Nucleon form factors with $N_F = 2$ twisted mass fermions

P. Guichon[∗,†], R. Baron[∗], C. Alexandrou[∗∗,‡], T. Korzec[∗∗], G. Koutsou[∗∗], M. Brinet[§,¶], J. Carbonell[§], P.-A. Harraud[§] and K. Jansen[‖,††]

[∗]*CEA-Saclay, IRFU/SPhN, 91191 Gif-sur-Yvette, France*
[†]*pierre.guichon@cea.fr, remi.baron@cea.fr*
[∗∗]*Department of Physics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus*
[‡]*alexand@ucy.ac.cy, korzec@ucy.ac.cy, i.koutsou@fz-juelich.de*
[§]*Laboratoire de Physique Subatomique et Cosmologie, UJF/CNRS/IN2P3, 53 avenue des Martyrs, 38026 Grenoble, France*
[¶]*mariane@lpsc.in2p3.fr,Jaume.Carbonell@lpsc.in2p3.fr , harraud@lpsc.in2p3.fr*
[‖]*NIC, DESY, Platanenallee 6, D-15738 Zeuthen, Germany*
[††]*Karl.Jansen@desy.de*

Abstract. We present results on the nucleon form factors using two degenerate flavors of twisted mass fermions on $24^3 \times 48$ and 32×64 lattices with pion masses in the range $290 \div 485$ MeV.

INTRODUCTION

The nucleon form factors are an interesting challenge to lattice simulations as they are controlled by the spatial extension of the quark-gluon structure. In particular one can expect a specific sensitivity to the finite volume of the lattice when the pion mass approaches its physical value. To perform simulations in large physical volumes with the available computing power one needs relatively large lattice spacings ($\sim 0.05 \div 0.1$ fm) which implies the use of improved action and operators. In this respect twisted mass fermions [1] is an attractive formulation of lattice QCD since it allows for automatic $\mathcal{O}(a)$ improvement by tuning only one parameter, requiring no further improvements of the operators. Important physical results are emerging using gauge configurations generated with two degenerate flavors of twisted quarks ($N_F = 2$) in both the meson [2] and baryon [3] sectors. An example is the accurate determination, using precise results in the meson sector, of low energy constants of great relevance to phenomenology. Currently, $N_F = 2$ simulations are available for pion mass in the range $290 \div 485$ MeV for three lattice spacings $a < 0.1$ fm. In this work we discuss high-statistics results on the nucleon form factors obtained at one value of the lattice spacing corresponding to $\beta = 3.9$.

The action for two degenerate flavors of quarks in twisted mass QCD is given by

$$S = S_g + a^4 \sum_x \bar{\chi}(x) \left[\frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla^*_\mu) - \frac{ar}{2} \nabla_\mu \nabla^*_\mu + m_{\rm crit} + i\gamma_5 \tau^3 \mu \right] \chi(x) \quad , \qquad (1)$$

where we use the tree-level Symanzik improved gauge action S_g. The quark fields χ are in the so-called "twisted basis" obtained from the "physical basis" at maximal twist by the transformation $\psi = \frac{1}{\sqrt{2}}[\mathbf{1} + i\tau^3\gamma_5]\chi$ and $\bar{\psi} = \bar{\chi}\frac{1}{\sqrt{2}}[\mathbf{1} + i\tau^3\gamma_5]$. A crucial advantage is the fact that by tuning a single parameter, namely the bare untwisted quark mass to its critical value $m_{\rm cr}$, physical observables are automatically $\mathscr{O}(a)$ improved. A disadvantage is the explicit flavor symmetry breaking. In a recent paper we have checked that this breaking is small for baryon observables for the lattice spacing discussed here [5].

