Role of five-quark components in radiative and strong decays of the $\Lambda(1405)$ resonance

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Abstract

Within an extended chiral constituent quark model, three- and five-quark structure of the S_{01} resonance $\Lambda(1405)$ is investigated. Helicity amplitudes for the electromagnetic decays ($\Lambda(1405) \to \Lambda(1116)\gamma$, $\Sigma(1194)\gamma$), and transition amplitudes for strong decays ($\Lambda(1405) \to \Sigma(1194)\pi$, K^-p) are drived, as well as the relevant decay widths. The experimental value for the strong decay width, $\Gamma_{\Lambda(1405)\to(\Sigma\pi)^\circ} = 50 \pm 2$ MeV, is well reproduced with about 50% of five-quark admixture in the $\Lambda(1405)$. Important effects due to the configuration mixings among $\Lambda_1^2 P_A$, $\Lambda_8^2 P_M$ and $\Lambda_8^4 P_M$ are found. In addition, transitions between the three- and five-quark components in the baryons turn out to be significant in both radiative and strong decays of the $\Lambda(1405)$ resonance.

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I. INTRODUCTION

The structure and properties of the S_{01} resonance $\Lambda(1405)$, discovered in 1960's, is still one of the puzzling issues in hadron physics. In the literature the $\Lambda(1405)$ is considered as an s-channel resonance [1] or as a quasi-bound $(\bar{K}N, \Sigma\pi)$ state [2–13]. In the quark-model approaches, this hyperon is treated as a pure $|qqq\rangle$-state [14–20], or still as an admixture of $|q^3{+}q^4\overline{q}\rangle$ configuration [21, 22]. Other approaches takes this hyperon as an "elementary" field [23] or as a quasi-bound state [24] using chiral perturbation theory, or consider it as composed by an SU(2) soliton and a kaon bound in an S-wave [25].

In recent years, possible unconventional or exotic structure for that resonance has received significant attention, suggesting the presence of states other than pure three-quark configuration.

QCD-sum Rules framework has been applied to investigate [26–30] the nature of the $\Lambda(1405)$. Using the $\Sigma^\circ\pi^\circ$ multiquark interpolation field the mass of that resonance is overestimated by about 100 MeV [27]. Introducing [28] coupling between positive- and negative-parity baryons within the flavor-octet hyperons leads to the conclusion that the $\Lambda(1405)$ is not the parity partner of the Λ and may be a flavor-singlet or exotic state. Mixing of three- and five-quark Fock components attributes [29] to this latter 90% of occupations, emplying a non-unique flavor-singlet operator for it, composed of two flavor diquarks and one antiquark. Moreover, a recent work [30] predicts that resonance as an exotic $[udsg]$ strange hybrid and the mass of the lowest strange hybrid with $IJ^P = 0(1/2)^-$ turns out to be 1407 MeV.

Various lattice QCD calculations [31–36] have been devoted to predict the mass of $\Lambda(1405)$ and come up with masses higher than the observed one by 300-400 MeV. An interesting outcome of those works is nevertheless the need for five-quark components in $\Lambda(1405)$.

Investigations of the radiative and strong decays processes of baryons offer an appropriate case study in getting reliable insights to their internal structure. Several authors have studied the decay properties of the $\Lambda(1405)$ within constituent quark models [15–20]. However, the calculated strong decay width of the $\Lambda(1405)$ in the traditional constituent quark model turns out to be much smaller than the value $\Gamma = 50 \pm 2$ MeV reported by Particle Data Group (PDG) [37].

Recently, extended constituent quark models, which includes higher Fock components, have been developed to describe the properties of baryon resonances [38–43]. Those approaches support strongly the existence of significant genuine non-perturbative five-quark components in baryons (for a recent concise review see Ref. [44]) and provide much better descriptions for the electromagnetic and strong decays of $\Delta(1232)$ [38, 39], $N(1440)$ [40, 41] and $N(1535)$ [42, 43].

Here we investigate the relevance of five-quark components in $\Lambda(1405)$, within an extended chiral constituent quark approach. The orbital-flavor-spin configuration for the four-quark subsystem of the five-quark components in $\Lambda(1405)$, with lowest energy being $[31]_{XFS}[4]_X[211]_F[22]_S$ [45, 46], allows for $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ components in this resonance, while the lowest energy five-quark component in the S_{11} nucleon resonance $N(1535)$ can only be the $s\bar{s}$ component [42, 45]. Those features shed a light on the observed mass ordering of $\Lambda(1405)$ and $N(1535)$, which cannot be described wihtinin conventional constituent quark models.

In this work we focus on the radiative and strong decays widths of the $\Lambda(1405)$ in a truncated Fock space, which includes three- and five-quark components, as well as configuration mixings among them, namely, $qqq \leftrightarrow qqqq\bar{q}$ transitions (here, we have omitted the γ^* or the meson, π and K, which intervene in those transitions). We find that the mixing mechanism contributes significantly to both strong and radiative decays.

The manuscript is organized in the following way. The wave functions for the three- and five-quark components in $\Lambda(1405)$ and that in the $SU(3)$ octet baryons are given in Section II. In Section III, we give a brief account of to the formalism for the radiative and strong decays in the extended chiral constituent quark model. The numerical results are presented and discussed in Section IV. Finally, Section V contains our conclusions.

II. WAVE FUNCTION MODEL

In our extended chiral constituent quark model, we assume that the wave function for a baryon can be expressed as

$$|B\rangle \ = \ A_{(B)3q}|qqq\rangle + A_{(B)5q}\sum_i A_i|qqqq_i\bar{q}_i\rangle + \cdots . \tag{1}$$

Here $A_{(B)3q}$ and $A_{(B)5q}$ are the amplitudes for the 3-quark and 5-quark components, respectively, in the corresponding baryon. If we neglect higher Fock components, then $A^2_{(B)3q}+A^2_{(B)5q} = 1$. The sum over i runs over all the possible $qqqq_i\bar{q}_i$ components ($i = u, d, s$), and the factors A_i denote the coefficient for the corresponding $qqqq_i\bar{q}_i$ component, implying $\sum_i A_i^2 = 1$.

In this paper, we consider the S_{01} resonance $\Lambda(1405)$ to be an admixture of the configurations $\Lambda_1^2 P_A$, $\Lambda_8^2 P_M$. and $\Lambda_8^4 P_M$. we also assume the $SU(3)$ octet baryons to be an admixture of $B_8^2 S_S$, $B_8^2 S_S'$, and $B_8^2 S_M$ configurations. Concerning the mixing probability amplitudes for these latter configurations, we employ, for simplicity, the ones proposed in Refs. [15, 47]

$$|\Lambda(1405)\rangle = 0.90|\Lambda_1^2 P_A\rangle - 0.43|\Lambda_8^2 P_M\rangle + 0.06|\Lambda_8^4 P_M\rangle\,, \tag{2}$$

$$|\Lambda(1116)\rangle = 0.93|\Lambda_8^2 S_S\rangle + 0.30|\Lambda_8^2 S_s'\rangle - 0.20|\Lambda_8^2 S_M\rangle\,, \tag{3}$$

$$|\Sigma(1193)\rangle = 0.95|\Sigma_8^2 S_S\rangle + 0.18|\Sigma_8^2 S_S'\rangle - 0.16|\Sigma_8^2 S_M\rangle\,, \tag{4}$$

$$|N(939)\rangle = 0.90|N_8^2 S_S\rangle + 0.34|N_8^2 S_S'\rangle - 0.27|N_8^2 S_M\rangle. \tag{5}$$

Note that the signs of the second and third coefficients in the above equations are different from those in Ref. [15], due to our definitions for the spin states $|\frac{1}{2}, \pm\frac{1}{2}\rangle_{(\rho,\lambda)}$, the orbital state Φ_{200}^s and the configuration $|\Lambda_1^2 P_A\rangle$. We give the explicit wave functions for the components in $\Lambda(1405)$ in the following two subsections.

A. Wave functions for the three-quark components

Here we take the flavor-spin-orbital wave functions for the three-quark components in the considered configurations of $\Lambda(1405)$ ($\equiv \Lambda^*$) to be of the following forms:

$$|\Lambda(1405)\ {}_1^2P_A, \frac{1}{2}^-\rangle = \frac{1}{\sqrt{6}}|\Lambda\rangle_A X_A \Phi_{\Lambda^*}(\vec{q}_\lambda, \vec{q}_\rho)\,, \tag{6}$$

$$|\Lambda(1405)\ {}_8^2P_M, \frac{1}{2}^-\rangle = -\frac{1}{2\sqrt{3}}(|\Lambda\rangle_\lambda X_\lambda + |\Lambda\rangle_\rho X_\rho)\Phi_{\Lambda^*}(\vec{q}_\lambda, \vec{q}_\rho)\,, \tag{7}$$

$$|\Lambda(1405)\ {}_8^4P_M, \frac{1}{2}^-\rangle = \frac{1}{2\sqrt{3}}(|\Lambda\rangle_\lambda X_\lambda' + |\Lambda\rangle_\rho X_\rho')\Phi_{\Lambda^*}(\vec{q}_\lambda, \vec{q}_\rho)\,, \tag{8}$$

where $|\Lambda\rangle_A$ and $|\Lambda\rangle_{\rho(\lambda)}$ are the totally anti-symmetric (the flavor singlet) and mixed symmetric (the flavor octet) flavor wave functions; X_A, $X_{\rho(\lambda)}$, and $X'_{\rho(\lambda)}$ denote the completely anti-symmetric and mixed symmetric spin-orbital coupled wave functions, respectively; $\Phi_{\Lambda^*}(\vec{q}_\lambda, \vec{q}_\rho)$ the symmetric orbital wave function, and the Jacobi momenta are related to those of the quarks by

$$\vec{q}_\rho = \frac{1}{\sqrt{2}}(\vec{q}_1 - \vec{q}_2)\ ,\ \vec{q}_\lambda = \frac{1}{\sqrt{6}}(\vec{q}_1 + \vec{q}_2 - 2\vec{q}_3). \tag{9}$$

For the considered configurations of the octet baryons, we employ the following flavor-spin-orbital wave functions:

$$|B_8^2S_S,\frac{1}{2}^+\rangle_{s_z} = \frac{1}{\sqrt{2}}(|B\rangle_\lambda|\frac{1}{2},s_z\rangle_\lambda+|B\rangle_\rho|\frac{1}{2},s_z\rangle_\rho)\Phi_{000}(\vec{q}_\lambda,\vec{q}_\rho)\,, \tag{10}$$

$$|B_8^2S_S',\frac{1}{2}^+\rangle_{s_z} = \frac{1}{\sqrt{2}}(|B\rangle_\lambda|\frac{1}{2},s_z\rangle_\lambda+|B\rangle_\rho|\frac{1}{2},s_z\rangle_\rho)\Phi^S_{200}(\vec{q}_\lambda,\vec{q}_\rho)\,, \tag{11}$$

$$|B_8^2S_M,\frac{1}{2}^+\rangle_{s_z} = \frac{1}{2}[(|B\rangle_\lambda|\frac{1}{2},s_z\rangle_\rho+|B\rangle_\rho|\frac{1}{2},s_z\rangle_\lambda)\Phi^\rho_{200}(\vec{q}_\lambda,\vec{q}_\rho)-(|B\rangle_\lambda|\frac{1}{2},s_z\rangle_\lambda+|B\rangle_\rho|\frac{1}{2},s_z\rangle_\rho)\Phi^\lambda_{200}(\vec{q}_\lambda,\vec{q}_\rho)]\,. \tag{12}$$

Here $|B\rangle_{\rho(\lambda)}$ denotes the mixed symmetric flavor wave function for the corresponding baryon, and $|\frac{1}{2},s_z\rangle_{\rho(\lambda)}$ the mixed symmetric spin wave function. $\Phi_{000}(\vec{q}_\lambda,\vec{q}_\rho)$, $\Phi^S_{200}(\vec{q}_\lambda,\vec{q}_\rho)$, $\Phi^\rho_{200}(\vec{q}_\lambda,\vec{q}_\rho)$ and $\Phi^\lambda_{200}(\vec{q}_\lambda,\vec{q}_\rho)$ are the harmonic orbital wave functions with the subscripts being the corresponding nlm quantum numbers. The explicit forms for all of the flavor, spin, and orbital wave functions are given in Appendix A.

