DOES THE COMPRESSION OR THE EXPANSION OF A SIMPLE TOPOLOGY POTENTIAL MAGNETIC FIELD LEAD TO THE DEVELOPMENT OF CURRENT SHEETS?

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ABSTRACT

Janse & Low (2009) have most recently addressed the following question. Consider a cylindrical domain containing a simple topology potential magnetic field threading its lower and upper horizontal faces, and a perfectly conducting plasma. Suppose that this domain is made to slowly contract or expand in the vertical direction, so driving the field into a quasi-static evolution through a series of force-free configurations. Then are these configurations smooth, or do they contain current sheets? We reexamine here their three steps argument leading to the conclusion that current sheets form most generally. We prove analytically that the field has to evolve through "topologically untwisted" and "nonpotential" configurations, thus confirming the two first steps. However, we find the third step – leading to the conclusion that a smooth untwisted field is necessarily potential – to be very disputable.

Subject headings: MHD — Sun: corona — Sun: magnetic fields

1. INTRODUCTION

Parker's mechanism of plasma heating through resistive dissipation of spontaneously formed current concentrations in a slowly evolving magnetic field (e.g., Parker (1994), and references therein) has attracted the attention of many authors because of its potentiality to explain the high temperature of the solar corona. Much work has thus been devoted to elucidate its precise nature, and significant progress has been achieved in the last decade on both numerical and analytical sides. On the numerical side, large scale computing resources have led to the possibility of undertaking rather complete simulations allowing to resolve the details of the processes occurring

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in a field brought into evolution by boundary motions. For instance, the most recent simulations performed by Rappazzo et al. (2008) have shown quantitatively how the energy injected by granulation through footpoint twisting is transferred via a turbulent nonlinear cascade to very small scales where it is dissipated in continuously formed current layers. On the analytical side, efforts have been mostly concentrated on a question which has been the subject of a considerable amount of debate for forthy years: in the absence of resistivity and for a quasi-statically evolving equilibrium with a simple topology, are the current concentrations predicted by Parker true zero thickness current sheets (CS) associated with field discontinuities, or only finite thickness layers? Recently Parker's claim that CSs actually form (his *magnetostatic theorem* (Parker 1994)) has received new support from the work by Ng & Bhattacharjee (1998) who coined an argument suggesting that CSs appearance is inescapable indeed once the field deformation becomes sufficiently large (see also Aly (2005, 2009)).

On the other hand, Low et al. (Low 2006; Low & Janse 2009; Janse & Low 2009) have started developing a new analytical approach with the hope of obtaining a rigorous answer to the theoretical question recalled above. Their basic idea is to introduce a particular class of magnetic fields, qualified as *topologically untwisted*, and to imagine situations in which it is possible to conclude to CSs formation by using general topological arguments, thus bypassing the need of computing the complicated evolution of an equilibrium. They analyze in particular (Janse & Low 2009, JL hereafter) the following situation. An initially simple topology potential (and then untwisted) field is contained in a cylindrical domain, threading its lower and upper horizontal faces, and it is submitted to a slow compression in the vertical direction. Their conclusion is that CSs thus form in most cases. It is based, however, on some arguments which, although physically plausible, have not yet been given a formal proof, and we thus feel that it is worth revisiting them from different points of view. It is the aim of this Letter to contribute to such a re-examination.

2. STATEMENT OF THE PROBLEM

Let D^λ be the cylinder bounded by the horizontal domains $S_0 \subset \{(x, y, 0)\}$ and $S_{\lambda h} \subset \{(x, y, \lambda h)\}$ (not necessarily circular), and the vertical surface S^λ parallel to the z-axis (Fig. 1). The length h is kept fixed, while $\lambda > 0$ is a free parameter. We denote as $\mathcal{H}^\lambda$ the set of the magnetic fields contained in D^λ and satisfying the boundary conditions $B_n(x, y, z) = 0$ on S^λ (with $\hat{\mathbf{n}}$ the unit external normal to S^λ) and $B_z(x, y, 0/\lambda h) = F_{0/h}(x, y) > 0$. Here $F_{0/h}$ are two given functions independent of λ and satisfying (for flux consistency) $\int F_0 \, \mathrm{d}s = \int F_h \, \mathrm{d}s$ $(= \Phi)$. Clearly a magnetic field in $\mathcal{H}^\lambda$ has all its lines entering D^λ through S_0 and leaving it through $S_{\lambda h}$. $\mathcal{H}^\lambda$ contains a unique potential field, $\mathbf{B}^\lambda_\pi$, with the latter having a simple topology (one has indeed necessarily $B^\lambda_{\pi z}(x, y, z) > 0$ (Aly 2009)).

