STUDIES OF DIPOLE FIELD QUALITY FOR THE BETA-BEAM DECAY RING*

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Abstract

The aim of the beta-beams is to produce highly energetic beams of pure electron neutrino and anti-neutrino, coming from β-decays of the $^{18}Ne^{10+}$ and $^{6}He^{2+}$, both at $\gamma = 100$, directed toward experimental halls situated in the Fréjus tunnel. The high intensity ion beams are stored in a ring until the ions decay. The β-decay products have a magnetic rigidity different from the one of the parent ions and are differently deflected in the 6 T superconducting dipoles. Consequently, all the injected ions are lost anywhere in the ring, generating a high level of irradiation. So, the dipole apertures need to be large enough to avoid the decay products hitting their walls, which may worsen the field quality.

A study on its tolerances has been carried out. Since the decay ring has to accept the beam during a large number of turns, the chosen criteria is the size of the dynamic aperture that the multipolar defects in the dipoles may shrink. Tolerances on the systematic errors of these defects have been investigated. In order to relax the tolerances, a routine was written which automatically enlarges the dynamic aperture in presence of field errors.

INTRODUCTION

In [1], a first optics of the decay ring was presented. Six sextupole families in the decay ring were needed in the arcs in order to correct the chromaticity and to enlarge the dynamic aperture. The enlargement of the dynamic aperture was realized with *BETA* [2] by minimizing the third order resonance terms and the first derivatives of the tune with the amplitude. Finally, the obtained dynamic aperture is given in Fig. 1. The dynamic aperture is then large enough to accept the stored beam in the decay ring.

Nevertheless, the structure was assumed with perfect magnetic elements. In fact, since the dipole half-aperture is large (around 8 cm), the design cannot avoid strong multipole components inside. So, it is necessary to study the impact of these non linearities on the beam dynamics and thus on the dynamic aperture.

DEFECTS IN THE DIPOLES

A first estimation of the systematic multipolar components in the superconducting dipoles in the beta-beam decay ring is given in Table 1 [3]. The multipolar components b_n in the 6 T dipoles of angle $\theta = \pi/86$ rad are calculated at a reference radius R_{ref} of 60 mm. In the third column, the integrated field K_nL of each multipolar component in the dipole has been calculated according to the Eq. 1:

$$K_nL = \frac{b_n\theta}{R_{\text{ref}}^{n-1}} \tag{1}$$

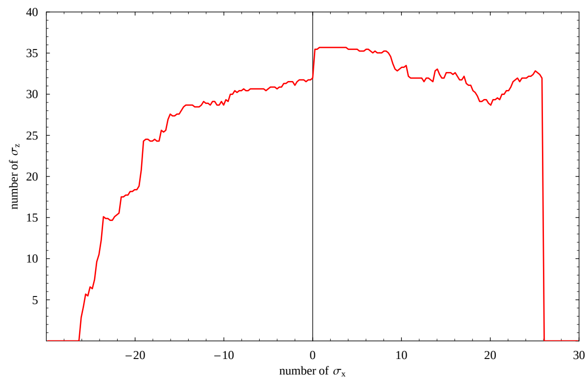

Figure 1: Dynamic aperture for 1000 turns without a defect in the dipoles. The horizontal and vertical emittances are taken equal to 0.22 π mm.mrad

Table 1: Multipolar components in the dipoles of the arcs. The reference radius R_{ref} is taken equal to 60 mm

Multipole order n	$\mathbf{b_n}$ (10^{-4})	$\mathbf{K_nL}$ (m^{1-n})
1 (main field)	10^4	π/86 rad
2	0	0
3	-1.685	-0.00171
4	0	0
5	33.018	9.307
6	0	0
7	-50.123	-3924.5
8	0	0
9	29.583	643400

In order to estimate the influence of each multipole component on the beam dynamics, the dynamic aperture has been plotted for the same structure as in [1] by considering only one of the multipolar components each time. The results are given on Fig. 2. The multipole components have been simulated by the insertion of multipolar lenses into the dipoles. The dipoles are assumed to be curved. Therefore, the reference orbit passes by the centre of the lenses. As expected, the dynamic aperture shrinks in presence of the multipolar defects and the components which have the largest effect are $n = 5$ and $n = 7$. The sextupole components in the dipoles do not significantly decrease the dynamic aperture. Its interior radius in presence of the mul-

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tipolar defects in the dipoles is less than 6 σ, which is not enough for the allowed loss amount. Therefore, it is necessary to enlarge the dynamic aperture.