B. Wave functions for the five-quark components

Flavor-spin-orbital configurations of the four-quark subsystems in the five-quark components, with lowest energy for the $J^p = \frac{1}{2}^-$ resonances, are [45, 46] $[31]_{FSX}[4]_X[31]_{FS}[211]_F[22]_S$, with the hyperfine interaction between the quarks (anti-quark) assumed to depend either on flavor and spin [48] or on color and spin [49]. Accordingly, the octet baryon is $[31]_{FSX}[31]_X[4]_{FS}[22]_F[22]_S$.

Wave functions for the five-quark components in the $\Lambda(1405)$ resonance, and for the octet baryons can be written, respectively, in the following general forms:

$$|\Lambda(1405),s_z\rangle_{5q} = \sum_{abc} C^{[1^4]}_{[31]_a[211]_a} C^{[31]_a}_{[211]_b[22]_c}[4]_X[211]_F(b)[22]_S(c)[211]_C(a)\bar{\chi}_{s_z}\Psi(\vec{\kappa}_i)\,, \tag{13}$$

$$|B_{octet},s_z\rangle_{5q} = \sum_{a,b,c}\sum_{m,s} C^{\frac{1}{2}s_z}_{1m,\frac{1}{2}s} C^{[1^4]}_{[31]_a[211]_a} C^{[4]}_{[22]_b[22]_c}[211]_C(a)[31]_{X,m}(a)[22]_F(b)[22]_S(c)\bar{\chi}_s \times\psi(\vec{\kappa}_i)\,. \tag{14}$$

Here the color, space, and flavor-spin wave functions of 4-qurak subsystem are denoted in their Young patterns. The sum over a runs over the 3 configurations of the $[211]_C$ and $[31]_{XFS}$, those over b and c run over all the configurations of the [22] and [211] representations of S_4, respectively. $C^{[1^4]}_{[31]_a[211]_a}$ and $C^{[31]_a}_{[211]_b[22]_c}$ are the Clebsch-Gordan coefficients of the S_4 permutation group, the values of which are $C^{[1^4]}_{[31]_1[211]_1} = -\ C^{[1^4]}_{[31]_2[211]_2} = C^{[1^4]}_{[31]_3[211]_3} = \frac{1}{\sqrt{3}}$, $C^{[4]}_{[22]_b[22]_c} = \frac{1}{\sqrt{2}}\delta_{bc}$ and the coefficients $C^{[31]_a}_{[211]_b[22]_c}$ are shown in the decompositions of the $|[31]_{FS}\rangle$ configurations in Appendix B 1. The orbital, flavor, spin, and color wave functions are denoted by the Weyl tableau, and we give the explicit forms for those wave functions in Appendix B. $\Psi(\vec{\kappa}_i)$ and $\psi(\vec{\kappa}_i)$ in Eqs. (13) and (14) are the orbital symmetric wave functions for the five-quark components in $\Lambda(1405)$ and the octet baryons, respectively, with the Jacobi momenta

$$\vec{\kappa}_1 = \sqrt{\frac{1}{2}}(\vec{q}_1 - \vec{q}_2), \vec{\kappa}_2 = \sqrt{\frac{1}{6}}(\vec{q}_1 + \vec{q}_2 - 2\vec{q}_3)\,, \tag{15}$$

$$\vec{\kappa}_3 = \sqrt{\frac{1}{12}}(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 - 3\vec{q}_4), \vec{\kappa}_4 = \sqrt{\frac{1}{20}}(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4 - 4\vec{q}_5)\,. \tag{16}$$

The orbital configuration $[4]_X$ for $\Lambda(1405)$ is completely symmetric, which means all of the quarks and anti-quark should be in their orbital ground states, and the explicit form of the mixed symmetric orbital configuration $[31]_X$ for the octet baryons is

$$|[31]\rangle_{X1} = \sqrt{\frac{1}{12}}\{3|0001\rangle - |0010\rangle - |0100\rangle - |1000\rangle\}\,, \tag{17}$$

$$|[31]\rangle_{X2} = \sqrt{\frac{1}{6}}\{2|0010\rangle - |0100\rangle - |1000\rangle\}\,, \tag{18}$$

$$|[31]\rangle_{X3} = \sqrt{\frac{1}{2}}\{|0100\rangle - |1000\rangle\}\,, \tag{19}$$

where 0 and 1 correspond to the quark in its ground or first orbitally excited state, respectively. The explicit orbital wave function is the combination of the orbital configuration, Eqs. (17)-(19), and the symmetric wave function $\psi(\vec{\kappa}_i)$. Explicit color-orbital coupled wave function are reported in Appendix B.

In Table I we give for decomposition of baryon states the relevant flavor-spin configurations, as well as the Coefficients A_i, Eq. 1. The corresponding five-quark components in $\Lambda(1405)^2_8P_M$ and $B^2_8S_S$ are taken from Ref. [50], and those for the other configurations are obtained by employing the weight diagram method [51]. In this latter case, one can also apply the $SU(3)$ uppering and lowering operators in the flavor space.

TABLE I: Five-quark components in $\Lambda(1405)$, Λ, Σ^0, the proton and the corresponding coefficients.

Baryon	Flavor-spin configuration	A_u	A_d	A_s
$\Lambda(1405)\ {}^2_1P_A$	$[211]_F$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{1}{3}}$
$\Lambda(1405)\ {}^2_8P_M$	$[211]_F$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{2}{3}}$
$\Lambda(1405)\ {}^4_8P_M$	$[211]_F$	$-\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{2}{3}}$
$\Lambda(1116)\ {}^2_8S_S$	$[22]_F$	$-\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	0
$\Lambda(1116)\ {}^2_8S'_S$	$[22]_F$	$-\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	0
$\Lambda(1116)\ {}^2_8S_M$	$[22]_F$	$-\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	0
$\Sigma(1194)\ {}^2_8S_S$	$[22]_F$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{2}{3}}$
$\Sigma(1194)\ {}^2_8S'_S$	$[22]_F$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{2}{3}}$
$\Sigma(1194)\ {}^2_8S_M$	$[22]_F$	$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{2}{3}}$
$N(939)\ {}^2_8S_S$	$[22]_F$	0	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$
$N(939)\ {}^2_8S'_S$	$[22]_F$	0	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$
$N(939)\ {}^2_8S_M$	$[22]_F$	0	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$

III. FORMALISM FOR THE RADIATIVE AND STRONG DECAYS

Taking into account the five-quark components, the decays of a baryon embodies three types of possible transitions: *i*) between the three-quark, *ii*) between the five-quark, *iii*) between three- and five-quark. The first two processes are the so-called diagonal, and the last one nondiagonal transitions.

In the next two subsections, we describe briefly the formalism for radiative and strong decays of the baryons in a non-relativistic quark model.

A. Formalism for radiative decay

It is established that the radiative decay of baryons can be described by the helicity amplitudes for the electromagnetic transitions. For $\gamma^* Y \to \Lambda(1405)$, with $Y \equiv \Lambda(1116),\ \Sigma(1193)$, they are defined as follows:

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{K}} \frac{1}{e} \langle \Lambda(1405), S^*_z | = \frac{1}{2} | \epsilon^+_\mu J^\mu | B, S_z = -\frac{1}{2} \rangle. \tag{20}$$

Here ϵ_μ^+ is the polarization vector for the right-handed photon, J^μ denotes the electromagnetic current, and K the real photon three-momentum magnitude in the centre-of-mass frame of the $\Lambda(1405)$ resonance. For the $\Lambda(1405) \to \Lambda(1116)\gamma$, $\Sigma(1194)\gamma$ radiative decays, the values for K are about 259 MeV/c and 195 MeV/c, respectively.

The diagonal electromagnetic transition operator in the non-relativistic constituent quark model takes [20, 52] the following form:

$$\hat{T}_d = \sum_i^{nq} \sqrt{2}\hat{\mu}_i \phi_z^{i'\dagger} \begin{pmatrix} \sqrt{2}q_{i+} & k \\ 0 & \sqrt{2}q_{i+} \end{pmatrix} \phi_z^i . \tag{21}$$

Here the sum over i runs over the quark contents of the corresponding components, i.e. $nq = 3$ for the three-quark and $nq = 5$ for the five-quark components. $\hat{\mu}_i = \frac{e_i}{2m_i}$ denotes the magnetic moment operator of the i^{th} quark, $\phi_z^{i'}$ and ϕ_z^i are the i^{th} quark spin operators for the initial and final states, respectively, and $q_{i+} = \frac{1}{\sqrt{2}}(q_{ix} + iq_{iy})$ with $\vec{q}_i$ being the momentum of the i^{th} quark. Finally, k is the z-component of the photon momentum. Note that we have taken the photon momentum to be $\vec{k} = (0, 0, k)$, and it is related to the square of the four-momentum transfer Q^2

$$k^2 = Q^2 + \frac{(M^2_{\Lambda(1405)} - m^2_Y - Q^2)^2}{4M^2}, \tag{22}$$

where $Y \equiv \Lambda(1160),\ \Sigma(1193)$.

For the nondiagonal transitions, taking the $q\bar{q} - \gamma$ vertices to have the elementary forms $\bar{u}(q_i)\gamma^\mu v(\bar{q})$ ($3q \to 5q$) and $\bar{v}(\bar{q})\gamma^\mu u(q_i)$ ($5q \to 3q$), then the transition operators in the non-relativistic constituent quark model can be derived

$$\hat{T}_{35} = \sum_i^4 \sqrt{2} e_i \phi_z^{i\dagger} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \phi_z^{\bar{q}} , \tag{23}$$

$$\hat{T}_{53} = \sum_i^4 \sqrt{2} e_i \phi_z^{\bar{q}\dagger} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \phi_z^{i} . \tag{24}$$

Here $\hat{T}_{35}$ and $\hat{T}_{53}$ are the operators for the $\gamma^* qqq \to qqqq\bar{q}$ and $\gamma^* qqqq\bar{q} \to qqq$ transitions, respectively.

