quantities C_{zz}^l, C_{xx}^l, C_{xz}^l, C_{xy}^l and C_{yz}^l are the correlation coefficients due to the tensor polarization of the deuteron target, when the photon is linearly polarized (they can be non–zero even if the reaction amplitudes are real). The quantities C_{xy}^c and C_{yz}^c are the correlation coefficients which are determined by the tensor polarization of the deuteron target and the circular polarization of the photon (they are completely determined by the reaction mechanism beyond the impulse approximation, for example, by the final–state interaction effects). All these polarization observables can be expressed in terms of the structure functions c_i, $(i=1-10)$ as

$$\begin{aligned}
&\frac{d\sigma_{un}}{d\Omega}A_{zz}=\frac{N}{2}[2\gamma_1^2(c_1+c_2)-c_3-c_4],\ \frac{d\sigma_{un}}{d\Omega}A_{xx}=\frac{N}{2}(c_3+c_4),\\
&\frac{d\sigma_{un}}{d\Omega}A_{xz}=2N\gamma_1(c_5+c_6),\frac{d\sigma_{un}}{d\Omega}C^l_{zz}=\frac{N}{2}[2\gamma_1^2(c_1-c_2)-c_3+c_4],\\
&\frac{d\sigma_{un}}{d\Omega}C^l_{xx}=\frac{N}{2}(c_3-c_4),\quad \frac{d\sigma_{un}}{d\Omega}C^l_{xz}=2N\gamma_1(c_5-c_6),\quad \frac{d\sigma_{un}}{d\Omega}C^l_{xy}=4Nc_9,\\
&\frac{d\sigma_{un}}{d\Omega}C^l_{yz}=4N\gamma_1c_7,\quad \frac{d\sigma_{un}}{d\Omega}C^c_{xy}=4Nc_{10},\quad \frac{d\sigma_{un}}{d\Omega}C^c_{yz}=4N\gamma_1c_8,\\
&\qquad\gamma_1=\frac{W^2+M^2}{2MW}.
\end{aligned}\tag{41}$$

6.4 Polarization of the nucleon

Taking into account the expression of the quantity J_i we may write the nucleon polarization, in the $\gamma+d\to\Delta+N$ reaction, in the following form:

$$\frac{d\sigma_{un}}{d\Omega}\vec{P}=N\rho_{ij}\vec{P}_{ij},\tag{42}$$

where $\vec{P}_{ij}=Tr\rho^{\Delta}_{lk}G_{ki}\vec{\sigma}G^+_{lj}$ (ρ^{Δ}_{lk} is the Δ–isobar spin–density matrix). The general structure of this tensor, for the case of unpolarized Δ–isobar and deuteron target, can be represented in the following form

$$\begin{aligned}
\vec{P}_{ij}=&\hat{\vec{k}}(d_1\{m,n\}_{ij}+id_2[m,n]_{ij})+\vec{m}(d_3\{m,n\}_{ij}+id_4[m,n]_{ij})+\\
&+\vec{n}(d_5m_im_j+d_6n_in_j),
\end{aligned}\tag{43}$$

where d_i $(i=1-6)$ are the structure functions and their expressions in terms of the reaction scalar amplitudes are given in Appendix E. The nucleon polarization in the $\gamma+d\to\Delta+N$ reaction, is completely determined by six structure functions, when the photon is arbitrarily polarized and the other particles are unpolarized.

The vector components of the nucleon polarization are

$$\begin{aligned}
&\frac{d\sigma_{un}}{d\Omega}P_y=P^0_y+\cos2\beta P^l_y,\\
&\frac{d\sigma_{un}}{d\Omega}P_x=\sin2\beta(\cos\delta P^l_x+\sin\delta P^c_x),
\end{aligned}$$

$$\frac{d\sigma_{un}}{d\Omega}P_z = \sin 2\beta(\cos\delta P_z^l + \sin\delta P_z^c), \tag{44}$$

where P_y^0 is the y–component of the nucleon polarization when all other particles are unpolarized, whereas P_y^l, P_x^l and P_z^l are the components of the nucleon polarization when the photon is linear polarized. All these observables arise due to reaction mechanisms beyond the impulse approximation. The quantities P_x^c and P_z^c are the x– and z–components of the nucleon polarization when the photon is circularly polarized and in general, they can be non–zero in the impulse approximation. The expressions of these observables in terms of the structure functions d_i are

$$\begin{aligned} &P_y^0 = \frac{N}{2}(d_5 + d_6),\ P_y^l = \frac{N}{2}(d_5 - d_6), P_x^l = Nd_3,\ P_z^l = Nd_1, \\ &P_x^c = Nd_4,\ P_z^c = Nd_2. \end{aligned} \tag{45}$$

7 Conclusions

We developed a relativistic approach to the calculation of the differential cross section and various polarization observables for the Δ–isobar production in deuteron photo– and electrodisintegration processes, $\gamma + d \to \Delta + N$ and $e^- + d \to e^- + \Delta + N$.

A general analysis of the structure of the differential cross section and polarization observables for the Δ–isobar excitation in the scattering of the electrons by the deuteron target, $\gamma^* + d \to \Delta + N$ was derived. Our formalism is based on the most general symmetry properties of the hadron electromagnetic interaction, such as gauge invariance (the conservation of the hadronic and leptonic electromagnetic currents) and P–invariance (invariance with respect to the space reflections) and does not depend on the deuteron structure and on the details of the reaction mechanism for $\gamma^* + d \to \Delta + N$. This general analysis was done with the help of the structure function formalism which is especially convenient for the investigation of the polarization phenomena in this reaction.

The observables related to the cases of an arbitrary polarized deuteron target, longitudinally polarized electron beam, polarization of the outgoing nucleon, as well as the polarization transfer from electron to final nucleon, and the correlation of the electron and deuteron polarizations were considered in detail. We derived the expressions for polarization effects which are absent in the impulse approximation and due to the strong $\Delta N-$ interaction in the final state.

A particular case of the process of the photoproduction of the Δ–isobar on the deuteron target has been considered in details. The differential cross section and various polarization observables have been derived in terms of the reaction amplitudes. The polarization observables due to the linear and circular polarizations of the photon provided the deuteron target is arbitrarily polarized have been derived in terms of the reaction amplitudes. The polarization of the final nucleon is also considered.

General properties of these observables have been derived and underlined. Such properties should be fulfilled by any model calculation. In this respect, the present approach is important, as it gives on one side, guidelines for models and, on the other side, defines the strategy (the observables and the kinematical conditions) for experiments.

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9 Appendix A

In this Appendix, we present the formulas for the structure functions which determine the hadronic tensor H_{ij} for various polarization states of the deuteron target. The functions are written in terms of the scalar amplitudes f_i ($i = 1, ..., 36$) determining the $\gamma^* + d \to \Delta + N$ reaction.

- Unpolarized deuteron target.

The hadronic tensor $H_{ij}(0)$ is determined by the structure functions α_i, ($i = 1, .., 5$)

$$\begin{aligned}
\alpha_1 = \frac{2}{3}\Big\{ & Ax_1\Big[|f_{13}|^2 + |f_{15}|^2 + |f_{17}|^2 + |f_{18}|^2 + z|f_{14}|^2 + z|f_{16}|^2\Big] + \\
& Ax_2\Big[|f_{31}|^2 + |f_{33}|^2 + |f_{35}|^2 + |f_{36}|^2 + z|f_{32}|^2 + z|f_{34}|^2\Big] + \\
& 2Ax_3Re(f_{13}f_{31}^* + f_{18}f_{36}^* + f_{17}f_{35}^* + f_{15}f_{33}^* + zf_{14}f_{32}^* + zf_{16}f_{34}^*) + \\
& 2BRe(f_{18}f_{35}^* - f_{17}f_{36}^* + f_{33}f_{13}^* - f_{31}f_{15}^* + zf_{34}f_{14}^* - zf_{32}f_{16}^*)\Big\}, \\
\alpha_2 = \frac{2}{3}\Big\{ & Ax_1\Big[|f_7|^2 + |f_8|^2 + |f_9|^2 + |f_{10}|^2 + z|f_{11}|^2 + z|f_{12}|^2\Big] +
\end{aligned}$$

$$Ax_2\Big[|f_{25}|^2+|f_{26}|^2+|f_{27}|^2+|f_{28}|^2+z|f_{29}|^2+z|f_{30}|^2\Big]+$$
$$2Ax_3Re(f_7f_{25}^*+f_{10}f_{28}^*+f_9f_{27}^*+f_8f_{26}^*+zf_{11}f_{29}^*+zf_{12}f_{30}^*)+$$
$$2BRe(f_{10}f_{27}^*-f_9f_{28}^*+f_{26}f_7^*-f_{25}f_8^*+zf_{12}f_{29}^*-zf_{11}f_{30}^*)\Big\},$$

$$\alpha_3=\frac{2}{3}\Big\{Ax_1\Big[|f_1|^2+|f_3|^2+|f_5|^2+|f_6|^2+z|f_2|^2+z|f_4|^2\Big]+$$
$$Ax_2\Big[|f_{19}|^2+|f_{21}|^2+|f_{23}|^2+|f_{24}|^2+z|f_{20}|^2+z|f_{22}|^2\Big]+$$
$$2Ax_3Re(f_1f_{19}^*+f_3f_{21}^*+f_5f_{23}^*+f_6f_{24}^*+zf_2f_{20}^*+zf_4f_{22}^*)+$$
$$2BRe(f_6f_{23}^*-f_5f_{24}^*+f_{21}f_1^*-f_{19}f_3^*+zf_{22}f_2^*-zf_{20}f_4^*)\Big\},$$

$$\alpha_4=\frac{2}{3}ReC,\ \alpha_5=-\frac{2}{3}ImC,$$

$$C=Ax_1\Big[f_1f_{13}^*+f_3f_{15}^*+f_5f_{17}^*+f_6f_{18}^*+zf_2f_{14}^*+zf_4f_{16}^*\Big]+$$
$$Ax_2\Big[f_{19}f_{31}^*+f_{21}f_{33}^*+f_{23}f_{35}^*+f_{24}f_{36}^*+zf_{20}f_{32}^*+zf_{22}f_{34}^*\Big]+$$
$$Ax_3\Big[f_1f_{31}^*+f_3f_{33}^*+f_5f_{35}^*+f_6f_{36}^*+f_{19}f_{13}^*+f_{21}f_{15}^*+f_{23}f_{17}^*+$$
$$f_{24}f_{18}^*+zf_2f_{32}^*+zf_4f_{34}^*+zf_{20}f_{14}^*+zf_{22}f_{16}^*\Big]+$$
$$B\Big[f_6f_{35}^*+f_{23}f_{18}^*-f_5f_{36}^*-f_{14}f_{17}^*+f_{21}f_{13}^*-f_{19}f_{15}^*+f_1f_{33}^*-$$
$$f_3f_{31}^*-zf_{20}f_{16}^*+zf_{22}f_{14}^*+zf_2f_{34}^*-zf_4f_{32}^*\Big].$$

We use here the notation

$$x_1=1+\frac{(\vec{m}\cdot\vec{p})^2}{M_\Delta^2},\ x_2=1+\frac{(\hat{\vec{k}}\cdot\vec{p})^2}{M_\Delta^2},\ x_3=\frac{\vec{m}\cdot\vec{p}\hat{\vec{k}}\cdot\vec{p}}{M_\Delta^2},$$
$$A=\frac{2M_\Delta^2}{3M_\Delta^2+\vec{p}^2},\ B=\frac{M_\Delta E_\Delta}{3M_\Delta^2+\vec{p}^2},\ z=\frac{\omega^2}{M^2},$$

where M_Δ is the Δ–isobar mass, $\vec{p}$ (E_Δ) and ω are the momentum (energy) and energy of the Δ–isobar and deuteron in CMS of the $\gamma^*+d\rightarrow\Delta+N$ reaction, which are expressed in term of the total energy and of the masses of the particles as:

$$\omega=\frac{W^2+M^2-k^2}{2W},\ E_\Delta=\frac{W^2+M_\Delta^2-m^2}{2W},$$
$$|\vec{p}|=\frac{1}{2W}\sqrt{(W^2+M_\Delta^2-m^2)^2-4W^2M_\Delta^2}.$$

