Towards a new model of atmospheric tides: from Venus to super-Earths

Pierre Auclair-Desrotour

DDays – Journées des thésards,
July 3rd and 4th

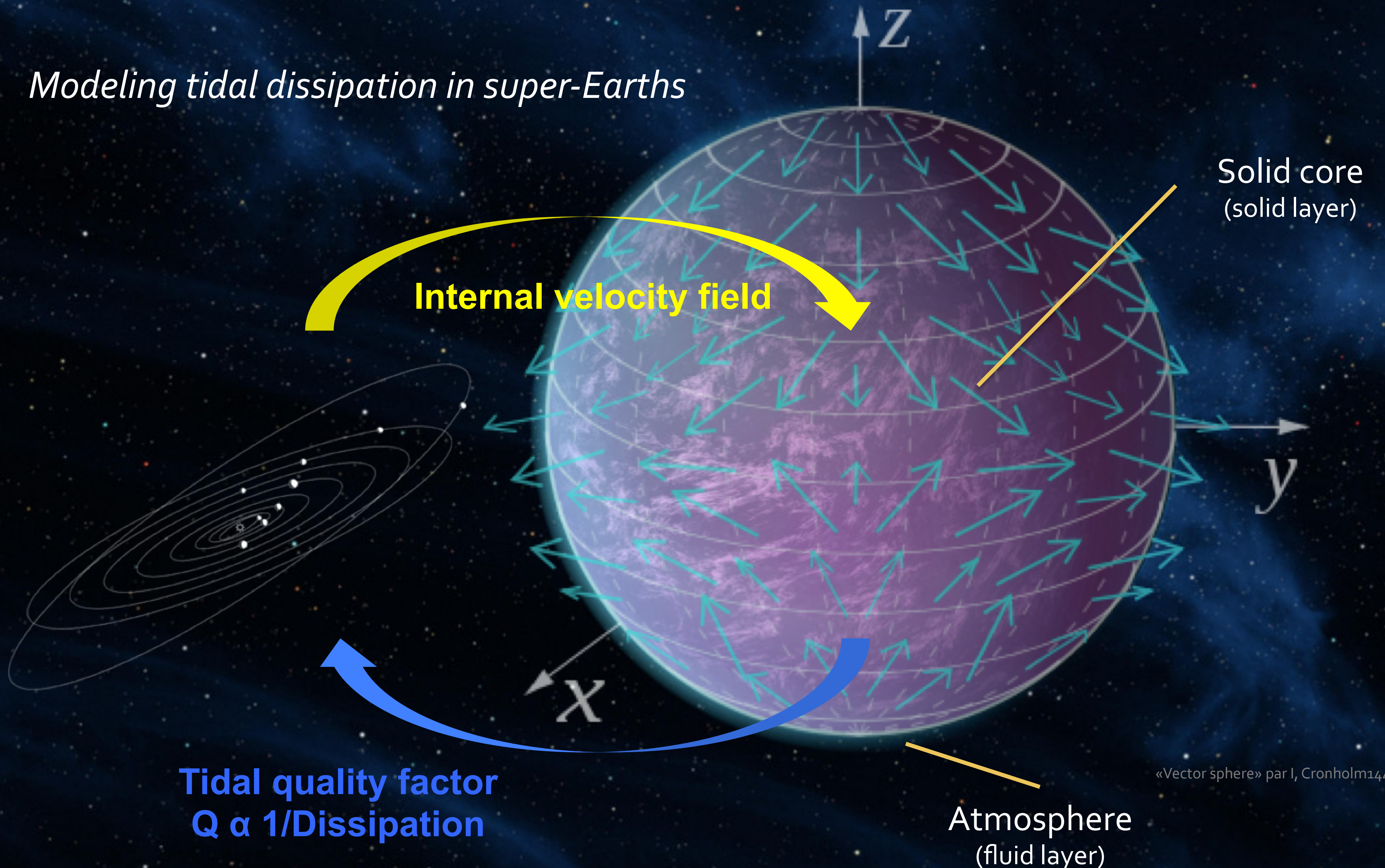
Phd advisors: Jacques Laskar (IMCCE/Observatoire de Paris),
Stéphane Mathis (AIM/CEA)

Introduction

- Auclair-Desrotour Pierre
- <u>University course:</u>
  - Ecole des Ponts et Chaussées, ParisTech, *Department of Mechanical Engineering*
  - *Master of Astronomy & Astrophysics* of Observatoire de Paris, specialty *Gravitational Systems Dynamics*
- <u>Contact:</u> Master's thesis
- <u>Motivations:</u>
  - Interdisciplinary topic valorizing knowledge acquired in engineering school and master
    - ➢ solid and fluid mechanics, celestial mechanics, astrophysics culture, scientific computing
  - Theoretical physics problem
  - Dynamic teams
  - Research training

Introduction

Modeling tidal dissipation in super-Earths



The revolution of exoplanets

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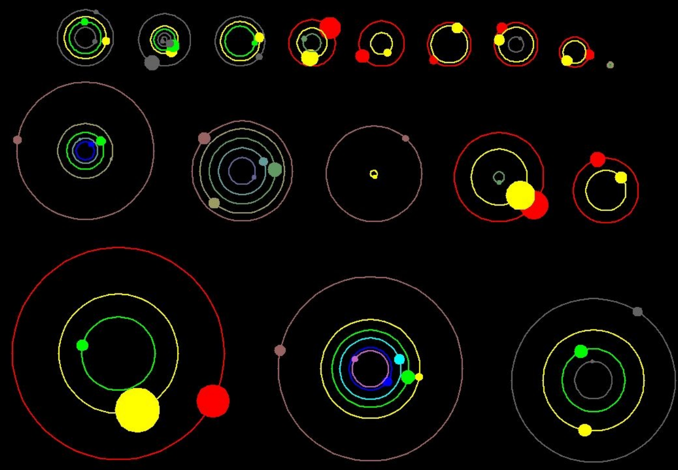

Fabricky et al. (2012)

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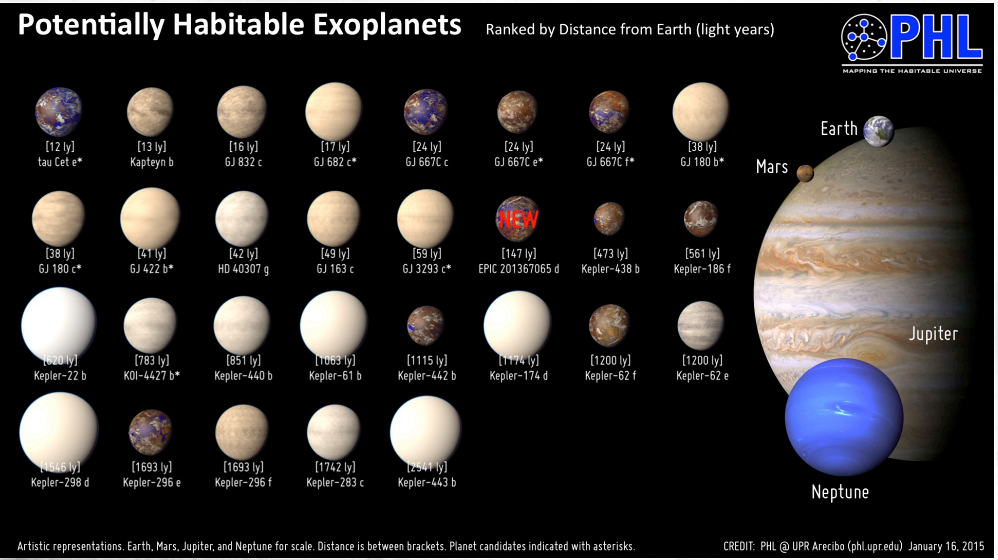