Thus, the helicity amplitude $A_{1/2}$ for the electromagnetic transition $\gamma^* Y \to \Lambda(1405)$ can be written in the following form:

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{K}} \frac{1}{e} \langle \Lambda(1405), \frac{1}{2} | (\hat{T}_d + \hat{T}_a) | Y, -\frac{1}{2} \rangle, \tag{25}$$

TABLE II: Helicity amplitude $A^{\Lambda}_{1/2}$ for electromagnetic transition $\gamma^*\Lambda \to \Lambda(1405)$. Note that the full amplitudes in columns 2 to 4 are obtained by multiplying each term by the following expressions: $\sqrt{\frac{2\pi\alpha}{K}}A^{\Lambda}_{3q}A^{\Lambda^*}_{3q}\exp\{-\frac{k^2}{6\omega_3^2}\}$ for $3q \to 3q$, $\sqrt{\frac{2\pi\alpha}{K}}A^{\Lambda}_{5q}A^{\Lambda^*}_{5q}\frac{1}{48}(\frac{1}{m}+\frac{2}{m_s})\omega_5\exp\{-\frac{k^2}{5\omega_3^2}\}$ for $5q \to 5q$, and $\sqrt{\frac{2\pi\alpha}{K}}A^{\Lambda}_{3q}A^{\Lambda^*}_{5q}C_{35}\exp\{-\frac{3k^2}{20\omega_3^2}\}$ for $N-D$.

	$3q \to 3q$	$5q \to 5q$	$N-D$
$\Lambda^2_8 S_S \to \Lambda^2_1 P_A$	$\frac{1}{18}(\frac{1}{m}+\frac{2}{m_s})\omega_3(1+\frac{k^2}{2\omega_3^2})$	$1/\sqrt{3}$	$\frac{1}{6}$
$\Lambda^2_8 S_S \to \Lambda^2_8 P_M$	$\frac{1}{36}[(\frac{1}{m}-\frac{2}{m_s})\frac{k^2}{\omega_3}-(\frac{1}{m}+\frac{2}{m_s})2\omega_3]$	$-1/\sqrt{6}$	$\frac{\sqrt{2}}{12}$
$\Lambda^2_8 S_S \to \Lambda^4_8 P_M$	$\frac{1}{36m}\frac{k^2}{\omega_3}$	$-1/\sqrt{6}$	$\frac{\sqrt{2}}{12}$
$\Lambda^2_8 S'_S \to \Lambda^2_1 P_A$	$-\frac{1}{18\sqrt{3}}(\frac{1}{m}+\frac{2}{m_s})\omega_3[(1+\frac{k^2}{6\omega_3^2})-(1-\frac{k^2}{6\omega_3^2})\frac{k^2}{2\omega_3^2}]$	$1/\sqrt{3}$	0
$\Lambda^2_8 S'_S \to \Lambda^2_8 P_M$	$\frac{1}{54\sqrt{3}}[(\frac{1}{2m}-\frac{1}{m_s})\frac{k^2}{\omega_3}(1-\frac{k^2}{6\omega_3^2})+(\frac{1}{m}-\frac{2}{m_s})2\omega_3(1+\frac{k^2}{6\omega_3^2})]$	$-1/\sqrt{6}$	0
$\Lambda^2_8 S'_S \to \Lambda^4_8 P_M$	$\frac{1}{36\sqrt{3}m}(1-\frac{k^2}{6\omega_3^2})\frac{k^2}{\omega_3}$	$-1/\sqrt{6}$	0
$\Lambda^2_8 S_M \to \Lambda^2_1 P_A$	$-\frac{\sqrt{6}}{54}(\frac{1}{m}+\frac{2}{m_s})\omega_3(1-\frac{k^2}{12\omega_3^2}+\frac{k^4}{24\omega_3^4})$	$1/\sqrt{3}$	0
$\Lambda^2_8 S_M \to \Lambda^2_8 P_M$	$-\frac{\sqrt{6}}{108}\omega_3[(\frac{1}{m}+\frac{1}{m_s})\frac{k^2}{\omega_3^2}-\frac{k^4}{6m_s\omega_3^4}]$	$-1/\sqrt{6}$	0
$\Lambda^2_8 S_M \to \Lambda^4_8 P_M$	$-\frac{\sqrt{6}}{162}\omega_3[(\frac{1}{m}-\frac{1}{m_s})\frac{k^2}{\omega_3^2}-\frac{k^4}{8m\omega_3^4}]$	$-1/\sqrt{6}$	0

where we have defined $\hat{T}_a = \hat{T}_{35} + \hat{T}_{53}$, which correspond to nondiagonal transitions.

Taking into account the configurations mixing effects and the contributions of the five-quark components, we need to calculate 36 transition amplitudes for each decay. For the diagonal transitions ($3q \to 3q$ and $5q \to 5q$) the calculations are similar to that in Refs. [15, 49, 53]. Explicit calculations of the nondiagonal ($N-D$) electromagnetic transitions elements in our approach are similar to the one in Ref. [42] for the $\gamma^*N \to N(1535)$ process. Amplitudes for $\gamma^*\Lambda(1116) \to \Lambda(1405)$ and $\gamma^*\Sigma^\circ(1194) \to \Lambda(1405)$ are given in Tables II and III, respectively.

Notice that, in Tables II and III we have defined

$$C_{35} = \langle\varphi_{00}(\vec{\kappa}_1)\varphi_{00}(\vec{\kappa}_2)|\varphi_{00}(\vec{\kappa}_1)\varphi_{00}(\vec{\kappa}_2)\rangle \;= (\frac{2\omega_3\omega_5}{\omega_3^2+\omega_5^2})^3\,, \qquad (26)$$

which is the orbital overlap integral factor in the matrix elements of the nondiagonal transitions. Here, ω_3 and ω_5 are the oscillator frequencies for the qqq and $qqqq\bar{q}$ systems, respectively.

Finally, the radiative decay width of $\Lambda(1405)$ in terms of the helicity amplitudes $A_{1/2}$ at

TABLE III: Helicity amplitude $A^{\Sigma}_{1/2}$ for electromagnetic transition $\gamma^*\Sigma^0 \to \Lambda(1405)$. Note that the full amplitudes in columns 2 to 4 are obtained by multiplying each term by the following expressions: $\sqrt{\frac{2\pi\alpha}{K}} A^{\Sigma}_{3q} A^{\Lambda^*}_{3q} \exp\{-\frac{k^2}{6\omega_3^2}\}$ for $3q \to 3q$, $\sqrt{\frac{2\pi\alpha}{K}} A^{\Sigma}_{5q} A^{\Lambda^*}_{5q} \frac{\omega_5}{m} \exp\{-\frac{k^2}{5\omega_3^2}\}$ for $5q \to 5q$, and $\sqrt{\frac{2\pi\alpha}{K}} A^{\Sigma}_{3q} A^{\Lambda^*}_{5q} C_{35} \exp\{-\frac{3k^2}{20\omega_3^2}\}$ for $N-D$.

	$3q \to 3q$	$5q \to 5q$	$N-D$
$\Sigma^2_8 S_S \to \Lambda^2_1 P_A$	$-\frac{1}{2\sqrt{3}}\frac{\omega_3}{m}(1+\frac{k^2}{2\omega_3^2})$	$-\frac{1}{16}$	$-\frac{1}{2\sqrt{3}}$
$\Sigma^2_8 S_S \to \Lambda^2_8 P_M$	$-\frac{1}{4\sqrt{3}}\frac{\omega_3}{m}(2+\frac{k^2}{3\omega_3^2})$	$-\frac{3}{16\sqrt{2}}$	$\frac{1}{2\sqrt{6}}$
$\Sigma^2_8 S_S \to \Lambda^4_8 P_M$	$\frac{\sqrt{3}}{36m}\frac{k^2}{\omega_3}$	$-\frac{3}{16\sqrt{2}}$	$\frac{1}{2\sqrt{6}}$
$\Sigma^2_8 S'_S \to \Lambda^2_1 P_A$	$\frac{\omega_3}{6m}[(1+\frac{k^2}{6\omega_3^2})-(1-\frac{k^2}{6\omega_3^2})\frac{k^2}{2\omega_3^2}]$	$-\frac{1}{16}$	0
$\Sigma^2_8 S'_S \to \Lambda^2_8 P_M$	$-\frac{1}{4m}[\frac{k^2}{9\omega_3}(1-\frac{k^2}{6\omega_3^2})-\frac{2\omega_3}{3}(1+\frac{k^2}{6\omega_3^2})]$	$-\frac{3}{16\sqrt{2}}$	0
$\Sigma^2_8 S'_S \to \Lambda^4_8 P_M$	$\frac{1}{36m}(1-\frac{k^2}{6\omega_3^2})\frac{k^2}{\omega_3}$	$-\frac{3}{16\sqrt{2}}$	0
$\Sigma^2_8 S_M \to \Lambda^2_1 P_A$	$\frac{\sqrt{2}}{6}\frac{\omega_3}{m}(1-\frac{k^2}{12\omega_3^2}+\frac{k^4}{24\omega_3^4})$	$-\frac{1}{16}$	0
$\Sigma^2_8 S_M \to \Lambda^2_8 P_M$	$-\frac{\sqrt{2}}{72}\frac{\omega_3}{m}[\frac{k^2}{\omega_3^2}-\frac{k^4}{6\omega_3^4}]$	$-\frac{3}{16\sqrt{2}}$	0
$\Sigma^2_8 S_M \to \Lambda^4_8 P_M$	$-\frac{\sqrt{2}\omega_3}{72m}\frac{k^4}{\omega_3^4}$	$-\frac{3}{16\sqrt{2}}$	0

the real photon point is [53]

$$\Gamma_{Y\gamma} = \frac{k^2 m_Y}{\pi M}|A_{1/2}(Q^2=0)|^2. \tag{27}$$

B. Formalism for strong decay

In the chiral constituent quark model, the coupling of the light quarks (u, d, s) to the octet of light pseudoscalar mesons takes the form

$$\mathcal{L}_{Mqq} = i\frac{g_A^q}{2f_M}\bar{\psi}_q\gamma_5\gamma_\mu\partial^\mu m_a\lambda_a\psi_q\,. \tag{28}$$

Here, g_A^q denotes the axial coupling constant for the constituent quarks, f_M is the decay constant of meson M (π, K). ψ_q is the quark field and m_a the meson field. Finally, λ_as are the $SU(3)$ Gell-Mann matrices. Combination of Eq. (28) with the representation

$$m_a\lambda_a = \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^\circ + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^\circ + \frac{1}{\sqrt{6}}\eta & K^\circ \\ K^- & \bar{K}^\circ & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \tag{29}$$

leads to the following quark-meson-quark chiral coupling in the momentum space

$$\mathcal{L}_{Mqq} = \frac{g_A^q}{2f_M}\bar{\psi}_q\gamma_5\gamma_\mu k^\mu X_M^q \psi_q \,, \tag{30}$$

where X_M^q is the flavor operator for emission of meson M from the corresponding quark q

$$X_{\pi^\circ}^q = \lambda_3 \,, \tag{31}$$

$$X_{K^\pm}^q = \frac{1}{\sqrt{2}}(\lambda_4 + \lambda_5) \,, \tag{32}$$

$$X_{K^\circ}^q = -\frac{1}{\sqrt{2}}(\lambda_6 - i\lambda_7) \,, \tag{33}$$

$$X_\eta^q = cos\theta\lambda_8 - sin\theta\mathcal{I}, X_{\eta'}^q = cos\theta\lambda_8 + sin\theta\mathcal{I}, \tag{34}$$

with $\mathcal{I}$ the unit operator in the $SU(3)$ flavor space.