• <u>Vector polarized deuteron target.</u>

The hadronic tensor $H_{ij}(\xi)$ is determined by the structure functions β_i ($i = 1,..,13$)

$$
\begin{aligned}
\beta_1 =& -2\frac{\omega}{M}ImD_1,\ \ \beta_2 = -2\frac{\omega}{M}ImD_2,\\
D_1 =& Ax_1(f_{14}f_{13}^* + f_{16}f_{15}^*) + Ax_2(f_{32}f_{31}^* + f_{34}f_{33}^*) + Ax_3(f_{14}f_{31}^* - f_{13}f_{32}^* +\\
& +f_{16}f_{33}^* - f_{15}f_{34}^*) - B(f_{13}f_{34}^* + f_{16}f_{31}^* - f_{14}f_{33}^* - f_{15}f_{32}^*),\\
D_2 =& Ax_1(f_2f_1^* + f_4f_3^*) + Ax_2(f_{20}f_{19}^* + f_{22}f_{21}^*) + Ax_3(f_2f_{19}^* - f_1f_{20}^* +\\
& f_4f_{21}^* - f_3f_{22}^*) - B(f_1f_{22}^* + f_4f_{19}^* - f_2f_{21}^* - f_3f_{20}^*),\\
\beta_3 =& -2\frac{\omega}{M}ImD_3,\\
D_3 =& Ax_1(f_{11}f_9^* + f_{12}f_{10}^*) + Ax_2(f_{29}f_{27}^* + f_{30}f_{28}^*) + Ax_3(f_{12}f_{28}^* +\\
& f_{30}f_{10}^* + f_{11}f_{27}^* - f_9f_{29}^*) + B(f_{12}f_{27}^* + f_9f_{30}^* - f_{10}f_{29}^* - f_{11}f_{28}^*),\\
\beta_4 =& -\frac{\omega}{M}ImD_4,\ \ \beta_5 = -\frac{\omega}{M}ReD_4,\\
\beta_6 =& -ImD_5,\ \ \beta_8 = -ReD_5,\\
\beta_7 =& -ImD_6,\ \ \beta_9 = -ReD_6,\\
D_4 =& Ax_1(f_2f_{13}^* + f_4f_{15}^* - f_1f_{14}^* - f_3f_{16}^*) +\\
& Ax_2(f_{20}f_{31}^* + f_{22}f_{33}^* - f_{19}f_{32}^* - f_{21}f_{34}^*) +\\
& Ax_3(f_2f_{31}^* + f_{20}f_{13}^* - f_1f_{32}^* - f_{19}f_{14}^* + f_4f_{33}^* - f_3f_{34}^* + f_{22}f_{15}^* - f_{21}f_{16}^*) -\\
& B(f_{21}f_{14}^* + f_1f_{34}^* - f_{22}f_{13}^* - f_2f_{33}^* + f_{20}f_{15}^* + f_4f_{31}^* - f_{19}f_{16}^* - f_3f_{22}^*),\\
D_5 =& Ax_1(f_9f_{17}^* + f_{10}f_{18}^* - f_7f_{13}^* - f_8f_{15}^*) +\\
& Ax_2(f_{27}f_{35}^* + f_{28}f_{36}^* - f_{25}f_{31}^* - f_{26}f_{33}^*) +\\
& Ax_3(f_{10}f_{36}^* + f_{28}f_{18}^* - f_7f_{31}^* - f_{25}f_{13}^* + f_{27}f_{17}^* - f_8f_{33}^* + f_9f_{35}^* - f_{26}f_{15}^*) +\\
& B(f_{10}f_{35}^* + f_{27}f_{18}^* - f_9f_{36}^* - f_{28}f_{17}^* + f_8f_{31}^* + f_{25}f_{15}^* - f_7f_{33}^* - f_{26}f_{13}^*),\\
D_6 =& Ax_1(f_9f_5^* + f_{10}f_6^* - f_7f_1^* - f_8f_3^*) +\\
& Ax_2(f_{27}f_{23}^* + f_{28}f_{24}^* - f_{25}f_{19}^* - f_{26}f_{21}^*) +\\
& Ax_3(f_{10}f_{24}^* + f_{28}f_6^* - f_7f_{19}^* - f_{25}f_1^* + f_{27}f_5^* - f_8f_{21}^* + f_9f_{23}^* - f_{26}f_3^*) +\\
& B(f_{10}f_{23}^* + f_{27}f_6^* - f_9f_{24}^* - f_{28}f_5^* + f_8f_{19}^* + f_{25}f_3^* - f_7f_{21}^* - f_{26}f_1^*),\\
\beta_{10} =& -\frac{\omega}{M}ImD_7,\ \ \beta_{12} = -\frac{\omega}{M}ReD_7,\\
\beta_{11} =& -\frac{\omega}{M}ImD_8,\ \ \beta_{13} = -\frac{\omega}{M}ReD_8,\\
D_7 =& Ax_1(f_7f_{14}^* + f_8f_{16}^* - f_{11}f_{17}^* - f_{12}f_{18}^*) + Ax_2(f_{25}f_{32}^* + f_{26}f_{34}^* -\\
& f_{29}f_{35}^* - f_{36}f_{30}^*) + Ax_3(f_7f_{32}^* + f_{25}f_{14}^* -\\
& f_{12}f_{36}^* - f_{30}f_{18}^* + f_8f_{34}^* - f_{29}f_{17}^* + f_{26}f_{16}^* - f_{11}f_{35}^*) +\\
& B(f_{30}f_{17}^* + f_{11}f_{36}^* - f_{29}f_{18}^* - f_{12}f_{35}^* + f_{26}f_{14}^* + f_7f_{34}^* - f_{25}f_{16}^* - f_8f_{32}^*),\\
D_8 =& Ax_1(f_7f_2^* + f_8f_4^* - f_{11}f_5^* - f_{12}f_6^*) +\\
& Ax_2(f_{25}f_{20}^* + f_{26}f_{22}^* - f_{29}f_{23}^* - f_{30}f_{24}^*) +\\
& Ax_3(f_7f_{20}^* + f_{25}f_2^* - f_{30}f_6^* - f_{12}f_{24}^* + f_8f_{22}^* - f_{29}f_5^* + f_{26}f_4^* - f_{11}f_{23}^*) +\\
& B(f_{30}f_5^* + f_{11}f_{24}^* - f_{29}f_6^* - f_{12}f_{23}^* + f_{26}f_2^* + f_7f_{22}^* - f_{25}f_4^* - f_8f_{20}^*).
\end{aligned}
$$

• <u>Tensor polarized deuteron target.</u>

The hadronic tensor $H_{ij}(S)$ is determined by the structure functions γ_i ($i = 1,..,23$)

$$\begin{aligned}
\gamma_1 = {} & 2Ax_1\Big[|f_{14}|^2+|f_{16}|^2-\frac{M^2}{\omega^2}(|f_{17}|^2+|f_{18}|^2)\Big]+2Ax_2\Big[|f_{32}|^2+|f_{34}|^2- \\
& \frac{M^2}{\omega^2}(|f_{35}|^2+|f_{36}|^2)\Big]+4Ax_3Re\Big[f_{16}f_{34}^*+f_{14}f_{32}^*-\frac{M^2}{\omega^2}(f_{17}f_{35}^*+ \\
& f_{18}f_{36}^*)\Big]-4BRe\Big[f_{32}f_{16}^*-f_{34}f_{14}^*-\frac{M^2}{\omega^2}(f_{17}f_{36}^*-f_{18}f_{35}^*)\Big], \\
\gamma_2 = {} & 2Ax_1\Big[|f_2|^2+|f_4|^2-\frac{M^2}{\omega^2}(|f_5|^2+|f_6|^2)\Big]+2Ax_2\Big[|f_{20}|^2+|f_{22}|^2- \\
& \frac{M^2}{\omega^2}(|f_{23}|^2+|f_{24}|^2)\Big]+4Ax_3Re\Big[f_4f_{22}^*+f_2f_{20}^*-\frac{M^2}{\omega^2}(f_5f_{23}^*+ \\
& f_6f_{24}^*)\Big]-4BRe\Big[f_{20}f_4^*-f_{22}f_2^*-\frac{M^2}{\omega^2}(f_5f_{24}^*-f_6f_{23}^*)\Big], \\
\gamma_3 = {} & 2Ax_1\Big[|f_{11}|^2+|f_{12}|^2-\frac{M^2}{\omega^2}(|f_7|^2+|f_8|^2)\Big]+2Ax_2\Big[|f_{29}|^2+|f_{30}|^2- \\
& \frac{M^2}{\omega^2}(|f_{25}|^2+|f_{26}|^2)\Big]+4Ax_3Re\Big[f_{11}f_{29}^*+f_{12}f_{30}^*-\frac{M^2}{\omega^2}(f_8f_{26}^*+ \\
& +f_7f_{25}^*)\Big]+4BRe\Big[f_{12}f_{29}^*-f_{11}f_{30}^*-\frac{M^2}{\omega^2}(f_{26}f_7^*-f_{25}f_8^*)\Big], \\
\gamma_4 = {} & 2Ax_1Re\Big[f_2f_{14}^*+f_4f_{16}^*-\frac{M^2}{\omega^2}(f_6f_{18}^*+f_5f_{17}^*)\Big]+2Ax_2Re\Big[f_{20}f_{32}^*+ \\
& f_{22}f_{34}^*-\frac{M^2}{\omega^2}(f_{24}f_{36}^*+f_{23}f_{35}^*)\Big]+2Ax_3Re\Big[f_4f_{34}^*+f_{22}f_{16}^*+f_2f_{32}^*+ \\
& f_{20}f_{14}^*-\frac{M^2}{\omega^2}(f_5f_{35}^*+f_{23}f_{17}^*+f_6f_{36}^*+f_{24}f_{18}^*)\Big]+2BRe\Big[f_{34}f_2^*+ \\
& f_{14}f_{22}^*-f_{32}f_4^*-f_{16}f_{20}^*-\frac{M^2}{\omega^2}(f_6f_{35}^*+f_{23}f_{18}^*-f_5f_{36}^*-f_{24}f_{17}^*)\Big], \\
\gamma_5 = {} & -2Ax_1Im\Big[f_2f_{14}^*+f_4f_{16}^*-\frac{M^2}{\omega^2}(f_6f_{18}^*+f_5f_{17}^*)\Big]- \\
& 2Ax_2Im\Big[f_{20}f_{32}^*+f_{22}f_{34}^*-\frac{M^2}{\omega^2}(f_{24}f_{36}^*+f_{23}f_{35}^*)\Big]- \\
& 2Ax_3Im\Big[f_4f_{34}^*+f_{22}f_{16}^*+f_2f_{32}^*+f_{20}f_{14}^*- \\
& \frac{M^2}{\omega^2}(f_5f_{35}^*+f_{23}f_{17}^*+f_6f_{36}^*+f_{24}f_{18}^*)\Big]-
\end{aligned}$$

$$2BIm\Big[-f_{34}f_2^* - f_{14}f_{22}^* + f_{32}f_4^* + f_{16}f_{20}^* -$$
$$\frac{M^2}{\omega^2}(f_6f_{35}^* + f_{23}f_{18}^* - f_5f_{36}^* - f_{24}f_{17}^*)\Big],$$

$$\gamma_6 = 2Ax_1\Big[|f_{13}|^2 + |f_{15}|^2 - |f_{17}|^2 - |f_{18}|^2\Big] + 2Ax_2\Big[|f_{31}|^2 + |f_{33}|^2 -$$
$$|f_{35}|^2 - |f_{36}|^2\Big] + 4Ax_3Re\Big[f_{15}f_{33}^* + f_{13}f_{31}^* - f_{17}f_{35}^* - f_{18}f_{36}^*\Big] -$$
$$4BRe\Big[f_{18}f_{35}^* - f_{17}f_{36}^* + f_{31}f_{15}^* - f_{33}f_{13}^*\Big],$$

$$\gamma_7 = 2Ax_1\Big[|f_1|^2 + |f_3|^2 - |f_5|^2 - |f_6|^2\Big] + 2Ax_2\Big[|f_{19}|^2 + |f_{21}|^2 - |f_{23}|^2 -$$
$$|f_{24}|^2\Big] + 4Ax_3Re\Big[f_3f_{21}^* + f_1f_{19}^* - f_5f_{23}^* - f_6f_{24}^*\Big] -$$
$$4BRe\Big[f_6f_{23}^* - f_5f_{24}^* + f_{19}f_3^* - f_{21}f_1^*\Big],$$

