Compilation and analysis of charge asymmetry measurements from electron and positron scattering on nucleon and nuclei

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Abstract

World data of elastic and inelastic scattering of electrons and positrons on nucleon and nuclei are compiled and discussed. The evidence of an experimental charge asymmetry different from zero, being a pure quantum effect, would be a signature of contributions beyond the Born approximation. After reviewing the published results, we compare the elastic data to a calculation which includes the box diagram due to two-photon exchange. We show that all the data on the cross section ratio, in the limit of their precision, do not show evidence of sizable hard two-photon contribution, which however should be present if it constitutes the explanation for the difference between Rosenbluth and polarization measurements of the proton electromagnetic form factors.

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I. INTRODUCTION

Although elastic electron hadron scattering is one of the simplest elementary reactions, it is the object of large experimental and theoretical effort since many decades. Most of the interpretation of the observables, in polarized and unpolarized scattering, is based on the assumption that the interaction of the electron with a hadron (nucleon or nucleus):

$$e(k) + h(p) \rightarrow e(k') + h(p'), \tag{1}$$

(in brackets are the four momenta of the corresponding particles) occurs through the exchange of one virtual photon, with four-momentum $q = k - k'$ ($q^2 < 0$, and $-q^2 = Q^2$).

Since the early sixties, it was noted in the literature, that two (n)- photon exchange could also contribute, although the size of the amplitude is scaled by the factor $Z\alpha$ ($(Z\alpha)^n$) ($\alpha = 1/137$ is the fine structure constant of the electromagnetic interaction, and Z the target charge number). It was theoretically predicted [1] that a possible large effect could arise from 2γ exchange (TPE) when Q^2 increases. A reaction mechanism where the transferred momentum is equally shared between the two photons could compensate the scaling in $Z\alpha$ due to the steep decreasing of the form factors (FFs). Therefore, it is expected that TPE would become more important

1. when Q^2 increases;

2. when the charge Z of the target increases.

Indeed, FFs decrease more steeply with Q^2 as the number of constituents increases. From quark counting rules the number of constituents goes into an exponent, and asymptotically one expects FFs to be proportional to $(Q^2)^{-(n-1)}$, with $n = 3(6)$ for a proton(deuteron) [2].

Experimentally there are interesting observables which contain information on the presence of TPE. In this work we focus our attention on the observables which are sensitive to the real part of the TPE amplitude.

The simplest observables are those which vanish in Born approximation, such as the charge asymmetry in electron and positron scattering on a nucleus in the same kinematical conditions.

The most reliable predictions were based on model independent statements, derived from symmetry properties of the strong and electromagnetic interactions [3]. Due to C-parity conservation, TPE would induce

- for elastic electron-proton scattering: non linearities in the Rosenbluth fit, i.e., in the unpolarized cross section versus ϵ at fixed Q^2, where $\epsilon^{-1} = 1 + 2(1+\tau)\tan^2(\theta/2)$ is the linear polarization of the virtual photon, $\tau = Q^2/(4M^2)$, M is the proton mass, and θ is the angle of the scattered electron in the laboratory (lab) system.

- for the crossed channels (the annihilation channels $e^+ + e^- \leftrightarrow p + \bar{p}$): the presence of terms in the unpolarized angular distribution, which are odd with respect to $\cos\tilde{\theta}$ (where $\tilde{\theta}$ is the center of mass (cms) angle of the produced particle).

- for $e^{\pm}p$ scattering a non vanishing charge–asymmetry:

$$A^{odd} = \frac{\sigma(e^+p \to e^+p) - \sigma(e^-p \to e^-p)}{\sigma(e^+p \to e^+p) + \sigma(e^-p \to e^-p)}. \tag{2}$$

In the 70's the presence of a possible TPE contribution was an object for extended experimental and theoretical investigations. As a conclusion of a series of measurements, detailed below, no experimental evidence was found, in limits of data precision, and, since that time, the one photon exchange approximation was assumed *a priori* .

For *ed* scattering, only one dedicated polarization experiment was done in the past on unpolarized deuteron target. The vector deuteron polarization, $\vec{P}$ was measured, with the value $|P|$ =0.075 ±0.088 at Q^2=0.72 GeV^2 [4], showing no evidence for the presence of a TPE contribution in the limits of precision of the experiment.

The possibility of TPE contribution was forgotten for many years, but it becomes timely with the advent of high duty cycle electron accelerators which provide very precise data.

In 1998 [3] TPE was suggested as a possibility to reconcile two sets of data on electron deuteron elastic scattering. The analysis concluded to a probable systematic error on the experimental side, but the possibility of an observable contribution of TPE over Q^2=1 GeV^2 was not excluded from the arguments given above. More recently a number of papers was devoted to this subject, due to discrepancy in the ratio of the electric and magnetic FFs of the proton, measured by polarized and unpolarized experiments (for a review, see [5]).

In the space-like region, the analysis of the data in terms of deviation from the Rosenbluth plot was done in Refs. [6], and later on in Ref. [7]. The TPE contribution was estimated to be lower than one percent. In that work it was also pointed out that radiative corrections (RC), as they were applied to the data, may induce important effects on the relevant observables.

In particular RC change the slope of the Rosenbluth plot, and even its sign, when Q^2 increases.

Recently, the GEp collaboration has measured the angular dependence of the ratio of longitudinal to transverse polarization, more exactly as a function of ϵ [8]. The very precise results, although preliminary, show a constant behavior as a function of ϵ, in agreement with the one photon exchange expectation.

Due to the lack of statistics, few data exist on angular distributions in the annihilation region. Recently the process $e^+ + e^- \to p + \bar{p} + \gamma$ has been measured by the BABAR collaboration [9]. The initial state radiation, when the photon is sufficiently hard, allows to factorize out the kinematical terms associated to the photon and to extract the differential cross section of the elementary process $e^+ + e^- \to p + \bar{p}$. The analysis of these data in terms of angular asymmetry was done in Ref. [10] and showed no visible TPE effect in the limit of the errors.

The presence of a sizable TPE contribution, if experimentally found, would be a serious complication for precision experiments which aim to extract the nucleon properties from electron scattering. The interpretation of present experiments and the proposals foreseen after the upgrade of JLab would be seriously affected. The simple and elegant formalism which allows to access properties of hadrons would not apply anymore. For example, for ep elastic scattering, it has been shown that instead of two amplitudes, real functions of one variable, Q^2, in case of one photon exchange, one has to deal with three complex amplitudes, functions of two variables (q^2 and the total energy s) in case of TPE. It has also been proved, in a model independent way [11], that hadron electromagnetic FFs can still be extracted, at the price of difficult experiments involving double and triple spin observables, which need to be measured with a precision of the order of α. Similar analysis has been done for the crossed channels [12].

This work is devoted to charge asymmetry between electron and positron scattering on a nucleon (nucleus) in the same kinematical conditions. We discuss results from past experiments: elastic data (which we compare to a recent calculation of charge asymmetry [16]), as well as inelastic and deep-inelastic data for unpolarized electron and positron scattering on proton and nuclei.