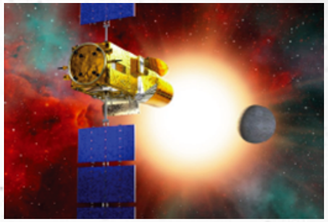
- **Orbital dynamics:**
  - Semi-major axis
  - Eccentricity
  - Orbital inclination
- **Rotational dynamics:**
  - Obliquity
  - Rotation (magnetic dynamo)
  - Internal heating (evolution)

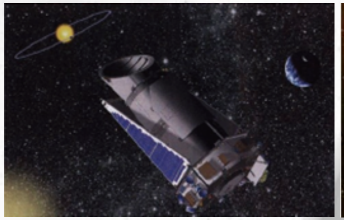
Affected by tidal effects

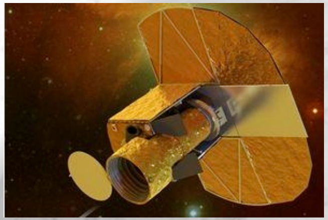
Tidal interactions must be understood and quantified!

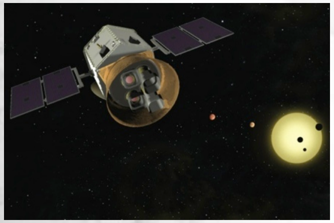
State of the art

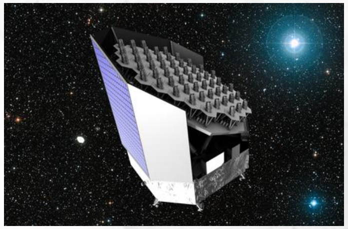
A defined observational roadmap


CoRoT (2006)


Kepler (2009)


CHEOPS (2017)


TESS (2017)


PLATO (2024)

Tidal dissipation little understood and poorly quantified!

Recent important theoretical progresses:

→ **Fluid layers**

e.g. Remus, Mathis & Zahn (2012) ; Ogilvie & Lin (2004) ; Ogilvie (2009 – 2013)

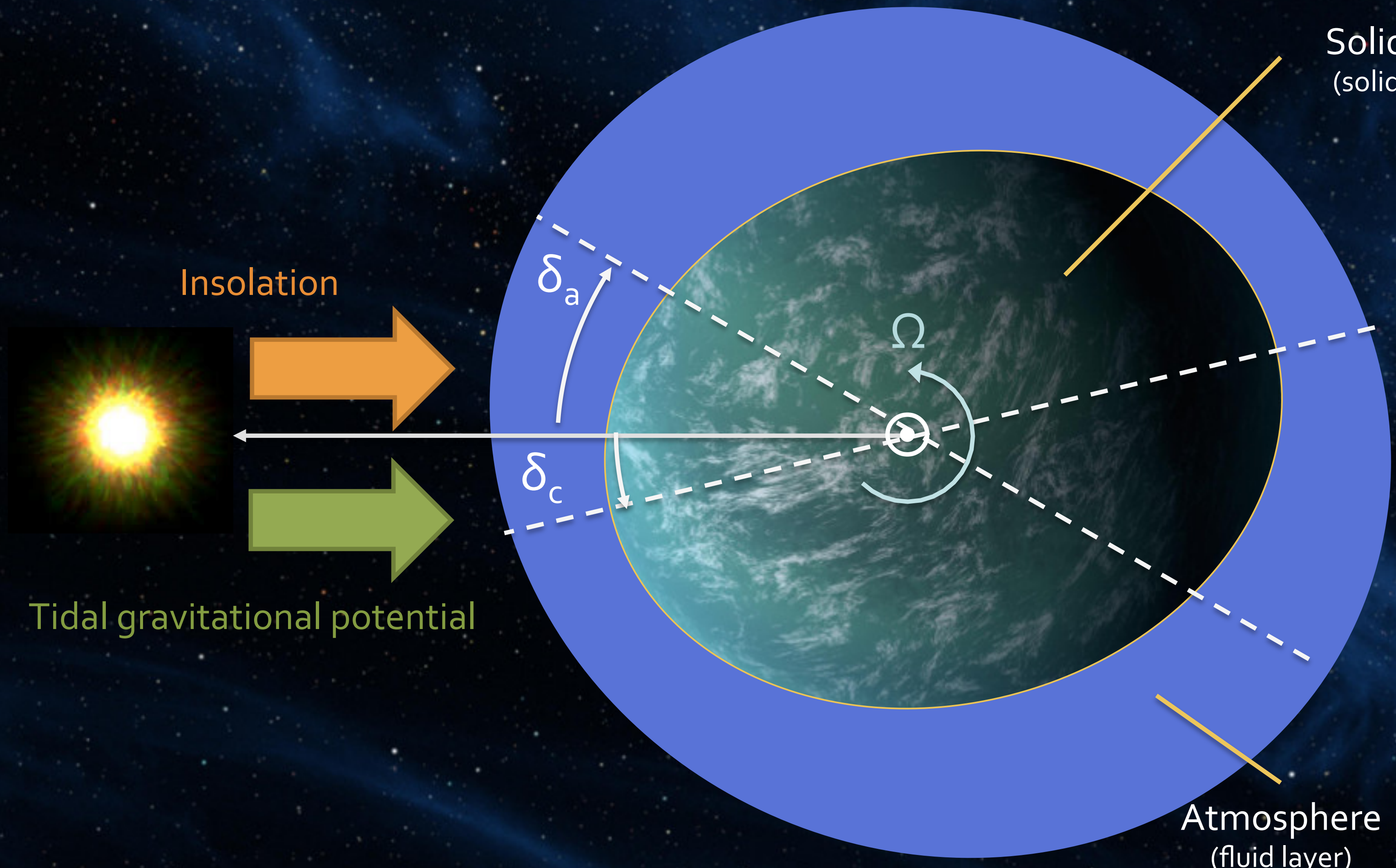
→ **Rocky/icy layers**

e.g. Correia, Levrard, Laskar (2008), Efroimsky (2012) ; Remus, ..., Lainey (2012, 2015)

→ **super-Earths atmospheres**

e.g. Forget & Leconte (2014)

Tidal effects in super-Earths



Equilibrium states: a torques balance

Spin equation:

$$\boxed{\frac{dL}{dt} = -\frac{\partial U_g}{\partial \vartheta} - \frac{\partial U_a}{\partial \vartheta}}$$