$$\gamma_8 = 2Ax_1\Big[|f_9|^2 + |f_{10}|^2 - |f_7|^2 - |f_8|^2\Big] + 2Ax_2\Big[|f_{27}|^2 + |f_{28}|^2 - |f_{25}|^2 -$$
$$|f_{26}|^2\Big] + 4Ax_3Re\Big[f_9f_{27}^* + f_{10}f_{28}^* - f_8f_{26}^* - f_7f_{25}^*\Big] +$$
$$4BRe\Big[f_{10}f_{27}^* - f_9f_{28}^* + f_{25}f_8^* - f_{26}f_7^*\Big],$$

$$\gamma_9 = 2Ax_1Re\Big[f_1f_{13}^* + f_3f_{15}^* - f_6f_{18}^* - f_5f_{17}^*\Big] + 2Ax_2Re\Big[f_{19}f_{31}^* + f_{21}f_{33}^* -$$
$$f_{24}f_{36}^* - f_{23}f_{35}^*\Big] + 2Ax_3Re\Big[f_3f_{33}^* + f_{21}f_{15}^* + f_1f_{31}^* + f_{19}f_{13}^* - f_5f_{35}^* -$$
$$f_{23}f_{17}^* - f_6f_{36}^* - f_{24}f_{18}^*\Big] - 2BRe\Big[f_6f_{35}^* + f_{23}f_{18}^* - f_5f_{36}^* - f_{24}f_{17}^* +$$
$$f_{15}f_{19}^* + f_{31}f_3^* - f_{13}f_{21}^* - f_{33}f_1^*\Big],$$

$$\gamma_{10} = -2Ax_1Im\Big[f_1f_{13}^* + f_3f_{15}^* - f_6f_{18}^* - f_5f_{17}^*\Big] - 2Ax_2Im\Big[f_{19}f_{31}^* +$$
$$f_{21}f_{33}^* - f_{24}f_{36}^* - f_{23}f_{35}^*\Big] - 2Ax_3Im\Big[f_3f_{33}^* + f_{21}f_{15}^* + f_1f_{31}^* + f_{19}f_{13}^* -$$
$$f_5f_{35}^* - f_{23}f_{17}^* - f_6f_{36}^* - f_{24}f_{18}^*\Big] + 2BRe\Big[f_6f_{35}^* + f_{23}f_{18}^* - f_5f_{36}^* -$$
$$f_{24}f_{17}^* - f_{15}f_{19}^* - f_{31}f_3^* + f_{13}f_{21}^* + f_{33}f_1^*\Big],$$

$$\gamma_{11} = 2Ax_1Re\Big[f_{13}f_{14}^* + f_{15}f_{16}^*\Big] + 2Ax_2Re\Big[f_{31}f_{32}^* + f_{33}f_{34}^*\Big] +$$
$$2Ax_3Re\Big[f_{15}f_{34}^* + f_{16}f_{33}^* + f_{13}f_{32}^* + f_{14}f_{31}^*\Big] -$$
$$2BRe\Big[f_{31}f_{16}^* + f_{32}f_{15}^* - f_{33}f_{14}^* - f_{34}f_{13}^*\Big],$$

$$\gamma_{12} = 2Ax_1Re\Big[f_1f_2^* + f_3f_4^*\Big] + 2Ax_2Re\Big[f_{19}f_{20}^* + f_{21}f_{22}^*\Big] + 2Ax_3Re\Big[f_3f_{22}^* +$$

$$f_4f_{21}^* + f_2f_{19}^* + f_1f_{20}^*\Big] - 2BRe\Big[f_{19}f_4^* + f_{20}f_3^* - f_{21}f_2^* - f_{22}f_1^*\Big],$$

$$\gamma_{13} = 2Ax_1Re\Big[f_9f_{11}^* + f_{10}f_{12}^*\Big] + 2Ax_2Re\Big[f_{27}f_{29}^* + f_{28}f_{30}^*\Big] +$$
$$2Ax_3Re\Big[f_9f_{29}^* + f_{10}f_{30}^* + f_{11}f_{27}^* + f_{12}f_{28}^*\Big] +$$
$$2BRe\Big[f_{10}f_{29}^* + f_{12}f_{27}^* - f_9f_{30}^* - f_{11}f_{28}^*\Big],$$

$$\gamma_{14} = Ax_1Re\Big[f_1f_{14}^* + f_2f_{13}^* + f_3f_{16}^* + f_4f_{15}^*\Big] + Ax_2Re\Big[f_{19}f_{32}^* + f_{20}f_{31}^* +$$
$$f_{21}f_{34}^* + f_{22}f_{33}^*\Big] + Ax_3Re\Big[f_1f_{32}^* + f_2f_{31}^* + f_{22}f_{15}^* + f_3f_{34}^* +$$
$$f_4f_{33}^* + f_{19}f_{14}^* + f_{20}f_{13}^* + f_{21}f_{16}^*\Big] - BRe\Big[f_{15}f_{20}^* + f_{16}f_{19}^* + f_{31}f_4^* +$$
$$f_{32}f_3^* - f_{13}f_{22}^* - f_{14}f_{21}^* - f_{33}f_2^* - f_{34}f_1^*\Big],$$

$$\gamma_{15} = -Ax_1Im\Big[f_1f_{14}^* + f_2f_{13}^* + f_3f_{16}^* + f_4f_{15}^*\Big] - Ax_2Im\Big[f_{19}f_{32}^* + f_{20}f_{31}^* +$$
$$f_{21}f_{34}^* + f_{22}f_{33}^*\Big] - Ax_3Im\Big[f_1f_{32}^* + f_2f_{31}^* + f_{22}f_{15}^* + f_3f_{34}^* +$$
$$f_4f_{33}^* + f_{19}f_{14}^* + f_{20}f_{13}^* + f_{21}f_{16}^*\Big] - BIm\Big[f_{15}f_{20}^* + f_{16}f_{19}^* + f_{31}f_4^* +$$
$$f_{32}f_3^* - f_{13}f_{22}^* - f_{14}f_{21}^* - f_{33}f_2^* - f_{34}f_1^*\Big],$$

$$\gamma_{16} = Ax_1Re\Big[f_7f_{14}^* + f_{11}f_{17}^* + f_8f_{16}^* + f_{12}f_{18}^*\Big] + Ax_2Re\Big[f_{25}f_{32}^* + f_{26}f_{34}^* +$$
$$f_{29}f_{35}^* + f_{30}f_{36}^*\Big] + Ax_3Re\Big[f_7f_{32}^* + f_{25}f_{14}^* + f_{18}f_{30}^* + f_{36}f_{12}^* +$$
$$f_{17}f_{29}^* + f_{35}f_{11}^* + f_8f_{34}^* + f_{26}f_{16}^*\Big] + BRe\Big[f_{12}f_{35}^* + f_{29}f_{18}^* - f_{11}f_{36}^* -$$
$$f_{30}f_{17}^* + f_{14}f_{26}^* + f_{34}f_7^* - f_{16}f_{25}^* - f_{32}f_8^*\Big],$$

$$\gamma_{17} = Ax_1Re\Big[f_2f_7^* + f_4f_8^* + f_5f_{11}^* + f_6f_{12}^*\Big] + Ax_2Re\Big[f_{20}f_{25}^* + f_{22}f_{26}^* +$$
$$f_{23}f_{29}^* + f_{24}f_{30}^*\Big] + Ax_3Re\Big[f_5f_{29}^* + f_{23}f_{11}^* + f_4f_{26}^* + f_{22}f_8^* +$$
$$f_2f_{25}^* + f_{20}f_7^* + f_6f_{30}^* + f_{24}f_{12}^*\Big] + BRe\Big[f_6f_{29}^* + f_{23}f_{12}^* - f_5f_{30}^* -$$
$$f_{24}f_{11}^* + f_{26}f_2^* + f_7f_{22}^* - f_{25}f_4^* - f_8f_{20}^*\Big],$$

$$\gamma_{18} = -Ax_1Im\Big[f_7f_{14}^* + f_{11}f_{17}^* + f_8f_{16}^* + f_{12}f_{18}^*\Big] - Ax_2Im\Big[f_{25}f_{32}^* + f_{26}f_{34}^* +$$
$$f_{29}f_{35}^* + f_{30}f_{36}^*\Big] + Ax_3Im\Big[-f_7f_{32}^* - f_{25}f_{14}^* + f_{18}f_{30}^* + f_{36}f_{12}^* +$$
$$f_{17}f_{29}^* + f_{35}f_{11}^* - f_8f_{34}^* - f_{26}f_{16}^*\Big] + BIm\Big[-f_{12}f_{35}^* - f_{29}f_{18}^* +$$

$$f_{11}f_{36}^* + f_{30}f_{17}^* + f_{14}f_{26}^* + f_{34}f_7^* - f_{16}f_{25}^* - f_{32}f_8^*\Big],$$

$$\gamma_{19} = Ax_1Im\Big[f_2f_7^* + f_4f_8^* + f_5f_{11}^* + f_6f_{12}^*\Big] + Ax_2Im\Big[f_{20}f_{25}^* +$$

$$f_{22}f_{26}^* + f_{23}f_{29}^* + f_{24}f_{30}^*\Big] + Ax_3Im\Big[f_5f_{29}^* + f_{23}f_{11}^* + f_4f_{26}^* +$$

$$f_{22}f_8^* + f_2f_{25}^* + f_{20}f_7^* + f_6f_{30}^* + f_{24}f_{12}^*\Big] + BIm\Big[f_6f_{29}^* +$$

$$f_{23}f_{12}^* - f_5f_{30}^* - f_{24}f_{11}^* - f_{26}f_2^* - f_7f_{22}^* + f_{25}f_4^* + f_8f_{20}^*\Big],$$

$$\gamma_{20} = Ax_1Re\Big[f_7f_{13}^* + f_{10}f_{18}^* + f_8f_{15}^* + f_9f_{17}^*\Big] + Ax_2Re\Big[f_{25}f_{31}^* +$$

$$f_{26}f_{33}^* + f_{28}f_{36}^* + f_{27}f_{35}^*\Big] + Ax_3Re\Big[f_7f_{31}^* + f_{25}f_{13}^* + f_{18}f_{28}^* +$$

$$f_{36}f_{10}^* + f_{17}f_{27}^* + f_{35}f_9^* + f_8f_{33}^* + f_{26}f_{15}^*\Big] + BRe\Big[f_{10}f_{35}^* +$$

$$f_{27}f_{18}^* - f_9f_{36}^* - f_{28}f_{17}^* + f_{13}f_{26}^* + f_{33}f_7^* - f_{15}f_{25}^* - f_{31}f_8^*\Big],$$

$$\gamma_{21} = Ax_1Re\Big[f_1f_7^* + f_3f_8^* + f_5f_9^* + f_6f_{10}^*\Big] + Ax_2Re\Big[f_{19}f_{25}^* + f_{21}f_{26}^* +$$

$$f_{23}f_{27}^* + f_{24}f_{28}^*\Big] + Ax_3Re\Big[f_5f_{27}^* + f_{23}f_9^* + f_3f_{26}^* + f_{21}f_8^* +$$

$$f_1f_{25}^* + f_{19}f_7^* + f_6f_{28}^* + f_{24}f_{10}^*\Big] + BRe\Big[f_6f_{27}^* + f_{23}f_{10}^* - f_5f_{28}^* -$$

$$f_{24}f_9^* + f_{26}f_1^* + f_7f_{21}^* - f_{25}f_3^* - f_8f_{19}^*\Big],$$

$$\gamma_{22} = -Ax_1Im\Big[f_7f_{13}^* + f_{10}f_{18}^* + f_8f_{25}^* + f_9f_{17}^*\Big] - Ax_2Im\Big[f_{25}f_{31}^* +$$

$$f_{26}f_{33}^* + f_{28}f_{36}^* + f_{27}f_{35}^*\Big] + Ax_3Im\Big[-f_7f_{31}^* - f_{25}f_{13}^* + f_{18}f_{28}^* +$$

$$f_{36}f_{10}^* + f_{17}f_{27}^* + f_{35}f_9^* - f_8f_{33}^* - f_{26}f_{15}^*\Big] + BIm\Big[-f_{10}f_{35}^* - +$$

$$f_{27}f_{18}^* + f_9f_{36}^* + f_{28}f_{17}^* + f_{13}f_{26}^* + f_{33}f_7^* - f_{15}f_{25}^* - f_{31}f_8^*\Big],$$

$$\gamma_{23} = Ax_1Im\Big[f_1f_7^* + f_3f_8^* + f_5f_9^* + f_6f_{10}^*\Big] + Ax_2Im\Big[f_{19}f_{25}^* + f_{21}f_{26}^* +$$

$$f_{23}f_{27}^* + f_{24}f_{28}^*\Big] + Ax_3Im\Big[f_5f_{27}^* + f_{23}f_9^* + f_3f_{26}^* + f_{21}f_8^* +$$

$$f_1f_{25}^* + f_{19}f_7^* + f_6f_{28}^* + f_{24}f_{10}^*\Big] + BIm\Big[f_6f_{27}^* + f_{23}f_{10}^* - f_5f_{28}^* -$$

$$f_{24}f_9^* - f_{26}f_1^* - f_7f_{21}^* + f_{25}f_3^* + f_8f_{19}^*\Big].$$

10 Appendix B

Here we present the expressions for the structure functions P_i, $(i = 1 - 13)$, which determine the tensor $\vec{P}_{ij}$. These structure functions, defining the nucleon polarization vector $\vec{P}$, are written in terms of the scalar amplitudes f_i $(i = 1, ..., 36)$ determining the $\gamma^* + d \to \Delta + N$ reaction.