We adopt after Low (2006) the following definitions applying to any field $\mathbf{B}$ in $\mathcal{H}^\lambda$: (i) The *topological class of* $\mathbf{B}$, $\mathcal{T}[\mathbf{B}]$, is the set of the fields in $\mathcal{H}^\lambda$ which are accessible from $\mathbf{B}$ by an ideal deformation (i.e., one constrained by the frozen-in law) keeping fixed the footpoints on S_0 and $S_{\lambda h}$. (ii) $\mathbf{B}$ is an *everywhere untwisted field* if the associated current density has an identically vanishing z-component, i.e., $\hat{\mathbf{z}}\cdot\nabla\times\mathbf{B} = 0$. The circulation of such a $\mathbf{B}$ around an horizontal contour encircling any flux tube vanishes. This seems to preclude a winding of the lines around each other, thus justifying the use of the term "untwisted". Clearly, $\mathbf{B}_\pi^\lambda$ is an everywhere untwisted field. (iii) $\mathbf{B}$ is a *topologically untwisted field* if its topological class contains an everywhere untwisted field. Then an everywhere untwisted field is also topologically untwisted.

We now state in precise terms the problem addressed in JL, and sketch out their general argument. We assume that $\lambda = 1$ at some initial time $t = 0$, and take D to contain the potential field $\mathbf{B}_\pi$ embedded in a low beta perfectly conducting plasma – we use here the shorthand X for X^1. For $0 \leq t$, we slowly deform the domain into a sequence D^λ, with $\lambda = \lambda(t)$ being an arbitrary monotonic function such that $\lambda(0) = 1$. The compression (when $\lambda < 1$) or the expansion (when $\lambda > 1$) are assumed to be slow in that sense that the speed $|\lambda'(t)|h$ is much smaller than the Alfven speed in D^λ. The field is thus supposed to evolve quasi-statically through a sequence of force-free configurations $\mathbf{B}(\lambda)$ (for convenience we use λ rather than t for labelling the configurations in that sequence. λ can be replaced anywhere by the arbitrary $\lambda(t)$ because time does not play any role in a quasi-static evolution, no dynamics being involved). The main question is then to determine if the fields $\mathbf{B}(\lambda)$ are smooth, or if they contain CSs. JL favor the second alternative, and they use the following argument to show that it should hold true. (i) As the compression or the expansion of the domain does not change the "twist" of the field – no rotation is imposed to the footpoints on $S_{\lambda h}$, which are just moved parallel to the z-axis –, the initially untwisted field has to stay untwisted. (ii) In general the field cannot stay potential, as the magnetic mapping of $\mathbf{B}(\lambda)$ and that one of $\mathbf{B}_\pi^\lambda$ (which can be easily numerically computed) turn out most often to be different (we may have $\mathbf{B}(\lambda) = \mathbf{B}_\pi^\lambda$ only in some special situations, for instance when the cylinder is circular and the field $\mathbf{B}_\pi$ is axisymmetric). Then $\mathbf{B}(\lambda)$ is an untwisted force-free field with non-vanishing electric currents. (iii) A smooth force-free field with $\alpha \not\equiv 0$ possesses "twist", and then, if $\mathbf{B}(\lambda)$ were smooth, it would have to be potential in contradiction with the conclusion of the second step. Therefore $\mathbf{B}(\lambda)$ is unsmooth and CSs appear for $t > 0$. Note the similarity (at least in the principle) between this argument and the classical one leading to the well-established conclusion that the deformation of a 2D potential field with an X-critical point leads to the appearance of a CS.

The problem with JL argument is that *it does appeal in the first and third steps to some intuitive properties of a topologically untwisted field rather than to the precise definition we have recalled above*. In fact, JL seem to attribute without proof to an untwisted field the properties of a field which would be "not twisted", with this last term being taken with its usual but somewhat imprecise meaning of "having its lines not winding around each other". Then there is clearly the need to check as rigorously as possible if these properties are really contained in the mathematical definition. We contribute to this task in the rest of this Letter, where we stay at a purely formal level to avoid introducing some unwanted implicit features.