Recently a re-analysis of a selected sample of the existing data concluded in evidence for two photon contribution [13], and that two photon effects suppress the cross section at low

ϵ and low Q^2. In Refs. [14, 15] predictions were done for charge asymmetry measurements, under specific assumptions and parametrization on the TPE contribution extracted from elastic ep experiments.

In Born approximation, which corresponds to the lowest order diagram for one photon exchange (OPE), the elastic lepton-proton scattering is symmetric with respect to the change of the lepton charge. As indicated above, the presence of TPE, more exactly the interference between the Born and the two photon exchange box diagram, including the one with crossed legs, induces charge odd (C-odd) contributions in the matrix element. But C-odd terms arise also from the higher order QED terms of elastic amplitude like radiative corrections. Radiative corrections to OPE contain two contributions: real soft photon bremsstrahlung with photon energies below the experimental resolution ΔE and virtual corrections. Only in the sum of these radiative corrections contributions and TPE amplitudes there is cancellation of infrared singularities. The global effect is proportional to $\log(\Delta E/E')$, where ΔE is the maximum energy of a soft photon, which escapes the detection and E' is the scattered electron energy. For the sake of simplicity, we define the "soft photon" contribution as the term dependent on the maximum photon energy ΔE, while all the rest of the asymmetry is identified with the TPE contribution. The term "hard box" define the contributions where both photons carry virtuality. The results from Ref. [16] show that the charge asymmetry may be measurable, when sizable contributions arise from the latter term, while the hard box is small in all the investigated kinematical range.

An exact calculation of charge asymmetry is possible only if the target is structureless as μ or e [17]. The case of a lepton target it constitutes an upper limit (in absolute value) for composite targets, when the intermediate state is the ground state [18]. The reason is that proton form factors are smaller than unity in almost all the q^2 range. Considering possible nucleon excitations, there are indications that a compensation exists among inelastic and elastic intermediate states. Such indications are based on model calculations [19, 20], as well as on analytical considerations [16].

It has also been shown in the annihilation channel [21], that hard photon emission largely compensates soft photon emission, giving an overall contribution to the charge asymmetry which is independent from ΔE and of the order of 1%. It is expected that a similar effect occurs in the scattering channel.

At high momentum transfer, the interference between one photon and Z boson exchange plays a large role. Data from muon scattering at 120 and 200 GeV [22] have been analyzed assuming one-photon exchange. The results, in agreement with the Standard Model, allowed the extraction of the muon neutral couplings.

II. DEFINITIONS

The unpolarized cross section $d\sigma_B$, for lepton hadron elastic scattering, assuming one photon exchange at the lowest order of perturbation theory, can be expressed in general in terms of two structure functions, A and B, which depend only on the momentum squared of the transferred photon, Q^2:

$$d\sigma_B(e^\pm h\to e^\pm h)=d\sigma_{Mott}\left[A(Q^2)+B(Q^2)\tan^2\frac{\theta}{2}\right], \tag{3}$$

where $d\sigma_{Mott}$ is the cross section for point-like particles. This is a very general expressions that holds for any hadron of any spin S. The structure functions depend on the $2S+1$ electromagnetic form factors of the hadron. In the Born approximation, the elastic cross section is identical for positrons and electrons. A deviation of the ratio:

$$R=\frac{\sigma(e^+h\to e^+h)}{\sigma(e^-h\to e^-h)}=\frac{1+A^{odd}}{1-A^{odd}} \tag{4}$$

from unity would be a clear signature of processes beyond the Born approximation. Those processes include the interference of one photon and two photon exchanges, and all the photon emissions which bring a C-odd contribution to the cross section. In Ref. [16], an exact QED calculation was performed for $e^\pm\mu$ scattering, and related to the crossed process in the annihilation channel. Due to C-parity conservation, one can show that the corresponding C-odd terms, in the annihilation channel, would change sign for $\tilde\theta\to\pi-\tilde\theta$ [3]. A model for two photon exchange was derived in Ref. [16]. The charge asymmetry:

$$\begin{aligned}A^{odd}\;&=\;\frac{d\sigma^{e+p}-d\sigma^{e^-p}}{2d\sigma^B}=\frac{2\alpha}{\pi}\left[\ln\frac{1}{\rho}\ln\frac{(2\Delta E)^2}{ME}+\ln x\ln\rho+\right.\\&\left.\mathrm{Li}_2\left(1-\frac{1}{\rho x}\right)-\mathrm{Li}_2\left(1-\frac{\rho}{x}\right)\right],\ \mathrm{Li}_2\left(z\right)=-\int_0^z\frac{dx}{x}\ln(1-x),\end{aligned} \tag{5}$$

with

$$\rho=\left(1-\frac{Q^2}{s}\right)^{-1}=1+2\frac{E}{M}\sin^2\frac{\theta}{2},\ x=\frac{\sqrt{1+\tau}+\sqrt{\tau}}{\sqrt{1+\tau}-\sqrt{\tau}},$$

was expressed as the sum of the contribution of two virtual photon exchange, (more exactly the interference between the Born amplitude and the box-type amplitude) and a term which depends on ΔE, the maximum energy of the soft photon which escapes the detection [1]. One can write $\Delta E = (1-c)E/\rho$, where $c \leq 1$ is the inelasticity cut, E is the initial energy and ρ is the fraction of the initial energy carried by the scattered electron, $\rho = E/E'$. It turns out that it is namely this term which gives the largest contribution to the asymmetry and contains a large ϵ dependence. Note that Eq. (5) holds at first order in α and does not include multi-photon emission. As shown in Fig. 4 of Ref. [16], the results are in good agreement (within 1%) with the results of Ref. [23].

Let us note that a C-odd effect is enhanced in the ratio R, Eq. (4), with respect to the asymmetry, Eq. (5). Moreover, the experimental cross section contains also C-even radiative correction terms, δ^{even}. Therefore, for the comparison with the experimental data, in the denominator of Eq. (5) the replacement $\sigma^B \to \sigma^B(1+\delta^{even})$ has to be done, where δ^{even} can be calculated, for example, from Ref. [23], which is also a first order calculation. However, the effect of this correction on the ratio (4) is $\leq 1\%$.

III. COMPILATION OF $e^{\pm}+p$ SCATTERING DATA

The unpolarized cross section of electron and positron scattering on hadronic targets was extensively studied in the 70's, in dedicated experiments. Besides a series of measurements, recently reviewed in [13], few works in inelastic and deep inelastic scattering, including heavy targets, achieved a much better precision and spanned a larger range in Q^2, confirming no detectable deviation of R from unity. The world data, concerning elastic and inelastic scattering, on proton target as well as on heavy ions, are summarized in Fig. 1. Most of the data concern electron and positron beams, few data correspond to muon beams. The measured values of the ratio R are drawn as a function of a serial number, which enumerates the results in chronological order. Let us review the findings and the main conclusions of these works:

1. The data from [28] (filled circles, black) were radiatively corrected according to [25] with $\Delta E = 0.03\ E$, which corresponds to an inelasticity cut $c = 0.94 \div 0.98$. In this case, we can apply the calculation [16] to the raw data taking into account all the other experimental corrections, which include energy loss, beam monitoring, and detection

[1] Note a difference of sign in the definition, Eq. (2) in Ref. [16].

efficiency.

