CP VIOLATION : A MATTER OF (ANTI)GRAVITY ?

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Abstract

We reexamine the possibility that CP violation observed in the neutral kaon system may be explained by "antigravity" —a different acceleration of matter and antimatter in a gravitational field. We criticize the impossibility arguments which led, thirty years ago, to the rejection of "antigravity". In particular, the reexpression of the Good argument suggests a simple approximate expression for ε , the CP violating parameter. Studying the compatibility of "antigravity" with the first law of thermodynamics provides a link with the Unruh and Bekenstein-Hawking effects. Requiring the compatibility with the second law indicates that the fundamental quantity in this problem is entropy or information. Some predictions are presented and two experimental tests are proposed.

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Introduction

The origin of time-asymmetry and the "master arrow of time" is still an open question, despite considerable theoretical effort. In their reviews devoted to timeasymmetry, Davies [1], Penrose [2] and Zeh [3], considering the various arrows of time — decay of the K^o-meson, quantum-mechanical observations, general entropy increase, retardation of radiation, psychological time, cosmological expansion, black holes versus white holes — all conjecture that gravitation must be at the origin of the "master arrow of time". The only evidence for a microscopic time-asymmetry — the neutral kaon system— then appears curiously isolated since, unlike the other "arrows", it incorporates, in the context of local field theories, neither an absolute past-future distinction [4], nor a means to explain by itself the predominance of matter in the Universe, as was first stressed by Sakharov [5]. Following Penrose [2], "it is hard to believe that Nature is not, so to speak, 'trying to tell us something' through the results of this delicate and beautiful experiment [6], which has been confirmed several times."

In this letter, we propose to reconsider the possibility that CP violation observed in the neutral kaon system can be explained by a violation of the equivalence principle "à la Morrison" [7]. We criticize the historical argument of Good [8] using the neutral kaon system to exclude "antigravity". Reexpressing this argument, we show that "antigravity" would provide, on the contrary, just the amount of anomalous regeneration that we call CP violation. This fascinating numerical coincidence does not constitute a theory, and we consider next the classical impossibility argument raised against Morrison's antigravity that it must violate energy conservation. We show qualitatively that the evaporation of the vacuum induced by antigravity appears to be related to the Unruh and the Bekenstein-Hawking effect, which is known to introduce the second law of thermodynamics (and therefore time-asymmetry) in the realm of general relativity. It then remains to be seen whether antigravity might allow a Maxwell-Wheeler demon [9] to violate the second law of thermodynamics. Using the Kolmogorov-Sinai entropy, we find some confirmation that such a violation is impossible. Finally, we consider two experiments that may test the idea that CP violation in the kaon system is due to "antigravity".

Antigravity and the impossibility arguments

Proposed by Morrison [7] in a celebrated paper, antigravity is known to violate the sacrosanct CPT symmetry, contradict the results of the Eötvös-Dicke-Panov experiments [10], exclude the existence of the long-lived component in the neutral kaon system in the presence of the Earth gravitational field [8], violate energy conservation and imply vacuum instability [7,11]. The arguments against antigravity then appear to be so compelling that it may seem presomptuous or even foolish to reconsider them [12]. However, Nieto and Goldman have recently reviewed critically these arguments [13] and we refer the reader to their recent review for a thorough discussion and an historical perspective on these impossibility arguments (see also Ref. 14). Here, we will only insist on the points necessary to the following discussion and on those parts of Nieto and Goldman's review with which we disagree. Concerning the first two impossibility arguments, let it suffice to say that the CPT theorem has not generally been demonstrated on curved spacetime and that the ineluctability of a past singularity imposed by the theorems of Penrose and Hawking [15] make it doubtful that the CPT theorem can ever be demonstrated without modification for gravitation [16,17]. Similarly, Goldman and Nieto have repeatedly stressed [18] that Schiff's argument on the Eötvös-Dicke-Panov experiments [10] is invalid because of his incorrect renormalization procedure. Attempts have been made to consider some adjustable vector interaction which, added to the tensor (and therefore always attractive) interaction dictated by general relativity, would lead to a violation of the equivalence principle applied to matter and antimatter. This arbitrariness is aesthetically objectionable, but Morrison's original antigravity [7] appears even worse : Goldman and Nieto themselves reject the possibility of such a gross violation of the equivalence principle where antimatter would "fall up", the total force on a static $e^+ e^-$ pair, e.g., being zero, since it would lead to a violation of energy conservation. We will come back later to the argument of energy non-conservation and first turn to Good's argument which appears to impose the most stringent constraint on antigravity.