Spin

Tidal potential

$$U_g = -k_2 \frac{G M_*^2}{R} \left(\frac{R}{r_*}\right)^3 \left(\frac{R}{r}\right)^3 P_2(\cos S)$$

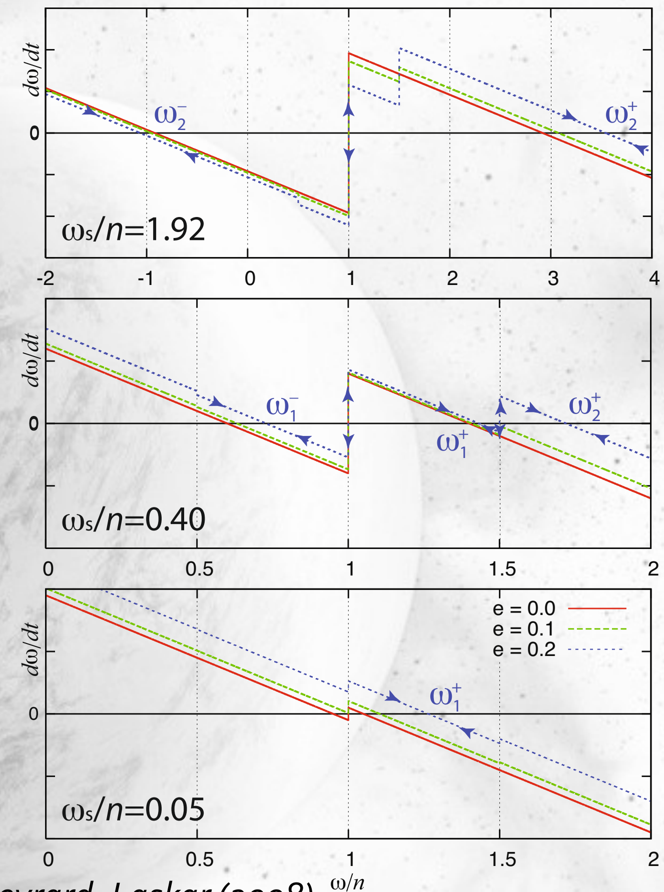
Tidal gravitational potential

$$U_a = -\frac{3}{5}\frac{\tilde{p}_2}{\bar{\rho}} \left(\frac{R}{r}\right)^3 P_2(\cos S)$$

Tidal atmospheric potential

Pressure oscillations at the ground !

$\boxed{\tilde{p}_2}$?

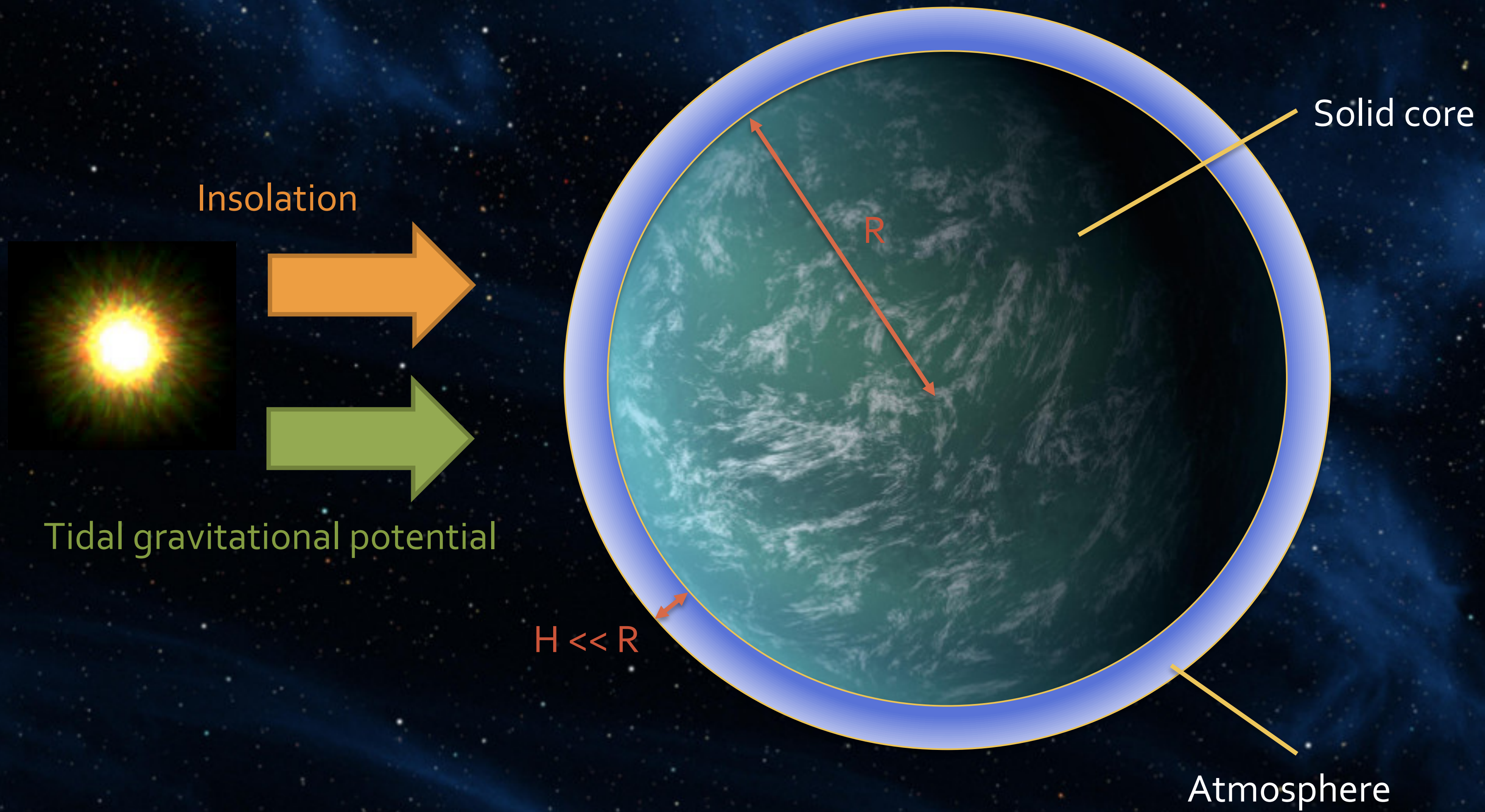


Correia, Levrard, Laskar (2008)