$$
\begin{aligned}
P_1 =& ImQ_1, \ P_2 = ImQ_2, \ P_3 = ReQ_1, \ P_4 = ReQ_2, \\
Q_1 =& \frac{2}{3}Ax_1\Big[f_{10}f_{13}^* + f_9 f_{15}^* - f_7 f_{18}^* - f_8 f_{17}^* + zf_{11}f_{16}^* + zf_{12}f_{14}^*\Big] + \\
& \frac{2}{3}Ax_2\Big[f_{27}f_{33}^* + +f_{28}f_{31}^* - f_{25}f_{36}^* - f_{26}f_{35}^* + zf_{29}f_{34}^* + zf_{30}f_{32}^*\Big] + \\
& \frac{2}{3}Ax_3\Big[f_9 f_{33}^* + f_{10}f_{31}^* + f_{27}f_{15}^* + +f_{28}f_{13}^* - f_7 f_{36}^* - f_8 f_{35}^* - f_{25}f_{18}^* - \\
& f_{26}f_{17}^* + zf_{11}f_{34}^* + zf_{12}f_{32}^* + zf_{29}f_{16}^* + zf_{30}f_{14}^*\Big] + \\
& \frac{2}{3}B\Big[f_8 f_{36}^* + f_{10}f_{33}^* + f_{25}f_{17}^* + f_{27}f_{13}^* - f_7 f_{35}^* - f_9 f_{31}^* - f_{26}f_{18}^* - \\
& f_{28}f_{15}^* + zf_{12}f_{34}^* + +zf_{29}f_{14}^* - zf_{11}f_{32}^* - zf_{30}f_{16}^*\Big], \\
Q_2 =& \frac{2}{3}Ax_1\Big[f_9 f_3^* + f_{10}f_1^* - f_7 f_6^* - f_8 f_5^* + zf_{11}f_4^* + zf_{12}f_2^*\Big] + \\
& \frac{2}{3}Ax_2\Big[f_{27}f_{21}^* + f_{28}f_{19}^* - f_{25}f_{24}^* - f_{26}f_{23}^* + zf_{29}f_{22}^* + zf_{30}f_{20}^*\Big] + \\
& \frac{2}{3}Ax_3\Big[f_9 f_{21}^* + f_{10}f_{19}^* + f_{27}f_3^* + +f_{28}f_1^* - f_7 f_{24}^* - f_8 f_{23}^* - f_{25}f_6^* - \\
& f_{26}f_5^* + zf_{11}f_{22}^* + zf_{12}f_{20}^* + zf_{29}f_4^* + zf_{30}f_2^*\Big] + \\
& \frac{2}{3}B\Big[f_8 f_{24}^* + f_{10}f_{21}^* + f_{25}f_5^* + f_{27}f_1^* - f_7 f_{23}^* - f_9 f_{19}^* - f_{26}f_6^* - \\
& f_{28}f_3^* + zf_{12}f_{22}^* + zf_{29}f_2^* - zf_{11}f_{20}^* - zf_{30}f_4^*\Big], \\
P_5 =& ImQ_3, \ P_6 = ImQ_4, \ P_7 = ReQ_3, \ P_8 = ReQ_4, \\
Q_3 =& \frac{2}{3}Ax_1\Big[f_9 f_{13}^* + f_8 f_{18}^* - f_7 f_{17}^* - f_{10}f_{15}^* + zf_{11}f_{14}^* - zf_{12}f_{16}^*\Big] + \\
& \frac{2}{3}Ax_2\Big[f_{27}f_{31}^* + f_{26}f_{36}^* - f_{25}f_{35}^* - f_{28}f_{33}^* + zf_{29}f_{32}^* + zf_{30}f_{34}^*\Big] + \\
& \frac{2}{3}Ax_3\Big[f_9 f_{31}^* + f_8 f_{36}^* + f_{26}f_{18}^* + f_{27}f_{13}^* - f_7 f_{35}^* - f_{10}f_{33}^* - f_{25}f_{17}^* - \\
& f_{28}f_{15}^* + zf_{11}f_{32}^* - zf_{12}f_{34}^* + zf_{29}f_{14}^* - zf_{30}f_{16}^*\Big] + \\
& \frac{2}{3}B\Big[f_7 f_{36}^* + f_8 f_{35}^* + f_9 f_{33}^* + f_{10}f_{31}^* - f_{25}f_{18}^* - f_{27}f_{15}^* - f_{26}f_{17}^* -
\end{aligned}
$$

$$f_{28}f_{13}^* + zf_{12}f_{32}^* - zf_{29}f_{16}^* + zf_{11}f_{34}^* - zf_{30}f_{14}^*\Big],$$

$$Q_4 = \frac{2}{3}Ax_1\Big[f_9f_1^* + f_8f_6^* - f_7f_5^* - f_{10}f_3^* + zf_{11}f_2^* - zf_{12}f_4^*\Big] +$$
$$\frac{2}{3}Ax_2\Big[f_{26}f_{24}^* + f_{27}f_{19}^* - f_{25}f_{23}^* - f_{28}f_{21}^* + zf_{29}f_{20}^* - zf_{30}f_{22}^*\Big] +$$
$$\frac{2}{3}Ax_3\Big[f_9f_{19}^* + f_8f_{24}^* + f_{26}f_6^* + f_{27}f_1^* - f_7f_{23}^* - f_{10}f_{21}^* - f_{25}f_5^* -$$
$$f_{28}f_3^* + zf_{11}f_{20}^* - zf_{12}f_{22}^* + zf_{29}f_2^* - zf_{30}f_4^*\Big] +$$
$$\frac{2}{3}B\Big[f_7f_{24}^* + f_8f_{23}^* + f_9f_{21}^* + f_{10}f_{19}^* - f_{25}f_6^* - f_{26}f_5^* - f_{27}f_3^* -$$
$$f_{28}f_1^* + zf_{12}f_{20}^* - zf_{29}f_4^* + zf_{11}f_{22}^* - zf_{30}f_2^*\Big],$$

$$P_9 = -\frac{4}{3}Im\Big\{Ax_1\Big[f_{13}f_{15}^* + f_{17}f_{18}^* + zf_{14}f_{16}^*\Big] +$$
$$Ax_2\Big[f_{31}f_{33}^* + f_{35}f_{36}^* + zf_{32}f_{34}^*\Big] +$$
$$Ax_3\Big[f_{13}f_{33}^* + f_{17}f_{36}^* + f_{31}f_{15}^* + f_{35}f_{18}^* + zf_{14}f_{34}^* + zf_{32}f_{16}^*\Big] +$$
$$B\Big[f_{18}f_{36}^* + f_{17}f_{35}^* + +f_{31}f_{13}^* + f_{33}f_{15}^* + zf_{32}f_{14}^* + zf_{34}f_{16}^*\Big]\Big\},$$

$$P_{10} = -\frac{4}{3}Im\Big\{Ax_1\Big[f_1f_3^* + f_5f_6^* + zf_2f_4^*\Big] + Ax_2\Big[f_{19}f_{21}^* + f_{25}f_{24}^* + zf_{20}f_{22}^*\Big] +$$
$$Ax_3\Big[f_1f_{21}^* + f_5f_{24}^* + f_{19}f_3^* + f_{23}f_6^* + zf_2f_{22}^* + zf_{20}f_4^*\Big] +$$
$$B\Big[f_6f_{24}^* + f_5f_{23}^* + f_{21}f_3^* + f_{19}f_1^* + zf_{20}f_2^* + zf_2f_4^*\Big]\Big\},$$

$$P_{11} = -\frac{4}{3}Im\Big\{Ax_1\Big[f_7f_8^* + f_9f_{10}^* + zf_{11}f_{12}^*\Big] +$$
$$Ax_2\Big[f_{25}f_{26}^* + f_{27}f_{28}^* + zf_{29}f_{30}^*\Big] +$$
$$Ax_3\Big[f_7f_{26}^* + f_9f_{28}^* + f_{25}f_8^* + f_{27}f_{10}^* + zf_{11}f_{30}^* + zf_{29}f_{12}^*\Big] +$$
$$B\Big[f_9f_{27}^* + f_{10}f_{28}^* + f_{23}f_7^* + f_{26}f_8^* + zf_{12}f_{30}^* + zf_{11}f_{29}^*\Big]\Big\},$$

$$P_{12} = -ImQ_5,\ \ P_{13} = -ReQ_5,$$

$$Q_5 = \frac{2}{3}\Big\{Ax_1\Big[f_1f_{15}^* + f_5f_{18}^* - f_3f_{13}^* - f_6f_{17}^* + zf_2f_{16}^* - zf_4f_{14}^*\Big] +$$
$$Ax_2\Big[f_{19}f_{33}^* + f_{23}f_{36}^* - f_{21}f_{31}^* - f_{24}f_{35}^* + zf_{20}f_{34}^* - zf_{22}f_{32}^*\Big] +$$
$$Ax_3\Big[f_1f_{33}^* + f_5f_{36}^* + f_{19}f_{15}^* + f_{23}f_{18}^* - f_3f_{31}^* - f_6f_{35}^* - f_{21}f_{13}^* -$$
$$f_{24}f_{17}^* + zf_2f_{34}^* - zf_4f_{32}^* - zf_{22}f_{14}^* + zf_{20}f_{16}^*\Big] +$$

$$B\Big[f_5f_{35}^* + f_6f_{36}^* + f_{19}f_{13}^* + f_{21}f_{15}^* - f_1f_{31}^* - f_3f_{33}^* - f_{23}f_{17}^* -$$
$$f_{24}f_{18}^* + zf_{20}f_{14}^* - zf_4f_{34}^* + zf_{22}f_{16}^* - zf_2f_{32}^*\Big].$$