The Good argument

In 1961, three years before the first experimental observation of CP violation, Good [8] observed that antigravity would impose that the K_L, a linear combination of K° and $\overline{K^{\circ}}$, would regenerate a K_S component. Good estimated the phase shift which would develop between the K° and $\overline{K^{\circ}}$ components from the energy difference due to the gravitational potential ϕ_G ; he supposed that the phase factor between the two components would oscillate as

$$\exp(2i m_K \phi_G t/h)$$
.

Noticing that the potential energy of a kaon in the Earth gravitational field is ≈ 0.4 eV, and that this energy is 10^5 larger than the energy splitting between the K_S and K_L eigenstates, Good concluded that antigravity was excluded. This argument suffers, however, from a severe criticism : as Good himself had noticed, there is no obvious reason why one should use the Earth potential; why not use instead the Sun, or the Galactic potential which would give even more stringent limits on the difference of acceleration between matter and antimatter ? In fact, as noted by Goldman and Nieto [18], gauge theories teach us that potential themselves are not observable, but only potential differences. The counterexample to the Good argument of an e⁺ e⁻ pair created in a strong electric field shows that the phase difference between the two particles builds up with their separation instead of being created instantaneously [13].

Let us then try to restate the Good argument independently of absolute potentials. A kaon has a rest energy of ≈ 500 MeV, its size is then $\approx \hbar/m_K c \approx 0.4$ fm. If there is a mechanism which separates the s quark from the \overline{s} quark (and u from \overline{u}) by more than the size of the kaon during the mixing time $\Delta \tau$ introduced by weak interactions, which continuously transform $s \leftrightarrow \overline{s}$, $\Delta \tau = a$ few ($\hbar/\Delta mc^2$), a large K_S component will be regenerated. Assuming antigravity, let us estimate the time needed for the s quark to separate from the \overline{s} to a distance that would induce a regeneration of the K_S component such as observed in CP violation. This time is given by the equation

$$gt^2 \approx \varepsilon \times (kaon \ size) = \varepsilon \times hm_K c.$$
 (1)

where $\boldsymbol{\epsilon}$ is the CP violating parameter. This gives numerically

$$t \approx 3 \ 10^{-10} \ s \approx 1.7 \ h\Delta mc^2$$

In other words, the time needed for antigravity to generate the amount of regeneration observed in CP violation is just about equal to the mixing time imposed by weak interactions. Expressed independently of absolute potentials, the Good argument, far from excluding the possibility of antigravity, provides an indication that CP violation may be explained by some kind of antigravity ! Replacing t in [1] by $\hbar/\Delta mc^2$ suggests the following approximate expression for the ε parameter

This expression has a simple physical interpretation : it means that the energy to lift a kaon by $\Delta z = \hbar/\Delta m c$ is of the order of $\varepsilon \Delta mc^2$. Note that this expression for ε is the only dimensionless quantity linear in the kaon mass which can be built from the above quantities; Good himself would probably have proposed the previous expression for ε if CP violation had been observed at that time. That the quantity $\hbar m_K g / \Delta m^2 c^3$ is of the order of the experimental value of ε has also been noted by Fischbach [19]. For a different expression of the argument, see also Ref. 14.

In their review, Nieto and Goldman [13] use the same potential independent argument but fail to obtain this result since they consider the spatial separation of a quark and antiquark over the whole beam line length of a kaon beam. This is incorrect since the proper time involved is much larger than the oscillation time of a few $h/\Delta mc^2$ imposed by weak interactions. Therefore, Nieto and Goldman obtain a separation distance which greatly exceeds the kaon size. They then argue that, even with such a separation, no anomalous regeneration would be observed due to the quantum-mechanical spread of the kaon wavepacket but this statement is in contradiction with measurements and calculations on neutrons [20, 21]. Recently, Kenyon [22] has also revived the Good argument but fails to notice the problems raised by the introduction of absolute potentials.