2. Ref. [26] (filled squares, red) is one of the two sets of data which claim to see a significant two photon effect, (4.0 + 1.5)% larger than predicted by RC calculated in Ref. [27]. The data show deviations from one at higher momentum transfer and backward angles. The data are affected by quite large errors, and the numerical value of RC is not explicitely given. However, one can note that the deviation of the ratio is of the same level as the applied RC. We may conclude that the measured asymmetry reflects the soft photon correction. These data were analyzed and discussed together with previous results, [28], which however lead to opposite conclusion by the authors. Note that the highest Q^2 point of the so called "first experiment" (as defined in the original paper) was strongly contaminated by a non-elastic background and was remeasured in a cleaner manner in the "second experiment". To avoid unknown systematic uncertainty due to knowledge of the contamination we excluded this point from our analysis.

3. In Ref. [29] (filled triangles, green) the ratio R, compatible with unity within 2% was measured at Cornell, for $0.35 \leq Q^2 \leq 0.93$ GeV^2. q^2-dependent radiative corrections were applied following [31].

4. Ref. [32] (triangles down, blue) reported on a measurement of the elastic ratio $R < 1.01$, for two values of $Q^2 \leq 1$. Radiative corrections, calculated from [27] were taken as constant and their contribution is as large as 4%.

5. In Ref. [30] (open circles, yellow) two values of R were measured at DESY, with the following results: $R = 1.012 \pm 0.032$ at Q^2 =0.45 GeV^2, and $R = 0.954 \pm 0.057$ at Q^2 =1.36 GeV^2. The error bars are quite large. Moreover, the results are radiatively corrected, according to [27], with $\Delta E = 0.01\ E$ estimated from the given beam acceptance and resolution.

6. In Ref. [33] (open squares, magenta) a series of measurements, from Q^2 =0.2 to 5 GeV^2, was performed and the results expressed as limits for $Re(A_2/A_1)$. The points, after radiative corrections, are consistent with $R = 1$, with errors ranging from 0.016 to 0.123. Two points concern the Δ region, where the ratio is also consistent with unity.

Again, the points which show largest deviation form unity (although compatible inside the errors) are those affected by larger radiative corrections, as large as 5%.

7. In Ref. [34] (open triangles, cyan) several measurements at $Q^2 \leq 1$ were summarized in two data points, at $\epsilon \sim 0.8$. The applied RC to the ratio were of the order of 2% or smaller.

8. In Ref. [35] (open lozenges, dark green) a specific study was devoted to backward angle, where, according to Gourdin [36], a larger effect of TPE is expected. The most recent results, taken for the present analysis, differ from earlier publication and can be found in the Thesis work by B. Bouquet. Two points, for $\epsilon \sim 1$, and Q^2 =0.31 and 1.24 GeV^2 show a deviation from unity, which is still present, but at a lesser extent, after applying all corrections. The applied RC were respectively 4% and 9%.

9. Ref. [37] (open crosses, dark blue) reports on an experiment with the muon beam of AGS (Brookhaven), where not only charge asymmetry, but also deviation from linearities of the Rosenbluth plot were measured in the range $0.15 \leq Q^2 \leq 0.85$ GeV^2. No evidence of mechanism beyond one photon exchange was found, in both kind of tests. Radiative corrections were at most 3%, independent from the charge of the beam.

10. A measurement on deep inelastic scattering [38] (filled stars, red), also done at AGS (Brookhaven), in the range for $Q^2 < 2.1$ GeV^2 and $\nu < 5$ GeV, concluded that TPE amplitudes contribute less that 0.17%. No radiative corrections were applied.

11. Elastic and inelastic scattering on protons at DESY has been reported in Refs. [39, 40], (open stars, gray) for θ =9 and 13°, giving a value of the ratio compatible with one, within an error of 4 and 5%. Radiative corrections from [27] were applied to elastic data. The inelastic region from $1.2 < W < 3.4$ was covered by several measurements in which no systematic trend was observed. No radiative corrections were applied to the data in case of inelastic scattering. The work in Ref. [41] (open stars, gray) deserves a particular discussion, since $e^{\pm}$ inelastic scattering on ^{12}C and ^{27}Al was investigated. The final result $R = 1.005 \pm 0.027$, was obtained in the region of momentum transfer $0.08 < Q^2 < 0.45$ GeV^2 and invariant mass $0.95 \leq W \leq 3.3$ GeV of the hadronic system. The final result has been averaged from several measurements, after verifying

that no dependence on the momentum transfer, on the inelasticity and on the charge of the target appeared in the limit of the experimental error. No radiative corrections were included in the data.

12. Ref. [42] (asterisks, green) reports on measurements on hydrogen and deuterium up to 15 GeV^2. The ratio is consistent with unity, within errors of a few percent. Specific settings of the spectrometer allowed to measure different charges, alternatively. No radiative corrections were applied. Note that the largest deviation from unity comes from the point which is affected by the largest error.

13. In Ref. [43] (crosses, blue) the main result, $R = 1.0027 \pm 0.0035$ was obtained as an average of four measurement in the range $1.2 < Q^2 < 3.3$ GeV^2 and $2 < \nu < 9.5$, after insuring that there was no systematic trend of the data in the spanned kinematical range. This measurement, which is quite precise, is especially interesting for our discussion, as no RC were applied. The difference for electron and positron cross sections was very small. The lepton scattering angle was $\theta = 8°$, and the measurements correspond to large $\epsilon \sim 0.98$. Here, soft photon emission is very small, whatever is the inelasticity cut, therefore inducing very small asymmetry.

This fast review shows that, globally, all the data are consistent with unity. The 145 data points can be fitted by a constant $< R >= 1.002 \pm 0.002$ with $\chi^2/ndf = 1$, where ndf is the number of degrees of freedom. As shown in Fig. 1, the general trend of the ratio is consistent with unity. This does not exclude that some subset of data may deviate from unity, or that there may be, locally, a specific trend as a function of a particular kinematical variable. Below we discuss separately elastic and inelastic data and their dependences on the relevant kinematical variables. Moreover, the elastic scattering data can be compared point by point with the calculation from Ref. [16].

IV. ANALYSIS OF $e^{\pm} + p$ ELASTIC SCATTERING DATA

We report in Table I the results of the different experiments for elastic electron and positron scattering off the nucleon, together with the values of the relevant kinematical variables Q^2, and ϵ as well as the mean value of the ratio R for each set of data. The published data are also illustrated in Figs. 2 and 3 as functions of ϵ and Q^2.

Fitting the values of R with a constant gives $R = 0.996 \pm 0.004$ with ($\chi^2/ndf = 41/51 = 0.8$). A linear fit in ϵ gives $R = -(4.4 \pm 1.4) \cdot 10^{-2}\epsilon + (1.03 \pm 0.01)$ ($\chi^2/ndf = 40/50 = 0.8$). An enhancement at small ϵ was pointed out in Ref. [13] and considered as an evidence for TPE. In our case such deviation is smaller, due to the extended and updated data set. This enhancement is due to the two points at backward angle, from Ref. [35]. Omitted these two points, one finds better compatibility with unity: $R = -(3.2 \pm 2.5) \cdot 10^{-2}\epsilon + (1.02 \pm 0.02)$ ($\chi^2/ndf = 40/50 = 0.8$). A two parameter linear fit of the data as function of Q^2 gives $R = (5.3 \pm 5.7) \cdot 10^{-3}Q^2 + (0.993 \pm 0.005)$ ($\chi^2/ndf = 48/50 = 1$).