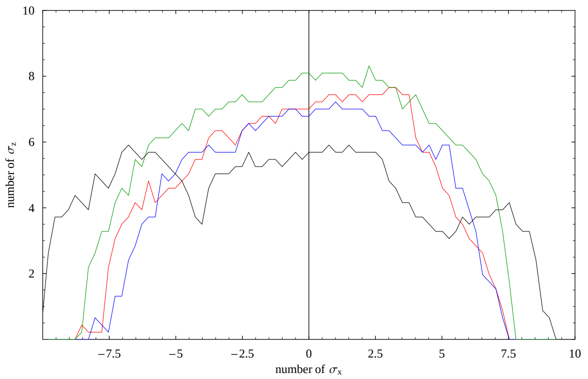

Figure 2: Dynamic aperture for 1000 turns for the same structure as in [1]. In red, $n = 5$ component, in blue $n = 7$ component and in green $n = 9$ component, in black with every systematic multipolar component

ENLARGEMENT PROCEDURE OF THE DYNAMIC APERTURE

The presence of high order multipole defects makes the compensation of the third order resonances and of the first derivatives of the tune with the amplitude insufficient. The first idea would be to compensate the higher order derivatives by introducing octupole and decapole lenses in the ring like in [4]. However, the cancellation of the derivatives of the tunes at the origin is not so valuable for ions with large amplitudes. The combination of the different chromatic sextupoles generates high order harmonics which could compensate these multipolar defects. Therefore, before inserting compensating multipole lenses, it would be interesting to use the chromatic sextupoles in the decay ring to enlarge the dynamic aperture. The used algorithm is presented in [5]. The principle is described below:

1. The user chooses the sextupole families to be considered and the number of iterations N in order to correct the chromaticity. Since the program lies on the progressive correction of the chromaticity, the used sextupoles must be in dispersive areas. The program stores the initial chromaticity of the lattice in each plane $\xi_{0,y}$ (with $y = x$ or z).
2. Only two sextupole families are needed to obtain any wanted chromaticity. Therefore, at each step i, the algorithm considers every sextupole couple and partially corrects the chromaticity to reach the value $\xi_{i,y} = \frac{N-i}{N}\xi_{0,y}$ in each plane. It calculates the dynamic aperture once. The couple which is kept for the next step is the one for which the dynamic aperture is the largest (or the interior radius).
3. The best sextupole couple is then applied and the program increments i by 1. It starts again the previous step until i is equal to N.

Finally, the chromaticity has been corrected and normally the dynamic aperture has been enlarged.The biggest advantage of this algorithm is to be automatic and quite quick, which enables to run the optimization procedure on a large number of structures. The behaviour with the momentum can be taken into account by adding the calculation of the dynamic aperture at different momenta during the step 2 of the algorithm and by choosing the sextupole couple according to the results.

CHOICE OF THE WORKING POINT

The different multipole components in the dipoles must be taken into account in order to choose the working point. Initially, the working point ($\nu_x = 22.228$, $\nu_z = 12.16$) was chosen to avoid the second and third order resonances. However, it is very near a fifth order coupling resonance, which can explain the strong decrease of the dynamic aperture. Therefore, other working points like ($\nu_x = 22.18$, $\nu_z = 12.16$) and ($\nu_x = 22.22$, $\nu_z = 12.24$) were tested because quite far from the different resonances (Fig. 3). In the following, we will call respectively each of these working points A, B and C. The algorithm described sooner was then applied at each configuration. Moreover, in order to have more freedom degrees, two sextupole families were added in the arcs.

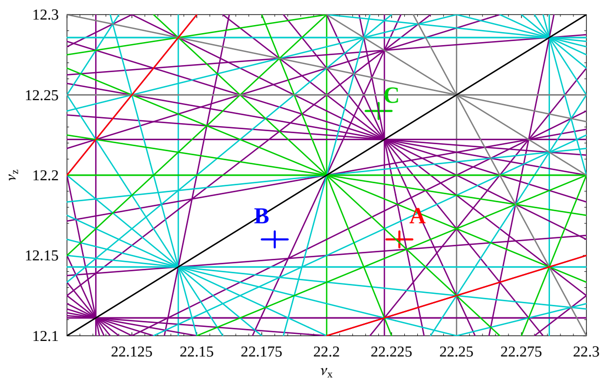

Figure 3: Tune diagram. In black, 2^{nd} order resonances, in red 3^{rd} order resonances, in gray 4^{th} order resonances, in green 5^{th} order resonances, in cyan 7^{th} order resonances and in purple 9^{th} order resonances. The different working points are represented by crosses on the diagram

The dynamic aperture obtained in the three cases is represented on Fig. 4. Compared to Fig. 2, the dynamic aperture has been enlarged. The interior radius is superior to 6 σ for the new working points. However, since the random errors or other sources of non linearities in the structure were not taken into account, it is still too small to enable the storage of the ions up to 6 σ. Moreover, the momentum acceptance shrinks a lot for the working point C because the tune strongly varies with the momentum for this working point (Table 2). Besides, the multipole components excite non destructive coupling resonances. Therefore, there is a motion transfer between the horizontal and vertical planes. The transportation of the ions is still stable but their maximum amplitude and thus the beam size in each plane may increase (Fig 5). Finally, the best compromise seems to be the working point B.