Energy (non-)conservation and vacuum instability

As noted previously, a classical argument against Morrison's antigravity, devised by the author himself [7], is that energy conservation must be violated by this process. But now, from Noether's theorem, a conserved quantity named energy can only be defined whenever invariance under time translations is respected. Any process defining an "arrow of time" may then forbid the construction of energy as a conserved quantity (instead, as we shall see, *entropy* usually becomes the relevant quantity). Therefore, an apparent violation of energy conservation is not unexpected if antigravity defines an arrow of time. This remark can be illustrated by the Bekenstein-Hawking radiation of black holes [22, 23] : when trying to find a static solution for a system including a black hole, it is found that no such solution can be found (strictly speaking, it is possible to find one particular size of the box containing a black hole for which a static solution is found [24]); instead, a flux of radiation is seen to emerge from the black hole which behaves like a grey body with a temperature $k_B T_{bh} = hg/2\pi c$, where g is the surface gravity of the black hole and k_B is the Boltzmann constant. A second example is provided by the Unruh effect [25], related to the previous effect : a uniformly accelerated detector in vacuum measures a nonzero temperature proportional to its acceleration, $k_B T_u = hg/2\pi c$. Here also, it may seem at first sight that there is violation of energy conservation [26] : a detector is excited and there is emission of photons, but the Unruh effect has been shown to be similar to the anomalous Doppler effect [26].

The apparent violation of energy conservation by antigravity can be evidenced with the aid of a gedanken experiment invented by Wigner [11]. The principle of the gedanken experiment can be summarized in the following way : assuming antigravity, let us consider an $e^+ e^-$ pair at rest in a constant gravitational field of intensity g at an altitude $z=z_0$.

Transporting this $e^+ e^-$ pair at a lower altitude $z=z_1$ can be done in two ways : we may choose to transport separately both the electron and the positron in the gravitational field, at zero energetic cost if antigravity is assumed and if the transport is made adiabatic. But we have also the possibility to transport the pair of particles in a different way: we may first annihilate the $e^+ e^-$ pair into two photons, propagate downwards the two photons to the altitude $z=z_1$, and there recreate the $e^+ e^-$ pair. For this purpose, we

use the ellipsoid depicted in Fig.1. Initially, each annihilation photon has an energy mc^2 , where *m* is the electron mass and *c* the velocity of light. After the fall in the gravitational field, however, it is known, from the experiments of Pound and Rebka [28] and Vessot and Levine [29], that the two photons have gained an energy $mg(z_0 - z_1)$. This means that recreating the e⁺ e⁻ pair at the lower altitude $z=z_1$ would result in an excited state and, for small values of $mg(z_0 - z_1)$, in releasing a deexcitation photon of this energy.

It then appears that we have the possibility to realize a cyclic process resulting in the extraction of a photon from the gravitational field. Namely, an $e^+ e^-$ pair is annihilated at the altitude $z=z_0$, the annihilation photons transported to the lower altitude $z=z_1$ where a pair is recreated in an excited state (emission of a photon) and the pair is carried back, at zero energetic cost, at the altitude $z=z_0$.



Fig.1: Schematic representation of a cyclic process in which a photon is extracted from the gravitational field.

This macroscopic argument (the construction of a perpetual engine) has a microscopic counterpart which results in the instability of the vacuum in the presence of a gravitational field, even if there are no "real" particles present. Let us find the typical energy of photons created in virtual creation-annihilation processes in the vacuum. From the Heisenberg inequalities, a virtual creation-annihilation process will probe spacetime over a length scale $\Delta z \approx \hbar/mc$, where *m* is the mass of the propagated particle. Assuming antigravity, the typical energy of a photon which can be extracted from the

gravitational field is then: $\Delta E \approx m g \Delta z \approx hg/c$ (this quantity is independent of the mass of the propagated particle). The associated wavelength is $\lambda \approx c^2/g$, approximately one light-year at the surface of the Earth, and is therefore almost unobservable. The fundamental point in our argument resides in the fact that, as we have noted previously, a similar expression of a "vacuum temperature" enters the Bekenstein-Hawking radiation of black holes. *The vacuum instability induced by antigravity then appears very similar to the very effect which introduces the second law and time-asymmetry in the realm of general relativity*.