In order to verify the effect of standard radiative corrections, and to compare to the theoretical predictions, one has to deconvoluate the raw data, *i.e.*, the data including all experimental corrections except radiative corrections. Different ansatz for first order radiative corrections [23, 25, 27] differ at most by 1.5%. The exercise of deconvoluting the raw data and applying to all of them the same prescription for the radiative corrections (taken, for example from [23]) gives the following result. For a linear fit in ϵ: $R^c = -(2.0 \pm 1.4) \cdot 10^{-2}\epsilon + (1.01 \pm 0.01)$ ($\chi^2/ndf = 41/50 = 0.8$). A fit with a constant would give $R^c = 0.994 \pm 0.004$ with ($\chi^2/ndf = 41/51 = 0.8$). This procedure does not change essentially the results: one obtains a slightly better compatibility of the ratio with a constant. The dispersion of the data, due to different types of radiative corrections is smoothed out. However this effect on the ratio is of the order of a percent.

A quantitative comparison of the data with theoretical expectations, can be done for the calculation of Ref. [16] based on radiative corrections at first order, which gives a definite and simple expression for soft photon emission and two photon exchange. The asymmetry, as well as the ratio R, depend on three quantities, Q^2, ϵ, and ΔE, among which ΔE affects only the soft contribution. In order to check the agreement of the theory from the data, and to unfold the role of each variable, let us define

$$D(Q^2, \epsilon, \Delta E) = \frac{R_i^{raw} - R^{th}(Q^2, \epsilon, \Delta E)}{R^{th}(Q^2, \epsilon, \Delta E)}, \tag{6}$$

where $R^{raw} \pm \Delta R^{raw}$ are the experimental data including all corrections besides radiative corrections, and R^{th} is built from Eqs. (5,4), for the corresponding experimental conditions. We do not attribute any error to the theoretical value, therefore the error ΔD is directly related to the error on the experimental data.

A careful and specific analysis is necessary for each set of data. Eq. (5) includes soft photon emission, which has to be unfolded from the published result, and a complete calculation of the two photon box, which was not or only partially included in the radiative corrections applied to the data. Not always the data have been radiatively corrected. Care should be taken in double counting for the ΔE dependent corrections. Finally, D is built with the difference of the calculation and the measured values, after including all other experimental corrections except radiative corrections.

Ref.	Q^2 [GeV^2]	ϵ	T	N	$< R \pm \Delta R >$	$< D \pm \Delta D >$	χ^2	χ^2_0	χ^2_1
[28]	1.2÷3.3	0.1÷0.87	H	5	0.994 ± 0.008	-0.007 ± 0.008	3.1	3.1	3.1
[26]	0.27÷0.76	0.29÷0.68	H	6	1.024 ± 0.015	0.007 ± 0.010	11.6	10.1	4.2
[29]	0.64	0.70	H	1	0.996 ± 0.020	$0. \pm 0.020$	0.09	2.87	1.13
[32]	0.85, 0.78	0.75, 1	H	2	0.981 ± 0.026	-0.003 ± 0.026	0.02	3.7	1.94
[30]	0.45, 1.37	0.95, 0.78,	H	2	0.998 ± 0.028	-0.008 ± 0.028	1.01	1.10	1.79
[34]	0.20÷ 0.85	0.44÷ 0.77	H	5	0.992 ± 0.012	-0.006 ± 0.012	2.68	0.67	7.17
[33]	0.2÷5	0.72÷0.99	H	8	0.993 ± 0.005	-0.010 ± 0.005	11.6	137.3	9.2
[35]	0.31, 1.24	0.	H	2	1.038 ± 0.015	0.009 ± 0.017	0.42	6.16	11.1
[37]	0.14÷ 0.75	0.98 ÷1	H	15	0.973 ± 0.020	-0.028 ± 0.020	14.9	12.9	14.8
[39]	0.22÷ 0.45	0.97÷ 0.99	H	3	0.992 ± 0.030	-0.009 ± 0.030	0.96	1.21	0.96
[41]	0.22÷O.45	0.97÷ 0.98	C,Al	3	1.005 ± 0.023	0.003 ± 0.023	4.49	5.56	4.57
					Total		χ^2/ndf		
				52	0.996 ± 0.004	-0.007 ± 0.003	1.0	3.7	1.2

TABLE I: Summary of experimental data on the ratio of positron to electron elastic scattering cross section off the nucleon.

Let us show the general trend of the calculation, as a function of ϵ and Q^2, separately because in the data the ϵ and Q^2 dependences can not be deconvoluated. Let us fix the inelasticity cut at 3% from the elastic peak position. The expected behavior of the asymmetry [16] is shown

- as a function of ϵ for 3 different values of Q^2: 1, 3, and 5 GeV^2 (Fig. 2).

- as a function of Q^2 for 3 different values of ϵ=0.2, 0.5, 0.8 (Fig. 3).

These plots are given to illustrate qualitatively the expected behavior of the asymmetry, as only the difference point by point takes into account the experimental values for both variables ϵ and Q^2.

In total, for the considered 52 experimental points in elastic scattering, the difference between the calculation and the experimental points is, in average, very small : $D = -0.005 \pm 0.003$, value to be compared to an experimental error on each point of the order of few percent. We can conclude that our calculation is in very good agreement with the data. This allows us to study separately the dependence on the relevant variables.

V. DISCUSSION

A. Comparison with theory

After being convinced that the model of Ref. [16] reproduces satisfactorily the data, let us look in more detail, the dependence of the ratio on Q^2, ϵ and on the inelasticity cut. In particular it is possible to look on the hard and soft contributions separately.

The behavior of the ratio as a function of Q^2 is shown in Fig. 4, for Q^2=3 GeV^2 and for two values of the inelasticity cut: $c = 0.97$ (thin lines) and $c = 0.99$ (thick lines). The hard box contribution does not depend on the inelasticity cut (red, dashed line). The solid lines correspond to the full contribution, and the dotted lines to the soft contribution. The deviation from one is expected to increase as Q^2 increases: the soft contribution is larger as $c \to 1$. The hard contribution, although less sizable, also increases with Q^2, and has an opposite effect with respect to the soft contribution, reducing the ratio.

In Fig. 5 the ratio R is reported as a function of ϵ. One can see that the soft contribution largely dominates, increases at small ϵ and it is larger as c approaches to one.

