# Sensitivity of the g-mode frequencies to pulsation codes and their parameters

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**Abstract**

From the recent work of the Evolution and Seismic Tools Activity (ESTA, Monteiro *et al.*, 2006; Lebreton *et al.*, 2008), whose Task 2 is devoted to compare pulsational frequencies computed using most of the pulsational codes available in the asteroseismic community, the dependence of the theoretical frequencies with non-physical choices is now quite well fixed. To ensure that the accuracy of the computed frequencies is of the same order of magnitude or better than the observational errors, some requirements in the equilibrium models and the numerical resolutions of the pulsational equations must be followed. In particular, we have verified the numerical accuracy obtained with the Saclay seismic model, which is used to study the solar g-mode region (60 to $140\mu Hz$). We have compared the results coming from the Aarhus adiabatic pulsation code (ADIPLS), with the frequencies computed with the Granada Code (GraCo) taking into account several possible choices. We have concluded that the present equilibrium models and the use of the Richardson extrapolation ensure an accuracy of the order of $0.01\mu Hz$ in the determination of the frequencies, which is quite enough for our purposes.

**Keywords:** Helioseismology, Modelling; Interior, Radiative zone, Core

## 1. Introduction

The interior of the Sun has been very well studied thanks to the information provided by pressure-driven modes (p modes). We have been able to determine some structural variables such as the sound-speed velocity till the very inner solar core (e.g. Basu *et al.*, 2009) but with less and less accuracy towards the deepest layers of the radiative zone. Besides, these modes are less sensitive to other structural variables like the

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density. In the case of the dynamics of the solar interior, due to the very small number of non-radial p modes penetrating inside the core, neither the rotation profile (e.g. Chaplin *et al.*, 1999; Thompson *et al.*, 2003; García *et al.*, 2004; García *et al.*, 2008) nor the dynamical process (e.g. Mathis and Zahn, 2004; Mathis and Zahn, 2005) are well constrained inside this region.

Gravity (g) modes are eigenmodes propagating inside the Sun while buoyancy is their restoring force. They can travel inside the radiative region but when they propagate in the convective zone they become evanescent reaching the solar surface with tiny amplitudes (Belkacem *et al.*, 2009 and references therein). These modes represent the "master key" that would give us a complete access to the solar core, in particular, to its dynamics (e.g. Mathur *et al.*, 2008; Mathur, Garcia, and Eff-Darwich, 2009).

Gravity modes have been searched for a long time, almost since the beginning of helioseismology (Hill *et al.*, 1991; Pallé, 1991). But there is currently no undisputed detection of individual g modes in the Sun (Appourchaux *et al.*, 2009). However, thanks to the high-quality observations provided by VIRGO[1] and GOLF[2] on board SoHO and the ground-based networks BiSON[3] and GONG[4] , some peaks (Gabriel *et al.*, 2002; Jiménez and García, 2009) and groups of peaks (Turck-Chièze *et al.*, 2004; García *et al.*, 2008) have been considered as reliable g-mode candidates as they have more than 90% of confidence level and they present several of their expected properties. Moreover, to increase the probability of detection, García *et al.* (2007) searched for the global signature of such modes instead of looking for individual g modes. They found the signature of the asymptotic dipole g modes with more than 99.99% confidence level. The detailed study of this asymptotic periodicity revealed a higher rotation rate in the core than in the rest of the radiative region and a better agreement with solar models computed with old surface abundances compared to the new ones (García, Mathur, and Ballot, 2008). However, it was not possible to identify the sequence of individual peaks generating the detected signal because of the very small signal-to-noise ratio. Thus, to go further it is necessary to use theoretical g-mode predictions to guide our search (García, 2010). For this purpose, we need to know the limits of the modeled physical processes and quantities as well as the internal numerical errors of the codes used to compute the predicted frequencies.

