2D Analytical Magnetic Optimizations for Accelerator Dipole Block Designs

E. Rochepault, P. Vedrine

***Abstract*—A new magnetic design method, which optimizes the position of the conductors in the cross-section of a racetrack-shaped magnet, has been set up. This method is based on analytical formulas of field, harmonics and forces. The conductors are modeled with rectangles, which is particularly well adapted to a block design. A first equation is used to calculate the contribution of the rectangles, as a function of their four spatial coordinates. All the block contributions are then added up to give the entire equation as a function of all the rectangles coordinates. Finally a non-linear optimization, using the analytic formulas, computes these coordinates to fulfill the magnet specifications. The objective can be either to maximize the central field, or to minimize a criterion, for instance the conductor volume, the harmonics, the electromagnetic forces. Various kinds of constraints can also be imposed: zeroed harmonics, blocks position and number, central field... Two magnets designs are proposed as examples: a 13 T Nb_3Sn magnet and a 20 T graded magnet.**

***Index Terms*—Magnet design, field optimization, cross-section, racetrack.**

I. Introduction

High energy physics requires particle accelerators with higher and higher energies. Consequently, accelerator magnet designers aim for superconducting magnets with high fields (> 12 T), high current densities (J_E ¿ 100 A/mm^2) and large apertures. These conditions lead to elevated Lorentz forces on the conductors (> 150 MPa). For instance, the Nb_3Sn is chosen to reach high fields but it doesn't sustain high stresses, so the design must deal with forces distribution. Recent high field magnets (HD2 [1], TAMU3 [2], FRESCA2 [3]) chose a "block design" rather than a "cosθ design" which allows a simpler manufacturing and a better stress management. To be stable, superconductors need to operate with margins, which are integrated in the specifications. Furthermore, superconductors price representing an significant part in the overall magnet manufacturing cost, the conductor volume must be as efficient as possible. In order to limit the beam losses, the field has to be homogeneous. The frequently used criterion is that each space harmonic has to be lower than 10^{-4} of the central field. There are also many constraints on the coil shape (minimum bending radius, maximum dimensions, inter-layer spacings...). All these constraints make superconducting high field magnets difficult to design and optimize. The principal interest of dipoles is to offer a long straight part (> 1 m), so a 2D analysis assuming infinitely long conductors is a good first approximation. This paper first describes an algorithm dedicated to the design of dipole cross sections. This algorithm is based on analytic formulas which compute the field and forces for rectangular conductors. This model fits well with a block design. Similar formulas have already been developed for sector coils [4]. Then case studies will be presented, to show an overview of the algorithm possibilities.

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II. Algorithm description

A. Magnetic field equations

It is useful to compute the magnetic field at any point of the space. Simple equations can be developed for a rectangular source block [5]. Let us first consider an infinite wire carrying a current $\vec{I} = I\vec{u}_z$, located at the source point Q with coordinates (a, b). With $x - a = x'$ and $y - b = y'$, the following distances are defined :

$$r = OP = \sqrt{x^2 + y^2} \tag{1}$$

$$r' = OQ = \sqrt{a^2 + b^2} \tag{2}$$

$$R = PQ = \sqrt{x'^2 + y'^2} \tag{3}$$

In a space free of magnetic material, the wire creates at point P(x, y) a magnetic vector potential :

$$\vec{A}_{z,wire} = -\frac{\mu_0 I}{2\pi} \ln\left(\frac{R}{r'}\right) \tag{4}$$

Considering a block limited by a_1, a_2, b_1, b_2, an integration in a and b gives the vector potential $\vec{A}_{z,block}$ for a current block. With $\vec{B} = \vec{rot}(\vec{A})$, the vector potential can be derived to obtain the two components of the field, assuming a current density J such as : $I = Jdadb$.

$$B_x(x, y) = -\frac{\mu_0 J}{2\pi}\left[\left[x' lnR^2 + 2y' \arctan\frac{x'}{y'}\right]_{a_1}^{a_2}\right]_{b_1}^{b_2} \tag{5}$$

$$B_y(x, y) = \frac{\mu_0 J}{2\pi}\left[\left[y' \ln R^2 + 2x' \arctan\frac{y'}{x'}\right]_{a_1}^{a_2}\right]_{b_1}^{b_2} \tag{6}$$