B. Comparison with experiment

As mentioned in the Introduction, FFs derived from polarized and unpolarized measurements are inconsistent. It has been argued that TPE may reconcile these measurements [45]. The experimental findings can be described by the following parametrizations, with the upper script $'D'$ for the dipole form, suggested by unpolarized cross section measurements whereas the upper script $'P'$ stands for the linear Q^2 dependence of the FFs ratio from polarization experiments (for simplicity, in this section we omit the explicit dependence on the kinematical variables for FFs and σ):

- the unpolarized cross section σ_u can be parametrized as in Eq. (3) with

$$G_E^D = G_M^D/\mu = (1 + Q^2/0.71)^{-2} \tag{7}$$

- polarization experiments are consistent with the following Q^2 dependence for the FFs ratio:

$$\begin{aligned} \mu G_E^P/G_M^P &= 1 \text{ for } Q^2 < 0.4 \text{ and} \\ \mu G_E^P/G_M^P &= 1.0587 - 0.14265\, Q^2 \; [(\mathrm{GeV/c})^2] \text{ for } Q^2 > 0.4, \end{aligned} \tag{8}$$

at larger Q^2, at least up to $Q^2 = 5.8$ GeV^2. Let us assume that experiments do not have any bias, *i.e.*, that the measured observables (cross section and polarization ratio) are both correct and that the difference is entirely due to a two-photon contribution which appears in the unpolarized cross section and cancel in the polarization ratio. Let us try to extract the TPE term from the difference between the data, and verify its compatibility with the data on the electron positron ratio, where such term should be enhanced.

Let us derive expressions for the (reduced) cross section, as our observable is $R = \sigma^+/\sigma^-$ and use a sign '-(+)' for the TPE term in electron(positron) scattering:

$$\begin{aligned} \sigma \equiv \sigma_u(e^\pm p \to e^\pm p) &= \epsilon G_E^2 + \tau G_M^2 \pm C_{2\gamma}(Q^2, \epsilon) = \\ & \qquad \sigma_B(Q^2, \epsilon) \pm C_{2\gamma}(Q^2, \epsilon). \end{aligned} \tag{9}$$

where σ_u is the reduced cross section in the unpolarized case,

$$\sigma_u^\pm = \epsilon (G_E^D)^2 + \tau (G_M^D)^2. \tag{10}$$

and the possible two photon contribution is parametrized by $C_{2\gamma}(Q^2, \epsilon)$, which is a function of the order of α, depending on both kinematical variables. Terms of order of α^2 are neglected.

We can consider that the cross section with the FFs extracted from polarization experiments coincides with the Born cross section (which is the same for electrons and positrons):

$$\sigma_B = \sigma_p^\pm = \epsilon (G_E^P)^2 + \tau (G_M^P)^2. \tag{11}$$

Let us assume that the difference between polarized and unpolarized electron scattering is fully due to the two photon term:

$$C_{2\gamma}(Q^2, \epsilon) = \sigma_p^- - \sigma_u^-. \tag{12}$$

Unfortunately, only the FFs ratio is determined by polarization experiments, demanding an additional assumption to obtain the individual FFs. Since no deviations from the linear dependence of the Rosenbluth plots were found [6], one may assume that nonlinearities induced by TPE are small, and may approximate the TPE contribution by a linear ϵ dependence. Two possible linear forms change the slope of the Rosenbluth plot, according two different assumptions:

1) the unpolarized cross section gives a reliable measurement of G_M (at low $Q^2 < 0.5$ GeV^2, G_E coincides with dipole in both cases, and at large Q^2 the magnetic term constitutes a very large part of the cross section) therefore one can take $G_M^P = G_M^D$, and then, deduce G_E^P from Eq. (8). This is equivalent to change the slope of the Rosenbluth plot, and keep the same intercept:

$$C_{2\gamma}^0 = \epsilon[(G_E^P)^2 - (G_E^D)^2] \tag{13}$$

In this case the TPE term vanishes at $\epsilon = 0$ (backward scattering) and is maximum at $\epsilon = 1$ (forward angles).

2) On the opposite, the TPE term can be considered as maximum at $\epsilon = 0$ (backward scattering) and vanishing at $\epsilon = 1$ (forward angles), which is consistent with the assumptions of Ref. [14] and with the model calculation [46]. In this case, using the information on the ratio and on this normalization point, one finds:

$$C_{2\gamma}^1 = (\epsilon - 1)[(G_E^P)^2 - (G_E^D)^2] \tag{14}$$

In both cases, the TPE contribution increases with Q^2, as the difference from the cross sections based on FFs derived from Eqs. (7) and (8).

Note that the underlined assumptions (reality of FFs, linearity of the TPE term in ϵ...), usually assumed in phenomenological analysis, contradict a number of model independent statements required by symmetry properties of the strong and electromagnetic interactions [11].

The effect of $C_{2\gamma}$, extracted from polarized/unpolarized electron scattering, can be inserted into the ratio of positron to electron nucleus scattering cross section and into the charge asymmetry A.

$$R_{0,1} = \frac{\sigma^+}{\sigma^-} = \frac{\sigma^B + C_{2\gamma}^{0,1}}{\sigma^B - C_{2\gamma}^{0,1}},\ A_{0,1} = \frac{C_{2\gamma}^{0,1}}{\sigma^B}, \tag{15}$$

For the comparison with the data, let us define χ^2

$$\chi^2_{0,1}(Q^2,\epsilon,\Delta E) = \frac{(R_i - R_{0,1})^2(Q^2,\epsilon,\Delta E)}{\Delta R_i^2(Q^2,\epsilon,\Delta E)}, \tag{16}$$

The quality of these assumptions compared to the data is summarized in the last two columns of Table I. One can see that in case of normalization of the TPE term at $\epsilon = 0$, the TPE contribution extracted from the difference of polarized and unpolarized experiments is totally inconsistent with the charge asymmetry data. In the second case (corresponding to the normalization at $\epsilon = 1$) one finds a better consistency among the data, which is of the same order as with the theoretical approach [16]. Such approach explains the difference by a large ΔE dependent contribution to radiative corrections.

Ref.	Q^2[GeV^2]	W[GeV]	X_{Bj}	T	N	$< R \pm \Delta R >$	χ^2/ndf
[33]	0.2, 0.7	1.24	0.23, 0.52	H	2	1.015 ± 0.011	1.59
[43]	1.2÷3.3	1.2÷3.7	0.11÷0.86	H	4	$R = 1.002 \pm 0.002$	1.6
[42]	5.2, 14.9	3.08,3.38	0.33, 0.64	D	2	$R = 0.999 \pm 0.013$	2.0
[42]	2.4÷14.9	2.4÷4.46	0.11÷ 0.64	H	9	$R = 1.002 \pm 0.003$	0.7
[41]	0.15÷ 0.39	1.2÷2.2	0.04÷0.34	^{12}C	10	$R = 1.004 \pm 0.010$	0.8
[41]	0.15÷ 0.39	1.2÷3.3	0.02÷0.23	^{27}Al	19	$R = 1.032 \pm 0.012$	1.1
[40]	0.08÷ 0.39	1.2÷3.3	0.02÷0.39	H	38	$R = 0.981 \pm 0.012$	0.8
[38]	0.08÷ 0.39	1.2÷3.3	0.02÷0.37	H	9	$R = 0.990 \pm 0.034$	0.9
	Total				93	1.004 ± 0.002	0.8

TABLE II: Summary of experimental data on the ratio of positron to electron inelastic scattering cross section off the nucleon.