Up to now, the physical processes and quantities included in the solar models have been improved thanks to the constraints brought by observations. Many physical inputs have been added or changed such as the microscopic diffusion, the diffusion in the tachocline or the chemical composition. Besides, more studies on these models have been done since the release of the solar composition of Asplund, Grevesse, and Sauval (2005) leading to a decrease of the metallicity. The models including the latter composition presented larger discrepancies in the sound-speed profile (Bahcall, Serenelli, and Basu, 2005; Turck-Chièze *et al.*, 2004) than those based in the former solar surface abundances (Grevesse, Noels, and Sauval, 1993). Recently, the decrease of the metallicity has been reviewed (Asplund *et al.*, 2009) and the differences between the models and the observations are slightly lower (Serenelli *et al.*, 2009).

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[1]Variability of solar IRradiance and Gravity Oscillations (Frohlich *et al.*, 1995)

[2]Global Oscillations at Low Frequency (Gabriel *et al.*, 1995)

[3]Birmingham Solar Oscillation Network (Chaplin *et al.*, 1996)

[4]Global oscillation Network Group (Harvey *et al.*, 1996)

The accuracy of the model has already been studied in Mathur *et al.*, 2007; Zaatri *et al.*, 2007. They showed that models with different physical inputs and fixed surface abundances present differences in the frequencies of the g modes that are below 1 $\mu$Hz in the range [60,140] $\mu$Hz.

In the present paper we study the numerical errors introduced by the approaches followed by the oscillation codes used to compute the g-mode frequencies of the Sun. It is a direct application of the study done in the ESTA group to the solar case and the calculation of the g-mode frequencies. The main aim of the Task 2 of this group (Moya *et al.*, 2008) is to compare different pulsational codes (a total of nine in this case) when a fixed equilibrium model is provided, the same for all. Therefore, any differences between the results provided by these codes are due to non-physical reasons. This comparison makes it possible to fix the global uncertainties of the numerical schemes used. The final goal of the group is to provide some recomendations to ensure that these differences are lower than the observational accuracies. In this work, we will start by recalling the methodology followed by the ESTA group in section 2. Then, in Section 3 we will briefly describe the solar-structure model and we will finish by discussing the results in Section 4.

## 2. The ESTA group

Within the CoRoT Seismology Working Group, the ESTA group (Monteiro *et al.*, 2006;Lebreton *et al.*, 2008) has been set up with the aim to extensively test, compare and optimise the numerical tools used to calculate stellar models and their oscillation frequencies. Its goals are 1) to be able to produce theoretical seismic predictions by means of different numerical codes and to understand the possible differences between them, and 2) to bring stellar models at the level of accuracy required to interpret the seismic data from space.

Nine pulsational codes joined the sub-group devoted to the eigenmode-frequency comparison. The first step of this sub-group was to test the accuracy of the results provided by these codes under the different mathematical schemes and/or algorithms used. To do so, an equilibrium model was fixed and distributed to the group. Therefore, any eventual difference could only be due to non-physical differences. To understand and minimize these differences compared to the observational uncertainties was the main goal of that work. A $1.5M_\odot$ model was used, and all the frequency ranges investigated.

The main sources of non-physical differences are the following:

- Set of eigenfunctions: Use of the Lagrangian or the Eulerian perturbation of the pressure ($\delta P$ or $P'$).
- Order of the integration scheme: Most of the codes use a second-order scheme, but some others have implemented a fourth-order scheme.
- Richardson extrapolation: Some of the codes, using a second-order scheme, have the possibility of using Richardson extrapolation (Shibahashi and Osaki, 1981) to decrease the truncation error.
- Integration variable: Two integration variables are used: 1) the radius ($r$), or 2) the ratio $r/P$.

The main conclusions of the work done by this group are the following (Moya *et al.*, 2008): 1) if the code uses a second order integration scheme, then the Richardson

extrapolation must be used, 2) the numerical description of the $\nabla_\mu$ zone must not have any numerical unaccuracy for a correct description of the g modes and the modes in avoided crossing, 3) at least 2000 mesh points are neccesary, with an optimal mesh of 4000 points and, 4) the same value of the gravity constant $G$ must be used to obtain the equilibrium model and the frequencies.

