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Impact of Large-Scale Magnetic Fields on Stellar Structure and Evolution

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Abstract. We study the impact on the stellar structure of a large-scale magnetic field in stellar radiation zones. The field is in magneto-hydrostatic (MHS) equilibrium and has a non force-free character, which allows us to study its influence both on the mechanical and and on the energetical balances. This approach is illustrated in the case of an A_p star where the magnetic field matches at the surface with an external potential one. Perturbations of the stellar structure are semi-analytically computed. We deduce the limits of validity of a linear derivation and the order of magnitude of the different terms. Their relative importance is discussed and a hierarchy, aiming at distinguishing various refinement degrees in the implementation of a large-scale magnetic field in a stellar evolution code, is established. This treatment also allows us to deduce the gravitational multipolar moments and the change in effective temperature associated with the presence of a magnetic field.

Keywords. stars: interiors, stars: magnetic fields, stars: evolution

1. Introduction

Nowadays it is well known from spectropolarimetry measurements that a non negligible fraction of the A-type stars, the peculiar ones (representing about 5 % of the population) exhibit magnetic fields organised over large scales, and whose strengths can reach several kG at their surface. Moreover, it has been shown by Alecian, *et al.* (2008) (see also her contribution in these proceedings) that some so-called Herbig Ae/Be stars are magnetic. Hence it is likely that a fossil magnetic field, already present before the early stages of stellar evolution, could have survived during the pre-main sequence phase and influenced the evolutionary track of their hosts.

We propose here to look at the effects that such a large-scale fossil field could have on the stellar structure by considering a magneto-hydrostatic (MHS) equilibrium in an A_p type star, based on a Grad-Shafranov model. This magnetic field in non force-free, presents a mixed poloidal-toroidal (twisted) configuration and spreads across the whole volume of the star; at its surface it matches with a potential, dipolar field with a 8 kG strength.

Based on a simplified stability analysis, we provide some elements tending to prove that the configuration found is likely to be stable.

The physical quantities likely to modify the stellar structure are then semi-analytically derived and illustrated in the case of interest.

Then, perturbations of the gravitational potential, density, pressure, temperature and radius are in both cases computed throughout the whole radius up to the surface. In particular, the gravitational multipolar moments induced by the presence of a magnetic field are obtained.

Finally, we establish the change in temperature owing to the perturbation of pressure and density; we investigate the energetical quantities perturbations generated by ohmic heating, Poynting's flux, and by the change of nuclear reaction rates induced by modification of the hydrostatic balance.

This allows us to propose a hierarchy of the various effects associated with the magnetic field and likely to act over evolution timescales.

2. The Non Force-Free Magneto-Hydrostatic Equilibrium

We here look for a large-scale magnetic field geometry likely to exist in the stellar radiative zone of A_p-type stars, where has been observed (see Wade *et al.* , 2000) dipolar, roughly axisymmetric configurations which are probably remnants of a fossil magnetic field.
Since we know from Tayler (1973) that purely toroidal fields are unstable, and from Markey & Tayler (1973, 1974) that purely poloidal fields are also unstable, a mixed poloidal-toroidal (twisted) configuration is needed for the field to survive over evolution timescales.
Furthermore, if force-free MHS equilibria are currently observed in plasma experiments, especially in spheromaks experiments, the conditions of pressure in stellar interiors make the problem quite different: in the former case the plasma is in the low-β regime. In the latter, as the magnetic field is a perturbation compared with the gravitational potential and the gaseous pressure gradient (high-β regime), the magnetic field is constrained to be in non force-free equilibrium.
Owing to these facts, we focus on magnetic field configurations such that the field is dipolar, in magneto-hydrostatic equilibrium and non force-free.

2.1. *The Axisymmetric Magnetic Field*

Let us express the magnetic field $\boldsymbol{B}(r,\theta)$ in the axisymmetric case as a function of a poloidal flux $\Psi(r,\theta)$ and a toroidal potential $F(r,\theta)$ such that it remains automatically divergenc-free :

$$\boldsymbol{B} = \frac{1}{r\sin\theta}\nabla\Psi\times\hat{\mathbf{e}}_\varphi + \frac{1}{r\sin\theta}\,F\,\hat{\mathbf{e}}_\varphi. \tag{2.1}$$

where in spherical coordinates the poloidal direction is in the meridional plane $(\hat{\mathbf{e}}_\mathbf{r},\hat{\mathbf{e}}_\theta)$ and the toroidal direction is along the azimuthal one $\hat{\mathbf{e}}_\varphi$.