VI. INELASTIC AND DEEP INELASTIC SCATTERING

Let us analyze here the inelastic and deep inelastic data on the cross section for electron and positron scattering off nucleons and nuclei. The relevant variables are, in this case, Q^2, the square root of the total energy, W, and the Bjorken variable $x_{Bj} = Q^2/(2M\nu)$, with $\nu = E - E' = (W^2 - M^2 + q^2)/(2M)$. The data are summarized in Table II and shown as a

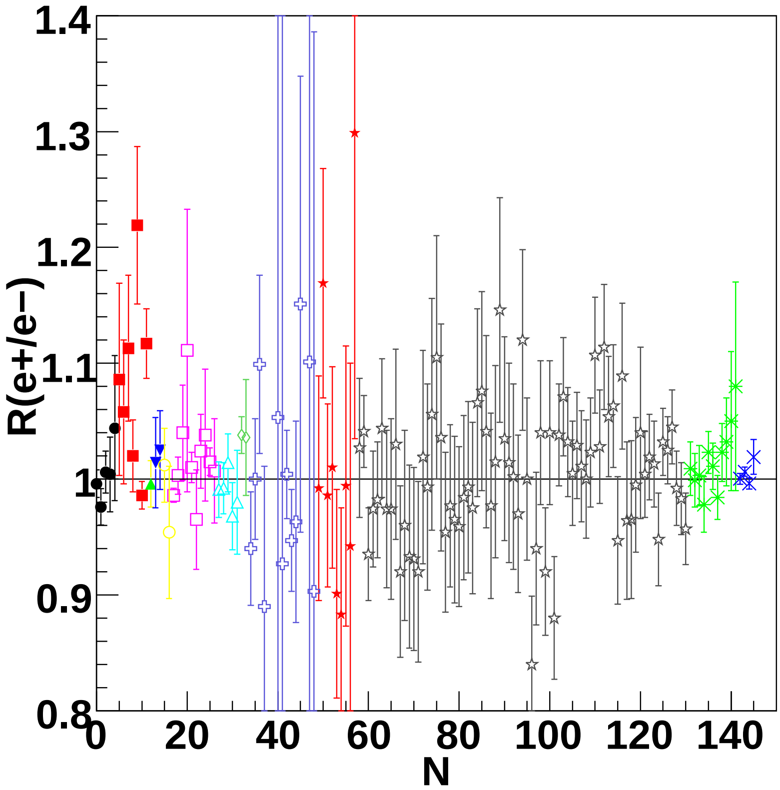

FIG. 1: (Color online) Ratio of cross sections $R = \sigma(e^+p)/\sigma(e^-p)$, as a function of N, a number attributed to each point, following a chronological order from Refs. [28] (solid circles, black), [26] (solid squares, red), [29] (solid triangles, green), [32] (solid triangles down, blue) [30] (open circles, yellow), [33] (open squares, magenta), [34] (open triangles, cyan), [35] (open lozenges, dark green), [37] (open crosses, dark blue), [38] (solid stars, red), [39–41] (open stars, gray), [42] (asterisks, green), [43] (crosses, blue).

function of Q^2 and x_{Bj} in Figs. 6 and 7, respectively. No systematic deviation of the ratio from unity is seen.

Note that the χ^2 given in table II does not imply a minimization, as there are no free parameters to be adjusted, but it is defined as

$$\chi^2 = \frac{(R-1)^2}{\Delta R^2} \tag{17}$$

A linear fit as a function of q^2 gives $R = -(-0.4 \pm 0.001) \cdot 10^{-3} q^2 + (1.004 \pm 0.003)$ and as a function of x_{Bj}: $R = (0.006 \pm 0.015) x_{Bj} + (1.002 \pm 0.004)$ both with $\chi^2/ndf = 77/91 = 0.84$. Note that a fit with a constant would give $R = 1.003 \pm 0.002$, with $\chi^2/ndf = 77/92 = 0.83$.

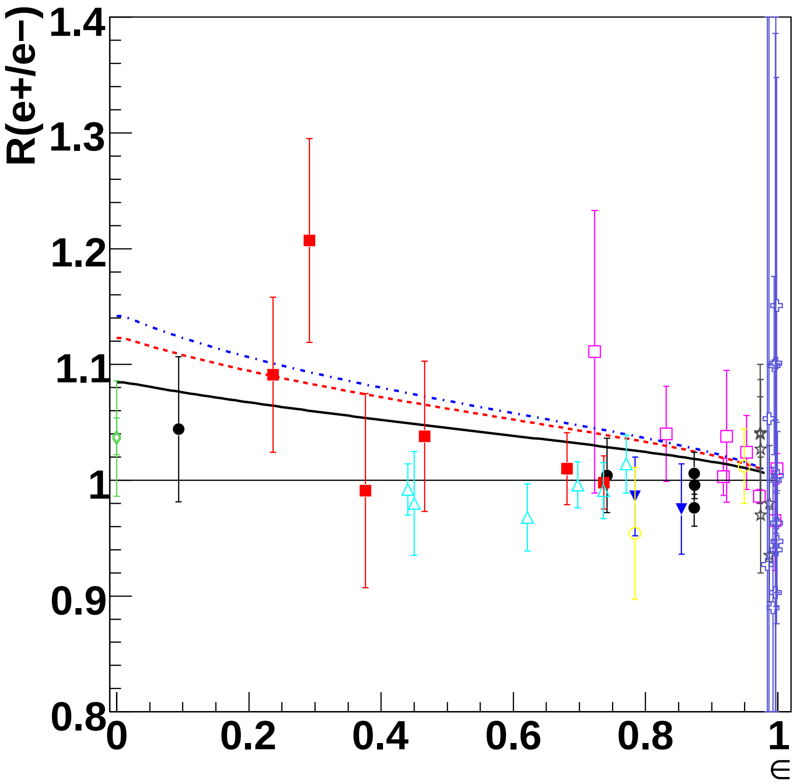

FIG. 2: (Color online) Ratio of cross sections $R = \sigma(e^+p)/\sigma(e^-p)$, as a function of ϵ, for $c = 0.97$ and different values of Q^2: $Q^2 = 1$ GeV^2 (solid line, black) $Q^2 = 3$ GeV^2 (dotted line, red) and $Q^2 = 5$ GeV^2 ((dash-dotted line, blue).

It is interesting that inelastic data do not give evidence for a deviation from unity, although they extend to quite large Q^2.