3. TWO BASIC PROPERTIES OF THE EVOLVING FIELD $\mathbf{B}(\lambda)$

We first show that $\mathbf{B}(\lambda)$ stays topologically untwisted. For that we introduce the field

$$\mathbf{B}_{\pi\lambda}(x, y, z) = (1/\lambda)\mathbf{B}_{\pi\perp}(x, y, z/\lambda) + B_{\pi z}(x, y, z/\lambda)\hat{\mathbf{z}} \quad \in \mathcal{H}^{\lambda}, \tag{1}$$

with the subscript "$\perp$" labelling the horizontal component of a vector. Any line of $\mathbf{B}_{\pi\lambda}$ is obtained by applying a vertical scaling to a line of $\mathbf{B}_{\pi}$, with the footpoint of the latter on S_0 being kept fixed, while its footpoint at (x, y, h) on S_h is moved to the new position $(x, y, \lambda h)$ on $S_{\lambda h}$. Then $\mathbf{B}_{\pi\lambda}$ has the same connectivity as $\mathbf{B}(\lambda)$ and belongs to its topological class, $\mathcal{T}[\mathbf{B}(\lambda)]$. On the other hand, by using the fact that $\nabla \times \mathbf{B}_{\pi} = 0$ in D, we obtain

$$(\nabla \times \mathbf{B}_{\pi\lambda})(x, y, z) = [1 - (1/\lambda^2)](\nabla_{\perp} B_{\pi z})(x, y, z/\lambda) \times \hat{\mathbf{z}}, \tag{2}$$

which implies that the electric current associated with $\mathbf{B}_{\pi\lambda}$ has an identically vanishing z-component. Therefore $\mathbf{B}_{\pi\lambda}$ is an everywhere untwisted field which belongs to the topological class of $\mathbf{B}(\lambda)$, and the latter is topologically untwisted indeed. The first JL assertion is established by keeping on with the formal definition of an untwisted field.

Next we compute analytically the relative helicity, $H[\mathbf{B}(\lambda)]$, of the field $\mathbf{B}(\lambda)$, and show that it is most generally nonzero, with the obvious consequence that $\mathbf{B}(\lambda) \neq \mathbf{B}_{\pi}^{\lambda}$ for $t > 0$ (but note that the condition $H[\mathbf{B}(\lambda)] \neq 0$ is not necessary for $\mathbf{B}(\lambda)$ to be nonpotential, it is just sufficient). As all the fields in the class $\mathcal{T}[\mathbf{B}(\lambda)]$ have the same helicity, we can write (Finn & Antonsen 1985)

$$H[\mathbf{B}(\lambda)] = H[\mathbf{B}_{\pi\lambda}] = \int_{D^{\lambda}} (\mathbf{A}_{\lambda} + \mathbf{A}^{\lambda}) \cdot (\mathbf{B}_{\pi\lambda} - \mathbf{B}_{\pi}^{\lambda})\, \mathrm{d}v, \tag{3}$$

where $\mathbf{A}^{\lambda}$ and $\mathbf{A}_{\lambda}$ are arbitrary vector potentials for $\mathbf{B}_{\pi}^{\lambda}$ and $\mathbf{B}_{\pi\lambda}$, respectively. For $\mathbf{A}^{\lambda}$, we choose the gauge (DeVore 2000)

$$\mathbf{A}^{\lambda}(x, y, z) = \mathbf{A}_0(x, y) + \nabla_{\perp} \int_0^z V^{\lambda}(x, y, z')\, \mathrm{d}z' \times \hat{\mathbf{z}}. \tag{4}$$