## 3. Modeling the Sun interior: computing p and g modes

### 3.1. Solar model

We have computed one solar model, which is the Seismic model developed in Saclay (Turck-Chièze *et al.*, 2001; Couvidat, Turck-Chièze, and Kosovichev, 2003). It is a 1D model computed with the *Code d'Evolution Stellaire Adaptatif et Modulaire* (CE-SAM, Morel, 1997). This model was tuned to better match helioseismic observations (e. g. the sound-speed profile), specially in the radiative region. Another goal of this model was to predict more accurately the neutrino fluxes.

The most commonly solar model used today is the Standard Solar Model (SSM), such as model S (Christensen-Dalsgaard *et al.*, 1996). Compared to this model, the Saclay-Seismic model includes the treatment of the diffusion in the tachocline prescribed by Spiegel and Zahn (1992), while the opacity and p-p reaction rates have been modified. Concerning the chemical composition, it uses the abundances table given by Grevesse and Noels (1993). This model is calibrated in terms of surface metallicity, luminosity, and radius at the age of 4.6 Gyrs with an accuracy of $10^{-5}$.

As one of the main aims of this study consists of calculating the frequencies of g modes, and these modes are mainly confined in the inner regions of the Sun, we have computed the model using the highest resolution in the core of the Sun (below $0.5$ R$_\odot$).

Finally, to calculate the p- and g-mode frequencies we have used two oscillation codes: The Granada Code (GraCo) and the Aarhus adiabatic pulsation code (ADIPLS), both being part of the ESTA group.

### 3.2. Pulsation codes brief description: Aarhus & GraCo

GraCo (Moya, Garrido, and Dupret, 2004; Moya and Garrido, 2008) is a non-radial non-adiabatic linear pulsational code using a second order integration scheme to solve the set of differential equations. The code also provides frequencies in the adiabatic approximation. This code has been used as a reference code for the ESTA study. Thus all the possible numerical schemes and algorithms in the literature have been implemented. Therefore, this code makes it possible to study the numerical accuracy of the frequencies obtained with any singular equilibrium model. In adition, this code have been used to study several g mode pulsatiors (Rodríguez *et al.*, 2006; Rodríguez *et al.*, 2006).

ADIPLS (Christensen-Dalsgaard, 2008) is one of the first and most used adiabatic pulsational codes in the world. It is also based on a second order integration scheme, and it uses the Eulerian variation of the pressure as eigenfunction. We have used the relaxation method where the equations are solved together with a normalization condition. The frequencies are found by iterating on the outer boundary condition. The frequencies have not been estimated using the Cowling approximation. Finally, to compute these frequencies, we have remeshed the model onto 2400 points and extrapolated the parameters bellow $0.05$ R$_\odot$.

## 4. Results

Using these two codes, we have obtained the adiabatic frequency spectrum of the Saclay equilibrium model. A complete comparison following the work done by the ESTA group has been carried out. For this purpose, the global characteristics of the numerical resolution for the ADIPLS code's results have been fixed, that is: the use of the Richardson extrapolation, $P'$ as eigenfunction, $\ln r$ as integration variable and $\delta P = 0$ as mechanical outer boundary condition. The frequencies obtained with ADIPLS are our reference frequencies.

On the other hand, the frequencies with GraCo have been obtained using all the possible options for the numerical resolution described in Sec. 2. An additional source of difference has been studied here: the outer mechanical boundary condition. We have studied the influence of using $\delta P = 0$ or the boundary conditions described in eq. 18.47 of Unno *et al.* (1989), where the following assumptions were used: 1) continuity of the eigenfunctions between the interior and a quasi-isothermal atmosphere and, 2) there is no flux coming from outside in the star. This is a physical source of difference, but its influence is of the order of the rest of the non-physical choices here studied, and it has never been tested, up to our knowledge, for solar g modes.

We first show, in Fig. 1, an overview of the complete frequency spectrum from 60 to 4000 $\mu$Hz. The differences for all the options except the absence of the Richardson extrapolation are presented in this comparison. If the Richardson extrapolation is not used, we reach differences up to 10 $\mu$Hz for the largest frequencies. In this figure, the main differences are observed with different outer mechanical boundary conditions, followed by the use of a different integration variable. Nevertheless, this last choice and the rest of the non-physical choices show differences in the range $[-0.2, 0.3]$ $\mu Hz$, similar to those obtained by the ESTA group using a 2000 mesh points model. Note that, if the same set of non-physical choices are used with both codes, the differences between the results related to the way in which the codes solve the equations, are similar to the change of one of these options.