2.2. *Non Force-Free Condition*

Let us now write the magneto-hydrostatic (MHS) equilibrium as follows:

$$\rho\,\boldsymbol{g} - \nabla P_{gas} + \boldsymbol{F}_\mathcal{L} = \boldsymbol{0} \tag{2.2}$$

where ρ is the density, $\boldsymbol{g}$ the local gravity field, P_gas the gas pressure, and $\boldsymbol{F}_\mathcal{L} = \boldsymbol{j}\times\boldsymbol{B}$ the Lorentz force, $\boldsymbol{j}$ being the current density.
In the toroidal direction, the Lorentz force $F_{\mathcal{L}_\varphi}$ vanishes everywhere, since in lack of rotation there is no other force in this direction to compensate for the equilibrium deviation. This condition writes as $\partial_r\Psi\partial_\theta F - \partial_\theta\Psi\partial_r F = 0$. The non trivial values for F are obtained by setting $F(r,\theta) = F(\Psi)$. Looking at the first order case, regular for the azimuthal magnetic field we have $F(\Psi) = \lambda_1\,\Psi$ where λ_1 is a real constant. According to (2.1) and to the Ampère's law $\nabla\times\boldsymbol{B} = \mu_0\,\boldsymbol{j}$ (in the classical approximation; μ_0 being the vacuum permeability), the Lorentz force can finally be concisely stated as†

$$\boldsymbol{F}_\mathcal{L} = \mathcal{A}(r,\theta)\,\nabla\Psi \quad \text{where} \quad \mathcal{A}(r,\theta) = -\frac{1}{\mu_0\,r^2\,\sin^2\theta}\left(\lambda_1^2\Psi + \Delta^*\Psi\right). \tag{2.3}$$

and where we introduce the so-called Grad-Shafranov operator in spherical coordinates

$$\Delta^*\Psi \equiv \frac{\partial^2\Psi}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\Psi}{\partial\theta}\right). \tag{2.4}$$

Taking the curl of the MHS equation divided by the equilibrium density ρ_0 we have

$$\nabla\times\left(\frac{1}{\rho_0}\nabla P_\mathrm{gas} - \boldsymbol{g}\right) = \nabla\times\left(\frac{1}{\rho_0}\boldsymbol{F}_\mathcal{L}\right), \tag{2.5}$$

which, assuming that the Lorentz force is a weak perturbation to the density and assuming the barotropic equilibrium, vanishes. We can then write, using eq. (2.3)

$$\nabla\left(\frac{\mathcal{A}}{\rho_0}\right)\times\nabla\Psi = \boldsymbol{0}. \tag{2.6}$$

† Notice that written in this way, we see immediatly that when $\lambda_1^2\Psi = -\Delta^*\Psi$, the field is forcee-free and corresponds to the solution described by Chandrasekhar (1956), and generalized later by Marsh (1992).

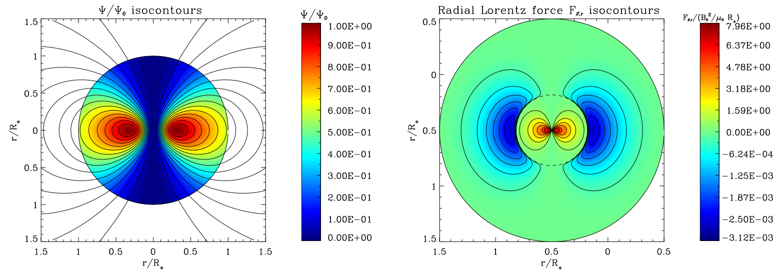

Figure 1. (Left): Ψ (normalized) and (right): radial Lorentz force (normalized) isocontours