11 Appendix C

The relations between the helicity amplitudes and the scalar amplitudes are given here:

$$h_1 = -\frac{1}{2\sqrt{6}}\Big\{\cos\vartheta(f_3+f_8) - \sin\vartheta(f_{21}+f_{26}) + \cos^2\vartheta(f_9-f_5) +$$
$$\sin^2\vartheta(f_{28}-f_{24}) - \sin\vartheta\cos\vartheta(f_{10}-f_6+f_{27}-f_{23}) -$$
$$2\frac{E_1}{M_\Delta}\Big[\cos\vartheta(f_{19}+f_{25}) + \sin\vartheta(f_1+f_7) + \cos^2\vartheta(f_{28}-f_{24}) +$$
$$\sin^2\vartheta(f_9-f_5) + \sin\vartheta\cos\vartheta(f_{10}-f_6+f_{27}-f_{23})\Big]\Big\},$$
$$h_2 = -\frac{1}{2\sqrt{6}}\Big\{-\cos\vartheta(f_3+f_8) + \sin\vartheta(f_{21}+f_{26}) + \cos^2\vartheta(f_9-f_5) +$$
$$\sin^2\vartheta(f_{28}-f_{24}) - \sin\vartheta\cos\vartheta(f_{10}-f_6+f_{27}-f_{23}) +$$
$$2\frac{E_1}{M_\Delta}\Big[\cos\vartheta(f_{19}+f_{25}) + \sin\vartheta(f_1+f_7) - \cos^2\vartheta(f_{28}-f_{24}) -$$
$$\sin^2\vartheta(f_9-f_5) - \sin\vartheta\cos\vartheta(f_{10}-f_6+f_{27}-f_{23})\Big]\Big\},$$
$$h_3 = -\frac{1}{2\sqrt{3}}\frac{\omega}{M}\Big\{\cos\vartheta(f_4+\cos\vartheta f_{11}) - \sin\vartheta(f_{22}-\sin\vartheta f_{30}) -$$
$$\sin\vartheta\cos\vartheta(f_{12}+f_{29}) - 2\frac{E_1}{M_\Delta}\Big[\cos\vartheta(f_{20}+\cos\vartheta f_{30}) +$$
$$\sin\vartheta(f_2+\sin\vartheta f_{11}) + \sin\vartheta\cos\vartheta(f_{12}+f_{29})\Big]\Big\},$$
$$h_4 = -\frac{1}{2\sqrt{3}}\frac{\omega}{M}\Big\{-\cos\vartheta(f_4-\cos\vartheta f_{11}) + \sin\vartheta(f_{22}+\sin\vartheta f_{30}) -$$
$$\sin\vartheta\cos\vartheta(f_{12}+f_{29}) + 2\frac{E_1}{M_\Delta}\Big[\cos\vartheta(f_{20}-\cos\vartheta f_{30}) +$$
$$\sin\vartheta(f_2-\sin\vartheta f_{11}) - \sin\vartheta\cos\vartheta(f_{12}+f_{29})\Big]\Big\},$$
$$h_5 = -\frac{1}{2\sqrt{6}}\Big\{\cos\vartheta(f_8-f_3) + \sin\vartheta(f_{21}-f_{26}) - \cos^2\vartheta(f_9+f_5) -$$
$$\sin^2\vartheta(f_{28}+f_{24}) + \sin\vartheta\cos\vartheta(f_{10}+f_6+f_{27}+f_{23}) +$$
$$2\frac{E_1}{M_\Delta}\Big[\cos\vartheta(f_{19}-f_{25}) + \sin\vartheta(f_1-f_7) + \cos^2\vartheta(f_{28}+f_{24}) +$$

$$\sin^2\vartheta(f_9+f_5)+\sin\vartheta\cos\vartheta(f_{10}+f_6+f_{27}+f_{23})\Big]\Big\},$$

$$h_6=-\frac{1}{2\sqrt{6}}\Big\{\cos\vartheta(f_3-f_8)-\sin\vartheta(f_{21}-f_{26})-\cos^2\vartheta(f_9+f_5)-$$
$$\sin^2\vartheta(f_{28}+f_{24})+\sin\vartheta\cos\vartheta(f_{10}+f_6+f_{27}+f_{23})+$$
$$2\frac{E_1}{M_\Delta}\Big[\cos\vartheta(f_{25}-f_{19})+\sin\vartheta(f_7-f_1)+\cos^2\vartheta(f_{28}+f_{24})+$$
$$\sin^2\vartheta(f_9+f_5)+\sin\vartheta\cos\vartheta(f_{10}+f_6+f_{27}+f_{23})\Big]\Big\},$$

$$h_7=-\frac{1}{2\sqrt{6}}\Big\{-\cos\vartheta(f_1+f_7)+\sin\vartheta(f_{19}+f_{25})+\cos^2\vartheta(f_{10}-f_6)-$$
$$\sin^2\vartheta(f_{27}-f_{23})+\sin\vartheta\cos\vartheta(f_9-f_5+f_{24}-f_{28})+$$
$$2\frac{E_1}{M_\Delta}\Big[-\cos\vartheta(f_{21}+f_{26})-\sin\vartheta(f_3+f_8)+\cos^2\vartheta(f_{27}-f_{23})-$$
$$\sin^2\vartheta(f_{10}-f_6)+\sin\vartheta\cos\vartheta(f_9-f_5+f_{24}-f_{28})\Big]\Big\},$$

$$h_8=-\frac{1}{2\sqrt{6}}\Big\{-\cos\vartheta(f_1+f_7)+\sin\vartheta(f_{19}+f_{25})-\cos^2\vartheta(f_{10}-f_6)+$$
$$\sin^2\vartheta(f_{27}-f_{23})+\sin\vartheta\cos\vartheta(f_5-f_9+f_{28}-f_{24})-$$
$$2\frac{E_1}{M_\Delta}\Big[\cos\vartheta(f_{21}+f_{26})+\sin\vartheta(f_3+f_8)+\cos^2\vartheta(f_{27}-f_{23})-$$
$$\sin^2\vartheta(f_{10}-f_6)-\sin\vartheta\cos\vartheta(f_5-f_9+f_{28}-f_{24})\Big]\Big\},$$

$$h_9=-\frac{1}{2\sqrt{3}}\frac{\omega}{M}\Big\{-\cos\vartheta(f_2-\cos\vartheta f_{12})+\sin\vartheta(f_{20}-\sin\vartheta f_{29})+$$
$$\sin\vartheta\cos\vartheta(f_{11}-f_{30})+2\frac{E_1}{M_\Delta}\Big[-\cos\vartheta(f_{22}-\cos\vartheta f_{29})-$$
$$\sin\vartheta(f_4+\sin\vartheta f_{12})+\sin\vartheta\cos\vartheta(f_{11}-f_{30})\Big]\Big\},$$

$$h_{10}=-\frac{1}{2\sqrt{3}}\frac{\omega}{M}\Big\{-\cos\vartheta(f_2+\cos\vartheta f_{12})+\sin\vartheta(f_{20}+\sin\vartheta f_{29})+$$
$$\sin\vartheta\cos\vartheta(f_{30}-f_{11})+2\frac{E_1}{M_\Delta}\Big[-\cos\vartheta(f_{22}+\cos\vartheta f_{29})-$$
$$\sin\vartheta(f_4-\sin\vartheta f_{12})-\sin\vartheta\cos\vartheta(f_{11}-f_{30})\Big]\Big\},$$

$$h_{11}=-\frac{1}{2\sqrt{6}}\Big\{\cos\vartheta(f_1-f_7)+\sin\vartheta(f_{25}-f_{19})-\cos^2\vartheta(f_{10}+f_6)+$$
$$\sin^2\vartheta(f_{27}+f_{23})-\sin\vartheta\cos\vartheta(f_5+f_9-f_{28}-f_{24})+$$
$$2\frac{E_1}{M_\Delta}\Big[\cos\vartheta(f_{21}-f_{26})+\sin\vartheta(f_3-f_8)-\cos^2\vartheta(f_{27}+f_{23})+$$
$$\sin^2\vartheta(f_{10}+f_6)-\sin\vartheta\cos\vartheta(f_5+f_9-f_{28}-f_{24})\Big]\Big\},$$

$$h_{12} = -\frac{1}{2\sqrt{6}}\Big\{\cos\vartheta(f_1 - f_7) + \sin\vartheta(f_{25} - f_{19}) + \cos^2\vartheta(f_{10} + f_6) -$$
$$\sin^2\vartheta(f_{27} + f_{23}) + \sin\vartheta\cos\vartheta(f_5 + f_9 - f_{28} - f_{24}) +$$
$$2\frac{E_1}{M_\Delta}\Big[\cos\vartheta(f_{21} - f_{26}) + \sin\vartheta(f_3 - f_8) + \cos^2\vartheta(f_{27} + f_{23}) -$$
$$\sin^2\vartheta(f_{10} + f_6) + \sin\vartheta\cos\vartheta(f_5 + f_9 - f_{28} - f_{24})\Big]\Big\},$$
$$h_{13} = -\frac{Q}{2\sqrt{3}}\Big\{\cos\vartheta f_{15} - \sin\vartheta f_{33} - \sin^2\vartheta f_{36} - \cos^2\vartheta f_{17} +$$
$$\sin\vartheta\cos\vartheta(f_{18} + f_{35}) + 2\frac{E_1}{M_\Delta}\Big[-\cos\vartheta f_{31} + \cos^2\vartheta f_{36} -$$
$$\sin\vartheta f_{13} + \sin^2\vartheta f_{17} + \sin\vartheta\cos\vartheta(f_{18} + f_{35})\Big]\Big\},$$
$$h_{14} = \frac{Q}{\sqrt{6}}\frac{\omega}{M}\Big[-\cos\vartheta f_{16} + \sin\vartheta f_{34} + 2\frac{E_1}{M_\Delta}\Big(\cos\vartheta f_{32} + \sin\vartheta f_{14}\Big)\Big],$$
$$h_{15} = \frac{Q}{2\sqrt{3}}\Big\{\cos\vartheta(f_{15} + \cos\vartheta f_{17}) - \sin\vartheta(f_{33} - \sin\vartheta f_{36}) -$$
$$\sin\vartheta\cos\vartheta(f_{18} + f_{35}) - 2\frac{E_1}{M_\Delta}\Big[\cos\vartheta(f_{31} + \cos\vartheta f_{36}) +$$
$$\sin\vartheta(f_{13} + \sin\vartheta f_{17}) + \sin\vartheta\cos\vartheta(f_{18} + f_{35})\Big]\Big\},$$
$$h_{16} = -\frac{Q}{2\sqrt{3}}\Big\{-\cos\vartheta(f_{13} - \cos\vartheta f_{18}) + \sin\vartheta(f_{31} - \sin\vartheta f_{35}) +$$
$$\sin\vartheta\cos\vartheta(f_{17} - f_{36}) + 2\frac{E_1}{M_\Delta}\Big[-\cos\vartheta(f_{33} - \cos\vartheta f_{35}) -$$
$$\sin\vartheta(f_{15} + \sin\vartheta f_{18}) + \sin\vartheta\cos\vartheta(f_{17} - f_{36})\Big]\Big\},$$
$$h_{17} = -\frac{Q}{\sqrt{6}}\frac{\omega}{M}\Big[-\cos\vartheta f_{14} + \sin\vartheta f_{32} - 2\frac{E_1}{M_\Delta}\Big(\cos\vartheta f_{34} + \sin\vartheta f_{16}\Big)\Big],$$
$$h_{18} = \frac{Q}{2\sqrt{3}}\Big\{-\cos\vartheta(f_{13} + \cos\vartheta f_{18}) + \sin\vartheta(f_{31} + \sin\vartheta f_{35}) +$$
$$\sin\vartheta\cos\vartheta(f_{36} - f_{17}) + 2\frac{E_1}{M_\Delta}\Big[-\cos\vartheta(f_{33} + \cos\vartheta f_{35}) -$$
$$\sin\vartheta(f_{15} - \sin\vartheta f_{18}) + \sin\vartheta\cos\vartheta(f_{36} - f_{17})\Big]\Big\},$$
$$h_{19} = -\frac{1}{2\sqrt{2}}\Big[\cos\vartheta(f_1 + f_7) - \sin\vartheta(f_{19} + f_{25}) +$$
$$\sin\vartheta\cos\vartheta(f_9 - f_5 + f_{24} - f_{28}) + \cos^2\vartheta(f_{10} - f_6) +$$
$$\sin^2\vartheta(f_{23} - f_{27})\Big],$$
$$h_{20} = -\frac{1}{2\sqrt{2}}\Big[\cos\vartheta(f_1 + f_7) - \sin\vartheta(f_{19} + f_{25}) +$$
$$\sin\vartheta\cos\vartheta(f_5 - f_9 + f_{28} - f_{24}) - \cos^2\vartheta(f_{10} - f_6) +$$

$$\sin^2\vartheta(f_{27}-f_{23})\Big],$$

$$h_{21}=\frac{1}{2}\frac{\omega}{M}\Big[-\cos\vartheta f_2+\sin\vartheta f_{20}-\cos^2\vartheta f_{12}+\sin^2\vartheta f_{29}+$$
$$\sin\vartheta\cos\vartheta(f_{30}-f_{11})\Big],$$

$$h_{22}=-\frac{1}{2}\frac{\omega}{M}\Big[\cos\vartheta f_2-\sin\vartheta f_{20}-\cos^2\vartheta f_{12}+\sin^2\vartheta f_{29}+$$
$$\sin\vartheta\cos\vartheta(f_{30}-f_{11})\Big],$$