With this model, we can notice a peak that appears around 300 $\mu Hz$. In Fig. 2 large separations are depicted as a function of the frequency for modes with $\ell = 1$ and 2 and frequencies in the region [150,500] $\mu$Hz. We see who the peak is coincident with a minimum followed of a maximum in the large separtions. This use to be a signature of the avoided-crossing fenomenum. This peak was also present in the ESTA studies, showing a difference of 0.1 $\mu Hz$. This overview of all the spectrum shows that, if the observational accuracy is 0.1 $\mu Hz$ or lower, we must increase the number of mesh points in the equilibrium model to ensure that the theoretical frequencies have a dependancy on non-physical choices lower than this accuracy.

Fig. 3 displays the results of the comparisons in the g-mode region from 60 to 140 $\mu Hz$ for the modes $\ell = 1$ and 2, covering the radial orders in the ranges n=[4,10] for $\ell = 1$ and n=[7,18] for $\ell = 2$. The results using $\delta P = 0$ as mechanical outer boundary condition are not plotted since it provides, for this frequency range, the same result as using the Unno *et al.* (1989) boundary condition.

In this figure we see that:

- The comparison of frequencies for the modes $\ell = 1$ and $\ell = 2$ provides similar results.
- When both codes use the same configuration for the numerical resolution, the differences found are in the range $[-0.02, 0.02]$ $\mu Hz$. These values are much

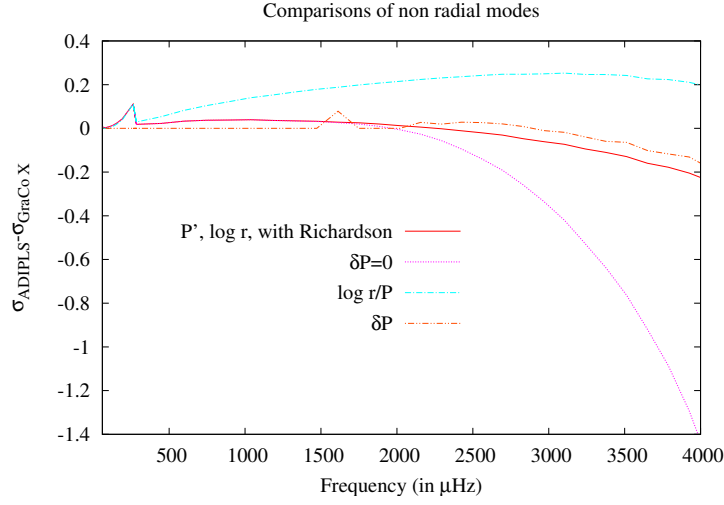Comparisons of non radial modes



**Figure 1.** Overview of the differences found along the complete frequency spectrum. The ADIPLS frequencies are the reference ones. The differences obtained with all the possibilities explained in the text except the case not using of the Richardson extrapolation are shown (see text for details).
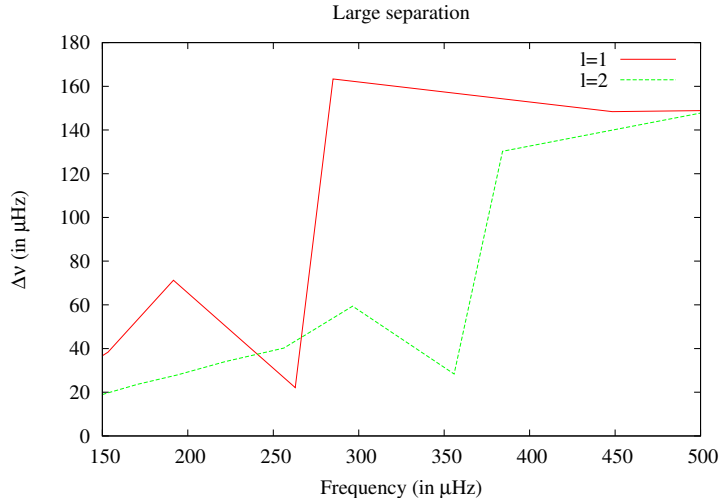
Large separation



**Figure 2.** Large separations as a function of the frequency for modes with $\ell = 1$ and 2 in the region around the fundamental radial mode.

lower than the observational accuracy. The reason of these differences must be searched in the use or not of the re-mesh. ADIPLS frequencies have been obtained using a re-mesh adapted to modes mainly propagating in the stellar interior, and GraCo has not this option.