This projects only along $\hat{\mathbf{e}}_\varphi$ as $\partial_r \left(\mathcal{A}/\rho_0\right) \partial_\theta \Psi - \partial_\theta \left(\mathcal{A}/\rho_0\right) \partial_r \Psi = 0$ so that there exists a function G of Ψ such that $\mathcal{A}/\rho_0 = G\left(\Psi\right)$, that reduces in the simplest, linear case to $G(\Psi) = \beta_0$. Then, Eq. (2.3) leads to the Grad-Shafranov-like linear differential equation that can be solved analytically

$$\Delta^* \Psi + \left(\frac{\lambda_1}{R}\right)^2 \Psi = -\mu_0 r^2 \sin^2\theta \, \rho_0 \, \beta_0. \tag{2.7}$$

Using Green's function method (Morse & Feschbach, 1953; Payne & Melatos, 2004), the expression for Ψ in terms of the density profile considered is found to be:

$$\Psi\left(r,\theta\right) = -\mu_0 \, \beta_0 \frac{\lambda_1}{R} \sin^2\theta \left\{ r j_1\!\left(\lambda_1 \frac{r}{R}\right) \int_r^R \left[y_1\!\left(\lambda_1 \frac{\xi}{R}\right) \rho_0 \xi^3 \right] \mathrm{d}\xi + r \, y_1\!\left(\lambda_1 \frac{r}{R}\right) \int_0^r \left[j_1\!\left(\lambda_1 \frac{\xi}{R}\right) \rho_0 \xi^3 \right] \mathrm{d}\xi \right\} \tag{2.8}$$

where j_1 and y_1 are respectively the spherical Bessel and Neumann functions of latitudinal order $l = 1$; the eigenvalue λ_1 is given by the boundary conditions at $r = R$ and the constant parameter β_0 is constrained by the magnetic field strength. The iso-Ψ surfaces (normalized to its maximum), tangent to the poloidal magnetic field, and the corresponding radial component of the Lorentz force (normalized to $B_0^2/\mu_0 R_*$) are represented in Fig. 1 in the meridional plane, in the case of a dipolar surface field with a mean surface magnetic field of 8 kG presenting a potential behaviour ($\lambda_1 = \pi/2$). It shows that the field has a centrifugal behaviour below $0.3\,R_*$ and a centripetal, but much weaker in the external part of the radius.

3. Stability Analysis

Follow Reisenegger (2008), we perform a first-order stability analysis. The variational principle of minimizing the magnetic energy is introduced (see Bernstein *et al.* , 1958). The variation of magnetic energy under an arbitrary lagrangian displacement ξ is given by:

$$\delta W_B = \frac{1}{2\mu_0} \delta \left[\int_V \boldsymbol{B}^2 \mathrm{d}V \right] = \frac{1}{\mu_0} \int_V \left[\boldsymbol{B} \cdot \nabla \times \left(\xi \times \mathbf{B}\right) \right] \mathrm{d}V \tag{3.1}$$

In the case of stellar radiation zones, due to the strong stable stratification, the anelastic approximation can be adopted for ξ so that $\nabla \cdot \left(\rho_0 \, \xi\right) = 0$. Then, it is possible to introduce an arbitrary vector field $\mathbf{a}$ such that: $\nabla \times \mathbf{a} = \rho_0 \, \xi$. Eq. (3.1) then becomes

$$\delta W_B = \int_V \nabla \cdot \left[\frac{\left(\mathbf{j} \times \mathbf{B}\right) \times \mathbf{a}}{\rho_0} \right] \mathrm{d}V - \int_V \left[\mathbf{a} \cdot \nabla \times \left(\frac{\mathbf{j} \times \mathbf{B}}{\rho_0} \right) \right] \mathrm{d}V. \tag{3.2}$$

Furthermore, since the anelastic approximation is used, we assume that $\hat{\mathbf{n}} \cdot \xi = 0$ at the surface of the star, giving $\hat{\mathbf{n}} \times \mathbf{a} = \boldsymbol{0}$. The first integral thus vanishes while the second one vanishes by construction. The considered equilibrium seems therefore to be stable since the total magnetic energy variation is nul.

4. Influence on the Stellar Structure

4.1. *Mechanical Balance*

4.1.1. *Magnetic Pressure Force vs. Magnetic Tension Force*

We can write the Lorentz force as the sum of the gradient of a magnetic pressure and of a magnetic tension force:

$$\boldsymbol{F}_{\mathcal{L}} = \boldsymbol{F}_{\mathcal{T}} - \nabla P_{\text{mag}}. \tag{4.1}$$

From Fig. 2 (Left panel), it appears that the magnetic pressure gradient has a predominant role in the internal part of the star over the magnetic tension. However, the latter's strength is of the order of the former in particular on the symmetry axis and in the vicinity of the surface, where both ones counterbalance each other. This leads to a force-free state, that cannot be achieved by considering the magnetic pressure as the only perturbative effect.