$$h_{23}=-\frac{1}{2\sqrt{2}}\Big[\cos\vartheta(f_7-f_1)+\sin\vartheta(f_{19}-f_{25})+\sin\vartheta\cos\vartheta(f_{24}+f_{28}$$
$$-f_5-f_9)-\cos^2\vartheta(f_{10}+f_6)+\sin^2\vartheta(f_{27}+f_{23})\Big],$$

$$h_{24}=-\frac{1}{2\sqrt{2}}\Big[\cos\vartheta(f_7-f_1)+\sin\vartheta(f_{19}-f_{25})-\sin\vartheta\cos\vartheta(f_{24}+f_{28}$$
$$-f_5-f_9)+\cos^2\vartheta(f_{10}+f_6)-\sin^2\vartheta(f_{27}+f_{23})\Big],$$

$$h_{25}=-\frac{1}{2\sqrt{2}}\Big[\cos\vartheta(f_3+f_8)-\sin\vartheta(f_{21}+f_{26})+\sin\vartheta\cos\vartheta(f_{27}+f_{10}$$
$$-f_6-f_{23})-\cos^2\vartheta(f_9-f_5)-\sin^2\vartheta(f_{28}-f_{24})\Big],$$

$$h_{26}=-\frac{1}{2\sqrt{2}}\Big[-\cos\vartheta(f_3+f_8)+\sin\vartheta(f_{21}+f_{26})+\sin\vartheta\cos\vartheta(f_{27}+f_{10}$$
$$-f_6-f_{23})-\cos^2\vartheta(f_9-f_5)-\sin^2\vartheta(f_{28}-f_{24})\Big],$$

$$h_{27}=-\frac{1}{2}\frac{\omega}{M}\Big[\cos\vartheta f_4-\sin\vartheta f_{22}-\cos^2\vartheta f_{11}-\sin^2\vartheta f_{30}+$$
$$\sin\vartheta\cos\vartheta(f_{12}+f_{29})\Big],$$

$$h_{28}=-\frac{1}{2}\frac{\omega}{M}\Big[-\cos\vartheta f_4+\sin\vartheta f_{22}-\cos^2\vartheta f_{11}-\sin^2\vartheta f_{30}+$$
$$\sin\vartheta\cos\vartheta(f_{12}+f_{29})\Big],$$

$$h_{29}=-\frac{1}{2\sqrt{2}}\Big[-\cos\vartheta(f_3-f_8)+\sin\vartheta(f_{21}-f_{26})-\sin\vartheta\cos\vartheta(f_{27}+f_{10}$$
$$+f_6+f_{23})+\cos^2\vartheta(f_9+f_5)+\sin^2\vartheta(f_{28}+f_{24})\Big],$$

$$h_{30}=-\frac{1}{2\sqrt{2}}\Big[\cos\vartheta(f_3-f_8)-\sin\vartheta(f_{21}-f_{26})-$$
$$\sin\vartheta\cos\vartheta(f_{27}+f_{10}+f_6+f_{23})+\cos^2\vartheta(f_9+f_5)+$$
$$\sin^2\vartheta(f_{28}+f_{24})\Big],$$

$$h_{31}=\frac{Q}{2}\Big[-\cos\vartheta f_{13}+\sin\vartheta f_{31}+\cos^2\vartheta f_{18}-\sin^2\vartheta f_{35}+$$

$$\sin\vartheta\cos\vartheta(f_{17}-f_{36})\Big],$$

$$h_{32}=\frac{Q}{\sqrt{2}}\frac{\omega}{M}\Big(\sin\vartheta f_{32}-\cos\vartheta f_{14}\Big),$$

$$h_{33}=\frac{Q}{2}\Big[\cos\vartheta f_{13}-\sin\vartheta f_{31}+\cos^2\vartheta f_{18}-\sin^2\vartheta f_{35}+$$
$$\sin\vartheta\cos\vartheta(f_{17}-f_{36})\Big],$$

$$h_{34}=\frac{Q}{2}\Big[\cos\vartheta f_{15}-\sin\vartheta f_{33}-\cos^2\vartheta f_{17}-\sin^2\vartheta f_{36}+$$
$$\sin\vartheta\cos\vartheta(f_{18}+f_{35})\Big],$$

$$h_{35}=-\frac{Q}{\sqrt{2}}\frac{\omega}{M}\Big(\sin\vartheta f_{34}-\cos\vartheta f_{16}\Big),$$

$$h_{36}=\frac{Q}{2}\Big[-\cos\vartheta f_{15}+\sin\vartheta f_{33}-\cos^2\vartheta f_{17}-\sin^2\vartheta f_{36}+$$
$$\sin\vartheta\cos\vartheta(f_{18}+f_{35})\Big],$$

where $Q=\sqrt{-k^2}/k_0$, $E_1=(W^2+M_\Delta^2-m^2)/2W$, and k_0, ω, E_1 are the energies of the virtual photon, deuteron, Δ– isobar, respectively, in the $\gamma^*+d\to\Delta+N$ reaction CMS, k^2 is the square of the virtual photon four–momentum, and ϑ is the angle between the virtual photon and Δ–isobar momenta.

Let us present here for completeness, the inverse relations, i.e., the expressions for the scalar amplitudes in terms of the helicity amplitudes:

$$f_1=-\frac{1}{\sqrt{2}}\Big\{\cos\vartheta(h_{19}+h_{20}-h_{23}-h_{24})+y\sin\vartheta\Big[\sqrt{3}(h_2-h_1+h_5-$$
$$h_6)+h_{25}-h_{26}-h_{29}+h_{30}\Big]\Big\},$$

$$f_2=\frac{M}{\omega}\Big[-\cos\vartheta(h_{21}+h_{22})+y\sin\vartheta(\sqrt{3}h_3-\sqrt{3}h_4+h_{28}-h_{27})\Big],$$

$$f_3=-\frac{1}{\sqrt{2}}\Big\{\cos\vartheta(h_{25}-h_{26}-h_{29}+h_{30})-y\sin\vartheta\Big[\sqrt{3}(h_7+h_8-h_{11}-$$
$$h_{12})+h_{19}+h_{20}-h_{23}-h_{24}\Big]\Big\},$$

$$f_4=\frac{M}{\omega}\Big[\cos\vartheta(h_{28}-h_{27})+y\sin\vartheta(\sqrt{3}h_9+\sqrt{3}h_{10}+h_{21}+h_{22})\Big],$$

$$f_5=\frac{1}{\sqrt{2}}\cos\vartheta\Big[\sin\vartheta(h_{19}-h_{20}+h_{23}-h_{24})-\cos\vartheta(h_{25}+h_{26}+h_{29}+$$
$$h_{30})\Big]+\frac{y}{\sqrt{2}}\sin\vartheta\Big\{\cos\vartheta\Big[\sqrt{3}(h_7+h_{11}-h_8-h_{12})+h_{20}+h_{24}-h_{19}-$$
$$h_{23}\Big]-\sin\vartheta\Big[\sqrt{3}(h_1+h_5+h_2+h_6)+h_{25}+h_{26}+h_{29}+h_{30}\Big]\Big\},$$

$$f_6 = \frac{1}{\sqrt{2}}\cos\vartheta\Big[\cos\vartheta(h_{19} - h_{20} + h_{23} - h_{24}) + \sin\vartheta(h_{25} + h_{26} + h_{29} +$$
$$h_{30})\Big] - \frac{y}{\sqrt{2}}\sin\vartheta\Big\{\cos\vartheta\Big[\sqrt{3}(h_1 + h_2 + h_5 + h_6) + h_{25} + h_{26} + h_{29} +$$
$$h_{30}\Big] + \sin\vartheta\Big[\sqrt{3}(h_7 + h_{11} - h_8 - h_{12}) + h_{20} + h_{24} - h_{19} - h_{23}\Big]\Big\},$$
$$f_7 = -\frac{1}{\sqrt{2}}\Big\{\cos\vartheta(h_{19} + h_{20} + h_{23} + h_{24}) - y\sin\vartheta\Big[\sqrt{3}(h_1 - h_2 + h_5 -$$
$$h_6) - h_{25} + h_{26} - h_{29} + h_{30}\Big]\Big\},$$
$$f_8 = \frac{1}{\sqrt{2}}\Big\{\cos\vartheta(h_{26} - h_{25} - h_{29} + h_{30}) + y\sin\vartheta\Big[\sqrt{3}(h_7 + h_8 + h_{11} +$$
$$h_{12}) + h_{19} + h_{20} + h_{23} + h_{24}\Big]\Big\},$$
$$f_9 = -\frac{1}{\sqrt{2}}\cos\vartheta\Big[\sin\vartheta(h_{19} - h_{20} - h_{23} + h_{24}) - \cos\vartheta(h_{25} + h_{26} - h_{29} -$$
$$h_{30})\Big] - \frac{y}{\sqrt{2}}\sin\vartheta\Big\{\cos\vartheta\Big[\sqrt{3}(h_7 - h_8 - h_{11} + h_{12}) + h_{20} - h_{19} +$$
$$h_{23} - h_{24}\Big] - \sin\vartheta\Big[\sqrt{3}(h_1 + h_2 - h_5 - h_6) + h_{25} + h_{26} - h_{29} - h_{30}\Big]\Big\},$$
$$f_{10} = -\frac{1}{\sqrt{2}}\cos\vartheta\Big[\cos\vartheta(h_{19} - h_{20} - h_{23} + h_{24}) + \sin\vartheta(h_{25} + h_{26} - h_{29} -$$
$$h_{30})\Big] + \frac{y}{\sqrt{2}}\sin\vartheta\Big\{\cos\vartheta\Big[\sqrt{3}(h_1 + h_2 - h_5 - h_6) + h_{25} + h_{26} - h_{29} -$$
$$h_{30}\Big] + \sin\vartheta\Big[\sqrt{3}(h_7 - h_{11} - h_8 + h_{12}) + h_{20} - h_{24} - h_{19} + h_{23}\Big]\Big\},$$
$$f_{11} = \frac{M}{\omega}\Big\{\cos\vartheta\Big[\cos\vartheta(h_{27} + h_{28}) + \sin\vartheta(h_{22} - h_{21})\Big] + y\sin\vartheta\Big[\sin\vartheta(\sqrt{3}h_3 +$$
$$\sqrt{3}h_4 + h_{27} + h_{28}) - \cos\vartheta(\sqrt{3}h_9 - \sqrt{3}h_{10} + h_{22} - h_{21})\Big]\Big\},$$
$$f_{12} = \frac{M}{\omega}\Big\{\cos\vartheta\Big[\cos\vartheta(h_{22} - h_{21}) - \sin\vartheta(h_{27} + h_{28})\Big] + y\sin\vartheta\Big[\cos\vartheta(\sqrt{3}h_3 +$$
$$\sqrt{3}h_4 + h_{27} + h_{28}) + \sin\vartheta(\sqrt{3}h_9 - \sqrt{3}h_{10} + h_{22} - h_{21})\Big]\Big\},$$
$$f_{13} = \frac{1}{Q}\Big[\cos\vartheta(h_{33} - h_{31}) + y\sin\vartheta(\sqrt{3}h_{13} - \sqrt{3}h_{15} - h_{36} + h_{34})\Big],$$
$$f_{14} = \frac{\sqrt{2}}{Q}\frac{M}{\omega}\Big[-\cos\vartheta h_{32} + y\sin\vartheta(\sqrt{3}h_{14} + h_{35})\Big],$$
$$f_{15} = \frac{1}{Q}\Big[\cos\vartheta(h_{34} - h_{36}) + y\sin\vartheta(\sqrt{3}h_{16} - \sqrt{3}h_{18} - h_{33} + h_{31})\Big],$$
$$f_{16} = \frac{\sqrt{2}}{Q}\frac{M}{\omega}\Big[\cos\vartheta h_{35} + y\sin\vartheta(\sqrt{3}h_{17} + h_{32})\Big],$$