Clearly, the thermal character of the radiation in the case of antigravity has not been demonstrated by our previous argument, but it can be qualitatively justified by the following remark : Boyer [30] had noted that if one applied "naively" the equivalence principle, there was an irreducible lowest temperature attainable for any experiment at the surface of the Earth. This temperature would be given by the Unruh expression $k_B T_u = \frac{hg}{2\pi c}$, since an object at rest on the Earth does not follow its geodesic path and is therefore accelerated. This argument has been criticized by several authors [31, 32] who show that, assuming general relativity and quantum field theory on curved spacetime, such a disaster does not occur which would prevent any really static situation to exist (since the thermal character of the excitation would decohere a system initially in a pure state). If the conventional viewpoint is adopted, the "naive" expression of the equivalence principle is lost and this is now the object following its geodesic path which measures a non zero temperature given by the Unruh expression. It then becomes possible to determine not only the Riemann curvature but also g by quasi-local experiments in free fall reference frames. Antigravity is just the tidal effect on the vacuum needed to induce the temperature suggested the "naive" expression of the equivalence principle first stated by Boyer [30].

Compatibility with the second law of thermodynamics

In Fig.1, our gedanken experiment is used to extract photons from the gravitational field and it represents the macroscopic version of the evaporation of the vacuum. It seems at first sight that the machine can be reversed and the process used to gradually remove photons initially present in the enclosure. If such a cyclic process could be realized at no cost, it would violate the Second Law of thermodynamics. We want here to suggest that when the information cost of following the annihilation photons on curved spacetime is taken into account, the process cannot be used to violate the Second Law.

Whereas for an integrable dynamical system two neighbouring orbits diverge linearly with time, in a chaotic system this divergence is exponential, resulting in an information loss rate which can be quantified by the Kolmogorov-Sinai (K-S) entropy [33], conjectured by Pesin [34] to be related to the sum of the positive Lyapounov exponents. It is well known that the orbits of photons in a Schwarzschild geometry are integrable, so that no K-S entropy takes place. Thus, it seems at first glance that the transport of photons can be ignored in the entropy balance. In fact, in the previous gedanken experiment, the orbits are not infinite but piecewise due to the reflexions needed to reconstruct the original $e^+ e^-$ pair. Performing the proposed experiment in the most efficient way requires a *passive* focusing system such as the ellipsoid of Fig.1 where the source of positronium is at the lower focus and the $e^+ e^-$ pair is reconstructed at the upper focus. From the point of view of dynamical system theory, we are left with a billiard enclosing a volume with negative curvature.

The properties of billiards as dynamical systems have been widely investigated [35]. The main point is that a small deviation from a billiard with a regular shape without chaos often leads to a chaotic billiard [36]. Here, the billiard without entropy production is the ellipsoid and the perturbation is the curvature of the piece of Schwarzschild metric enclosed by the focusing billiard. Whereas the backfolding of

trajectories is the ultimate source of stochasticity, the Lyapounov divergence will be dominated by the curvature.

Imagine a Maxwell-Wheeler demon trying to violate the Second Law using antigravity; he would have to take into account the following constraints

— the Heisenberg inequalities impose that the box be large enough to accommodate the wavelength of the photons that he is trying to remove

$$\Delta z_{Heis} \ge \sqrt{\frac{\hbar c}{2mg}} = r \sqrt{\frac{r_c}{r_s}} \quad (3)$$

noting $r_s = 2GM/c^2$, and $r_c = \hbar/m_e c$, the Schwarzschild and Compton radius, and *M* the mass of the gravitating object.

— the typical path length defining the Lyapounov exponent for the annihilation photons, given by the Riemann curvature,

$$\Delta z_{Lyap} = O(1) \sqrt{\frac{c^2 r^3}{GM}} = O(1) r \sqrt{\frac{r}{r_s}}$$
(4)

This parameter means that, given a semi-classical orbit of length *l*, the information cost is approximately given by $l/\Delta z_{Lvap}$ (in bits).