4.1.2. *Lorentz Force Perturbations on the Stellar Structure*

Let us then project the Lorentz force components on the Legendre polynomials $P_l(\cos\theta)$ (of order $l = 0$ and $l = 2$ in the case of a dipolar field), assuming it is a perturbation around the stellar non-magnetic state:

$$F_{\mathcal{L},r}(r,\theta) = \sum_l \mathcal{X}_{\boldsymbol{F}_{\mathcal{L}};l}(r)\, P_l(\cos\theta), \qquad F_{\mathcal{L},\theta}(r,\theta) = -\sum_l \mathcal{Y}_{\boldsymbol{F}_{\mathcal{L}};l}(r)\, \partial_\theta P_l(\cos\theta) \tag{4.2}$$

which gives us at the surface the gravitational potential $J_l = (R_*/GM_*)\,\widehat{\phi}_l\,(r = R_*)$. We can then deduce the gravitational potential perturbation $\widehat{\phi}_l$ to the non-magnetic state ϕ_0, from Sweet's equation†

$$\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}\left(r\widehat{\phi}_l\right) - \frac{l(l+1)}{r^2}\widehat{\phi}_l - \frac{4\pi G}{g_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}r}\widehat{\phi}_l = \frac{4\pi G}{g_0}\left[\mathcal{X}_{\boldsymbol{F}_{\mathcal{L}};l} + \frac{\mathrm{d}}{\mathrm{d}r}\left(r\mathcal{Y}_{\boldsymbol{F}_{\mathcal{L}};l}\right)\right]. \tag{4.3}$$

where g_0 is the equilibrium gravity and density and where we have $\phi(r,\theta) = \phi_0 + \sum_l \widehat{\phi}_l(r)\, P_l(\cos\theta)$. After numerical integration of the Sweet's equation, the density perturbation ρ_l and the pressure perturbation P_l for the mode l can respectively be computed according to

$$\widehat{\rho}_l = \frac{1}{g_0}\left[\frac{\mathrm{d}\rho_0}{\mathrm{d}r}\widehat{\phi}_l + \mathcal{X}_{\boldsymbol{F}_{\mathcal{L}};l} + \frac{\mathrm{d}}{\mathrm{d}r}\left(r\mathcal{Y}_{\boldsymbol{F}_{\mathcal{L}};l}\right)\right] \quad \text{and} \quad \widehat{P}_l = -\rho_0\widehat{\phi}_l - r\mathcal{Y}_{\boldsymbol{F}_{\mathcal{L}};l}. \tag{4.4}$$

Diagnosis from the stellar radius variation induced by the magnetic field can be established. The radius of an isobar is given by

$$r_P(r,\theta) = r\left[1 + \sum_{l\geqslant 0} c_l(r) P_l(\cos\theta)\right] \text{ with } c_l = -\frac{1}{r}\frac{\widehat{P}_l}{\mathrm{d}P_0/\mathrm{d}r} = \frac{\rho_0}{\mathrm{d}P_0/\mathrm{d}r}\left(\frac{1}{r}\widehat{\phi}_l + \frac{\mathcal{Y}_{\boldsymbol{F}_{\mathcal{L}};l}}{\rho_0}\right). \tag{4.5}$$

Finally, it can be interesting to look for temperature perturbations. Following Kippenhahn & Weigert (1990), we introduce the general equation of state for the stellar plasma $\mathrm{d}\rho/\rho = \alpha_s \mathrm{d}P/P - \delta_s \mathrm{d}T/T + \varphi_s \mathrm{d}\mu_s/\mu_s$ where $\alpha_s = (\partial\ln\rho/\partial\ln P)_{T,\mu_s}$, $\delta_s = -(\partial\ln\rho/\partial\ln T)_{P,\mu_s}$ and $\varphi_s = (\partial\ln\rho/\partial\ln\mu_s)_{P,T}$. For a perturbative Lorentz force, the stellar temperature (T) and the mean molecular weight (μ_s) can be expanded like P, ρ and ϕ according to $T(r,\theta,t) = \mu_{s;0}(r) + \sum_{l\geqslant 0}\widehat{\mu}_{s;l}(r,t)\, P_l(\cos\theta)$. Linearizing of the equation of state, we finally obtain

$$\widehat{T}_l = \frac{T_0}{\delta_s}\left[\alpha_s\frac{\widehat{P}_l}{P_0} - \frac{\widehat{\rho}_l}{\rho_0} + \varphi_s\frac{\widehat{\mu}_{s;l}}{\mu_{s;0}}\right]. \tag{4.6}$$