To extract the nucleon FFs we need to evaluate the nucleon matrix elements $\langle N(p_f,s_f)|j_\mu|N(p_i,s_i)\rangle$, where $|N(p_f,s_f)\rangle$, $|N(p_i,s_i)\rangle$ are nucleon states with final (initial) momentum $p_f(p_i)$ and spin $s_f(s_i)$ and j_μ is either the electromagnetic current $V_\mu^{EM}(x) = \frac{2}{3}\bar{u}(x)\gamma_\mu u(x) - \frac{1}{3}\bar{d}(x)\gamma_\mu d(x)$ or the axial current $A_\mu^a(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$. Whereas the matrix element of the axial current receives contributions only from the connected diagram the electromagnetic one has, in addition, disconnected contributions. In the isospin limit the matrix element of the isovector electromagnetic current $V_\mu^a(x) = \bar{\psi}(x)\gamma_\mu\frac{\tau^a}{2}\psi(x)$ has no disconnected contributions [6]. Therefore in this work we only evaluate the isovector nucleon FFs obtained from the connected diagram.

The electromagnetic matrix element of the nucleon can be expressed in terms of the Dirac and Pauli form factors, F_1 and F_2 defined as

$$\langle N(p_f,s_f)|V_\mu(0)|N(p_i,s_i)\rangle = \sqrt{\frac{m_N^2}{E_N(\vec{p}_f)E_N(\vec{p}_i)}}\bar{u}(p_f,s_f)\mathscr{O}_\mu u(p_i,s_i), \tag{2}$$

$$\mathscr{O}_\mu = \gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(Q^2) \tag{3}$$

with $q = p_f - p_i$ the momentum transfer and $Q^2 = -q^2$. These are related to the Sachs electric G_E and magnetic G_M FFs via: $G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{(2m_N)^2}F_2(Q^2)$ and $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$.

Similarly, the axial current matrix element of the nucleon $\langle N(p_f,s_f)|A_\mu^a(0)|N(p_i,s_i)\rangle$ can be expressed in terms of the form factors G_A and G_p with $\mathscr{O}_\mu$ given by

$$\mathscr{O}_\mu = \left[-\gamma_\mu\gamma_5 G_A(Q^2) + i\frac{q^\mu\gamma_5}{2m_N}G_p(Q^2)\right]\frac{\tau^a}{2}.$$

LATTICE EVALUATION

The nucleon interpolating field can be written in the twisted basis at maximal twist as $\tilde{J}(x) = \frac{1}{\sqrt{2}}[\mathbb{1} + i\gamma_5]\varepsilon^{abc}\left[\tilde{u}^{a\top}(x)\mathscr{C}\gamma_5\tilde{d}^b(x)\right]\tilde{u}^c(x)$. The transformation of the electromagnetic current, $V_\mu^a(x)$, to the twisted basis leaves the form of $V_\mu^{0,3}(x)$ unchanged. We use the Noether lattice current and therefore the renormalization constant $Z_V = 1$. The axial current A_μ^3 also has the same form in the two bases. In this case we use the local current and therefore we need the renormalization constant Z_A. The value of $Z_A = 0.76(1)$ [7] was determined non-perturbatively in the RI'-MOM scheme. This value is consistent with a recent analysis [8], which uses a perturbative subtraction of $\mathscr{O}(a^2)$ terms [9] for a better identification of the plateau yielding a value of $Z_A = 0.768(3)$ [8]. In order to increase overlap with the proton state we use Gaussian smeared quark fields [10] for the construction of the interpolating fields: $\mathbf{q}^a(t,\vec{x}) = \sum_{\vec{y}} F^{ab}(\vec{x},\vec{y};U(t))\; q^b(t,\vec{y})$ with $F = (\mathbb{1} + \alpha H)^n$ and $H(\vec{x},\vec{y};U(t)) = \sum_{i=1}^{3}[U_i(x)\delta_{x,y-\hat{\imath}} + U_i^\dagger(x-\hat{\imath})\delta_{x,y+\hat{\imath}}]$. In addition we apply APE-smearing to the gauge fields U_μ entering H. The smearing is the same as for our calculation of baryon masses with the smearing parameters α and n optimized for the nucleon ground state [3]. The present calculation has been performed at $\beta = 3.9$ which, according to the latest analyisis [4], corresponds to $a = 0.0790(26)$ fm. Note that this value differs from the one used in previous works [3], which explains why the pion masses in this work are somewhat larger.