When trying to reset the system, our demon should also care about the limited lifetime of the positronium $\tau_{\text{pos}} = 2r_c/c\alpha^5$ (for a disintegration in two photons) and, due to the Unruh effect, always manipulate it with an acceleration much less than

$$g_{max} = \frac{c^2}{\Delta z_{Heis}}$$
 (5)

In any case, it seems that using an ellipsoid with the minimal size imposed by Heisenberg inequalities, of typical volume $(\Delta z_{Heis})^3$, our demon would have to pay an information cost (in bits) to eradicate a photon in the fundamental mode (approx. one bit)

$$\Delta z_{Heis} \frac{(\Delta z_{Heis})^3}{(\alpha r_c)^3} / \Delta z_{Lyap} = \frac{r^{5/2}}{r_c r_s^{3/2} \alpha^3} > 1 \quad (6)$$

if one uses the two obvious conditions that $r \ge r_c$ and $r \ge r_s$.

Although we have been unable to build an information based conserved quantity and our argument remains contrived and unsatisfying, it suggests that the Second Law can be preserved when the dynamical information cost is taken into account.

Experimental tests

The previous three arguments suggest that we should reconsider the possibility of antigravity; they provide a new motivation for two types of experiments : firstly, direct measurements of the gravitational mass of antiparticles. The PS-200 experiment [37], at the Low-Energy Antiproton Ring (LEAR) at CERN, currently in the stage of installation, proposes to measure, with a precision of the order of 1%, the difference between the gravitational masses of proton and antiproton. However, since the quarks inside the proton carry only a small fraction of its mass, the most clear-cut test of an explicit violation of the equivalence principle would be measuring the gravitational mass of the positron, a tremendously difficult measurement advocated by Fairbank and Witteborn [38].

A second set of experiments involves the most precise interferometric system at our disposal : the neutral kaon system. Even in the absence of a consistent theoretical formalism, the approximate expression that we have proposed for ε suggests the measurement of this parameter in new experimental conditions. Indeed, *all the precision measurements of the \varepsilon parameter have involved ultrarelativistic horizontal kaon beams*,

with one notable exception. To our best knowledge, the only experiment for which the angle θ with the horizontal is such that $\gamma \sin \theta$ is not negligible compared to 1 (or, equivalently, for which the energy difference associated with the change in altitude of a kaon during the mixing time of weak interactions is comparable to $\varepsilon \Delta m c^2$) has led to a determination of the η_{+-} parameter equal to $(2.09 \pm 0.02) \ 10^{-3}$, in disagreement by 9 standard deviations from the world average [39]. In this last experiment, the neutral beam used made an angle of $6.25 \ 10^{-3}$ rad with the horizontal, at a typical energy of 70 GeV (average $\gamma \approx 150$). It seems therefore crucial to test whether this non standard result is just due to a coincidence or whether the ε parameter really depends on the direction and momentum of the neutral kaon beam relative to the Earth. In particular, the PS-195 experiment [40], also at LEAR, and currently taking data, should, for the first time, determine with a precision better than 1% the ε parameter for an isotropic kaon "beam" and using kaons in the momentum range [400, 700] MeV/c, generated from the annihilation at rest of a proton with an antiproton.

It should also be clear that our hypothesis requires that the ε' parameter is zero. In addition, the ε parameter in the B system is predicted to be of the order of $10^{-5}-10^{-6}$, from the $m/\Delta m^2$ scaling. These two predictions differ from the Standard Model predictions and provide two further tests of our hypothesis.

Conclusions

Following Goldman and Nieto [18], we have reconsidered the impossibility arguments against antigravity. Unlike them, we have tried to rehabilitate a form of antigravity where an apparent violation of energy conservation would be associated with entropy production and therefore a past/future distinction. Our approximate expression of the ε parameter, obtained from the reexpression of the Good argument, the link with the Unruh and Bekenstein-Hawking effects, and the perspective of defining a conserved quantity based on information which would relate the entropy produced by a gravitating system (through its thermal emission) to a dynamical entropy, hint to the possibility that "CP" violation in the kaon system may be explained by antigravity. The fundamental quantity then becomes entropy or information, an idea which follows naturally from the Bekenstein entropy formula [23], and strongly reemphasized by Wheeler [41]. Clearly, there remains to establish whether a relativistic viable expression can be obtained, but it is hoped that the perpective of defining a single arrow of time due to gravity and the fascinating possibility to test this idea through the properties of the kaon interferometric system may compensate, at least partially, the qualitative nature of our arguments.

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