V^{λ} is a scalar potential such that $\mathbf{B}_{\pi}^{\lambda} = B_0\hat{\mathbf{z}} + \nabla V^{\lambda}$, where $B_0 = \Phi/(\text{area of } S_0)$ (then ∇V^{λ} has zero flux through any horizontal cross-section of D^{λ}); V^{λ} satisfies Laplace equation in D^{λ} and Neumann conditions on ∂D^{λ}, which determines it uniquely along with the additional condition $\int_{S_0} V^{\lambda}\, \mathrm{d}s = 0$. $\mathbf{A}_0$ is required to satisfy $A_{0z} = 0$ and $\nabla_{\perp} \cdot \mathbf{A}_0 = 0$ on S_0, and $A_{0n} = 0$ on ∂S_0. An important point is that the quantities $\mathbf{A}^\lambda(z=0)=\mathbf{A}_0$, $\mathbf{A}^\lambda(z=\lambda h)$, and $\int_0^{\lambda h} V^\lambda\,\mathrm{d}z$, are independent of λ (this results from $F_{0/h}$ not depending on that parameter). As for $\mathbf{A}_\lambda$, it is readily checked that we can choose $\mathbf{A}_\lambda(x,y,z)=\mathbf{A}(x,y,z/\lambda)$. Moreover, we have $\mathbf{B}_{\pi\lambda\perp}(x,y,z)=(1/\lambda)(\boldsymbol{\nabla}_\perp V)(x,y,z/\lambda)$. Using these expressions in Eq. (3), we obtain after some algebra

$$H[\mathbf{B}(\lambda)]=\int_{S^\lambda}\left(\partial_l V^\lambda\int_0^z V^\lambda\,\mathrm{d}z'\right)\mathrm{d}s-\int_S\left(\partial_l V\int_0^z V\,\mathrm{d}z'\right)\mathrm{d}s, \tag{5}$$

where l denotes an horizontal coordinate on S^λ ($\mathrm{d}s=\mathrm{d}l\,\mathrm{d}z$). Note that B_0 does not appear in that formula, and that $H[\mathbf{B}(\lambda)]=0$ in the axisymmetric case, in agreement with a remark made in the previous section.

Using the above expression for H and the series expansions for V and V^λ given in JL, it is possible to conclude that $H[\mathbf{B}(\lambda)]$ either is nonzero, or can be made nonzero by effecting an arbitrarily small change of the functions $F_{0/h}$. Although straightforward, this general argument is however quite lengthy and heavy, and we shall content here to explicit it in a particular case studied in details in JL. We take D to be the circular cylinder of radius R and

$$V(r,\varphi,z)=B_1J_1(\mu r)\left[a\frac{\cosh[\mu(z-h/2)]}{\mu\sinh(\mu h/2)}\sin\varphi+b\frac{\sinh[\mu(z-h/2)]}{\mu\cosh(\mu h/2)}\cos\varphi\right], \tag{6}$$

where (r,φ,z) are standard cylindrical coordinates, B_1, a, and b, are arbitrary constants such that $F_{0/h}>0$, J_1 is the usual Bessel function, and μR is the first root of J_1' (Eq. (6) is given in JL with $a=b=1$). Then we obtain

$$H[\mathbf{B}(\lambda)]=\frac{2\pi B_1^2abJ_1^2(\mu R)}{\mu^4}\left(\frac{\lambda\mu h}{\sinh(\lambda\mu h)}-\frac{\mu h}{\sinh(\mu h)}\right), \tag{7}$$

and $H[\mathbf{B}(\lambda)]\neq 0$ for $\lambda\neq 1$ if $ab\neq 0$. Note that $H[\mathbf{B}(\lambda)]=0$ in the case where either $a=0$ or $b=0$, a result in accordance with the fact that $\mathbf{B}(\lambda)=\mathbf{B}_\pi^\lambda$ in that situation (JL).

Then we have reached a second conclusion – that $\mathbf{B}(\lambda)$ is in general nonpotential – in agreement with the analysis in JL. However, we see a problem starting pointing here: $\mathbf{B}(\lambda)$ is topologically untwisted, while a nonzero relative helicity is usually associated with the presence of ”twist” in a field. This is a first warning indicating that one should be very cautious with the vocabulary: the connotations carried on by the word ”untwisted” are not necessarily true properties of topologically untwisted fields.

At this stage, we should now reconsider JL conjectural claim that a smooth topologically untwisted force-free magnetic field is necessarily potential. However the latter seems to be quite difficult to prove or to disprove in a direct way, and we shall not attempt to do it here. Rather we shall reconsider in the next section the compression/expansion problem from a different point of view in which the concept of topologically untwisted field does not intervene anymore. This will allow us to show that this problem is likely to have a smooth solution, thus implying a contrario that JL claim is probably incorrect.