$$f_{17} = \frac{1}{Q}\Big\{\cos\vartheta\Big[\sin\vartheta(h_{31}+h_{33}) - \cos\vartheta(h_{34}+h_{36})\Big] -$$
$$y\sin\vartheta\Big[\sin\vartheta(\sqrt{3}h_{13}+\sqrt{3}h_{15}+h_{34}+h_{36}) +$$
$$\cos\vartheta(\sqrt{3}h_{16}+\sqrt{3}h_{18}+h_{31}+h_{33})\Big]\Big\},$$
$$f_{18} = \frac{1}{Q}\Big\{\cos\vartheta\Big[\cos\vartheta(h_{31}+h_{33}) + \sin\vartheta(h_{34}+h_{36})\Big] +$$
$$y\sin\vartheta\Big[-\cos\vartheta(\sqrt{3}h_{13}+\sqrt{3}h_{15}+h_{34}+h_{36}) +$$
$$\sin\vartheta(\sqrt{3}h_{16}+\sqrt{3}h_{18}+h_{31}+h_{33})\Big]\Big\},$$
$$f_{19} = \frac{1}{\sqrt{2}}\Big\{\sin\vartheta(h_{19}+h_{20}-h_{23}-h_{24}) - y\cos\vartheta\Big[\sqrt{3}(h_2-h_1+h_5-h_6) +$$
$$h_{25}-h_{26}-h_{29}+h_{30}\Big]\Big\},$$
$$f_{20} = \frac{M}{\omega}\Big[\sin\vartheta(h_{21}+h_{22}) + y\cos\vartheta(\sqrt{3}h_3-\sqrt{3}h_4+h_{28}-h_{27})\Big],$$
$$f_{21} = \frac{1}{\sqrt{2}}\Big\{\sin\vartheta(h_{25}-h_{26}-h_{29}+h_{30}) + y\cos\vartheta\Big[\sqrt{3}(h_7+h_8-h_{11}$$
$$-h_{12}) + h_{19}+h_{20}-h_{23}-h_{24}\Big]\Big\},$$
$$f_{22} = \frac{M}{\omega}\Big[\sin\vartheta(h_{27}-h_{28}) + y\cos\vartheta(\sqrt{3}h_9+\sqrt{3}h_{10}+h_{21}+h_{22})\Big],$$
$$f_{23} = -\frac{1}{\sqrt{2}}\sin\vartheta\Big[\sin\vartheta(h_{19}-h_{20}+h_{23}-h_{24}) - \cos\vartheta(h_{25}+h_{26}+h_{29}+$$
$$h_{30})\Big] - \frac{y}{\sqrt{2}}\cos\vartheta\Big\{\sin\vartheta\Big[\sqrt{3}(h_1+h_2+h_5+h_6) + h_{25}+h_{26}+h_{29}+$$
$$h_{30}\Big] - \cos\vartheta\Big[\sqrt{3}(h_7+h_{11}-h_8-h_{12}) + h_{20}+h_{24}-h_{19}-h_{23}\Big]\Big\},$$
$$f_{24} = -\frac{1}{\sqrt{2}}\sin\vartheta\Big[\cos\vartheta(h_{19}-h_{20}+h_{23}-h_{24}) + \sin\vartheta(h_{25}+h_{26}+h_{29}+$$
$$h_{30})\Big] - \frac{y}{\sqrt{2}}\cos\vartheta\Big\{\cos\vartheta\Big[\sqrt{3}(h_1+h_2+h_5+h_6) + h_{25}+h_{26}+h_{29}+$$
$$h_{30}\Big] + \sin\vartheta\Big[\sqrt{3}(h_7+h_{11}-h_8-h_{12}) + h_{20}+h_{24}-h_{19}-h_{23}\Big]\Big\},$$
$$f_{25} = \frac{1}{\sqrt{2}}\Big\{\sin\vartheta(h_{19}+h_{20}+h_{23}+h_{24}) + y\cos\vartheta\Big[\sqrt{3}(h_1+h_5-h_2-$$
$$h_6) + h_{26}+h_{30}-h_{25}-h_{29}\Big]\Big\},$$
$$f_{26} = -\frac{1}{\sqrt{2}}\Big\{\sin\vartheta(h_{26}+h_{30}-h_{25}-h_{29}) - y\cos\vartheta\Big[\sqrt{3}(h_2+h_6+h_{11}+$$
$$h_7) + h_{20}+h_{24}+h_{25}+h_{29}\Big]\Big\},$$

$$f_{27} = \frac{1}{\sqrt{2}} \sin\vartheta \Big[\sin\vartheta (h_{19} - h_{20} - h_{23} + h_{24}) - \cos\vartheta (h_{25} + h_{26} - h_{29} - h_{30}) \Big] + \frac{y}{\sqrt{2}} \cos\vartheta \Big\{ \sin\vartheta \Big[\sqrt{3}(h_1 + h_2 - h_5 - h_6) + h_{25} + h_{26} - h_{29} - h_{30} \Big] - \cos\vartheta \Big[\sqrt{3}(h_7 - h_8 - h_{11} + h_{12}) + h_{20} - h_{19} - h_{24} + h_{23} \Big] \Big\},$$

$$f_{28} = \frac{1}{\sqrt{2}} \sin\vartheta \Big[\cos\vartheta (h_{19} - h_{20} - h_{23} + h_{24}) + \sin\vartheta (h_{25} + h_{26} - h_{29} - h_{30}) \Big] + \frac{y}{\sqrt{2}} \cos\vartheta \Big\{ \cos\vartheta \Big[\sqrt{3}(h_1 + h_2 - h_5 - h_6) + h_{25} + h_{26} - h_{29} - h_{30} \Big] + \sin\vartheta \Big[\sqrt{3}(h_7 - h_8 - h_{11} + h_{12}) + h_{20} - h_{19} - h_{24} + h_{23} \Big] \Big\},$$

$$f_{29} = \frac{M}{\omega} \Big\{ \sin\vartheta \Big[\sin\vartheta (h_{21} - h_{22}) - \cos\vartheta (h_{27} + h_{28}) \Big] + y \cos\vartheta \Big[\sin\vartheta (\sqrt{3}h_3 + \sqrt{3}h_4 + h_{27} + h_{28}) - \cos\vartheta (\sqrt{3}h_9 - \sqrt{3}h_{10} + h_{22} - h_{21}) \Big] \Big\},$$

$$f_{30} = \frac{M}{\omega} \Big\{ \sin\vartheta \Big[\cos\vartheta (h_{21} - h_{22}) + \sin\vartheta (h_{27} + h_{28}) \Big] + y \cos\vartheta \Big[\cos\vartheta (\sqrt{3}h_3 + \sqrt{3}h_4 + h_{27} + h_{28}) + \sin\vartheta (\sqrt{3}h_9 - \sqrt{3}h_{10} + h_{22} - h_{21}) \Big] \Big\},$$

$$f_{31} = \frac{1}{Q} \Big[\sin\vartheta (h_{31} - h_{33}) + y \cos\vartheta (\sqrt{3}h_{13} - \sqrt{3}h_{15} - h_{36} + h_{34}) \Big],$$

$$f_{32} = \frac{\sqrt{2}}{Q} \frac{M}{\omega} \Big[\sin\vartheta h_{32} + y \cos\vartheta (\sqrt{3}h_{14} + h_{35}) \Big],$$

$$f_{33} = \frac{1}{Q} \Big[\sin\vartheta (h_{36} - h_{34}) + y \cos\vartheta (\sqrt{3}h_{16} - \sqrt{3}h_{18} - h_{33} + h_{31}) \Big],$$

$$f_{34} = \frac{\sqrt{2}}{Q} \frac{M}{\omega} \Big[- \sin\vartheta h_{35} + y \cos\vartheta (\sqrt{3}h_{17} + h_{32}) \Big],$$

$$f_{35} = \frac{1}{Q} \Big\{ \sin\vartheta \Big[\cos\vartheta (h_{34} + h_{36}) - \sin\vartheta (h_{31} + h_{33}) \Big] - y \cos\vartheta \Big[\sin\vartheta (\sqrt{3}h_{13} + \sqrt{3}h_{15} + h_{34} + h_{36}) + \cos\vartheta (\sqrt{3}h_{16} + \sqrt{3}h_{18} + h_{31} + h_{33}) \Big] \Big\},$$

$$f_{36} = \frac{1}{Q} \Big\{ - \sin\vartheta \Big[\sin\vartheta (h_{34} + h_{36}) + \cos\vartheta (h_{31} + h_{33}) \Big] - y \cos\vartheta \Big[\cos\vartheta (\sqrt{3}h_{13} + \sqrt{3}h_{15} + h_{34} + h_{36}) - \sin\vartheta (\sqrt{3}h_{16} + \sqrt{3}h_{18} + h_{31} + h_{33}) \Big] \Big\},$$

where $y = M_\Delta W/(W^2 + M_\Delta^2 - m^2)$.

12 Appendix D

In this Appendix, we present the formulas for the structure functions which determine the hadronic tensor H_{ij} for various polarization states of the deuteron target for the $\gamma+d \to \Delta+N$ reaction. The structure functions are expressed in terms of the scalar amplitudes g_i $(i = 1, ..., 24)$ determining the $\gamma+d \to \Delta+N$ reaction.

- Unpolarized deuteron target.

The hadronic tensor $H_{ij}(0)$ is determined by two real structure functions a_1 and a_2:

$$
\begin{aligned}
a_1 = \frac{2}{3}\Big\{ & r_1\Big[|g_1|^2 + |g_3|^2 + |g_5|^2 + |g_6|^2 + r|g_2|^2 + r|g_4|^2\Big] + \\
& r_2\Big[|g_{13}|^2 + |g_{15}|^2 + |g_{17}|^2 + |g_{18}|^2 + r|g_{14}|^2 + r|g_{16}|^2\Big] + \\
& 2r_3 Re(g_1 g_{13}^* + g_3 g_{15}^* + g_5 g_{17}^* + g_6 g_{18}^* + r g_2 g_{14}^* + r g_4 g_{16}^*) + \\
& 2r_4 Re(g_6 g_{17}^* - g_5 g_{18}^* + g_{15} g_1^* - g_{13} g_3^* + r g_{16} g_2^* - r g_{14} g_4^*)\Big\}, \\
a_2 = \frac{2}{3}\Big\{ & r_1\Big[|g_7|^2 + |g_8|^2 + |g_9|^2 + |g_{10}|^2 + r|g_{11}|^2 + r|g_{12}|^2\Big] + \\
& r_2\Big[|g_{19}|^2 + |g_{20}|^2 + |g_{21}|^2 + |g_{22}|^2 + r|g_{23}|^2 + r|g_{24}|^2\Big] + \\
& 2r_3 Re(g_7 g_{19}^* + g_{10} g_{22}^* + g_9 g_{21}^* + g_8 g_{20}^* + r g_{11} g_{23}^* + r g_{12} g_{24}^*) + \\
& 2r_4 Re(g_{10} g_{21}^* - g_9 g_{22}^* + g_{20} g_7^* - g_{19} g_8^* + r g_{12} g_{23}^* - r g_{11} g_{24}^*)\Big\},
\end{aligned}
$$

where we introduce the notations

$$
\begin{aligned}
& r_1 = 2\frac{1-(1-\gamma^2)\sin^2\vartheta}{2+\gamma^2}, \ r_2 = 2\frac{1-(1-\gamma^2)\cos^2\vartheta}{2+\gamma^2}, \ r_3 = \frac{\gamma^2-1}{2+\gamma^2}\sin 2\vartheta, \\
& r_4 = \frac{\gamma}{2+\gamma^2}, r = \frac{(W^2+M^2)^2}{4M^2W^2}, \ \gamma = \frac{W^2+M_\Delta^2-m^2}{2M_\Delta W},
\end{aligned}
\tag{46}
$$

where M_Δ, M and m are the masses of the Δ–isobar, deuteron and nucleon, respectively; W is the total energy of the ΔN pair in CMS of the $\gamma+d \to \Delta+N$ reaction, ϑ is the angle between Δ–isobar and photon momenta.

- Vector polarized deuteron target.