In order to calculate the aforementioned nucleon matrix elements we calculate respectively the two-point and three-point functions: $G(\vec{q},t_f) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\,\Gamma^0_{\beta\alpha}\,\langle J_\alpha(t_f,\vec{x}_f)\bar{J}_\beta(0)\rangle$ and $G^\mu(\Gamma^\nu,\vec{q},t) = \sum_{\vec{x},\vec{x}_f} e^{i\vec{x}\cdot\vec{q}}\,\Gamma^\nu_{\beta\alpha}\,\langle J_\alpha(t_f,\vec{x}_f)j^\mu(t,\vec{x})\bar{J}_\beta(0)\rangle$, where the projection matrices $\Gamma^0 = \frac{1}{4}(\mathbb{1}+\gamma_0)$ and $\Gamma^k = i\Gamma^0\gamma_5\gamma_k$. We create the nucleon at $t_i = 0$, $\vec{x} = 0$ (source) and annihilate it at $t_f/a = 12$, $\vec{p}_f = 0$ (sink). We checked that the sink-source time separation of $12a$ is sufficient for the isolation of the nucleon ground state by comparing the results with those obtained when $t_f/a = 14$ is used [6]. We insert the current j^μ at t carrying momentum $\vec{q} = -\vec{p}_i$. In this work we limit ourselves to the calculation of the connected diagram which in the isospin limit yields the isovector electromagnetic form factors. This is calculated by performing sequential inversions through the sink so that no new inversions are needed for different operator $j^\mu(t,\vec{q})$. However new inversions are necessary for a different choice of the projection matrices Γ^α. In this work, we consider the four choices given above, which are optimal for the form factors considered here and construct the ratio

$$R^\mu = \frac{G^\mu(\Gamma,\vec{q},t)}{G(\vec{0},t_f)}\sqrt{\frac{G(\vec{p}_i,t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(\vec{p}_i,t)G(\vec{p}_i,t_f)}} \overset{t_f-t,t\to\infty}{\longrightarrow} \Pi^\mu(\Gamma,\vec{q}) \quad . \qquad (4)$$

The leading time dependence and overlap factors cancel yielding as the plateau value $\Pi^\mu(\Gamma,\vec{q})$ from which we extract the form factors using the relations

$$\Pi^\mu(\Gamma^0,\vec{q}) = \frac{c}{2m}[(m+E)\delta_{0,\mu} + iq_k\delta_{k,\mu}]\; G_E(Q^2), \quad \Pi^i(\Gamma^k,\vec{q}) = \frac{c}{2m}\sum_{jl}\varepsilon_{jkl}q_j\delta_{l,i}\,G_M(Q^2)$$

and $\Pi^{5i}(\Gamma^k,\vec{q}) = \frac{ic}{4m}\left[\frac{q_k q_i}{2m}\,G_p(Q^2) - (E+m)\delta_{i,k}\,G_A(Q^2)\right]$, $k = 1,\cdots,3$, where $c = \sqrt{\frac{2m^2}{E(E+m)}}$.

RESULTS AND CONCLUSION

The form factors have been computed at $\beta = 3.9$ and $m_\pi = 290, 340, 420, 485$ MeV with a lattice volume $24^3\times 48$ and are shown on Figs.1-4. The calculation was also performed in the volume $32^3\times 64$ for the pion mass $m_\pi = 335$ MeV in order to quantify the finite volume effects. As expected G_P is more and more dominated by the pion