Results for the normalized perturbations of gravitational potential $\tilde{\Phi}_l$, density $\tilde{\rho}_l$, pressure $\tilde{P}_l$, temperature $\tilde{T}_l$ and radius c_l are shown in Fig. 2 for the modes $l = 0$ and $l = 2$ (resp. middle and right panel). At the surface the effective temperature change is found from the $l = 0$ temperature perturbation: it is $\widehat{T}_0 = +1.45 \times 10^{-4} T_{\text{eff}}$, i.e. for the considered case $T_{\text{eff}} = 8422$K instead of 8421K. The gravitational multipolar moments are $J_0 = -1.31 \times 10^{-7}$ and $J_2 = -2.54 \times 10^{-8}$.

† Let us recall that Sweet (1950) was the first to derive this result for the most general perturbing force, Moss (1974) having introduced the special case of the Lorentz force in the case of a poloidal field.

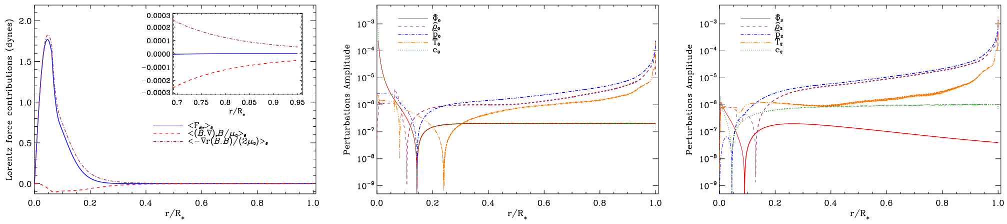

Figure 2. Left: radial Lorentz force (normalized), with the raltive contributions of the magnetic pressure gradient and the magnetic tension. Middle and Right : perturbations of mode $l = 0$ and $l = 2$ (in log. scale) respectively. Bold lines represent positive values whereas thin lines represent negative ones. The spikes corresponds to the vanishing of source terms for the equations 4.3, 4.4, 4.5 and 4.6.

4.2. *Energetic Balance*

4.2.1. *Poynting's Flux and Ohmic Heating*

The Ohmic heating is defined by

$$Q_{\rm Ohm}(r,\theta) = \mu_0\, \eta\, \boldsymbol{j}^2(r,\theta). \tag{4.7}$$

where η is the magnetic diffusivity that can be evaluated with the temperature-dependent law from Spitzer (1962)

$$\eta = 5.2 \times 10^{11}\ \log\Lambda\, T^{-3/2}\ {\rm cm^2 s^{-1}}. \tag{4.8}$$

The Poynting's flux is given by $F_{\rm Poynt} = \nabla\cdot(\boldsymbol{E}\times\boldsymbol{B}/\mu_0)$. In the static case, the simplified Ohm's law $\boldsymbol{j} = \sigma\boldsymbol{E}$ together with the identity $\eta = (\mu_0\sigma)^{-1}$, reduces the Poynting's flux expression to

$$F_{\rm Poynt} = \nabla\left(\eta\, \boldsymbol{F}_{\mathcal{L}}\right) \tag{4.9}$$