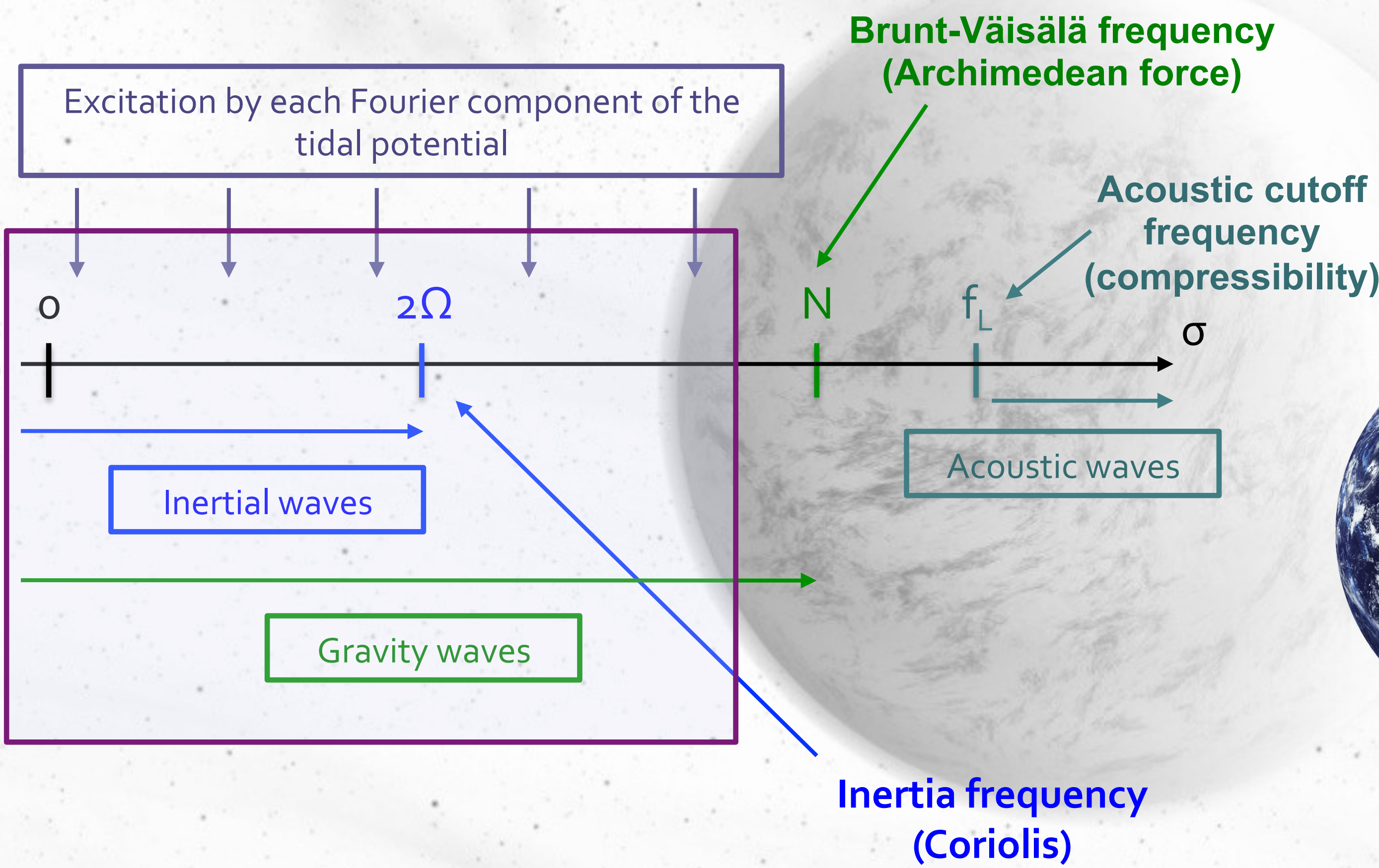
see also Gold & Soter (1969), Correia, Laskar, Néron de Surgy (2001), Correia & Laskar (2003)

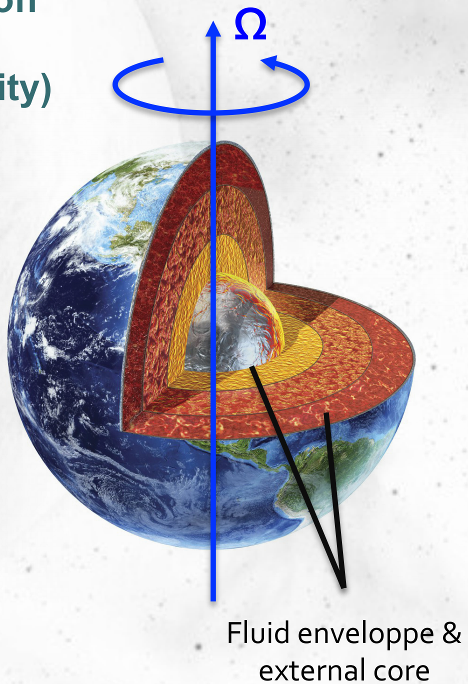
Need for a realistic physical modeling of atmospheric tides!

A global analytical model for thin atmospheres



Tidal waves properties





Atmospheric tides dynamics

Inertia frequency

$$\frac{\partial V_\theta}{\partial t} - 2\Omega V_\varphi \cos\theta = -\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\delta p}{\rho_0} + \boxed{U}\right),$$

Gravitational forcing

$$\frac{\partial V_\varphi}{\partial t} + 2\Omega\cos\theta V_\theta = -\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\frac{\delta p}{\rho_0} + \boxed{U}\right),$$

$$\rho_0\frac{\partial V_r}{\partial t} = -\frac{\partial \delta p}{\partial r} - g\delta\rho - \rho_0\boxed{\frac{\partial U}{\partial r}}.$$

$$\frac{\partial \delta\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\rho_0 V_r\right) + \frac{\rho_0}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta V_\theta) + \frac{\partial V_\varphi}{\partial\varphi}\right] = 0$$

Thermal forcing

$$\frac{1}{\Gamma_1 p_0}\left(\frac{\partial\delta p}{\partial t} + \Gamma_1\sigma_0\delta p\right) + \frac{N^2}{g}\frac{\partial \xi_r}{\partial t} = \frac{\kappa\rho_0}{p_0}\boxed{J} + \frac{1}{\rho_0}\left(\frac{\partial\delta\rho}{\partial t} + \sigma_0\delta\rho\right)$$

Brunt-Väisälä frequency

Thermal frequency

Reference model:

Chapman & Lindzen (1970)

Navier Stokes

Conservation of mass

Heat transport

Added terms

Horizontal structure

$$\delta p = \sum_{\sigma,s} \delta p^{\sigma,s}\left(\theta, x\right) e^{i(\sigma t + s\varphi)}$$

Expansion in Fourier series

$$\delta p^{\sigma,s} = \sum_{n} \delta p_n\left(x\right) \Theta_n\left(\theta\right)$$