B. Harmonics equations

2D field harmonics for a dipole are usually calculated on a circle centered on the axis and at a radius equal to 2/3 of the aperture. The expansion zone doesn't contain sources, so it is an "interior expansion" ($r < r'$). A Fourier analysis of the magnetic field would be too long for an optimization algorithm, especially since the accuracy depends on the number of points. A faster solution consists in developing a formula which computes directly the harmonic of a bloc [5]. An expansion of the wire vector potential (4) is described in [4]. Using the angles $\theta = (\vec{u}_x, \vec{OP})$, $\theta' = (\vec{u}_x, \vec{OQ})$, a similar expansion can be developed for a rectangular block. Then the expansion coefficients for B_x and B_y are deducted :

$$B_x(x,y) = \frac{\mu_0 J}{2\pi}\left[\left[r'\sum_{n=0}^{\infty}\left(\frac{r}{r'}\right)^n X_{Bxn}\cos\theta + Y_{Bxn}\sin\theta\right]_{a_1}^{a_2}\right]_{b_1}^{b_2} \quad (7)$$

$$X_{Bx0} = \left[b\arctan\left(\frac{a}{b}\right) + \frac{a}{2}\ln(a^2+b^2)\right] \quad (8)$$

$$X_{By0} = -\left[a\arctan\left(\frac{b}{a}\right) + \frac{b}{2}\ln(a^2+b^2)\right] \quad (9)$$

$$X_{Bx1} = -Y_{By1} = -\frac{1}{2}\ln(a^2+b^2) \quad (10)$$

$$Y_{Bx1} = X_{Bx1} = -\arctan\left(\frac{b}{a}\right) \quad (11)$$

$$X_{Bxn} = -Y_{Byn} = \frac{\cos(n-1)\theta'}{n(n-1)} \quad \forall n \geq 2 \quad (12)$$

$$Y_{Bxn} = X_{Byn} = \frac{\sin(n-1)\theta'}{n(n-1)} \quad \forall n \geq 2 \quad (13)$$

For multiple blocks, according to the superimposition principle, a summation over the dimensions $a_{i,j}$, $b_{i,j}$ ($i = 1, 2$; $j = blocknumber$) is required to have the overall contribution. For dipoles, thanks to the X and Y symmetries, only one quarter of the coil is calculated to compute the harmonics.

C. Forces equations

Another useful equations are the Lorentz forces, integrated over one block. Instead of computing the field at each point of a block and integrate numerically over this block, a formula is developed, which computes the integrated force on one block, created by all the sources. First of all, the Laplace's law gives the forces per unit length at point P(x,y) :

$$\frac{dF_x}{dl}(x,y) = -IB_y(x,y) \quad (14)$$

$$\frac{dF_y}{dl}(x,y) = IB_x(x,y) \quad (15)$$

A summation over all the blocks, and an integration over the block, lead to the following equations :

$$F_x = -\frac{\mu_0 J^2}{4\pi}\left[\left[\sum_{i=1}^{N_{blocks}}\left[[f_x(x,y,a,b)]_{a_{1i}}^{a_{2i}}\right]_{b_{1i}}^{b_{2i}}\right]_{a_1}^{a_2}\right]_{b_1}^{b_2} \quad (16)$$

$$f_x(x,y,a,b) = y'x'^2\arctan\frac{y'}{x'} + \frac{y'^3}{3}\arctan\frac{x'}{y'} + \frac{2}{3}x'^3\ln y' + \frac{x'}{2}\left\{y'^2 - \frac{x'^2}{2}\right\}\ln R^2 - \frac{5}{6}\left\{x'^2 + \frac{y'^2}{3}\right\} \quad (17)$$

$$f_y(x,y,a,b) = f_x(y,x,b,a) \quad (18)$$

D. Algorithm structure

All these analytic formulas are implemented in Maple®. First of all, the equations are analytically evaluated with, as fixed parameters, the number of blocks N_{blocks} and the current density J_i of each bloc i. Then the objective and the constraints are defined (see section III for examples). They are of different types : geometrical specifications, conductors cost, field, harmonics and forces. Note that in the case of a dipole, the symmetries impose $X_{Bxn} = Y_{Byn} = 0$. A peak field constraint can be calculated with the specified margin, the chosen J and the $J_c(B)$ fit of the conductor. These objective and constraints are entered in a Maple function of non-linear optimization with one objective. Finally the results are used for post-processing (margins computation, plots...).