4.2.2. *Perturbation of the Energetic Balance*

Here again a perturbative approach is adopted. The luminosity is expanded as

$$L = L_0 + \widehat{L}_{\rm tot}. \tag{4.10}$$

$\widehat{L}_{\rm tot}$ is the luminosity perturbation due to the magnetic terms :

$$\widehat{L}_{\rm tot}\left(r\right) = L_{\rm Ohm}\left(r\right) + L_{\rm Poynt}\left(r\right) + \widehat{L}_{\rm nuc}\left(r\right), \tag{4.11}$$

which are respectively the Ohmic heating contribution and the Poynting's flux one, and the induced modification of the specific energy production rate.
First, we integrate the Ohmic heating and the Poynting's flux over the volume delimited by r

$$L_{\rm Ohm}(r) = \int_0^r \int_\Omega Q_{\rm Ohm}(r',\theta')\, {\rm d}\Omega\, r'^2 {\rm d}r';\ L_{\rm Poynt}(r) = \int_0^r \int_\Omega F_{\rm Poynt}(r',\theta')\, {\rm d}\Omega\, r'^2 {\rm d}r', \tag{4.12}$$

where ${\rm d}\Omega = \sin\theta' d\theta' d\phi'$, r' thus ranging from 0 to r, θ' from 0 to π and ϕ' from 0 to 2π.
Then, to be able to conclude we finally consider the modification of the specific energy production rate (ε), which depends on ρ and T, due to magnetic field. First, the logarithmic derivative of ε is expanded like the one of ρ (cf. the equation of state, and see Mathis & Zahn 2004 and references therein): ${\rm d}\ln\varepsilon = \lambda\, {\rm d}\ln\rho + \nu\, {\rm d}\ln T$, where $\lambda = \left(\partial\ln\varepsilon/\partial\ln\rho\right)_T$ and $\nu = \left(\partial\ln\varepsilon/\partial\ln T\right)_\rho$. Then, like ρ and T, we expand ε on the Legendre polynomials so that we finally end up with

$$\varepsilon\left(r,\theta\right) = \varepsilon_0\left(r\right) + \sum_{l\geqslant 0} \widehat{\varepsilon}_l\left(r\right) P_l\left(\cos\theta\right) \quad \text{where} \quad \widehat{\varepsilon}_l = \varepsilon_0\left[\lambda\frac{\widehat{\rho}_l}{\rho_0} + \nu\frac{\widehat{T}_l}{T_0}\right]. \tag{4.13}$$

The luminosity perturbation induced by the MHS equilibrium over the nuclear reaction rates is

$$\widehat{L}_{\rm nuc}\left(r\right) = \int_0^r \int_\Omega \widehat{\varepsilon}_0\, \rho_0\, r'^2 {\rm d}r' {\rm d}\Omega = 4\pi \int_0^r \left\{\varepsilon_0\left[\lambda\frac{\widehat{\rho}_0}{\rho_0} + \nu\frac{\widehat{T}_0}{T_0}\right]\right\} \rho_0\, r'^2 {\rm d}r'. \tag{4.14}$$

The values found at the stellar surface are $\widehat{L}_{\rm nuc} = -6.06\times 10^{29}$erg.s, $L_{\rm Ohm} = 5.71\times 10^{23}$erg.s and $L_{\rm Poynt} = -5.97\times 10^{22}$erg.s, whereas the total luminosity is $L_0 = 1.59\times 10^{35}$erg.s.

5. Conclusion

We have shown that at a first glance the non force-free, barotropic MHS equilibria are stable. This type of configuration is thus relevant to model initial conditions for evolutionary calculations involving large-scale, long-time evolving fossil fields in stellar radiation zones as well as in degenerate objects such as white dwarfs or neutron stars (see Payne & Melatos, 2004). More particulary it can be used to initiate MHD rotational transport in dynamical stellar evolution codes where it is implemented (cf. Mathis & Zahn, 2005; Duez *et al.* , 2008) since axisymmetric transport equations that have been derived are devoted to the stable axisymmetric component of the magnetic field, the magnetic instabilities being treated using phenomenological prescriptions (see Spruit 1999; Maeder & Meynet, 2004) that have to be verified or improved by numerical experiments (see Braithwaite 2006 and subsequent works; Zahn, Brun & Mathis, 2007). In the context of implementing the magnetic field's effects in a stellar evolution code, the qualitative importance of the magnetic tension has been underlined.

In the case exposed here, the perturbative approach has shown that the direct contribution of the magnetic field to the change in the energetic balance through Ohmic heating or through Poynting's flux is weak compared with the indirect modification to the energetic balance induced by the change in pressure and density over the nuclear reaction rate. A first approach, consisting in limiting the impact of a large-scale magnetic field only to its impact upon the hydrostatic balance will therefore be justified.

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