Within the non-relativistic approximation, we can get the baryon-meson-baryon coupling in the chiral constituent quark model

$$\hat{T}_d^M = \sum_i^{nq} \frac{g_A^q}{2f_M}\phi_z^{i'\dagger}\begin{pmatrix} (1+\frac{k_0}{2m_f})k_M - \frac{k_0}{2\mu}q_{iz} & -\sqrt{2}\frac{k_0}{2\mu}q_{i-} \\ -\sqrt{2}\frac{k_0}{2\mu}q_{i+} & -(1+\frac{k_0}{2m_f})k_M + \frac{k_0}{2\mu}q_{iz} \end{pmatrix}\phi_z^i X_M^i \,, \tag{35}$$

$$\hat{T}_{53}^M = -\sum_i^4 \frac{g_A^q}{2f_M}(m_i + m_f)\phi_z^{\bar{q}\dagger}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\phi_z^i X_M^i \,, \tag{36}$$

$$\hat{T}_{35}^M = -\sum_i^4 \frac{g_A^q}{2f_M}(m_i + m_f)\phi_z^{i\dagger}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\phi_z^{\bar{q}} X_M^i. \tag{37}$$

Here m_i and m_f are the initial and final constituent masses of the quark which emits a meson, and $\mu = m_i m_f/(m_i + m_f)$. k_0 and k_M denote the energy and magnitude of the three-momentum of the final meson in the centre-of-mass frame of the initial baryon. Note that we have taken the meson three-momentum to be in the z-direction, $\vec{k_M} = (0, 0, k_M)$, and it is related to the masses of the initial and final hadrons

$$k_M = \{[M_i^2 - (M_f + m_M)^2][M_i^2 - (M_f - m_M)^2]\}^{1/2}/2M_i \,. \tag{38}$$

The transition amplitudes are obtained by the calculations of the following matrix elements

$$T^M = \langle\Lambda(1405), \frac{1}{2}|(\hat{T}_d^M + \hat{T}_a^M)|Y, \frac{1}{2}\rangle, \tag{39}$$

where we have defined $\hat{T}_a^M = \hat{T}_{35}^M + \hat{T}_{53}^M$.

Tables IV and V give the transition amplitudes for strong decay channel. We note that none of the diagonal transitions of the five-quark components contributes to the transition

TABLE IV: Transition amplitudes of the $\Lambda(1405) \to \Sigma(1194)\pi$ decay. Note that the full amplitudes in columns 2 and 3 are obtained by multiplying each term by the following expressions: $\frac{g_A^q}{2f_\pi} A_{3q}^{\Sigma^\circ} A_{3q}^{\Lambda^*} \omega_3 \exp\{-\frac{k^2}{6\omega_3^2}\}$ for column $3q \to 3q$, and $\frac{g_A^q}{f_\pi} A_{3q}^{\Sigma^\circ} A_{5q}^{\Lambda^*} m C_{35} \exp\{-\frac{3k^2}{20\omega_5^2}\}$ for column $N-D$. Here k denotes the π three-momentum magnitude k_π, and k_0 the energy of the π meson.

	$3q \to 3q$	N-D
$\Lambda_1^2 P_A \to \Sigma_8^2 S_S$	$-\frac{1}{\sqrt{6}}[(1+\frac{k_0}{6m})\frac{k^2}{\omega_3^2} - 3\frac{k_0}{m}]$	$\frac{1}{\sqrt{6}}$
$\Lambda_8^2 P_M \to \Sigma_8^2 S_S$	$-\frac{1}{3\sqrt{6}}[(1+\frac{k_0}{6m})\frac{k^2}{\omega_3^2} - 3\frac{k_0}{m}]$	$-\frac{1}{2\sqrt{3}}$
$\Lambda_8^4 P_M \to \Sigma_8^2 S_S$	$-\frac{2}{3\sqrt{6}}[(1+\frac{k_0}{6m})\frac{k^2}{\omega_3^2} - 3\frac{k_0}{m}]$	$-\frac{1}{2\sqrt{3}}$
$\Lambda_1^2 P_A \to \Sigma_8^2 S'_S$	$-\frac{1}{3\sqrt{2}}[3\frac{k_0}{m} + (1+\frac{k_0}{m})\frac{k^2}{\omega_3^2} - (1+\frac{k_0}{6m})\frac{k^4}{6\omega_3^4}]$	0
$\Lambda_8^2 P_M \to \Sigma_8^2 S'_S$	$-\frac{1}{9\sqrt{2}}[3\frac{k_0}{m} + (1+\frac{k_0}{m})\frac{k^2}{\omega_3^2} - (1+\frac{k_0}{6m})\frac{k^4}{6\omega_3^4}]$	0
$\Lambda_8^4 P_M \to \Sigma_8^2 S'_S$	$-\frac{\sqrt{2}}{9}[3\frac{k_0}{m} + (1+\frac{k_0}{m})\frac{k^2}{\omega_3^2} - (1+\frac{k_0}{6m})\frac{k^4}{6\omega_3^4}]$	0
$\Lambda_1^2 P_A \to \Sigma_8^2 S_M$	$-\frac{1}{9}[3\frac{k_0}{m} + (1+\frac{5k_0}{12m})\frac{k^2}{\omega_3^2} - (1+\frac{k_0}{6m})\frac{k^4}{4\omega_3^4}]$	0
$\Lambda_8^2 P_M \to \Sigma_8^2 S_M$	$-\frac{1}{18}[3\frac{k_0}{m} + (1+\frac{k_0}{2m})\frac{k^2}{\omega_3^2} - (1+\frac{k_0}{6m})\frac{k^4}{3\omega_3^4}]$	0
$\Lambda_8^4 P_M \to \Sigma_8^2 S_M$	$\frac{1}{9}[(5+\frac{11k_0}{6m})\frac{k^2}{6\omega_3^2} - (1+\frac{k_0}{6m})\frac{k^4}{6\omega_3^4}]$	0

amplitudes. Those null values can easily be understood, noticing that the spin configurations for $\Lambda(1405)$ and for the octet baryons are taken to be $[22]_S$, for which the total spin is $S=0$, and there are no spin-independent terms in the diagonal transition operator (which is not the case in the electromagnetic transition operator). However, the configuration mixing effects might be significant.

Finally, following Eq. (39), the strong decay width for $\Lambda(1405) \to (\Sigma(1194)\pi)^\circ$ reads

$$\Gamma_{\Lambda(1405)\to(\Sigma\pi)^\circ} = \frac{3}{4\pi}\frac{E' + m_\Sigma}{M}|\vec{k}_\pi||T^\pi|^2, \tag{40}$$

where E' is the energy of the final Σ hyperon

$$E' = \frac{M^2 - m_\pi^2 + m_\Sigma^2}{2M}. \tag{41}$$

In addition, taking the hadronic level Lagrangian for the $\Lambda(1405)BM$ coupling, with $B \equiv \Sigma,\ N$ and $M \equiv \pi,\ K$, to be of the following form:

$$\mathcal{L}_{\Lambda(1405)BM} = i\frac{f_{\Lambda(1405)BM}}{m_M}\bar{\psi}_B\gamma_\mu\partial^\mu\phi_M X_M \psi_{\Lambda(1405)} + h.c., \tag{42}$$

the transition coupling amplitude reads $f_{\Lambda(1405)BM}(M_{\Lambda(1405)} - m_B)/m_M$. Comparing the

TABLE V: Transition amplitudes of the $\Lambda(1405) \to K^-p$ decay. Note that the full amplitudes in columns 2 and 3 are obtained by multiplying each term by the following expressions: $\frac{g_A^q}{2f_K} A_{3q}^N A_{3q}^{\Lambda^*} \omega_3 \exp\{-\frac{k^2}{6\omega_3^2}\}$ for column $3q \to 3q$, and the factors $\frac{g_A^q}{f_K} A_{3q}^N A_{5q}^{\Lambda^*} (m+m_s) C_{35} \exp\{-\frac{3k^2}{20\omega_5^2}\}$ for column $N-D$. Here k denotes the three-momentum magnitude k_K, and k_0 the energy of the K meson.

	$3q \to 3q$	$N-D$
$\Lambda_1^2 P_A \to N_8^2 S_S$	$-\frac{1}{\sqrt{6}}[(1+\frac{k_0}{2m}-\frac{k_0}{6\mu})\frac{k^2}{\omega_3^2}-\frac{3k_0}{2\mu}]$	$-\frac{1}{2\sqrt{6}}$
$\Lambda_8^2 P_M \to N_8^2 S_S$	$\frac{1}{\sqrt{6}}[(1+\frac{k_0}{2m}-\frac{k_0}{6\mu})\frac{k^2}{\omega_3^2}-3\frac{k_0}{2\mu}]$	$-\frac{1}{4\sqrt{3}}$
$\Lambda_8^4 P_M \to N_8^2 S_S$	0	$-\frac{1}{4\sqrt{3}}$
$\Lambda_1^2 P_A \to N_8^2 S'_S$	$-\frac{1}{3\sqrt{2}}[\frac{3k_0}{2\mu}+(1+\frac{k_0}{2m}+\frac{k_0}{4\mu})\frac{k^2}{\omega_3^2}-(1+\frac{k_0}{2m}-\frac{k_0}{6\mu})\frac{k^4}{6\omega_3^4}]$	0
$\Lambda_8^2 P_M \to N_8^2 S'_S$	$\frac{1}{3\sqrt{2}}[\frac{3k_0}{2\mu}+(1+\frac{k_0}{2m}+\frac{k_0}{4\mu})\frac{k^2}{\omega_3^2}-(1+\frac{k_0}{2m}-\frac{k_0}{6\mu})\frac{k^4}{6\omega_3^4}]$	0
$\Lambda_8^4 P_M \to N_8^2 S'_S$	0	0
$\Lambda_1^2 P_A \to N_8^2 S_M$	$-\frac{1}{9}[\frac{3k_0}{2\mu}+(1+\frac{k_0}{2m}-\frac{k_0}{24\mu})\frac{k^2}{\omega_3^2}-(1+\frac{k_0}{2m}-\frac{k_0}{6\mu})\frac{k^4}{4\omega_3^4}]$	0
$\Lambda_8^2 P_M \to N_8^2 S_M$	$\frac{1}{9}[\frac{3k_0}{\mu}+(2+\frac{k_0}{m}-\frac{5k_0}{24\mu})\frac{k^2}{\omega_3^2}-(1+\frac{k_0}{2m}-\frac{k_0}{6\mu})\frac{k^4}{4\omega_3^4}]$	0
$\Lambda_8^4 P_M \to N_8^2 S_M$	$\frac{2}{9}[\frac{3k_0}{2\mu}+(1+\frac{k_0}{2m}-\frac{k_0}{6\mu})\frac{k^2}{\omega_3^2}]$	0

latter expression to the results obtained in the chiral quark model, one gets

$$\frac{f_{\Lambda(1405)BM}}{m_M} = \frac{\langle[\hat{T}_d^M + \hat{T}_{35}^M + \hat{T}_{53}^M]\rangle}{M_{\Lambda(1405)} - m_B}. \tag{43}$$

IV. NUMERICAL RESULTS AND DISCUSSION

Using the formalism developed in the previous section, here we present our numerical results for both electromagnetic and strong decays. Those results have been obtained with no adjustable parameters. In Table VI we give the input parameters used in our calculations and comment about the adopted values.