The hadronic tensor $H_{ij}(\xi)$ is determined by six structure functions b_i ($i = 1-6$)

$$\begin{aligned}
b_1 =& -2\sqrt{r}Im\Big[r_1(g_2g_1^* + g_4g_3^*) + r_2(g_{14}g_{13}^* + g_{16}g_{15}^*) + r_3(g_2g_{13}^* - g_1g_{14}^* + \\
& g_4g_{15}^* - g_3g_{16}^*) - r_4(g_1g_{16}^* + g_4g_{13}^* - g_2g_{15}^* - g_3g_{14}^*)\Big], \\
b_2 =& -2\sqrt{r}Im\Big[r_1(g_{11}g_9^* + g_{12}g_{10}^*) + r_2(g_{23}g_{21}^* + g_{24}g_{22}^*) + r_3(g_{12}g_{22}^* + \\
& g_{24}g_{10}^* - g_{11}g_{21}^* - g_9g_{23}^*) + r_4(g_{12}g_{21}^* + g_9g_{24}^* - g_{10}g_{23}^* - g_{11}g_{22}^*)\Big], \\
b_3 =& -ImE_1, \; b_4 = -ReE_1, \\
E_1 =& r_1(g_9g_5^* + g_{10}g_6^* - g_7g_1^* - g_8g_3^*) + r_2(g_{21}g_{17}^* + g_{22}g_{18}^* - g_{19}g_{13}^* - \\
& g_{20}g_{15}^*) + r_3(g_{10}g_{18}^* + g_{22}g_6^* - g_7g_{13}^* - g_{19}g_1^* + g_{21}g_5^* - g_8g_{15}^* + \\
& g_9g_{17}^* - g_{20}g_3^*) + r_4(g_{10}g_{17}^* + g_{21}g_6^* - g_9g_{18}^* - g_{22}g_5^* + g_8g_{13}^* + g_{19}g_3^* - \\
& g_7g_{15}^* - g_{20}g_1^*), \\
b_5 =& -\sqrt{r}ImE_2, \; b_6 = -\sqrt{r}ReE_2, \\
E_2 =& r_1(g_7g_2^* + g_8g_4^* - g_{11}g_5^* - g_{12}g_6^*) + r_2(g_{19}g_{14}^* + g_{20}g_{16}^* - g_{23}g_{17}^* - \\
& g_{24}g_{18}^*) + r_3(g_7g_{14}^* + g_{19}g_2^* - g_{24}g_6^* - g_{12}g_{18}^* + g_8g_{16}^* - g_{23}g_5^* + \\
& g_{20}g_4^* - g_{11}g_{17}^*) + r_4(g_{24}g_5^* + g_{11}g_{18}^* - g_{23}g_6^* - g_{12}g_{17}^* + g_{20}g_2^* + \\
& g_7g_{16}^* - g_{19}g_4^* - g_8g_{14}^*).
\end{aligned}$$

- <u>Tensor polarized deuteron target.</u>

The hadronic tensor $H_{ij}(S)$ is determined, in this case, by ten structure functions c_i ($i = 1-10$) which have the following expressions in terms of the $\gamma + d \to \Delta + N$ reaction amplitudes

$$\begin{aligned}
c_1 =& 2r_1\left[|g_2|^2 + |g_4|^2 - \frac{1}{r}(|g_5|^2 + |g_6|^2)\right] + 2r_2\left[|g_{14}|^2 + |g_{16}|^2 - \right. \\
& \left.\frac{1}{r}(|g_{17}|^2 + |g_{18}|^2)\right] + 4r_3Re\left[g_4g_{16}^* + g_2g_{14}^* - \frac{1}{r}(g_5g_{17}^* + g_6g_{18}^*)\right] - \\
& 4r_4Re\left[g_{14}g_4^* - g_{16}g_2^* - \frac{1}{r}(g_5g_{18}^* - g_6g_{17}^*)\right], \\
c_2 =& 2r_1\left[|g_{11}|^2 + |g_{12}|^2 - \frac{1}{r}(|g_7|^2 + |g_8|^2)\right] + 2r_2\left[|g_{23}|^2 + |g_{24}|^2 - \right. \\
& \left.\frac{1}{r}(|g_{19}|^2 + |g_{20}|^2)\right] + 4r_3Re\left[g_{11}g_{23}^* + g_{12}g_{24}^* - \frac{1}{r}(g_8g_{20}^* + g_7g_{19}^*)\right] + \\
& 4r_4Re\left[g_{12}g_{23}^* - g_{11}g_{24}^* - \frac{1}{r}(g_{20}g_7^* - g_{19}g_8^*)\right], \\
c_3 =& 2r_1\left[|g_1|^2 + |g_3|^2 - |g_5|^2 - |g_6|^2\right] + 2r_2\left[|g_{13}|^2 + |g_{15}|^2 - |g_{17}|^2 - \right.
\end{aligned}$$

$$|g_{18}|^2\Big] + 4r_3Re\Big[g_3g_{15}^* + g_1g_{13}^* - g_5g_{17}^* - g_6g_{18}^*\Big] -$$
$$4r_4Re\Big[g_6g_{17}^* - g_5g_{18}^* + g_{13}g_3^* - g_{15}g_1^*\Big],$$
$$c_4 = 2r_1\Big[|g_9|^2 + |g_{10}|^2 - |g_7|^2 - |g_8|^2\Big] + 2r_2\Big[|g_{21}|^2 + |g_{22}|^2 - |g_{19}|^2 -$$
$$|g_{20}|^2\Big] + 4r_3Re\Big[g_9g_{21}^* + g_{10}g_{22}^* - g_8g_{20}^* - g_7g_{19}^*\Big] +$$
$$4r_4Re\Big[g_{10}g_{21}^* - g_9g_{22}^* + g_{19}g_8^* - g_{20}g_7^*)\Big],$$
$$c_5 = 2r_1Re\Big[g_1g_2^* + g_3g_4^*\Big] + 2r_2Re\Big[g_{13}g_{14}^* + g_{15}g_{16}^*\Big] + 2r_3Re\Big[g_3g_{16}^* +$$
$$g_4g_{15}^* + g_2g_{13}^* + g_1g_{14}^*\Big] - 2r_4Re\Big[g_{13}g_4^* + g_{14}g_3^* - g_{15}g_2^* - g_{16}g_1^*\Big],$$
$$c_6 = 2r_1Re\Big[g_9g_{11}^* + g_{10}g_{12}^*\Big] + 2r_2Re\Big[g_{21}g_{23}^* + g_{22}g_{24}^*\Big] + 2r_3Re\Big[g_9g_{23}^* +$$
$$g_{10}g_{24}^* + g_{11}g_{21}^* + g_{12}g_{22}^*\Big] + 2r_4Re\Big[g_{10}g_{23}^* + g_{12}g_{21}^* - g_9g_{24}^* - g_{11}g_{22}^*\Big],$$
$$c_7 = ReE_3,\ c_8 = ImE_3,$$
$$E_3 = r_1(g_2g_7^* + g_4g_8^* + g_5g_{11}^* + g_6g_{12}^*) + r_2(g_{14}g_{19}^* + g_{16}g_{20}^* +$$
$$g_{17}g_{23}^* + g_{18}g_{24}^*) + r_3(g_5g_{23}^* + g_{17}g_{11}^* + g_4g_{20}^* + g_{16}g_8^* +$$
$$g_2g_{19}^* + g_{14}g_7^* + g_6g_{24}^* + g_{18}g_{12}^*) + r_4(g_6g_{23}^* + g_{17}g_{12}^* - g_5g_{24}^* -$$
$$g_{18}g_{11}^* + g_2g_{20}^* + g_{16}g_7^* - g_4g_{19}^* - g_{14}g_8^*\Big],$$
$$c_9 = ReE_4,\ c_{10} = ImE_4,$$
$$E_4 = r_1(g_1g_7^* + g_3g_8^* + g_5g_9^* + g_6g_{10}^*) + r_2(g_{13}g_{19}^* + g_{15}g_{20}^* +$$
$$g_{17}g_{21}^* + g_{18}g_{22}^*) + r_3(g_5g_{21}^* + g_{17}g_9^* + g_3g_{20}^* + g_{15}g_8^* +$$
$$g_1g_{19}^* + g_{13}g_7^* + g_6g_{22}^* + g_{18}g_{10}^*) + r_4(g_6g_{21}^* + g_{17}g_{10}^* - g_5g_{22}^* -$$
$$g_{18}g_9^* + g_1g_{20}^* + g_{15}g_7^* - g_3g_{19}^* - g_{13}g_8^*).$$

13 Appendix E

In this Appendix, we present the formulas for the structure functions which determine the hadronic tensor $\vec{P}_{ij}$ describing the nucleon polarization in the $\gamma + d \to \Delta + N$ reaction. The structure functions d_i, $(i = 1-6)$, are written in terms of the reaction scalar amplitudes:

$$d_1 = ImR_1,\ d_2 = ReR_1,$$
$$R_1 = \frac{2}{3}r_1\Big[g_9g_3^* + g_{10}g_1^* - g_7g_6^* - g_8g_5^* + rg_{11}g_4^* + rg_{12}g_2^*\Big] + \frac{2}{3}r_2\Big[g_{21}g_{15}^* +$$
$$g_{22}g_{13}^* - g_{19}g_{18}^* - g_{20}g_{17}^* + rg_{23}g_{16}^* + rg_{24}g_{14}^*\Big] + \frac{2}{3}r_3\Big[g_9g_{15}^* + g_{10}g_{13}^* +$$

$$g_{21}g_3^* + g_{22}g_1^* - g_7g_{18}^* - g_8g_{17}^* - g_{19}g_6^* - g_{20}g_5^* + rg_{11}g_{16}^* + rg_{12}g_{14}^* + rg_{23}g_4^* + rg_{24}g_2^*\Big] + \frac{2}{3}r_4\Big[g_8g_{18}^* + g_{10}g_{15}^* + g_{19}g_5^* + g_{21}g_1^* - g_7g_{17}^* - g_9g_{13}^* - g_{20}g_6^* - g_{22}g_3^* + rg_{12}g_{16}^* + rg_{23}g_2^* - rg_{11}g_{14}^* - rg_{24}g_4^*\Big],$$

$$d_3 = ImR_2,\ d_4 = ReR_2,$$

$$R_2 = \frac{2}{3}r_1\Big[g_9g_1^* + g_8g_6^* - g_7g_5^* - g_{10}g_3^* + rg_{11}g_2^* - rg_{12}g_4^*\Big] + \frac{2}{3}r_2\Big[g_{20}g_{18}^* + g_{21}g_{13}^* - g_{19}g_{17}^* - g_{22}g_{15}^* + rg_{23}g_{14}^* - rg_{24}g_{16}^*\Big] + \frac{2}{3}r_3\Big[g_9g_{13}^* + g_8g_{18}^* + g_{20}g_6^* + g_{21}g_1^* - g_7g_{17}^* - g_{10}g_{15}^* - g_{19}g_5^* - g_{22}g_3^* + rg_{11}g_{14}^* - rg_{12}g_{16}^* + rg_{23}g_2^* - rg_{24}g_4^*\Big] + \frac{2}{3}r_4\Big[g_7g_{18}^* + g_8g_{17}^* + g_9g_{15}^* + g_{10}g_{13}^* - g_{19}g_6^* - g_{20}g_5^* - g_{21}g_3^* - g_{22}g_1^* + rg_{12}g_{14}^* - rg_{23}g_4^* + rg_{11}g_{16}^* - rg_{24}g_2^*\Big],$$

$$d_5 = -\frac{4}{3}Im\Big\{r_1\Big[g_1g_3^* + g_5g_6^* + rg_2g_4^*\Big] + r_2\Big[g_{13}g_{15}^* + g_{19}g_{18}^* + rg_{14}g_{16}^*\Big] + r_3\Big[g_1g_{15}^* + g_5g_{18}^* + g_{13}g_3^* + g_{17}g_6^* + rg_2g_{16}^* + rg_{14}g_4^*\Big] + r_4\Big[g_6g_{18}^* + g_5g_{17}^* + g_{15}g_3^* + g_{13}g_1^* + rg_{14}g_2^* + rg_2g_4^*\Big]\Big\},$$

$$d_6 = -\frac{4}{3}Im\Big\{r_1\Big[g_7g_8^* + g_9g_{10}^* + rg_{11}g_{12}^*\Big] + r_2\Big[g_{19}g_{20}^* + g_{21}g_{22}^* + rg_{23}g_{24}^*\Big] + r_3\Big[g_7g_{20}^* + g_9g_{22}^* + g_{19}g_8^* + g_{21}g_{10}^* + rg_{11}g_{24}^* + rg_{23}g_{12}^*\Big] + r_4\Big[g_9g_{21}^* + g_{10}g_{22}^* + g_{17}g_7^* + g_{20}g_8^* + rg_{12}g_{24}^* + rg_{11}g_{23}^*\Big]\Big\}.$$

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