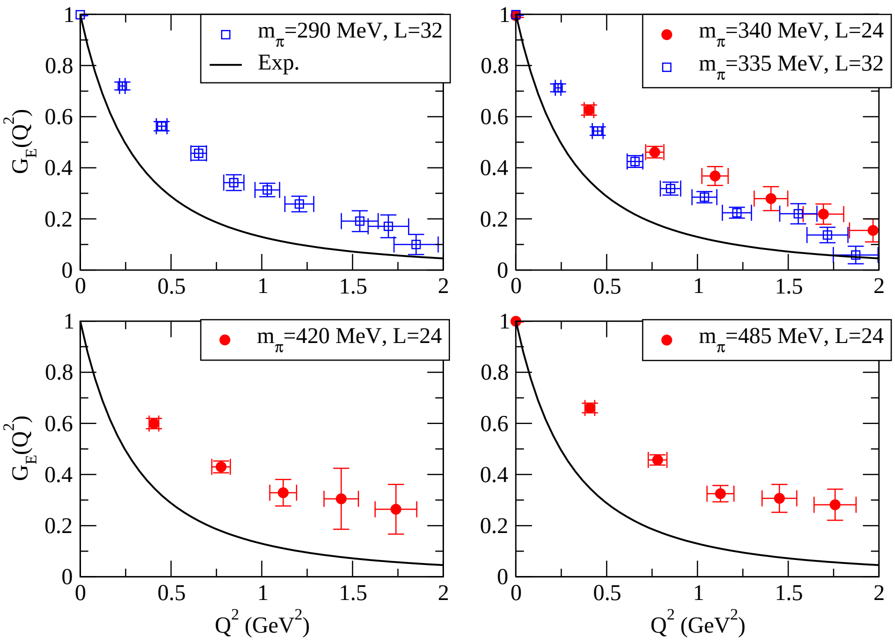

FIGURE 1. Electric form factor G_E.

pole approximation (continuous line in Fig.4) as the pion mass approaches the chiral limit. Though qualitative this approximate pion pole dominance in the space-like region is a non trivial check of the PCAC hypothesis. The situation is less satisfactory for G_A as shown on Fig.3. As the pion mass is decreased the lattice results have a shape which does not tend to the phenomenological curve (continuous line in Fig.3). Since the finite volume effects seem negligible, one may suspect a problem with the continuum limit. However a nasty combination of finite volume effect and chiral behaviour is not excluded, as already observed for the parton distributions [11]. The results look better for the (isovector) electromagnetic form factors. The shape of G_E and G_M approches the phenomenological curves (continuous line in Fig.1,2) as the pion mass is decreased. One interesting point is that G_M is little affected by finite volume effects, contrary to G_E. In the latter case the comparison of the $24^3 \times 48$ and $32^3 \times 64$ volumes shows a systematic difference, larger than the statistical errors, which indicates that finite volume effects need to be instigated before drawing any conclusion about the lattice form factors. The results at a smaller lattice spacings ($\beta = 4.05, 4.2$) will be soon available and this will allow to perform a quantitative analysis of volume effects and chiral extrapolation.

ACKNOWLEDGMENTS

This work was performed using HPC resources from GENCI Grant 2009-052271 and was partly supported by funding received by the DFG Sonderforschungsbereich/ Transregion SFB/TR9 and the Cyprus Research Promotion Foundation under contracts EPYAN/0506/08, KY-Γ/0907/11 and TECHNOLOGY/ΘΕΠΙΣ/0308(BE)/17.