TABLE VI: The input values used in this manuscript for non vanishing five-quark probability ($P_{5q} \neq 0$). Here, $m \equiv m_u = m_d$. For $P_{5q} = 0$ we used m =340 MeV. Values in columns 1 to 6 are in MeV.

m	m_s	ω_3	ω_5	f_π	f_K	g_A^q	A_{3q}^B	A_{5q}^B	$A_{3q}^{\Lambda^*}$	$A_{5q}^{\Lambda^*}$
290	430	340	600	93	113	0.82	$\sqrt{0.80}$	$\sqrt{0.20}$	$\sqrt{0.55}$	$\sqrt{0.45}$

Since we have introduced the five-quark components in the baryons, The constituent quark masses are slightly different from those used in the traditional constituent quark models. We take $m_u = m_d = 290$ MeV and $m_s = 430$ MeV, as suggested in Ref. [43] in order to reproduce the mass of the proton with 20% five-quark components, and to investigate successfully the electromagnetic transitions $\gamma^* N \to N^*(1535)$ and the strong decays of $N(1535)$. Values for the oscillator parameters ω_3 and ω_5 come also from this latter Reference [43].

The probability of five-quark components in proton leading to A^N_{5q}=$\sqrt{0.20}$ (see e.g. Ref. [43]) is also used for the lowest mass hyprons, A^Λ_{5q} and $A^{\Sigma^\circ}_{5q}$. Then the probabilities for $3q$ components are obtained within the used truncated Fock space, implying $(A^Y_{3q})^2 + (A^Y_{5q})^2$=1. For the $\Lambda(1405)$, our numerical results reported below (see sec. IV) favor $A^{\Lambda^*}_{5q} = \sqrt{0.45}$, and hence, $A^{\Lambda^*}_{3q} = \sqrt{0.55}$.

In Table VI, g^q_A denotes the axial coupling constant for the constituent quarks, and its extracted phenomenological values are [54–56] in the range $0.70 - 1.26$. Here, we have taken $g^q_A = 0.82$, which differs slightly from its value (0.88) in Ref. [55] , due to the fact that we have introduced the five-quark components.

Finally, for the decay constants of mesons, the empirical values are used ($f_\pi = 93$ MeV and $f_K = 113$ MeV).

A. Radiative decays of $\Lambda(1405)$

Helicity amplitudes $A^\Lambda_{1/2}$ and $A^\Sigma_{1/2}$ for the electromagnetic transitions $\gamma\Lambda(1116) \to \Lambda(1405)$ and $\gamma\Sigma(1194) \to \Lambda(1405)$ at the real photon point are given in Tables VII and VIII, respectively, showing that the configurations mixing effects are very important, and the diagonal transitions between the five-quark components have also non negligible contributions to the helicity amplitudes. Moreover, as we can see in the columns nondiagonal $(N - D)$, those transitions between the three- and five-quark components in $Y^2_8 S_S$ and $\Lambda(1405)^2_1 P_A$ contribute significantly to the helicity amplitudes $A^Y_{1/2}$: about 30% to $A^\Lambda_{1/2}$ and 37% to $A^\Sigma_{1/2}$.

Table IX shows our results for the radiative decays widths of $\Lambda(1405)$, employing Eq. (27). Column A contains the results obtained without five-quark admixture, i.e. $P_{5q} = 0\%$, columns B, C, D, and E correspond to $P_{5q} = 25\%$, 45%, 75% and 100%, respectively. The $\Lambda(1405) \to \Lambda\gamma$ channel does not show a significant sensitivity to the five-quark components, unless its weight exceeds ≈50%. For the $\Lambda(1405) \to \Sigma\gamma$ decay, in going from $P_{5q} = 0\%$ to $P_{5q} = 25\%$, the decay width increases by roughly 20%, and drops down with the increasing P_{5q} much slower than the width for $\Lambda(1405) \to \Lambda\gamma$ decay.

TABLE VII: Results for the helicity amplitude $A^{\Lambda}_{1/2}$ (in GeV$^{-1/2}$) for electromagnetic transition $\gamma\Lambda \to \Lambda(1405)$.

	$3q \to 3q$	$5q \to 5q$	N-D	total
$\Lambda^2_8 S_S \to \Lambda^2_1 P_A$	0.050	0.007	0.024	0.081
$\Lambda^2_8 S_S \to \Lambda^2_8 P_M$	-0.027	-0.003	0.011	-0.019
$\Lambda^2_8 S_S \to \Lambda^4_8 P_M$	0.011	-0.002	0.009	0.018
$\Lambda^2_8 S'_S \to \Lambda^2_1 P_A$	-0.022	0.008	0	-0.014
$\Lambda^2_8 S'_S \to \Lambda^2_8 P_M$	-0.005	-0.010	0	-0.015
$\Lambda^2_8 S'_S \to \Lambda^4_8 P_M$	0.002	-0.006	0	-0.004
$\Lambda^2_8 S_M \to \Lambda^2_1 P_A$	-0.018	0.004	0	-0.014
$\Lambda^2_8 S_M \to \Lambda^2_8 P_M$	-0.003	-0.007	0	-0.010
$\Lambda^2_8 S_M \to \Lambda^4_8 P_M$	0	-0.026	0	-0.026

TABLE VIII: Results for the helicity amplitude $A^{\Sigma}_{1/2}$ (in GeV$^{-1/2}$) of electromagnetic transitions $\gamma\Sigma \to \Lambda(1405)$.

	$3q \to 3q$	$5q \to 5q$	N-D	total
$\Sigma^2_8 S_S \to \Lambda^2_1 P_A$	-0.120	-0.018	-0.080	-0.218
$\Sigma^2_8 S_S \to \Lambda^2_8 P_M$	-0.073	-0.021	0.033	-0.061
$\Sigma^2_8 S_S \to \Lambda^4_8 P_M$	0.017	-0.013	0.027	0.031
$\Sigma^2_8 S'_S \to \Lambda^2_1 P_A$	0.038	-0.014	0	0.024
$\Sigma^2_8 S'_S \to \Lambda^2_8 P_M$	0.278	-0.176	0	0.102
$\Sigma^2_8 S'_S \to \Lambda^4_8 P_M$	0.002	-0.044	0	-0.042
$\Sigma^2_8 S_M \to \Lambda^2_1 P_A$	0.046	-0.010	0	0.036
$\Sigma^2_8 S_M \to \Lambda^2_8 P_M$	-0.001	-0.054	0	-0.053
$\Sigma^2_8 S_M \to \Lambda^4_8 P_M$	0	-0.102	0	-0.102

Table X summarizes the widths for the electromagnetic decay of the $\Lambda(1405)$ reported

TABLE IX: Results for the radiative decays widths of $\Lambda(1405) \to \Lambda(1116)\gamma$ ($\Gamma_{\Lambda\gamma}$), $\Lambda(1405) \to \Sigma(1194)\gamma$ ($\Gamma_{\Sigma\gamma}$) (in keV), and their ratio $R = \Gamma_{\Sigma\gamma}/\Gamma_{\Lambda\gamma}$.

	A	B	C	D	E
P_{5q} (%)	0	25	45	75	100
$\Gamma_{\Lambda\gamma}$	91	104	98	76	24
$\Gamma_{\Sigma\gamma}$	164	236	225	175	57
R	1.8	2.7	2.3	2.3	2.4

by several authors. One of the early extractions of those quantities is due to Burkhardt and Lowe [57], motivated by the advent of reliable K^-p atom data [58] published in late 80's. Since then, those results have been introduced in PDG, and are considered by some authors as "‘data"’, though Burkhardt and Lowe state clearly in their paper the highly phenomenological character of their investigation, e.g. *" There is some degree of arbitrariness in assigning values to the individual coupling constants required to calculate radiative decays"*. In other words, at the present time there are no reference values for those widths and various calculations put forward the relative importance of mechanisms considered in each approach. Moreover, given that the $\Lambda(1405)$ is 27 MeV below the K^-p threshold, in kaonic atom only the upper tail of that resonance intervenes.

Predictions for both channels decay widths (Table X) may vary by two orders of magnitude from one approach to another, making any conclusive comparisons pointless in the absence of data. Landberger [63] suggested that the predicted ratio $R = \Gamma_{\Sigma\gamma}/\Gamma_{\Lambda\gamma}$ might be more instructive. Inspection of that ratio for different approaches (fourth column in Table X) allows distinguishing three ranges: $R \gtrsim 1.0$ (present work and Refs. [57, 59, 60]), $0.4 \lesssim R \lesssim 0.6$ (Refs. [16, 18, 20, 25, 57, 60, 61]), and $R \lesssim 0.3$ (Refs. [18, 19, 25, 62, 64]).

Our model gives R=2.27, almost 70% larger than that obtained with algebric model [59], but about two times smaller than the ratio given by a very recent coupled channels unitary chiral perturbation theory ($U\chi PT$) [60]. This latter generates 2 poles corresponding to the nominal $\Lambda(1405)$, resulting in two different radiative decay widths. The low-energy pole lead to R=4.56, with $\Gamma_{\Lambda\gamma}$=16 keV, compatible with the value extracted within the above mentioned isobar model [57]. However, that model leads to a ratio compatible, within 1-σ, with both ≈1.2 and ≈0.5, so within the two first ranges. The results for $R \gtrsim 1.0$ lead then

TABLE X: Radiative decay widths (in keV) of the $\Lambda(1405) \to \Lambda\gamma,\ \Sigma\gamma$ decays in different approaches, and the corresponding ratios $R = \Gamma_{\Sigma\gamma}/\Gamma_{\Lambda\gamma}$.

Approach	$\Gamma_{\Lambda\gamma}$	$\Gamma_{\Sigma\gamma}$	R	Reference
χQM	104	236	2.27	Present work, with P_{5q}=45%
χQM	168	103	0.61	Yu *et al.* [20]
Algebric model	117	156	1.33	Bijker *et al.* [59]
$U\chi PT$	16	73	4.56	Geng *et al.* [60]
	65	33	0.51	Geng *et al.* [60]
Bonn CQM	912	233	0.26	Van Cauteren *et al.* [19]
NRQM	143	91	0.64	Darewych *et al.* [16]
NRQM	154	72	0.47	Kaxiras *et al.* [18]
	200	72	0.36	Kaxiras *et al.* [18]
RCQM	118	46	0.39	Warns *et al.* [61]
MIT bag	60	18	0.30	Kaxiras *et al.* [18]
	17	3	0.18	Kaxiras *et al.* [18]
Chiral bag	75	2	0.03	Umino - Myhrer [62]
Soliton	40	17	0.43	Schat *et al.* [25]
	44	13	0.30	Schat *et al.* [25]
Isobar model	27 ± 8	10 ± 4	0.37 ± 0.18	Burkhardt - Lowe [57]
	27 ± 8	23 ± 7	0.85 ± 0.36	Burkhardt - Lowe [57]

to two series with respect to the width $\Gamma_{\Lambda\gamma}\approx$100 keV (present work and Ref. [59]) and $\approx$20 keV [57, 60], while $\Gamma_{\Sigma\gamma}$ varies by two orders of magnitude.