4. REDUCTION TO A SHEARING PROBLEM AND LINEAR SOLUTION

To start with, we show that, for any fixed value of λ, $\mathbf{B}(\lambda)$ can be obtained as the final state of a standard shearing problem in the fixed domain D^λ. In that problem, an auxiliary field evolves from a potential state through a sequence of force-free configurations in D^λ as a consequence of its footpoints being moved horizontally on S_0 and $S_{\lambda h}$ in a prescribed way. For definiteness we suppose $\lambda < 1$. For each value of λ, we introduce the evolving field $\mathbf{b}(\lambda, \tau)$ (with $\tau \in [0, T]$ some auxiliary time) defined in the whole D, and satisfying $\mathbf{b}(\lambda, 0) = \mathbf{B}_\pi$. Its evolution is defined as follows (see Fig. 1). In the part of D above $S_{\lambda h}$, the field is passively advected by the slow velocity field

$$\mathbf{v}(x, y, z, \tau) = [(h - z)/T](\mathbf{B}_{\pi\perp}/B_{\pi z})(x, y, z + (h - z)\tau/T), \tag{8}$$

which deforms $\mathbf{B}_\pi$ into the vertical field $b(x, y, T)\hat{\mathbf{z}}$ at the large time $\tau = T$. The important point here is that this process does not add any overall twist. In the part of D under $S_{\lambda h}$ – i.e., in D^λ –, $\mathbf{b}(\lambda, \tau)$ evolves through a sequence of force-free configurations as its footpoints on $S_{\lambda h}$ are moved at the speed $\mathbf{v}(x, y, \lambda h, \tau)$ while its footpoints on S_0 are kept fixed. It should be clear that at time T, the field $\mathbf{b}(\lambda, T)$ in D^λ has gained the same connectivity as the field $\mathbf{B}(\lambda)$: then the sequence $\{\mathbf{b}(\lambda, T)\}$ and the sequence $\{\mathbf{B}(\lambda)\}$ are identical. In fact we can even argue that $\mathbf{B}(\lambda)$ can be obtained as the solution of a shearing problem in D^λ in which the initial field is taken to be the uniform field $B_0\hat{\mathbf{z}}$. The field $\mathbf{B}_\pi$ can be obtained indeed from $B_0\hat{\mathbf{z}}$ as the final state of a shearing problem in D (explicit examples of velocity field driving this transition are discussed in Aly (2009)), and we can effect successively this deformation and the one defined above. All these arguments can be adapted to treat the case $\lambda > 1$.

What conclusions can be drawn from our point: (i) The compression/expansion problem is not so particular as it may appear to be at first sight: it can be reduced to a standard problem which has long been studied in the literature. (ii) In general, shearing motions are thought to introduce some amount of "twist" in a force-free field – and then some amount of helicity, in accordance with the computation of the previous section –, and it would appear strange that this be not also the case in the conditions considered here. For the least, JL argument according to which the topologically untwisted field $\mathbf{B}(\lambda)$ has to be potential if it is smooth becomes difficult to understand when their problem is reformulated as a shearing problem, even if the boundary motions are somewhat peculiar. (iii) For the shearing problem, there are strong indications that CS do not appear at least as long as the deformation imposed to the field does not exceed some threshold. On the one hand, the numerous simulations which have been effected show the development of current concentrations – which are difficult to interpret either as CSs or as finite thickness layers owing to the finite resolution inherent to any numerical calculation. But these concentrations appear only at later times, while the field stays undoubtedly smooth at earlier ones (Craig & Sneyd 2005; Rappazzo et al. 2008; Wilmot-Smith et al. 2009). On the other hand analytical computations based on a perturbative approach show the existence of a smooth solution at the order ε when the displacements of the footpoints on $S_{0/\lambda h}$ are $O(\varepsilon)$ (Sakurai & Levine 1981; Zweibel & Li 1987; Craig & Sneyd 2005), and even at the arbitrary order ε^p (Aly 1987, 2009). As there is yet no published proof of the convergence of the perturbative series, this is not a complete proof that CSs are not present in the earlier phase of a solution of the full set of nonlinear magnetohydrostatic equations. But this may be taken as an indication that this should be the case if we remark that the necessity of the presence of CSs becomes obvious even at the linear level in some other situations which seem to be now well understood (e.g., in the Taylor problem considered in Hahm & Kulsrud (1985)).