Concerning electron scattering on a nucleus $A(Z, N)$ in the quasi-elastic region, the reaction can be approximately described as the incoherent sum of elastic scattering off individual nucleons, which is reasonable at sufficiently large Q^2 values:

$$\sigma^{exp}_{e^\pm A} = Z\sigma^{exp}_{e^\pm p} + N\sigma^{exp}_{e^\pm n} \tag{18}$$

Since the real photon emission from the neutron is unlikely, the neutrons do not contribute to the asymmetry, more exactly to the part which is due to interference between electron and target emission. On the other hand, neutrons do contribute to the hard box, as they have non zero FFs (although such contribution is expected to be smaller than for protons). We take this into account by averaging the asymmetry in case of nucleus target: $A^{odd}_A = A^{odd}/2$,

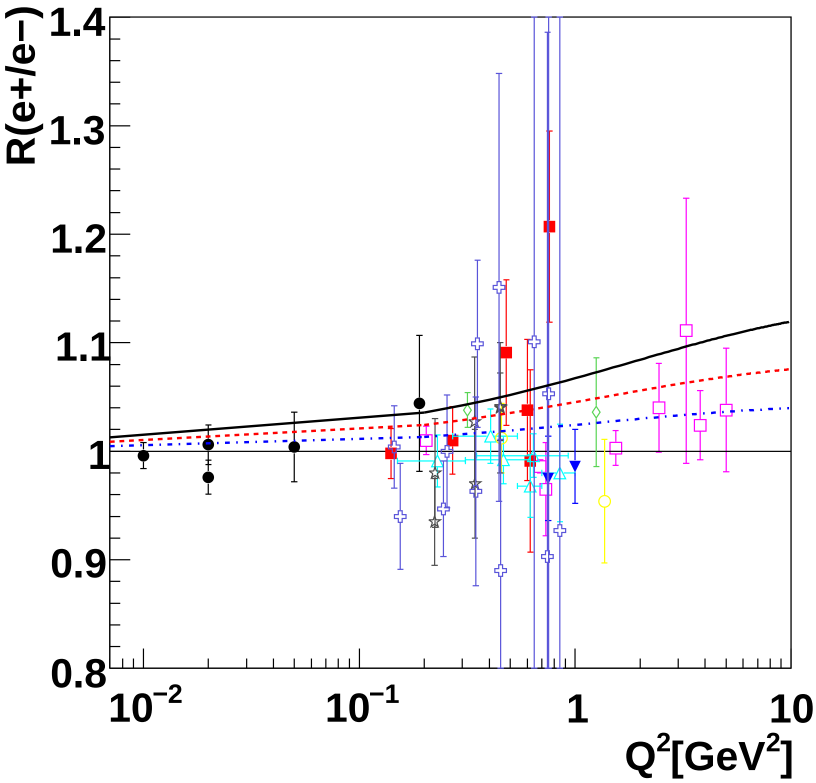

FIG. 3: (Color online)Ratio of cross sections $R = \sigma(e^+p)/\sigma(e^-p)$, as a function of Q^2. The lines are results of the calculation for $c = 0.97$, and different values of ϵ: $\epsilon = 0.2$ (solid line, black) $\epsilon = 0.5$ (dotted line, red) and $\epsilon = 0.8$ (dash-dotted line, blue).

where A^{odd} is the free proton asymmetry.

In the kinematical conditions of the experiment [41], one should note that the scattering angle is very small ($\epsilon \sim 1$), as well as Q^2. Although the Q^2 values are relatively small, one expects an enhancement of TPE effects by the strong Coulomb field of the nuclei. To give an order of magnitude, for the targets considered here which are relatively light, from Ref. [44] one expects an effect of $\sim 2\%$ from multiphoton exchange calculated in elastic kinematics, at E=3 GeV and for scattering angle $\theta = 9^0$.

VII. CONCLUSIONS

We have reanalyzed the existing data on electron and positron scattering off the nucleon (nucleus), in the same kinematical conditions. The deviation of the cross section ratio

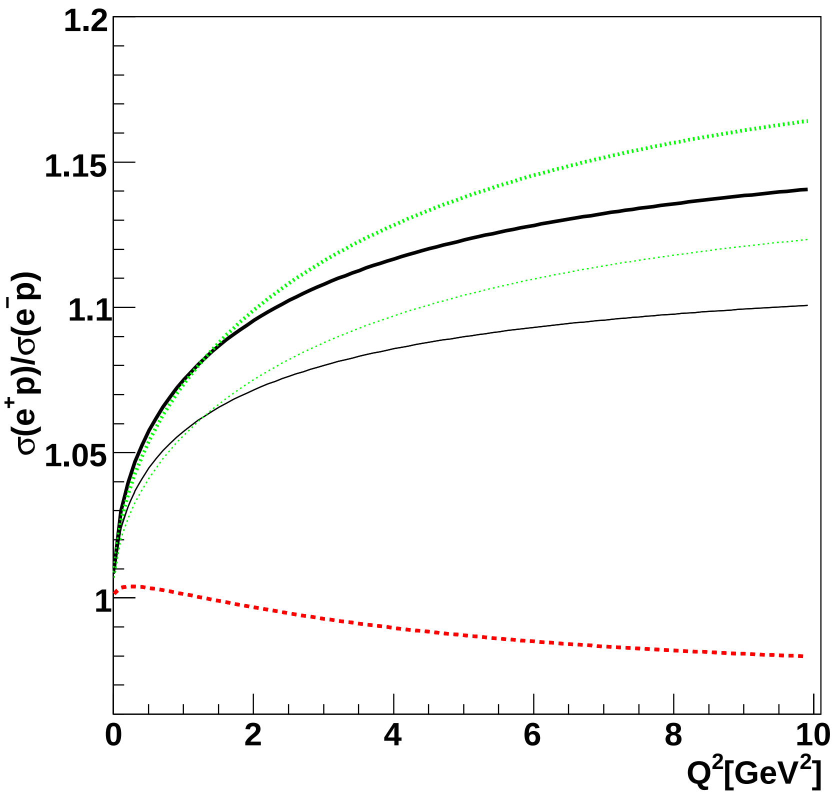

FIG. 4: (Color online) Ratio of cross sections $R = \sigma(e^+p)/\sigma(e^-p)$, as a function of Q^2, ($\epsilon = 0.2$), $c = 0.97$ (thick lines) and $c = 0.99$ (thin lines) form Ref. [16]: total contribution (solid line, black), soft contribution (dotted line, green). The hard contribution (dashed line, red) does not depend on the cut.

from unity would constitute an experimental evidence of contributions beyond the Born approximation. In order to make a quantitative analysis, we have compared the data to a calculation, which does not contain free parameters. Such calculation shows that the soft contribution arising from the interference between electron and target emission, can give rise to deviations from unity, strongly related to the inelasticity cut.