Radial profiles

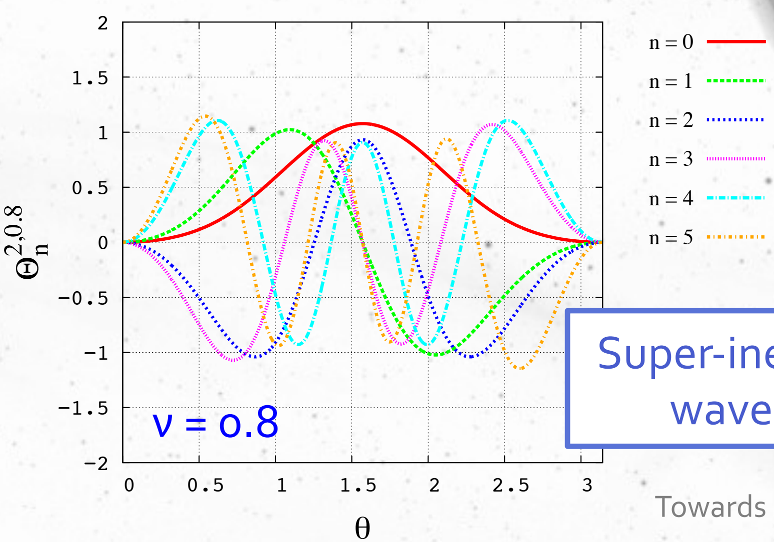
Hough functions

Expansion in Hough functions

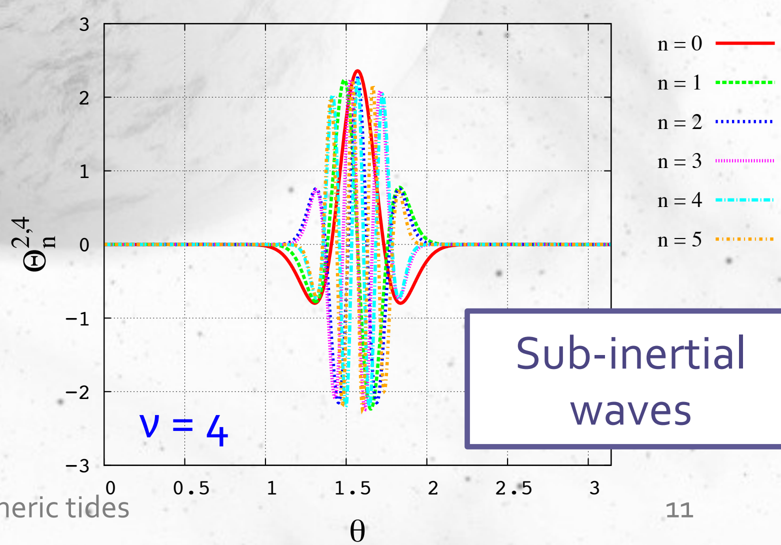
Laplace's tidal equation

$$\boxed{\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\nu^2\sin\theta}{1-\nu^2\cos^2\theta}\frac{\partial}{\partial\theta}\right) - \frac{\nu^2}{1-\nu^2\cos^2\theta}\left(s\nu\frac{1+\nu^2\cos^2\theta}{1-\nu^2\cos^2\theta} + \frac{s^2}{\sin^2\theta}\right)\right]\Theta_n = -\Lambda_n\Theta_n}$$

$$\nu = 2\Omega/\sigma$$



Super-inertial waves



Sub-inertial waves

Vertical structure

$$\delta p = \sum_{\sigma,s} \delta p^{\sigma,s}\left(\theta, x\right) e^{i(\sigma t + s\varphi)}$$

Expansion in Fourier series

$$\delta p^{\sigma,s} = \sum_{n} \delta p_n\left(x\right) \Theta_n\left(\theta\right)$$

Radial profiles

Hough functions

Expansion in Hough functions

Schrödinger-like equation

$$\boxed{\frac{d^2\Psi_n}{dx^2} + \lambda_n^2 \Psi_n = H^2 e^{-x/2} C\left(x\right)}$$

$$\lambda_n^2 = f\left(\sigma_0, \sigma\right)$$

Thermal frequency

VELOCITY FIELD

TEMPERATURE

DISPLACEMENT

DENSITY

PRESSURE

Frequency regimes: comparison with Chapman & Lindzen

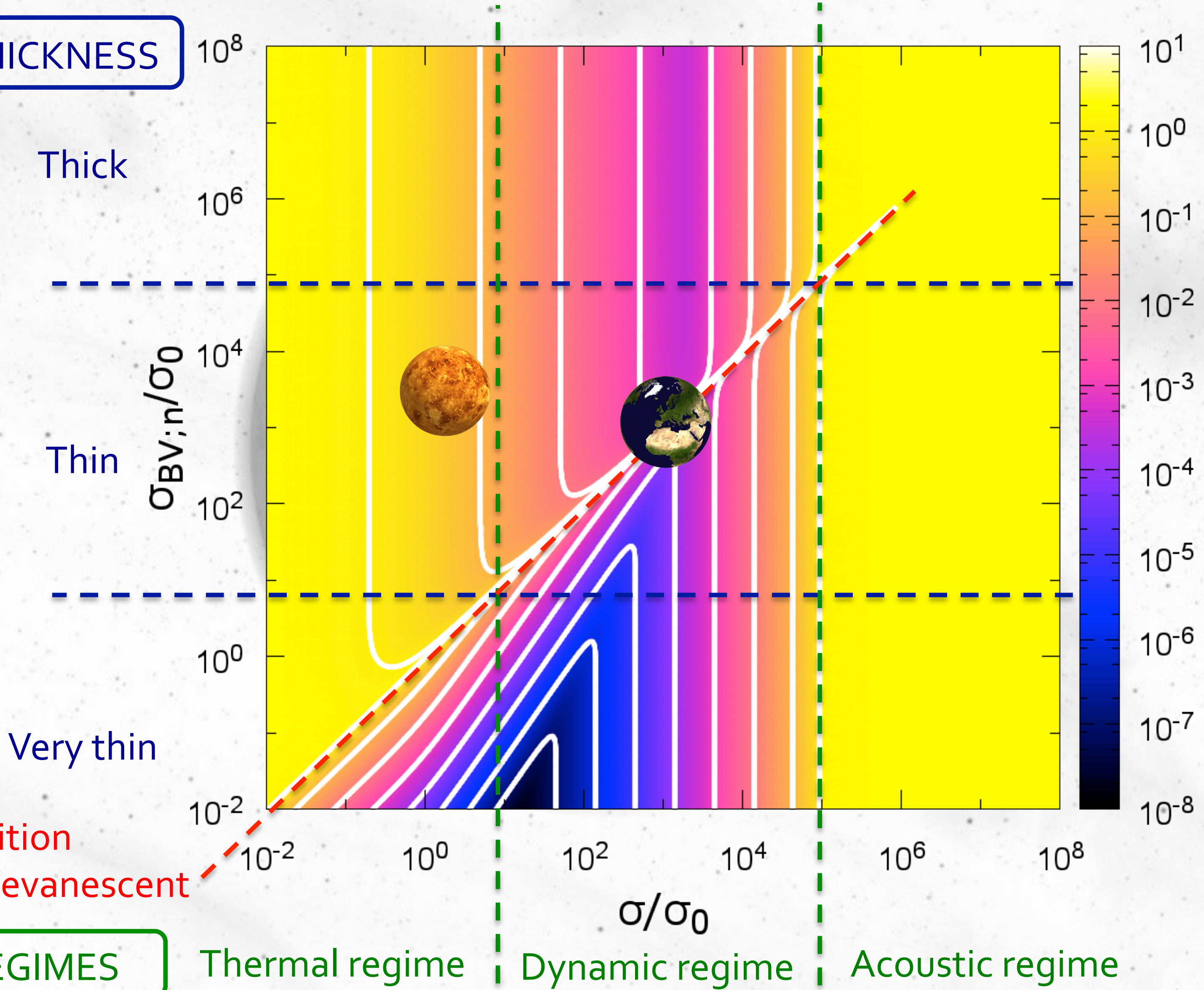
ATMOSPHERE THICKNESS

Relative difference between vertical wave numbers:

$k_v^{\text{Chapman-Lindzen}}$

VS

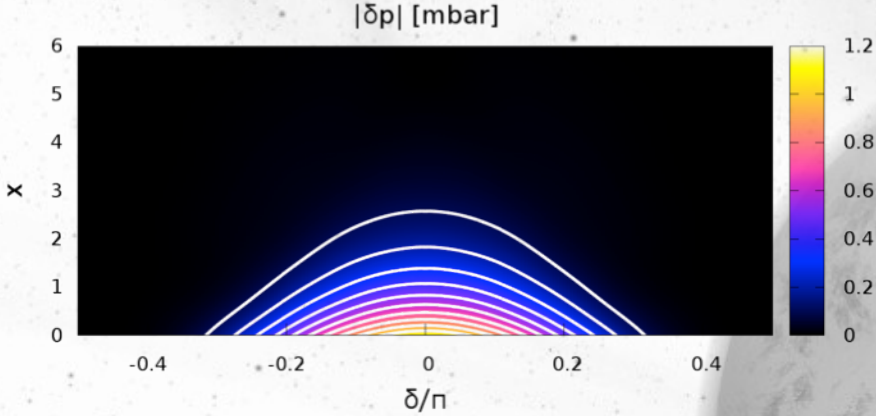
$k_v^{\text{this\_work}}$



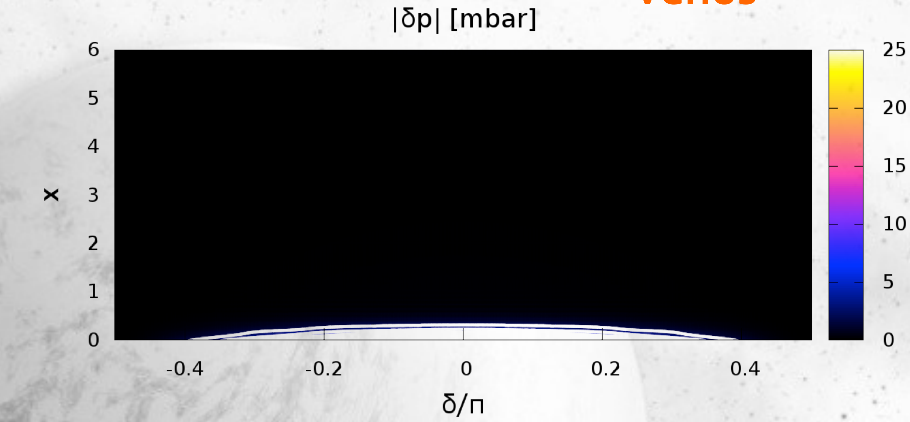
REGIMES

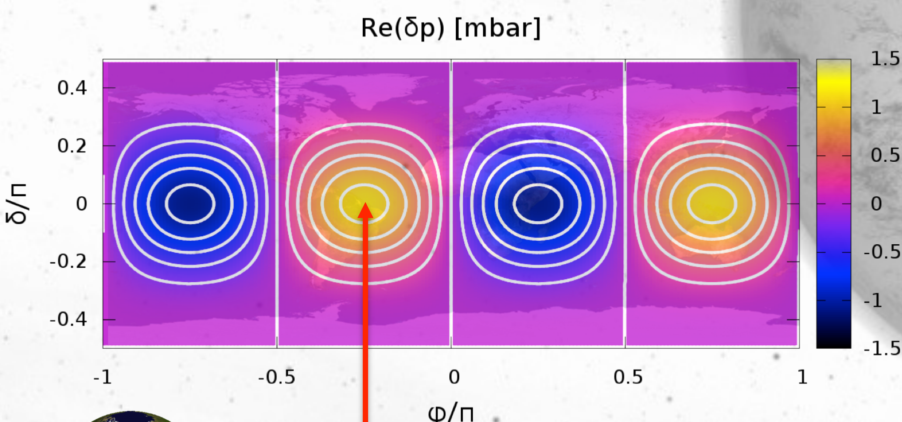
Spatial distribution of perturbed quantities