The higher-energy pole in the $U\chi PT$ [60] comes out in the second range $0.4 \lesssim R \lesssim 0.6$. It is worth noticing that various quark model based approaches [16, 18, 20, 61] predict ratios in the same interval, and three of them [16, 18, 61] give close enough predictions for both $\Gamma_{\Lambda\gamma}$ and $\Gamma_{\Sigma\gamma}$. It is however known that those approaches fail in describing the $\Lambda(1405)$. In the last part of this section will come back to the recent chiral quark approach [20].

Moreover, in the MIT bag model [18] there are two $J^P = 1/2^-$ Λ states at 1364 MeV and 1446 MeV, leading to R= 0.18 and 0.30, respectively, with decay widths much smaller than those predicted by quark models, but closer to the Soliton models [18, 25]. A more advanced chiral approach [62, 64] including gluon exchange mechanism, predicts a larger width for $\Gamma_{\Lambda\gamma}$ (=75 keV), but that for $\Gamma_{\Sigma\gamma}$ shrinks down to 2 keV. That work reproduces well enough the total width of $\Lambda(1520)$, but underestimates that for $\Lambda(1405)$.

Now, we would like to proceed to more detailed comparisons between our results set and that reported by Yu *et al.* [20], also within a chiral quark approach. Here, we need to go back to Eqs. (2) to (5). Table XI summarizes the state assignments used in the present work and those in Ref. [20], showing that in this latter work all resonances have been replaced by the lowest mass relevant baryon. The drawback of that approximation on numerical results is presented below.

TABLE XI: Resonance assignments (see Eqs. (2) to (5)).

State	Baryon Present work	Baryon Ref. [20]
$\Lambda_1^2 P_A$	$\Lambda^*(1405)$	$\Lambda^*(1405)$
$\Lambda_8^2 P_M$	$\Lambda^*(1670)$	$\Lambda^*(1405)$
$\Lambda_8^4 P_M$	$\Lambda^*(1800)$	$\Lambda^*(1405)$
$\Lambda_8^2 S_S$	$\Lambda(1116)$	$\Lambda(1116)$
$\Lambda_8^2 S_{S'}$	$\Lambda^*(1600)$	$\Lambda(1116)$
$\Lambda_8^2 S_M$	$\Lambda^*(1810)$	$\Lambda(1116)$
$\Sigma_8^2 S_S$	$\Sigma(1193)$	$\Sigma(1193)$
$\Sigma_8^2 S_{S'}$	$\Sigma^*(1660)$	$\Sigma(1193)$
$\Sigma_8^2 S_M$	$\Sigma^*(1770)$	$\Sigma(1193)$
$N_8^2 S_S$	$N(938)$	$N(938)$
$N_8^2 S_{S'}$	$N^*(1440)$	$N(938)$
$N_8^2 S_M$	$N^*(1710)$	$N(938)$

The hereafter called hybrid model (HM) results are obtained using our code, but state assignments of Yu *et al.* [20]. The resulting widths are reported in Table XII. The width $\Gamma_{\Lambda\gamma}$ comes out rather stable (for $P_{5q} \lesssim 45\%$), while $\Gamma_{\Sigma\gamma}$, and the ratio R both decrease drastically. Although the ratio found for P_{5q} =0% is very close to that obtained by Yu *et al.* [20], there are about 50% discrepancies among the widths. We will come back to this point.

In Tables XIII and XIV we report our results for helicity amplitudes for each state,

TABLE XII: Same as Table IX, but for hybrid model (using our formalism with resonance asignments of Ref. [20]).

	A	B	C	D	E
P_{5q} (%)	0	25	45	75	100
$\Gamma_{\Lambda\gamma}$	119	123	113	79	19
$\Gamma_{\Sigma\gamma}$	77	154	164	175	80
R	0.6	1.2	1.4	2.2	4.2

TABLE XIII: Numerical results for the helicity amplitude $A^{\Lambda}_{1/2}$ (in GeV$^{-1/2}$) for electromagnetic transition $\gamma\Lambda \to \Lambda(1405)$, with our results (2nd column), those from the hybrid model (HM, 3rd column), and from Yu *et al.* [20] (last column).

	total	HM	Ref. [20]
$\Lambda^2_8 S_S \to \Lambda^2_1 P_A$	0.081	0.071	-0.070
$\Lambda^2_8 S_S \to \Lambda^2_8 P_M$	-0.019	-0.058	0.062
$\Lambda^2_8 S_S \to \Lambda^4_8 P_M$	0.018	0.006	-0.004
$\Lambda^2_8 S'_S \to \Lambda^2_1 P_A$	-0.014	-0.026	0.030
$\Lambda^2_8 S'_S \to \Lambda^2_8 P_M$	-0.015	-0.006	-0.035
$\Lambda^2_8 S'_S \to \Lambda^4_8 P_M$	-0.004	0.003	-0.002
$\Lambda^2_8 S_M \to \Lambda^2_1 P_A$	-0.014	-0.004	-0.021
$\Lambda^2_8 S_M \to \Lambda^2_8 P_M$	-0.010	-0.009	-0.008
$\Lambda^2_8 S_M \to \Lambda^4_8 P_M$	-0.026	0	-0.002

including those obtained using the hybrid model, and compare them with values found in Ref [20]. Notice that there is an overall sign difference between our conventions and those used in Ref [20]. The first observation is that the state assignments of Ref [20], affect almost all the amplitudes for $\gamma^*\Lambda \to \Lambda(1405)$, bringing them close enough to those in Ref [20]. Then, the fact that the hybrid model and Ref. [20] produce different results for the decay width can be attributed on the one hand to small differences in some of the amplitudes and on the other hand to the input values.

The situation is very different for the $\gamma^*\Sigma \to \Lambda(1405)$ transition (Table XIV). Although the HM results show significant deviations from our original values, they also differ very

TABLE XIV: Results for the helicity amplitude $A^{\Sigma}_{1/2}$ (in GeV$^{-1/2}$) of electromagnetic transitions $\gamma\Sigma \to \Lambda(1405)$. Columns are as in Table XIII.

	total	HM	Ref. [20]
$\Sigma_8^2S_S \to \Lambda_1^2P_A$	-0.218	-0.154	-0.216
$\Sigma_8^2S_S \to \Lambda_8^2P_M$	-0.061	-0.139	-0.202
$\Sigma_8^2S_S \to \Lambda_8^4P_M$	0.031	0.007	0.007
$\Sigma_8^2S'_S \to \Lambda_1^2P_A$	0.024	0.069	0.196
$\Sigma_8^2S'_S \to \Lambda_8^2P_M$	0.102	0.077	0.109
$\Sigma_8^2S'_S \to \Lambda_8^4P_M$	-0.042	0.004	0.004
$\Sigma_8^2S_M \to \Lambda_1^2P_A$	0.036	0.105	-0.074
$\Sigma_8^2S_M \to \Lambda_8^2P_M$	-0.053	-0.003	0.005
$\Sigma_8^2S_M \to \Lambda_8^4P_M$	-0.102	0	0.003

significantly from values reported in Ref [20]. The main explanation for that feature might be due to a sign difference in their expression for $\Phi^{\rho}_{\Sigma^\circ}$ (Eq. (A1) in that reference) and $|\Sigma^\circ\rangle_\rho$ in the present manuscript (Eq. (A5)). This observation explains, at least partly, the differences between the values found for $\Gamma_{\Sigma\gamma}$ in the result coming from hybrid model and those reported in Ref [20]. Results from this latter work, after having corrected the sign, might allow more conclusive comparisons with our findings.

At this point, and having discussed results compiled in Table X, the main firm message is that decay widths measurements are mandatory in identifying the most reliable approaches. In the meanwhile, comparisons among outputs from those works with other observable constitute an alternative to progress. Accordingly, in the next Section we concentrate on the strong channels decay.

B. Strong decay of $\Lambda(1405)$

Using the formalism developed in Sec. III B and transition amplitudes reported in Tables IV and V, here we present our numerical results.

The transition amplitudes for $\Lambda(1405) \to \Sigma(1194)\pi$ and $\Lambda(1405) \to K^-p$ are given in Tables XV and XVI, respectively.

TABLE XV: Results for the amplitudes of the $\Lambda(1405) \to \Sigma(1194)\pi$ decay. Amplitudes for $5q \to 5q$ transitions vanish (see Sec. II B).

	$3q \to 3q$	N-D	total
$\Lambda_1^2 P_A \to \Sigma_8^2 S_S$	0.736	0.384	1.120
$\Lambda_8^2 P_M \to \Sigma_8^2 S_S$	0.287	-0.229	0.058
$\Lambda_8^4 P_M \to \Sigma_8^2 S_S$	0.491	-0.204	0.287
$\Lambda_1^2 P_A \to \Sigma_8^2 S'_S$	-0.722	0	-0.722
$\Lambda_8^2 P_M \to \Sigma_8^2 S'_S$	0.001	0	-0.001
$\Lambda_8^4 P_M \to \Sigma_8^2 S'_S$	-0.228	0	-0.228
$\Lambda_1^2 P_A \to \Sigma_8^2 S_M$	-0.511	0	-0.511
$\Lambda_8^2 P_M \to \Sigma_8^2 S_M$	-0.051	0	-0.051
$\Lambda_8^4 P_M \to \Sigma_8^2 S_M$	-0.016	0	-0.016

The nondiagonal terms, wherever relevant, play significant roles in both decay channels. For the $\Lambda(1405) \to \Sigma(1194)\pi$ transition (Table XV) the effect turns out constructive for the first transition, $\Lambda_1^2 P_A \to \Sigma_8^2 S_S$, enhancing its dominant character. For the two other transitions the destructive combinations of those terms with pure $3q$ transitions lead to

TABLE XVI: Results for the amplitudes of the $\Lambda(1405) \to K^- p$ decay. Amplitudes for $5q \to 5q$ transitions vanish (see Sec. II B).

	$3q \to 3q$	$N - D$	total
$\Lambda_1^2 P_A \to N_8^2 S_S$	1.478	-0.824	0.654
$\Lambda_8^2 P_M \to N_8^2 S_S$	-0.878	-0.228	-1.106
$\Lambda_8^4 P_M \to N_8^2 S_S$	0	-0.198	-0.198
$\Lambda_1^2 P_A \to N_8^2 S'_S$	0.745	0	0.745
$\Lambda_8^2 P_M \to N_8^2 S'_S$	-0.140	0	-0.140
$\Lambda_8^4 P_M \to N_8^2 S'_S$	0	0	0
$\Lambda_1^2 P_A \to N_8^2 S_M$	0.082	0	0.082
$\Lambda_8^2 P_M \to N_8^2 S_M$	-0.363	0	-0.363
$\Lambda_8^4 P_M \to N_8^2 S_M$	-0.194	0	-0.194

almost vanishing contribution from $\Lambda^2_8 P_M \rightarrow \Sigma^2_8 S_S$, suppressiny by a factor of 2, the mgnitude of $\Lambda^4_8 P_M \rightarrow \Sigma^2_8 S_S$ transition amplitude.