- When the Richardson extrapolation is not used, the differences obtained are in the range $[0.01, 0.08]$ $\mu Hz$, which can be up to four times bigger than the other comparisons.
- The rest of the possible choices for the numerical resolution provide differences in the range $[-0.02, 0.02]$ $\mu Hz$. This is similar to the range obtained when the same configuration is used in both codes.
- The different outer mechanical boundary conditions have no influence in this frequency range.
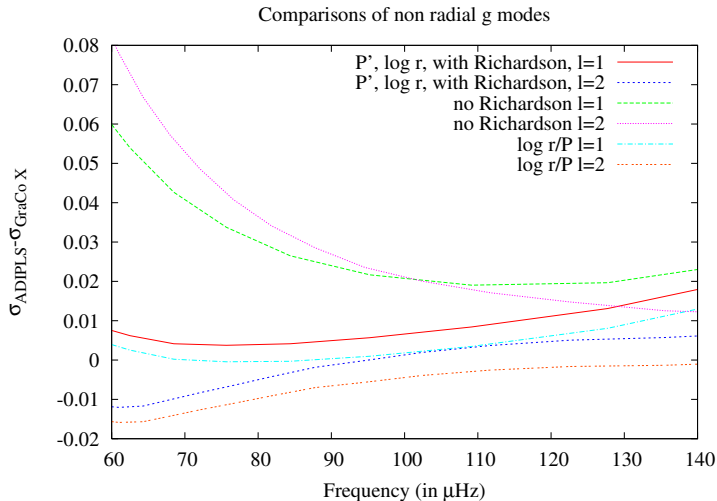


**Figure 3.** Differences between the g-mode frequencies from the GraCo code with different options and the reference frequencies from the ADIPLS code as a function of the ADIPLS frequencies in the range $[60, 140]$ $\mu Hz$.

## 5. Conclusions

The search for g modes has been a long quest as they are the best probes of the solar core, thus representing a huge potential to better constrain its structure and dynamics. Up to now, a few candidates have been detected and recently the global properties of dipole g modes have been detected with more than 99% of confidence level. The next step in the search for individual g modes would consist in being guided by the theoretical predictions of their frequencies obtained with an oscillation code for a given solar model.

This is the reason why the accuracy of the frequencies calculated with numerical algorithms is important. In this paper we have taken the advantage of the previous studies of the ESTA group and we have tested the accuracy of the equilibrium model used for the search of g modes in the Sun under changes of non-physical choices in the numerical integration of the pulsational equations. The Saclay-Seismic model has been used as an input of the pulsational codes ADIPLS and GraCo. Two comparisons have been studied: 1) an overview of the differences obtained along the complete frequency spectrum and, 2) a especial analysis of the g-mode region $[60, 140]$ $\mu Hz$.

This first comparison has shown that among the different non-physical choices in several zones of the spectrum, the present equilibrium model provides differences of the order of 0.1 $\mu Hz$ or larger than that : the avoided crossing and the p modes with large frequencies. This also happens when the same choices are used for both codes. This means that we need a larger number of mesh points if we want to accurately fit the observed frequencies in these regions.

On the other hand, the g-mode region presents an accuracy of the order of $\pm$ 0.02 $\mu Hz$ for any non-physical choice when the Richardson extrapolation is used which is much better than the uncertainties given by the physical prescriptions used in the models. This study has shown that, if we want to lead a search related to observed g modes in this region and based on theoretical models, the characteristics of the numerical choices in the Saclay-Seismic model and the use of either ADIPLS or GraCo code, in terms of number of mesh points and numerical accuracy of sentitive quantities, are enough. Thus, such a guided research will be mainly sensitive to uncertainties coming from the physical inputs of the models.

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