III. Results

A. Comparison of different configurations with the same specifications

1) Specifications: In order to explore the different options of the algorithm, several dipole cases have been studied. The same specifications, listed in Tab. I, are taken for all the examples. These specifications are representative of an experimental Nb_3Sn superconducting dipole, with a large aperture and bore field. The parameters are close for instance to those of the FRESCA2 magnet [3], currently in design. In this subsection, no cable insulation is taken into account. For a preliminary design, the coil can be modeled with a block per layer. This allows a fast resolution with an excellent approximation. A lot of cases have been tested, only three extreme cases are presented in this section.

TABLE I
Specifications used.

Name	Value	Unit
Aperture	=100	mm
Current density in the layer	=260	A/mm^2
Inter-layer insulation	=0.5	mm
Bending radius	>20	mm
Bore field B_0	=13	T
Margin	>10	%
Harmonics	$<10^{-4}$	B_0
Stress	<150	MPa
Coil height	<200	mm
Coil width	<300	mm
Volume (1/4 of the cross section)	<12 009	mm^2
Number of blocks	<10	-

2) Maximization of the central field: A first test of the algorithm can be to check if 10 blocks reach the maximum central field (optimization objective) in the maximum surface imposed by the specifications. Geometrical constraints between block are also imposed. For comparison, the maximum field is computed analytically with a rectangular conductor block and a circular hole. The field achieves 18.76 T with 11 457 mm^2, which corresponds to 94% of the maximum field with 95% of the maximum surface. Because of the limited number of blocks and the interlayer insulation, the 100 % cannot be reached. This design is not realistic for superconductors like Nb_3Sn because the field is far above the critical field, but it allows a first verification of the algorithm.

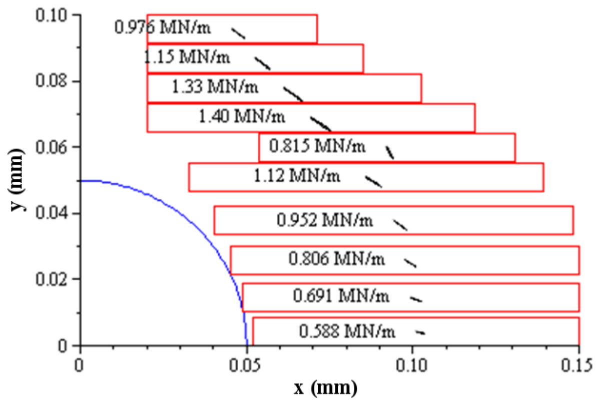

Fig. 1. Design which minimizes the conductor volume with space harmonics equal to zero up to order 40. Value and direction of the forces are indicated for each block.

3) Minimization of the conductor volume: Another concern of the magnet designers is the minimization of the conductor cost. In a 2D modeling, the conductor surface in the magnet cross-section is chosen as an objective. A coil is usually wound with a unique cable, in this example, all the layers have the same height. Dipole magnets also need to produce a homogeneous field in the bore. All the harmonics, up to the order 40, are set to zero. For comparison between the designs, a Total Harmonic Distortion (THD) is defined as follows :

$$THD_{40} = \frac{10^4}{B_0}\sqrt{\sum_{n=1}^{20} X_{By2n}^2} \tag{19}$$

Note that, thanks to the horizontal symmetry, all the odd orders are already equal to zero. Actually the THD is not exactly zero because harmonics are set to zero with an optimization tolerance. It is anyway very low and the harmonic specification is satisfied.

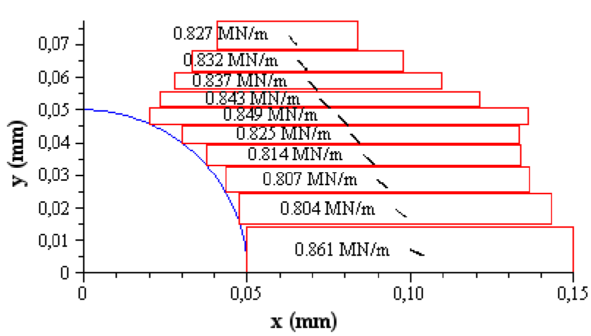

Fig. 2. Design which minimizes the force criterion on the conductors.

4) Minimization of the forces: For the Nb_3Sn case, the stress management is very important. The transverse pressure limit is for now fixed to 150 MPa [6], and can be much lower for some conductors. The objective of this design is to minimize a force criterion defined by :

$$F_{tot}^2 = \sum_{i=1}^{N_{blocs}} \left(F_{x,i}^2 + F_{y,i}^2\right) \tag{20}$$

Fig. 2 shows that all the forces on the layers are lower than 0.9 MN/m. The force criterion is much lower than the other examples.