To go deeper into the last point, let us assume that $|\ \mathbf{B}_\pi - B_0\hat{\mathbf{z}}\ | \ll B_0$ and compute the field $\mathbf{B}(\lambda)$ by solving at the linear level the shearing problem for $\mathbf{b}(\lambda, \tau)$ (we generalize here to the case of a field in a cylinder of arbitrary shape the results in Sakurai & Levine (1981); Zweibel & Li (1987); Aly (1987); Craig & Sneyd (2005)). Using one of the statement above, we suppose that we start from the uniform field $B_0\hat{\mathbf{z}}$, and write $\mathbf{b} = B_0[\hat{\mathbf{z}} + \nabla \times (\boldsymbol{\xi} \times \hat{\mathbf{z}})]$, with $\boldsymbol{\xi}(\lambda, \tau)$ denoting the horizontal displacement suffered by a plasma element. $\boldsymbol{\xi}$ is required to satisfy the boundary conditions $\hat{\mathbf{n}} \cdot \boldsymbol{\xi} = 0$ on S^λ, and $\boldsymbol{\xi} = \boldsymbol{\xi}_{0/\lambda h}$ on $S_{0/\lambda h}$, where $\boldsymbol{\xi}_{0/\lambda h}(\lambda, \tau) = O(\varepsilon)$ are the displacements imparted at time τ to the footpoints by the prescribed velocity fields. Moreover it is readily seen that $\boldsymbol{\xi}$ obeys the equation $\partial_{zz}\boldsymbol{\xi} + \nabla_\perp\Theta = 0$ as a consequence of the field being force-free. The quantity $\Theta = \nabla_\perp \cdot \boldsymbol{\xi}$ satisfies a Laplace equation in D^λ (just take the divergence of the previous equation) which can be solved in a unique way once we note that it satisfies boundary conditions fully determined by the data of the problem ($\Theta = \nabla_\perp \cdot \boldsymbol{\xi}_{0/h}$ on $S_{0/\lambda h}$ and $\partial_n\Theta = 0$ on S^λ). Once Θ has been so computed, we get after two simple integrations

$$\boldsymbol{\xi}(x, y, z, \tau) = \left(1 - \frac{z}{\lambda h}\right)\left(\boldsymbol{\xi}_0(x, y, \tau) + \int_0^z z'\nabla_\perp\Theta'\,\mathrm{d}z'\right) + \frac{z}{\lambda h}\left(\boldsymbol{\xi}_{\lambda h}(x, y, \tau) + \int_z^{\lambda h} (\lambda h - z')\,\nabla_\perp\Theta'\,\mathrm{d}z'\right) \quad (9)$$

for $0 \le \tau \le T$, where $\Theta' = \Theta(z')$, and we thus obtain a perfectly smooth $\mathbf{B}(\lambda) = \mathbf{b}(\lambda, T) = B_0[\hat{\mathbf{z}} + \nabla \times (\boldsymbol{\xi}(\lambda, T) \times \hat{\mathbf{z}})]$, in contradiction indeed with JL conjectural third claim.

It is interesting to also compute the force-free function associated to $\mathbf{b}$. Using Eq. (9) for determining $\nabla_\perp \times \boldsymbol{\xi}$, we obtain to the first order in ε

$$\alpha = \partial_z[\hat{\mathbf{z}} \cdot (\nabla_\perp \times \boldsymbol{\xi})] = \hat{\mathbf{z}} \cdot (\nabla_\perp \times \boldsymbol{\xi}_{\lambda h} - \nabla_\perp \times \boldsymbol{\xi}_0)/(\lambda h). \quad (10)$$

At the end of the shearing process, the displacements $\boldsymbol{\xi}_{0/\lambda h}(\lambda, T)$ are identical to the displacements $\boldsymbol{\xi}_{\pi 0/h}(T)$ necessary to deform the uniform field in D into $\mathbf{B}_\pi$, and then $\alpha(T) = \alpha_\pi(T)/\lambda$ (here, we have applied the formula for α – with $\lambda = 1$ – to the initial field $\mathbf{B}_\pi$ in D). But $\mathbf{B}_\pi$ being potential, $\alpha_\pi(T) = 0$, and therefore $\alpha(T) = 0$: the field $\mathbf{B}(\lambda)$ is still potential at order ε. Electric currents are $O(\varepsilon^2)$, and can be computed to that order by using the formalism established in Aly (1987, 2009).