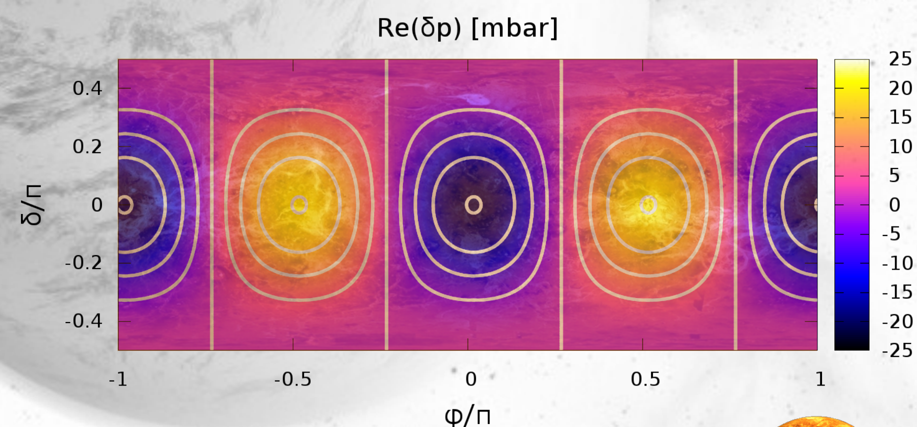
Earth



Venus

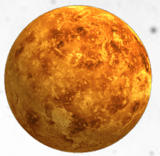






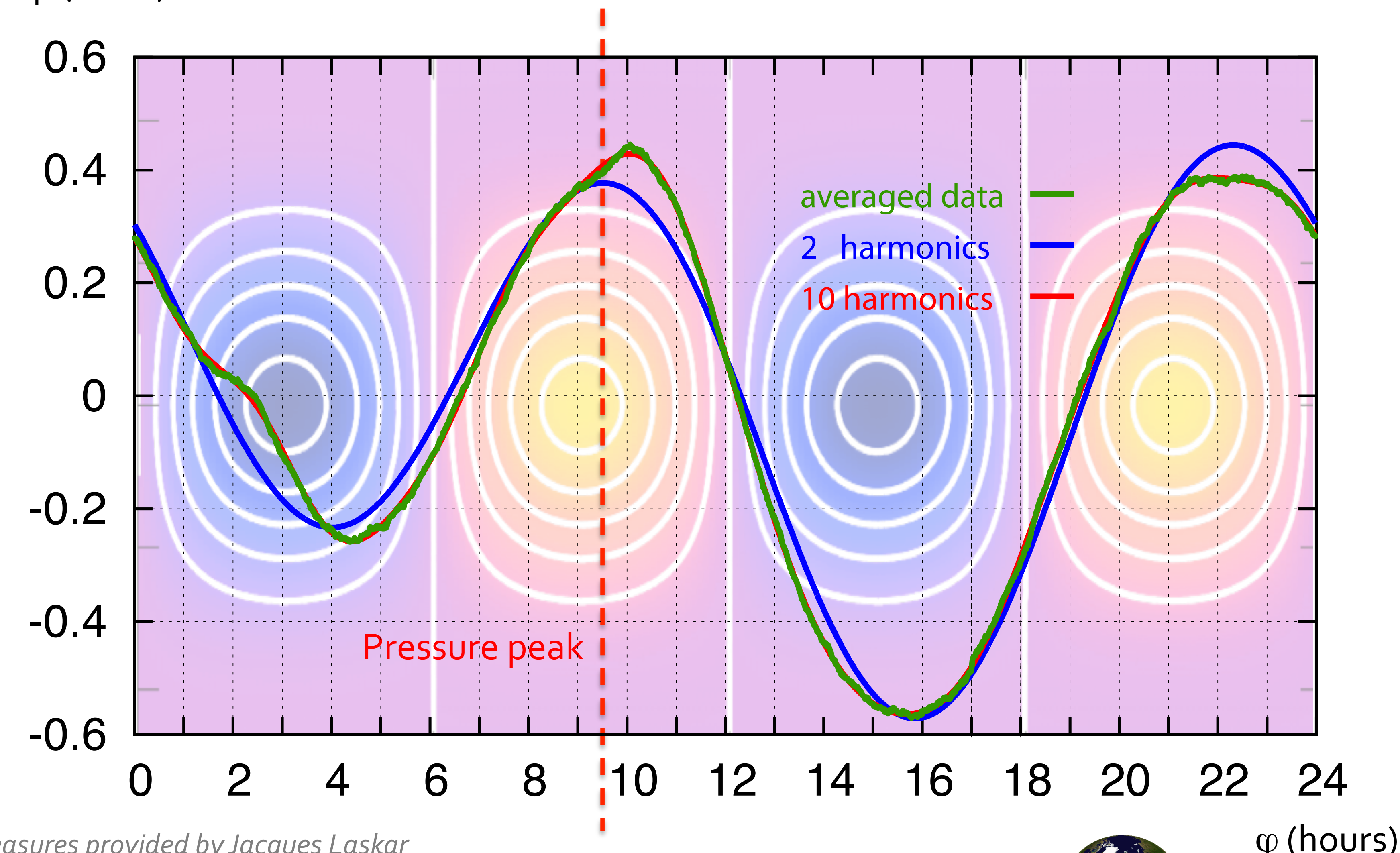
Pressure peak

Semi-diurnal tide



In good agreement with the GCM simulations of Leconte, Wu, Menou, Murray (2015)