5) Comparison: Tab. II compares the different results. Obviously a magnet design is a matter of compromise: (i) The bore field can be maximized to the detriment of the conductor volume, THD and forces. (ii) A very low THD can be reach with a 13 T bore field and reasonable volume and forces. (iii) The volume and forces can be optimized if the THD is not a constraint.

TABLE II
COMPARISON OF DIFFERENT DESIGNS.

Objective	Constraints	B_0 T	V mm^2	THD $10^{-4}B_0$	F_{tot} MN/m
B_0 max.	10 blocks	18.76	11457	229	14.96
V min.	6 blocks	13	6379	175	4.175
V min.	10 blocks, Same height, Harmonics=0.	13	7543	2.5 10^{-3}	3.210
V min.	6 blocks, Same height, Stress≤100 MPa.	13	6433	288	3.585
F min.	10	13	6429	128	2.625

B. Optimization example with various constraints

The following example deals with an optimization that takes into account several constraints. The objective is to minimize the conductor volume of a magnet wound with two pancakes of for layers each. The specifications listed in Tab. I are used. In the case of superconductors, a criterion of stability according to the critical properties must be integrated. Here, a margin of 10%, between the peak field and the critical field is chosen. The critical field is evaluated using a J_c(B,T) fit calculated with the FRESCA2 Nb_3Sn strand [3]. Its properties are a diameter of 1.25 mm, a copper to non-copper ratio of 1.25, and a J_c(12T,4.2K) of 1780 A/mm^2. The temperature is 4.2K and the fit assumes a 10% cabling degradation. With a current density of 260 A/mm^2, taken over the entire layer, this gives a maximum peak field $B_{peak} = 14.35$ T. So the first type of constraints is: $B(x,y) \le B_{peak}, \forall(x,y)$ in the conductor. Nb_3Sn is also very sensitive to transverse pressure, thus a second kind of constraints is proposed: $F_{x,i} \le h_i P_{x,max}$ and $F_{y,i} \le l_i P_{y,max}$, h_i and l_i being respectively the heigth and the length of the bloc i; $P_{x,max}$ and $P_{y,max}$ being the maximum allowed pressures. For accelerator magnets, the field homogeneity is also important. The third kind of constraints is: $X_{By2n} \le 10^{-4}B_0$ for n from 1 to 20. A first quick solution[1] can be obtained modeling each layer with one block. The solution is better than hoped, given that the maximum horizontal pressure is 104 MPa, the maximum vertical pressure is 27 MPa, and the margin is 16%. From this solution, the parameters of the cable used to wind the coil can be determined. The height of the blocks is 23.48 mm, if the width is conserved at 1.82 mm, this corresponds to a cable of 2 by 22 strands. Another option could have been to split the block into to layers, but the equivalent current density would have decreased due to the inter-layer insulation. The total width of the layers gives the number of turns. The "one block per layer" model is coarse, but gives a good and fast approximation. It also can serve as an input for a more accurate model: the "one block per cable" model. A simple computation, just replacing the layer blocks by cable blocks, will give little errors on the specifications. A second optimization is then needed to correct these errors. The current density is adapted to take the insulation into account. All the constraints are finally fulfilled. The results are summarized in Tab. III and the design is shown Fig. 3.

TABLE III
RESULTS FOR A MULTIPLE CONSTRAINTS OPTIMIZATION.

Name	Unit	One block per layer	One block per cable
J	A/mm^2	260	326
THD	10^{-4}B$_0$	2.244	1.779
S	mm^2	6 913	5 684
B$_{peak}$	T	13.38	13.39
Margin	%	16.08	16.02

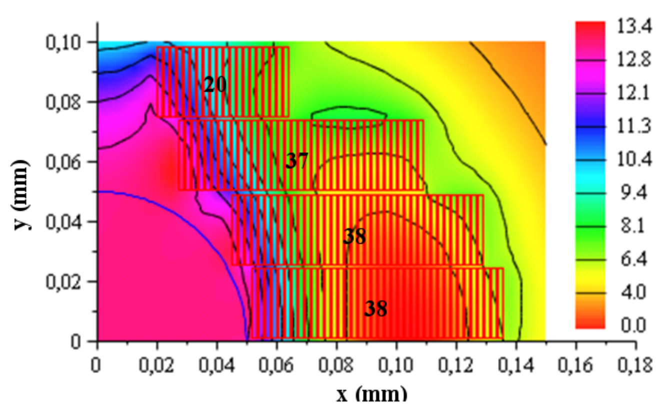

Fig. 3. Optimized design with multiple constraints. Isovalues are indicated in T. Number of turns are indicated on each layer.