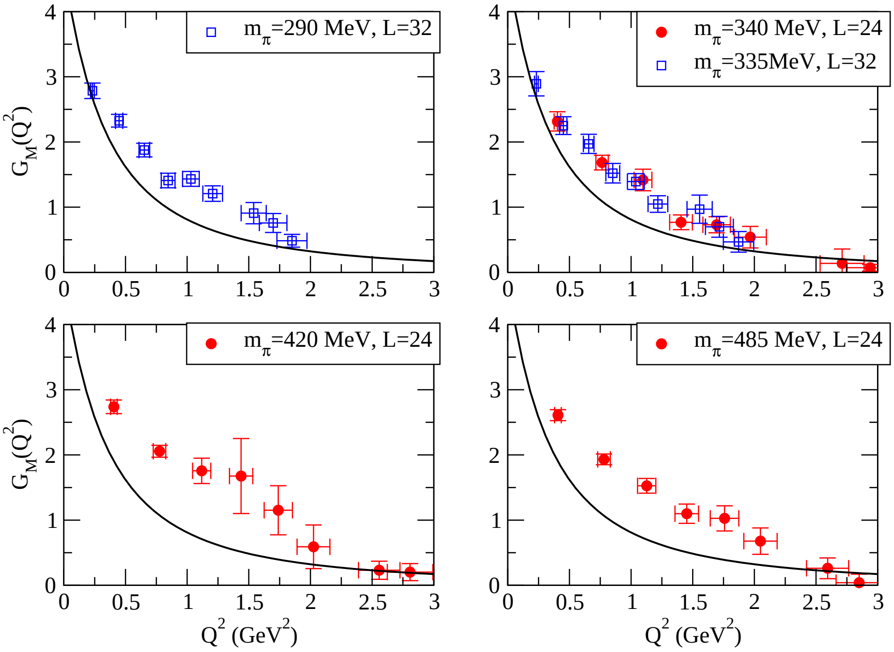

FIGURE 2. Magnetic form factor G_M

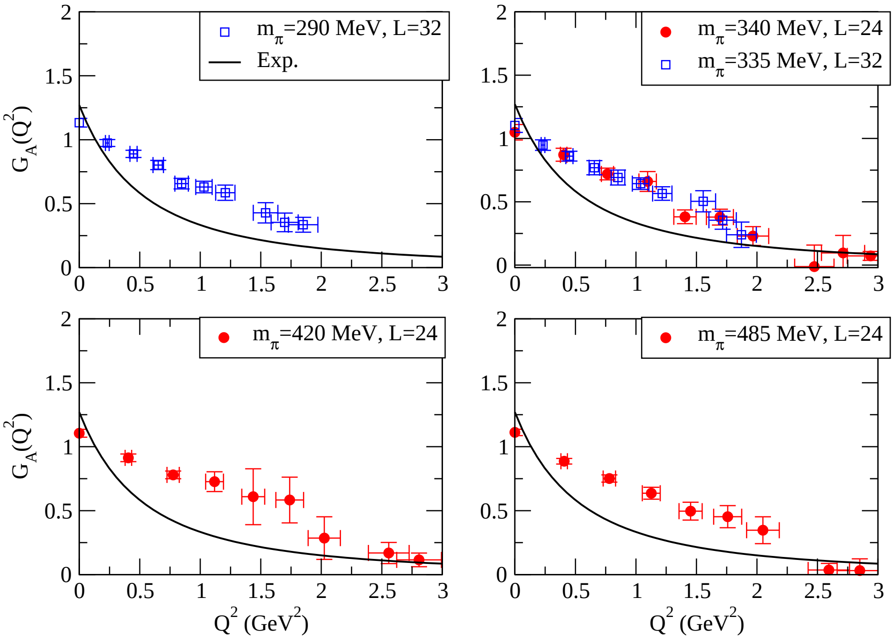

FIGURE 3. Axial form factor G_A

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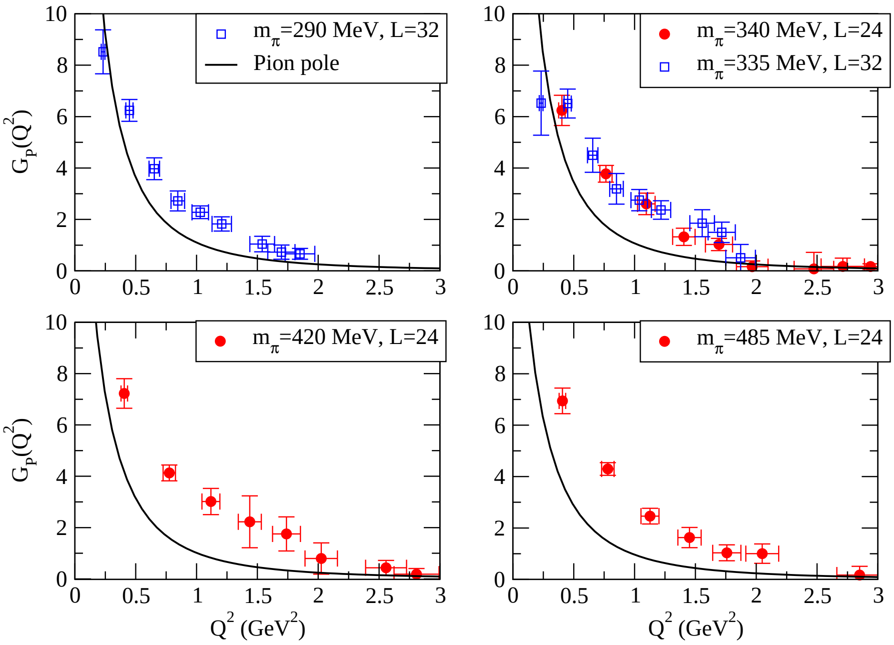

FIGURE 4. Pseudo-scalar from factor G_P

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