For the $\Lambda(1405) \rightarrow K^- p$ decay (Table XVI), the dominant term in pure $3q$ transition, $\Lambda^2_1 P_A \rightarrow N^2_8 S_S$, gets reduced by more than 50% due to nondiagonal term, while the magnitude of the second transition, $\Lambda^2_8 P_M \rightarrow N^2_8 S_S$, inreases by 20%. Finally, the nondiagonal terms attribute a significant role to the $\Lambda^4_8 P_M \rightarrow N^2_8 S_S$ transition, otherwise vanishing in pure $3q \rightarrow 3q$ scheme.

Using those transition amplitudes, we now move to numerical results for decay width and coupling constants. In Table XVII, we give the numerical results with $P_{5q} = 0\%, 25\%, 45\%, 75\%$ and 100% in columns A, B, C, D and E, respectively. By comparing results in columns A and B, we observe very significant effects arising from the nondigonal terms discussed above.

TABLE XVII: Results for the $\Sigma\pi$ decay width of $\Lambda(1405)$, and the $\Lambda(1405)\Sigma\pi$ and $\Lambda(1405)K^- p$ couplings.

	A	B	C	D	E
P_{5q} (%)	0	25	45	75	100
$\Gamma_{\Sigma\pi}$ (MeV)	24	43	50	45	23
$f_{\Lambda(1405)\Sigma\pi}/m_\pi$	3.0	4.0	4.2	4.1	2.9
$f_{\Lambda(1405)K^- p}/m_K$	11.3	6.5	5.4	1.9	-4.1

The most striking result is the predicted values for the width of $\Lambda(1405) \rightarrow \Sigma\pi$ decay. While a pure $3q$ constituent quark model underestimates that observable by a factor of 2, introduction of five-quark components in $\Lambda(1405)$ with P_{5q} $\approx$50%, leads to excellent agreement with the value, 50$\pm$2, reported in PDG [37], and coming from Ref. [65]. This latter work, published by Dalitz and Deloff in 1991, is an impulse approximation approach fitting a subset of data from Ref. [66], and discarding the only other data set [67] available in those days.

In Table XVIII, we summarize the relevant works on $\Gamma_{\Lambda(1405)\rightarrow\Sigma\pi}$. Recent data obtained at COSY by Zychor *et al.* [68] give a decay width of about 60 MeV, and a recent [69] phenomenological analysis leads to 40$\pm$8. Two other formalisms, based on Bethe-Salpeter coupled-channels [70] and chiral quark model [71], find values compatible with the findings by Dalitz and Deloff [65]. Our result is also in line with those reported values. Width determined within a unitary chiral perturbation theory [72] suggests a smaller value, within a doule-pole picture of $\Lambda(1405)$. Very recently Akaishi *et al.* [73], using a varational treatment, question that picture and advocate a single-pole nature for that resonance.

So, within our work with P_{5q}=45%, the $\Lambda(1405)$ resonance appears to favor a mixed structure of the three- and five-quark components.

TABLE XVIII: Results for the $\Sigma\pi$ decay width of $\Lambda(1405)$.

Approach	$\Gamma_{\Lambda(1405)\to(\Sigma\pi)^\circ}$	Reference
χQM	50	Present work with P_{5q} 45%
Bethe-Salpeter coupled-channels	50±7	Garcia-Recio *et al.* [70]
$U\chi PT$	38	Magas *et al.* [72]
χQM	48	Zhong - Zhao [71]
coupled-channels potential model	40±8	Esmaili *et al.* [69]
COSY experiment	≈ 60	Zychor *et al.* [68]
K-matrix	50±2	Dalitz - Deloff [65], PDG [37]

Finally, our results for the $\Lambda(1405)\Sigma\pi$ and $\Lambda(1405)K^-p$ couplings (Table XVII, reported without including isospin factors, show significantly different dependence on the structure of $\Lambda(1405)$, namely, in going from a pure $3q$ configuration to an admixture of the three- and five-quark components, the coupling $f_{\Lambda(1405)\Sigma\pi}$ gets increased by roughly 30%, while $f_{\Lambda(1405)K^-P}$ decreases by about 40%.

V. SUMMARY AND CONCLUSIONS

Within an extended chiral constituent quark model, we investigated the three- and five-quark structure of the S_{01} resonance $\Lambda(1405)$. The wave functions for this resonance and the octet baryons in our approach were reported explicitly. We derived the electro-excitation helicity amplitudes for $\gamma^*\Lambda(1116) \to \Lambda(1405)$, $\gamma^*\Sigma^\circ(1194) \to \Lambda(1405)$ processes, as well as transition amplitudes for the $\Lambda(1405) \to \Sigma(1194)\pi$, K^-p decays. Using those amplitudes, we gave expressions for the electromagnetic and strong decays widths, namely, $\Gamma_{\Lambda(1405)\to Y\gamma}$, with $Y \equiv \Lambda(1116),\ \Sigma(1194)$ and $\Gamma_{\Lambda(1405)\to(\Sigma\pi)^\circ}$, with $(\Sigma\pi)^\circ \equiv \Sigma^\circ\pi^\circ,\ \Sigma^+\pi^-,\ \Sigma^-\pi^+$.

The numerical values computed using those expressions were presented and the dependence of various decay widths on the percentage of the five-quark components were investigated and compared with other sources. For the photo-excitation helicity amplitudes $A^{\Lambda}_{1/2}$, we found good agreements with the only set of published results by Yu *et al.* [20], using their approximations. For the $A^{\Sigma}_{1/2}$, a seemingly sign problem in that paper did not allow us to proceed to meaningful comparisons. We also examined the situation with respect to the decay widths $\Gamma_{\Lambda(1405)\to\Lambda\gamma}$, $\Gamma_{\Lambda(1405)\to\Sigma\gamma}$ and their ratio. We argued that large discrepencies among a dozen of works [16, 18–20, 25, 57, 59–62, 64] devoted to that topic render impossible any conclusive comparisons. Then, among the quanties investigated here, the only firm ground is offered by the experimental results for the $\Lambda(1405) \to \Sigma(1194)\pi$ decay width ($\Gamma_{(\Sigma\pi)^\circ}$). Our formalism, embodying about 45% of five-quark components in the $\Lambda(1405)$ resonance and 20% in the octet baryons, allows reproducing $\Gamma_{(\Sigma\pi)^\circ} = 50\pm2$ reported in PDG and endorsed OUR other findings, especially with respect to the electromagnetic decay widths.

Our work hence favors a mixed structure of the three- and five-quark components in the $\Lambda(1405)$ resonance, with $[31]_{XFS}[4]_X[211]_F[22]_S$ scheme for the orbital-flavor-spin configuration of the four-quark subsystem. This configuration allows the presence of the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ components in $\Lambda(1405)$, while as shown by An *et al.* [42, 45], that configuration rules out the $u\bar{u}$ and $d\bar{d}$ components in $N(1535)$. Moreover, the probability of the five-quark components in $N(1535)$ turns out to be in the same range as that of $\Lambda(1405)$, making the $N(1535)$ heavier than $\Lambda(1405)$. In consequence, with respect to the five-quark components in baryons, our results complementing those published on the Roper [40, 41] and the first S_{11} resonances [42, 43], allows us to put forward an explanation for the mass ordering of the $N(1440)$, $\Lambda(1405)$, and $N(1535)$ resonances. Those issues have also been investigated in lattice QCD approaches [33, 74], an effective linear realization chiral $SU_L(2)$ x $SU_R(2)$ and $U_A(1)$ symmetric Lagrangian [75], and concisely reviewed in [76].

Finally, we wish to underline the importance of the mixing mechanism resulting from the present study. The presence of three- and five-qurak components in $\Lambda(1405)$ leads to nondiagonal terms arising from transitions among those components ($qqq \leftrightarrow qqqq_i\bar{q}_i$). In the case of photo-excitation helicity amplitudes, we find larger effects due to those transitions, than contributions from five-quark components. For the strong channels, not getting any contributions from those pure five-quark components, the nondiagonal terms turn out again crucial, increasing by about a factor of 2 the width for the $\Lambda(1405) \to \Sigma(1194)\pi$ decay and bringing it into agreement with the data. Comparable effects due to the mixing mechanism have also been reported for the electromagnetic transition $\gamma^* N \to N(1535)$ [42, 43], and the radiative and strong decays of the Roper resonance [40, 41]. This may reveal a new mechanism for the decay properties of baryons, i.e. $q\bar{q} \to \gamma^*,\ \pi,\ K$ transitions have significant contributions to the baryon resonance decays.

Appendix A: Wave functions for the three quark components

1. Flavor wave functions

The flavor wave functions for the baryons considered in this paper are as follows:

$$|\Lambda\rangle_A = \frac{1}{\sqrt{6}}\{|uds\rangle + |dsu\rangle + |sud\rangle - |usd\rangle - |dus\rangle - |sdu\rangle\}\,, \tag{A1}$$

$$|\Lambda\rangle_\rho = \frac{1}{2\sqrt{3}}\{|usd\rangle - |dsu\rangle - |sud\rangle + |sdu\rangle + 2|uds\rangle - 2|dus\rangle\}\,, \tag{A2}$$

$$|\Sigma^\circ\rangle_\lambda = -\frac{1}{2\sqrt{3}}\{|usd\rangle + |dsu\rangle + |sud\rangle + |sdu\rangle - 2|uds\rangle - 2|dus\rangle\}\,, \tag{A3}$$

$$|\Lambda\rangle_\lambda = \frac{1}{2}\{|usd\rangle + |sud\rangle - |sdu\rangle - |dsu\rangle\}\,, \tag{A4}$$

$$|\Sigma^\circ\rangle_\rho = \frac{1}{2}\{|usd\rangle + |dsu\rangle - |sud\rangle - |sdu\rangle\}\,, \tag{A5}$$

$$|p\rangle_\lambda = \frac{1}{\sqrt{6}}\{2|uud\rangle - |duu\rangle - |udu\rangle\}\,, \tag{A6}$$

$$|p\rangle_\rho = \frac{1}{\sqrt{2}}\{|udu\rangle - |duu\rangle\}. \tag{A7}$$

2. Spin wave functions

The spin-orbital coupled wave function read

$$X_A = -|\frac{1}{2},\frac{1}{2}\rangle_\lambda(\rho,0) + \sqrt{2}|\frac{1}{2},-\frac{1}{2}\rangle_\lambda(\rho,+1) + |\frac{1}{2},\frac{1}{2}\rangle_\rho(\lambda,0) - \sqrt{2}|\frac{1}{2},-\frac{1}{2}\rangle_\rho(\lambda,+1)\,, \tag{A8}$$