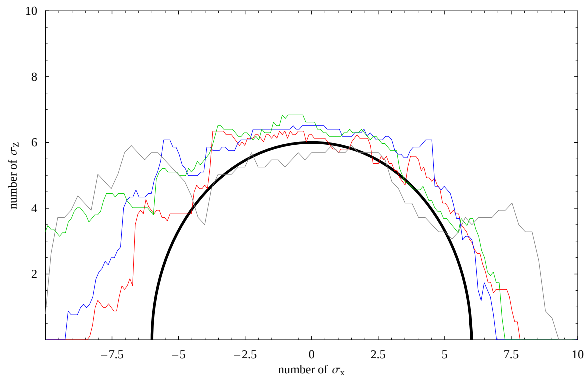

Figure 4: Dynamic aperture at 1000 turns for different working points after optimization (in gray for A before optimization, in red after, in blue for B and in green for C)

Table 2: Momentum acceptance of the decay ring for different working points with the multipolar defects in the dipoles given in Table 1

(ν_x, ν_z)	$\delta_{\text{min}}\ (10^{-3})$	$\delta_{\text{max}}\ (10^{-3})$
$(22.228, 12.16)$	-8	8.2
$(22.14, 12.16)$	-5.1	7.5
$(22.22, 12.24)$	-3.5	8.8

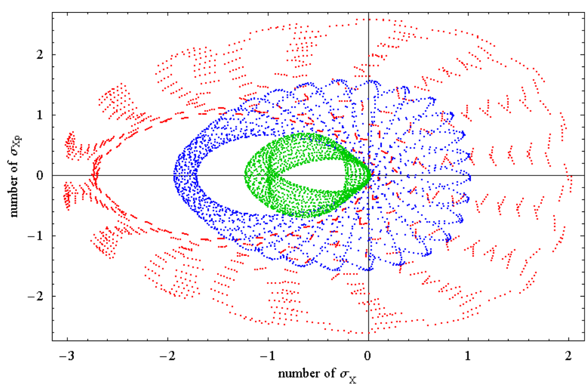

Figure 5: Representation in the horizontal normalized plane of the successive positions of an ion of the stored beam at $\delta = 0$ and with an initial amplitude $\left(x = 0,\ z = 5\sqrt{\beta_z \epsilon_z}\right)$ for different working points after optimization (in red for the working point A, in blue for B and in green for C)

Another solution to enlarge the dynamic aperture would be to decrease the multipole components in the dipoles by increasing the design constraints. On Fig. 6, the dynamic aperture for different multipole defect values is plotted. Almost 1 σ can be gained by dividing the multipoles components given in Table 1 by 1.5.

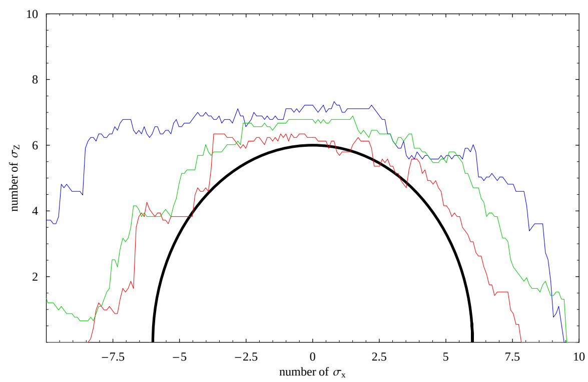

Figure 6: Dynamic aperture at 1000 turns for the working point A for different multipole defect values after optimization (in red for the reference values given in Table 1, in green for the reference values divided by 1.5 and in blue for divided by 2

CONCLUSION

A first estimation of the impact of the multipolar defects inside the decay ring dipoles on the beam dynamics was made. The dynamic aperture dramatically decreases, principally due to the $n = 5$ and $n = 7$ components. An automatic algorithm of the enlargement of the dynamic aperture using the chromatic sextupoles was implemented in *BETA* [2]. Different working points were compared with their dynamic aperture. Finally, the dynamic aperture has effectively been enlarged but the gain was not sufficient to keep the whole beam in the ring up to 6 σ. The insertion of higher order multipoles such as octupoles or decapoles could become useful in the future, which needs further studies. Otherwise, the constraints on the field quality in the dipoles will have to increase.

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