The argument reported in Huang et al. (2009) is similar to the previous one, but it is applied to a different setting in which the field occupies the whole layer comprised between the planes $\{(x, y, 0)\}$ and $\{(x, y, \lambda h)\}$, and is independent of x as well as periodic in y. When the upper plane is moved, the initial potential field is compressed or expanded, and it is found to pass, in the framework of the linear theory, through a sequence of smooth force-free configurations. In contrast with our result, however, there is a bulk electric current which is $O(\varepsilon)$. This is due to their magnetic field having an $O(1)$ horizontal component, while $B_\perp = O(\varepsilon)$ in our model.

5. CONCLUSION

In this Letter, we have revisited JL mechanism for CSs formation. We have found important parts of their theory to be undoubtedly correct and valuable, and even contributed to confort them. Unfortunately, we have also identified an important gap in their arguments: in our view, JL assertion according to which a smooth topologically untwisted force-free magnetic field is necessarily potential rests on intuitive properties of ”twist” rather than on the clear mathematical definition. Actually, our conclusions point towards the impossibility of filling the gap, casting doubt on the validity of their claim that CSs generally form spontaneously once an initially potential field in a cylinder is either compressed or expanded adiabatically. We emphasize that our results, which are complementary to those reported in Huang et al. (2009), just concern JL mechanism. They should not be considered in any way as a criticism of Parker’s general theory of plasma heating.

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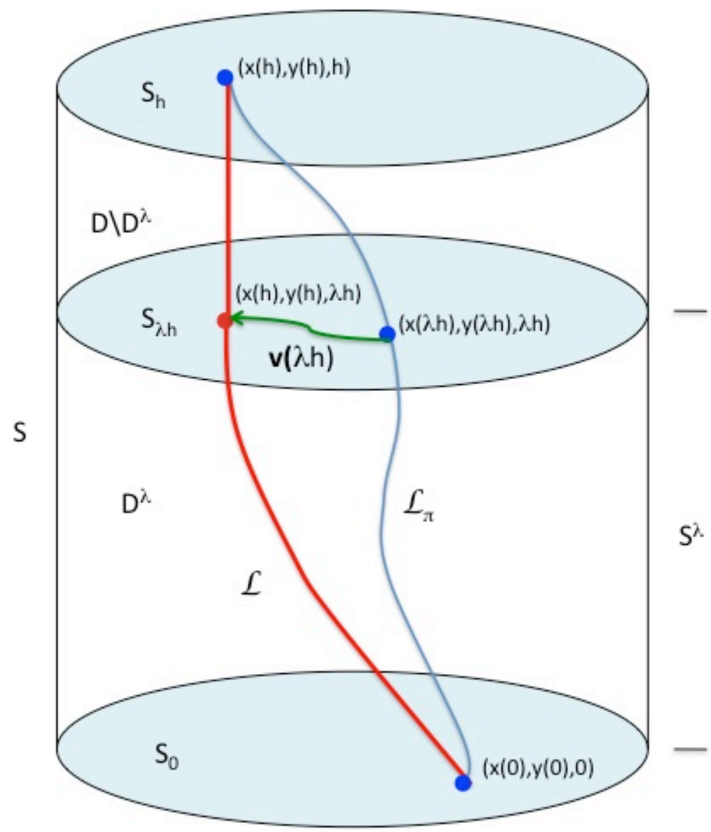

Fig. 1.— At time T, the line $\mathcal{L}_\pi = \{(x(z), y(z), z)\}$ of the initial potential field $\mathbf{B}_\pi$ in D has been deformed into a line whose upper part (for $\lambda h \leq z \leq h$) is the vertical segment $\{(x(h), y(h), z)\}$ resulting from the passive transport by the velocity field $\mathbf{v}$ (see Eq. (8)), and whose lower part (for $0 \leq z \leq \lambda h$) is the line $\mathcal{L}$ of $\mathbf{B}(\lambda)$ with footpoints at $(x(0), y(0), 0)$ and $(x(h), y(h), \lambda h)$. The latter is produced by a quasi-static evolution driven in D^λ by the velocities $\mathbf{v}\,|_{S_0} = 0$ and $\mathbf{v}\,|_{S_{\lambda h}} = \mathbf{v}(z = \lambda h)$.