The present analysis shows that the cross section ratio data, in the limit of their precision, are not sufficient to show sizable TPE effects, which should be present if the TPE explanation of the discrepancy of cross section and polarization FFs data is correct. The charge asymmetry calculation used for the present analysis is in agreement with previous theoretical studies and with the findings of the experimental papers reported here. Such con-

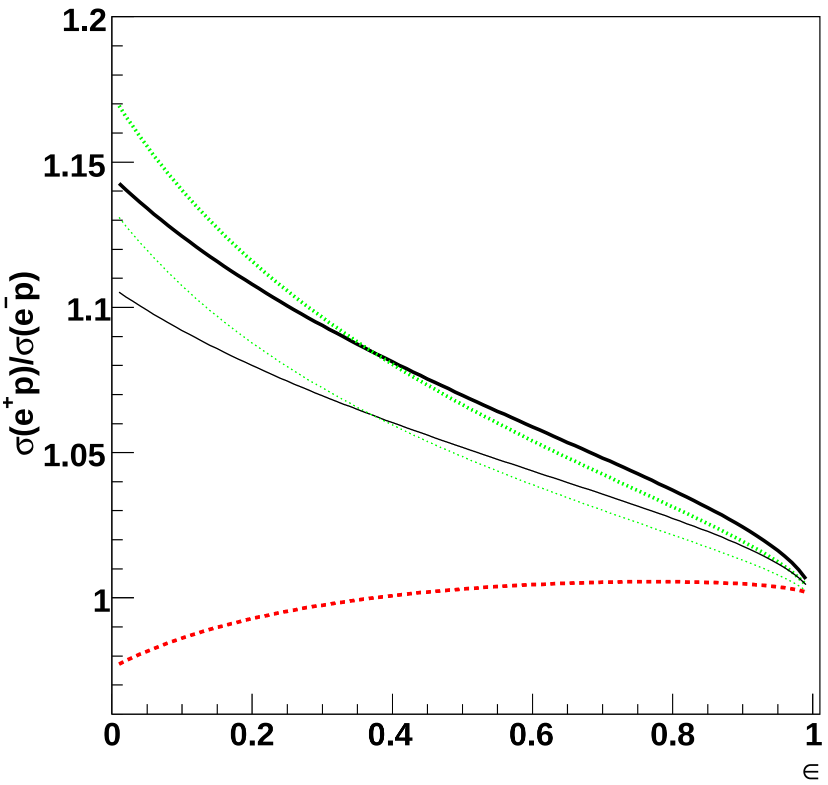

FIG. 5: (Color online) Ratio of cross sections $R = \sigma(e^+p)/\sigma(e^-p)$, as a function of ϵ, (Q^2=3 GeV^2). Notations as in Fig. 4.

clusion is, in our opinion, also supported by the fact that the few experimental points which show a signal for asymmetry are affected by large radiative corrections. In the experiments discussed here, radiative corrections have typically been calculated at first order.

Note that if the scattered electron energy is not measured, or the electron is detected in a calorimeter, it is experimentally not possible to remove the contribution of hard photons, which are emitted along the direction of the scattered electron. A compensation between soft and hard photons emitted along the scattered electron takes place [48], which results in a reduction of about a factor of two in the asymmetry. At our knowledge, this effect has not been always taken into account in experimental analysis.

A good understanding of the electron proton system and of the reaction mechanism is very important for different applications. Let us note that the equivalence of e^-p and $e^+\bar{p}$ scattering may be used for investigations on the mechanisms of polarization of an antiproton

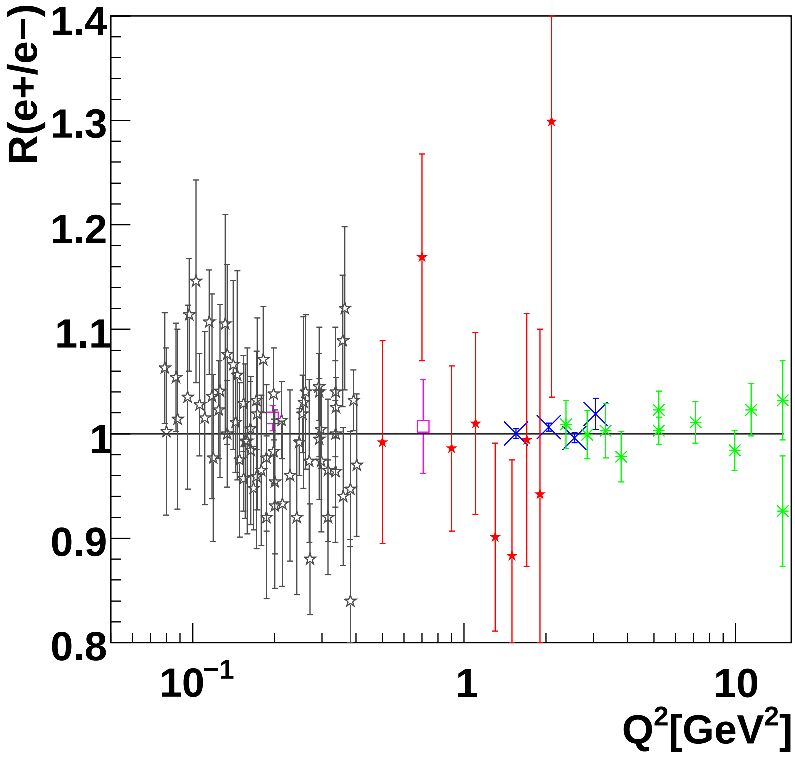

FIG. 6: (Color online) Ratio of inelastic cross sections $R = \sigma(e^+p)/\sigma(e^-p)$, as a function of Q^2. Notations for data symbols as in Fig. 1.

beam [47].

VIII. ACKNOWLEDGMENTS

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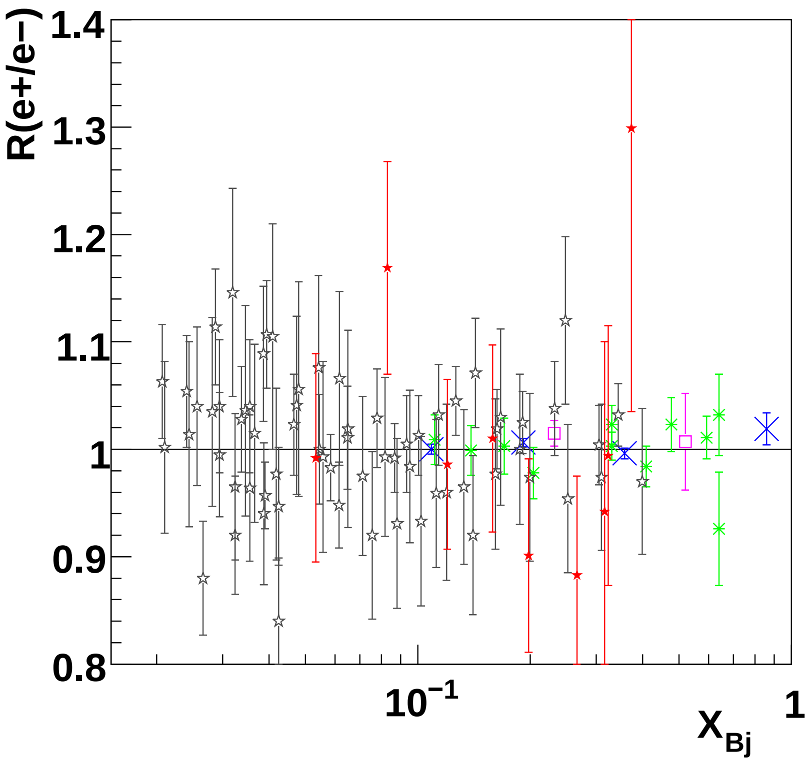

FIG. 7: (Color online) Ratio of inelastic cross sections $R = \sigma(e^+p)/\sigma(e^-p)$, as a function of x_Bj. Notations for data symbols as in Fig. 1.

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