C. Grading

In the context of increasing the magnets bore fields, grading seems to be the most promising solution. A block optimization is well adapted to this objective. This section presents an optimized design for a 20 T magnet. A current density of 380 A/mm^2, corresponding to the actual LHC magnets is assumed [7]. The aperture is kept low (40 mm) to reduce the cost and stored energy. Only HTS can withstand the high field around the bore. So Bi2212 is chosen for the very high field area because of its high critical field and its technical maturity. Nb_3Sn blocks with low current density are taken for high field area, and with normal current density for mid field area. For the low field area, a NbTi block is put. A 10% margin is assumed, considering the best conductors available. Different configurations have been tested. The objective is to minimize the conductor cost. Assuming that Nb_3Sn is 4 times expansive and Bi2212 15 times expansive than NbTi, a cost criterion is defined: $Cost = \sum_{i=1}^{N_{blocs}} Cost_i S_i$. With Cost$_i$ being the normalized cost of each conductor and S$_i$ being the surface of each block. The magnetic constraints are a central field of 20 T and $B_i(x,y) \leq B_{peak,i}$, with i the bloc number. Some geometrical constraints are added to get a realistic coil. The results are summarized in Tab. IV and the design is shown Fig. 4.

TABLE IV
RESULTS OF AN OPTIMIZED 20 T MAGNET USING GRADING.

Block	1	2	3	4	5	6
Conductor	Bi2212	Bi2212	Nb_3Sn	Nb_3Sn	NbTi	Nb_3Sn
Normalized cost	1	1	4/15	4/15	1/15	4/15
Current (A/mm^2)	380	380	190	380	380	380
B$_{peak}$ max. (T)	22	22	15	13	7	13

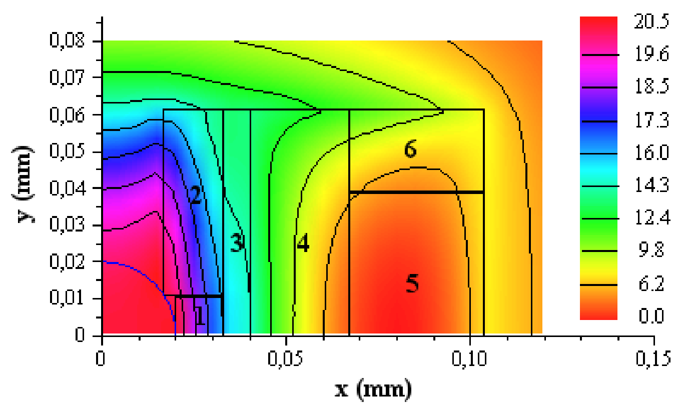

Fig. 4. Optimized design for a 20 T graded magnet. Isovalues are indicated in T. Number are indicated on each block.

IV. CONCLUSION

A series of analytic formulas (field, harmonics, forces) has been developed and integrated in an optimization method, based on a block model. The analytic form allows an accurate and direct evaluation of the harmonics. So there is no need for a Fourier decomposition or a FEM analysis. The few steps of calculation make this algorithm fast. Thanks to the formulas, a lot of various constraints can be handled. Three different extreme possibilities of designs were presented: (i) The algorithm can compute the maximum field achievable in a given space with geometrical constraints. (ii) The first 40 harmonics can

[1]only few minutes with a desk computer (Core^{TM}2 Duo, 2.66 GHz, 3 Go RAM).

be set to zero with 10 blocks and a 13 T bore field. (iii) The blocks can be re-arranged to limit the forces on the conductor. Moreover, thanks to this method, a 13 T Nb_3Sn magnet have been designed, with a margin of 16%, equivalent pressures lower than 150 MPa and harmonics lower than $10^{-4}B_0$. A 20 T magnet with grading and different supercondutors has also been designed, optimizing the conductor cost and considering 10% margins. Accelerator magnet designers can thus use this versatile algorithm to design the 2D geometry of any racetrack magnet, taking into account different kinds of constraints. A similar method should also been developed for sector coils, using existing formulas.

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