$$X_\lambda = -|\frac{1}{2},\frac{1}{2}\rangle_\lambda(\lambda,0) + \sqrt{2}|\frac{1}{2},-\frac{1}{2}\rangle_\lambda(\lambda,+1) + |\frac{1}{2},\frac{1}{2}\rangle_\rho(\rho,0) - \sqrt{2}|\frac{1}{2},-\frac{1}{2}\rangle_\rho(\rho,+1)\,, \tag{A9}$$

$$X_\rho = |\frac{1}{2},\frac{1}{2}\rangle_\rho(\lambda,0) - \sqrt{2}|\frac{1}{2},-\frac{1}{2}\rangle_\rho(\lambda,+1) + |\frac{1}{2},\frac{1}{2}\rangle_\lambda(\rho,0) - \sqrt{2}|\frac{1}{2},-\frac{1}{2}\rangle_\lambda(\rho,+1)\,, \tag{A10}$$

$$X'_\lambda = \sqrt{3}|\frac{3}{2},\frac{3}{2}\rangle(\lambda,-1) - \sqrt{2}|\frac{3}{2},\frac{1}{2}\rangle(\lambda,0) + |\frac{3}{2},-\frac{1}{2}\rangle(\lambda,1)\,, \tag{A11}$$

$$X'_\rho = \sqrt{3}|\frac{3}{2},\frac{3}{2}\rangle(\rho,-1) - \sqrt{2}|\frac{3}{2},\frac{1}{2}\rangle(\rho,0) + |\frac{3}{2},-\frac{1}{2}\rangle(\rho,1), \tag{A12}$$

with $(\lambda,0)=q_{\lambda,z}$, $(\lambda,+1)=-\frac{1}{\sqrt{2}}(q_{\lambda,x}+iq_{\lambda,y})$, $(\rho,0)=q_{\rho,z}$ and $(\rho,+1)=-\frac{1}{\sqrt{2}}(q_{\rho,x}+iq_{\rho,y})$.

The spin wave functions are

$$|\frac{1}{2},\frac{1}{2}\rangle_\rho = \frac{1}{\sqrt{2}}\{|\uparrow\downarrow\uparrow\rangle-|\downarrow\uparrow\uparrow\rangle\},|\frac{1}{2},\frac{1}{2}\rangle_\lambda=\frac{1}{\sqrt{6}}\{2|\uparrow\uparrow\downarrow\rangle-|\uparrow\downarrow\uparrow\rangle-|\downarrow\uparrow\uparrow\rangle\}\,, \quad \text{(A13)}$$

$$|\frac{1}{2},-\frac{1}{2}\rangle_\rho = \frac{1}{\sqrt{2}}\{|\uparrow\downarrow\downarrow\rangle-|\downarrow\uparrow\downarrow\rangle\},|\frac{1}{2},\frac{1}{2}\rangle_\lambda=-\frac{1}{\sqrt{6}}\{2|\downarrow\uparrow\uparrow\rangle-|\downarrow\uparrow\downarrow\rangle-|\uparrow\downarrow\downarrow\rangle\}\,, \quad \text{(A14)}$$

$$|\frac{3}{2},\frac{3}{2}\rangle = |\uparrow\uparrow\uparrow\rangle,|\frac{3}{2},\frac{1}{2}\rangle=\frac{1}{\sqrt{3}}\{|\uparrow\uparrow\downarrow\rangle+|\uparrow\downarrow\uparrow\rangle+|\downarrow\uparrow\uparrow\rangle\}\,, \quad \text{(A15)}$$

$$|\frac{3}{2},-\frac{1}{2}\rangle = \frac{1}{\sqrt{3}}\{|\uparrow\downarrow\downarrow\rangle+|\downarrow\uparrow\downarrow\rangle+|\downarrow\downarrow\uparrow\rangle\}. \quad \text{(A16)}$$

3. Orbital wave functions

Here we employ the harmonic oscillator wave functions

$$\Phi_{\Lambda^*}(\vec{q}_\lambda,\vec{q}_\rho) = \frac{\sqrt{2}}{\pi^{3/2}\omega_3^4}exp\{-\frac{q_\lambda^2+q_\rho^2}{2\omega_3^2}\}\,, \quad \text{(A17)}$$

$$\Phi_{000}(\vec{q}_\lambda,\vec{q}_\rho) = \frac{1}{(\pi\omega_3^2)^{3/2}}exp\{-\frac{q_\lambda^2+q_\rho^2}{2\omega_3^2}\}\,, \quad \text{(A18)}$$

$$\Phi^S_{200}(\vec{q}_\lambda,\vec{q}_\rho) = \frac{1}{\sqrt{3}(\pi\omega_3^2)^{3/2}}(3-\frac{q_\lambda^2+q_\rho^2}{\omega_3^2})exp\{-\frac{q_\lambda^2+q_\rho^2}{2\omega_3^2}\}\,, \quad \text{(A19)}$$

$$\Phi^\rho_{200}(\vec{q}_\lambda,\vec{q}_\rho) = \frac{2}{\sqrt{3}\pi^{3/2}\omega_3^5}(\vec{q}_\rho\cdot\vec{q}_\lambda)exp\{-\frac{q_\lambda^2+q_\rho^2}{2\omega_3^2}\}\,, \quad \text{(A20)}$$

$$\Phi^\lambda_{200}(\vec{q}_\lambda,\vec{q}_\rho) = \frac{1}{\sqrt{3}\pi^{3/2}\omega_3^5}(q_\rho^2-q_\lambda^2)exp\{-\frac{q_\lambda^2+q_\rho^2}{2\omega_3^2}\} \quad \text{(A21)}$$

Appendix B: Wave functions for the five quark components

1. Flavor and spin couplings

The decomposition of the flavor-spin configuration $[31]_{FS}[211]_F[22]_S$ is [77]

$$|[31]_{FS}\rangle_1 = \frac{1}{2}\{\sqrt{2}|[211]\rangle_{F1}|[22]\rangle_{S1}-|[211]\rangle_{F2}|[22]\rangle_{S1}+|[211]\rangle_{F3}|[22]\rangle_{S2}\}\,, \quad \text{(B1)}$$

$$|[31]_{FS}\rangle_2 = \frac{1}{2}\{\sqrt{2}|[211]\rangle_{F1}|[22]\rangle_{S2}+|[211]\rangle_{F2}|[22]\rangle_{S2}+|[211]\rangle_{F3}|[22]\rangle_{S1}\}\,, \quad \text{(B2)}$$

$$|[31]_{FS}\rangle_3 = \frac{1}{\sqrt{2}}\{-|[211]\rangle_{F2}|[22]\rangle_{S2}+|[211]\rangle_{F3}|[22]\rangle_{S1}\}\,, \quad \text{(B3)}$$

and that for $[4]_{FS}[22]_F[22]_S$

$$|[4]_{FS}\rangle = \frac{1}{\sqrt{2}}\{|[22]_{F1}\rangle|[22]_{S1}\rangle+|[22]_{F2}\rangle|[22]_{S2}\rangle\} \quad \text{(B4)}$$

2. Flavor wave functions

The flavor wave functions for $[22]_F$ in the $uuds\bar{s}$ component

$$
\begin{aligned}
|[22]_{F1}\rangle &= \frac{1}{\sqrt{24}}\{2|uuds\rangle + 2|uusd\rangle + 2|dsuu\rangle + 2|sduu\rangle - |duus\rangle - |udus\rangle - |sudu\rangle \\
&\quad -|usdu\rangle - |suud\rangle - |dusu\rangle - |usud\rangle - |udsu\rangle\}\,, \qquad \text{(B5)} \\
|[22]_{F2} &= \frac{1}{\sqrt{8}}\{|udus\rangle + |sudu\rangle + |dusu\rangle + |usud\rangle - |duus\rangle - |usdu\rangle - |suud\rangle \\
&\quad -|udsu\rangle\}\,. \qquad \text{(B6)}
\end{aligned}
$$

The flavor wave functions for $[211]_F$ in the $uuds\bar{s}$ component

$$
\begin{aligned}
|[211]\rangle_{F1} &= \frac{1}{4}\{2|uuds\rangle - 2|uusd\rangle - |duus\rangle - |udus\rangle - |sudu\rangle - |usdu\rangle + |suud\rangle \\
&\quad +|dusu\rangle + |usud\rangle + |udsu\rangle\}\,, \qquad \text{(B7)} \\
|[211]\rangle_{F2} &= \frac{1}{\sqrt{48}}\{3|udus\rangle - 3|duus\rangle + 3|suud\rangle - 3|usud\rangle + 2|dsuu\rangle - 2|sduu\rangle \\
&\quad -|sudu\rangle + |usdu\rangle + |dusu\rangle - |udsu\rangle\}\,, \qquad \text{(B8)} \\
|[211]\rangle_{F3} &= \frac{1}{\sqrt{6}}\{|sudu\rangle + |udsu\rangle + |dsuu\rangle - |usdu\rangle - |dusu\rangle - |sduu\rangle\}\,. \qquad \text{(B9)}
\end{aligned}
$$

All of the other flavor wave functions which are used in this paper are obtained by applying the lowering operator in the $SU(3)$ flavor space to the above functions.

3. Spin wave functions

Expressions for the spin wave functions $[22]_S$ are

$$
\begin{aligned}
|[22]\rangle_{S1} &= \frac{1}{\sqrt{12}}\{2|\uparrow\uparrow\downarrow\downarrow\rangle + 2|\downarrow\downarrow\uparrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\downarrow\uparrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle\}\,, \qquad \text{(B10)} \\
|[22]\rangle_{S2} &= \frac{1}{2}\{|\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle\}\,. \qquad \text{(B11)}
\end{aligned}
$$

4. Orbital wave functions

Orbital wave function of the five-quark components in $\Lambda(1405)$ reads

$$
[4]_X\Psi(\vec{\kappa}_i) = \frac{1}{\pi^3\omega_5^6}\exp\{-\frac{\sum_i \kappa_i^2}{2\omega_5^2}\}. \qquad \text{(B12)}
$$

The color-orbital coupled wave function for the five-quark components in the octet baryons is

$$\psi_C(\{\vec{\kappa}_i\}) = \frac{1}{\sqrt{3}}\{[211]_{C1}\varphi_{01m}(\vec{\kappa}_1)\varphi_{000}(\vec{\kappa}_2)\varphi_{000}(\vec{\kappa}_3) - [211]_{C2}\varphi_{000}(\vec{\kappa}_1)\varphi_{01m}(\vec{\kappa}_2)\varphi_{000}(\vec{\kappa}_3) + [211]_{C3}\varphi_{000}(\vec{\kappa}_1)\varphi_{000}(\vec{\kappa}_2)\varphi_{01m}(\vec{\kappa}_3)\}\varphi_{000}(\vec{\kappa}_4)\,. \tag{B13}$$

Here $[211]_{Ci}$ denote the three color configurations, $\varphi_{0lm}(\vec{\kappa}_i)$ the harmonic orbital wave function with the quantum number nlm and the oscillator frequency ω_5. Notice that the $\vec{\kappa}_i (i = 1, 2, 3)$ generate the 3 configurations of $[31]_X$ in Eqs. (17)-(19).

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