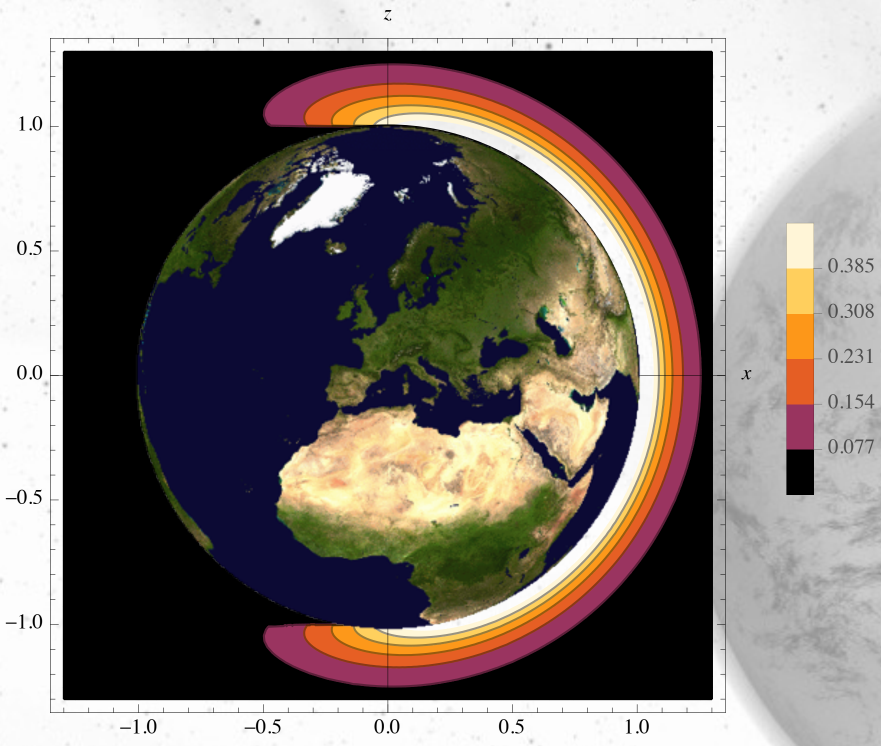
Comparison with measures

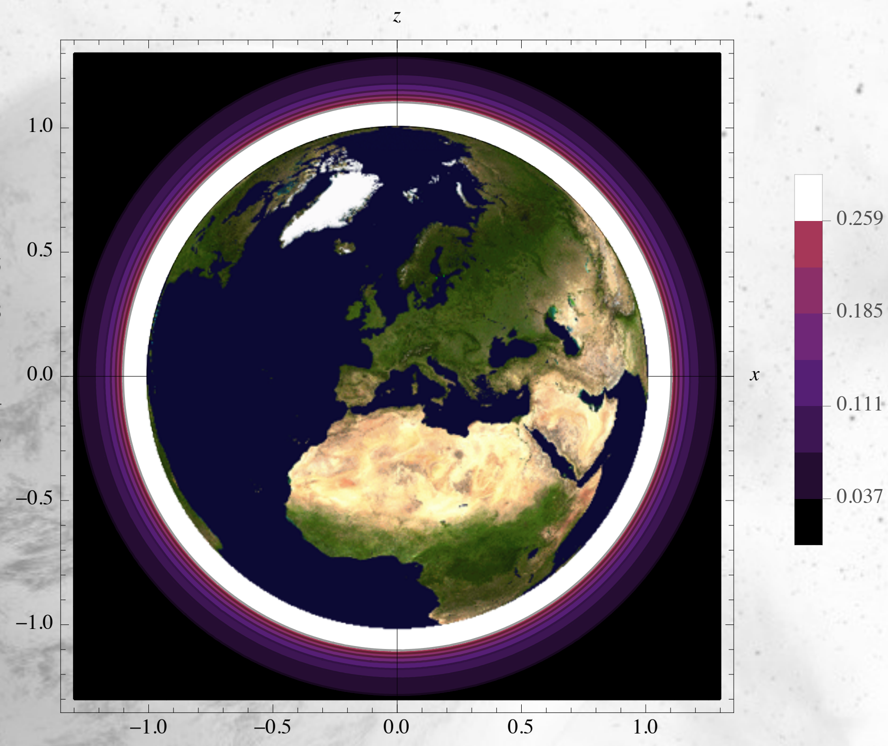


Measures provided by Jacques Laskar

Earth's semi-diurnal tide

Thermal forcings





SUN
Heating by the incident flux

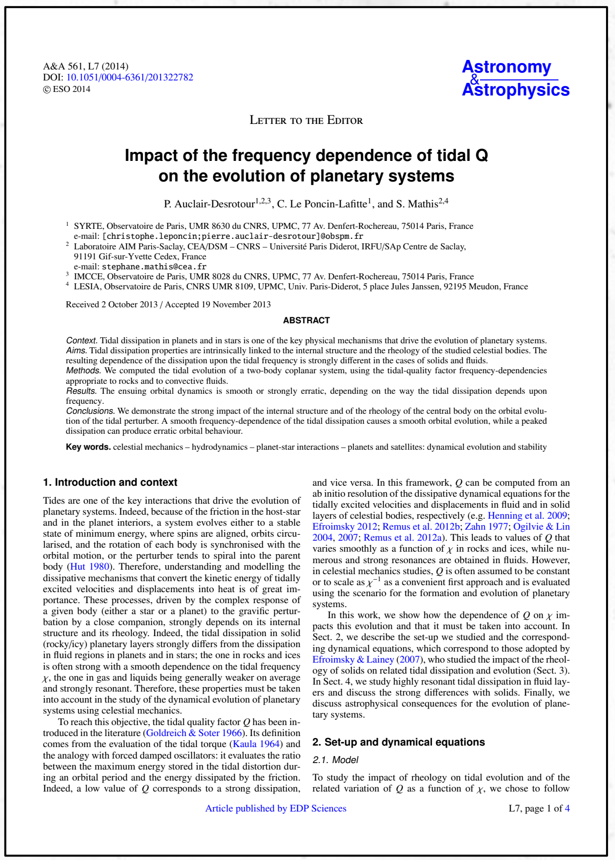
GROUND
Radiative heating
+ boundary layer turbulent diffusion

Conclusions and prospects

- Earth's semi-diurnal tide explained by the analytical model
- Identification of tidal regimes
- Dependence of the tidal torque on the tidal frequency
- Exploration of the domain of parameters
- Application to Venus and typical super-Earths
- Coupling with solid tides models (cf. Remus & al. 2012)

  - ➢ Publication A&A in preparation

Publication 1 (Master's thesis) - *Impact of the frequency dependence of tidal Q on the evolution of planetary systems*



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Astronomy & Astrophysics

Letter to the Editor

Impact of the frequency dependence of tidal Q on the evolution of planetary systems

P. Auclair-Desrotour[1,2,3], C. Le Poncin-Lafitte[1], and S. Mathis[2,4]

[1] SYRTE, Observatoire de Paris, UMR 8630 du CNRS, UPMC, 77 Av. Denfert-Rochereau, 75014 Paris, France
e-mail: [christophe.leponcin;pierre.auclair-desrotour]@obspm.fr
[2] Laboratoire AIM Paris-Saclay, CEA/DSM – CNRS – Université Paris Diderot, IRFU/SAp Centre de Saclay, 91191 Gif-sur-Yvette Cedex, France
e-mail: stephane.mathis@cea.fr
[3] IMCCE, Observatoire de Paris, UMR 8028 du CNRS, UPMC, 77 Av. Denfert-Rochereau, 75014 Paris, France
[4] LESIA, Observatoire de Paris, CNRS UMR 8109, UPMC, Univ. Paris-Diderot, 5 place Jules Janssen, 92195 Meudon, France

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ABSTRACT

Context. Tidal dissipation in planets and in stars is one of the key physical mechanisms that drive the evolution of planetary systems.
Aims. Tidal dissipation properties are intrinsically linked to the internal structure and the rheology of the studied celestial bodies. The resulting dependence of the dissipation upon the tidal frequency is strongly different in the cases of solids and fluids.
Methods. We computed the tidal evolution of a two-body coplanar system, using the tidal-quality factor frequency-dependencies appropriate to rocks and to convective fluids.
Results. The ensuing orbital dynamics is smooth or strongly erratic, depending on the way the tidal dissipation depends upon frequency.
Conclusions. We demonstrate the strong impact of the internal structure and of the rheology of the central body on the orbital evolution of the tidal perturber. A smooth frequency-dependence of the tidal dissipation causes a smooth orbital evolution, while a peaked dissipation can produce erratic orbital behaviour.

Key words. celestial mechanics – hydrodynamics – planet-star interactions – planets and satellites: dynamical evolution and stability

1. Introduction and context

Tides are one of the key interactions that drive the evolution of planetary systems. Indeed, because of the friction in the host-star and in the planet interiors, a system evolves either to a stable state of minimum energy, where spins are aligned, orbits circularised, and the rotation of each body is synchronised with the orbital motion, or the perturber tends to spiral into the parent body (Hut 1980). Therefore, understanding and modelling the dissipative mechanisms that convert the kinetic energy of tidally excited velocities and displacements into heat is of great importance. These processes, driven by the complex response of a given body (either a star or a planet) to the gravific perturbation by a close companion, strongly depends on its internal structure and its rheology. Indeed, the tidal dissipation in solid (rocky/icy) planetary layers strongly differs from the dissipation in fluid regions in planets and in stars; the one in rocks and ices is often strong with a smooth dependence on the tidal frequency χ, the one in gas and liquids being generally weaker on average and strongly resonant. Therefore, these properties must be taken into account in the study of the dynamical evolution of planetary systems using celestial mechanics.

To reach this objective, the tidal quality factor Q has been introduced in the literature (Goldreich & Soter 1966). Its definition comes from the evaluation of the tidal torque (Kaula 1964) and the analogy with forced damped oscillators: it evaluates the ratio between the maximum energy stored in the tidal distortion during an orbital period and the energy dissipated by the friction. Indeed, a low value of Q corresponds to a strong dissipation, and vice versa. In this framework, Q can be computed from an ab initio resolution of the dissipative dynamical equations for the tidally excited velocities and displacements in fluid and in solid layers of celestial bodies, respectively (e.g. Henning et al. 2009; Efroimsky 2012; Remus et al. 2012b; Zahn 1977; Ogilvie & Lin 2004, 2007; Remus et al. 2012a). This leads to values of Q that varies smoothly as a function of χ in rocks and ices, while numerous and strong resonances are obtained in fluids. However, in celestial mechanics studies, Q is often assumed to be constant or to scale as χ^{-1} as a convenient first approach and is evaluated using the scenario for the formation and evolution of planetary systems.

In this work, we show how the dependence of Q on χ impacts this evolution and that it must be taken into account. In Sect. 2, we describe the set-up we studied and the corresponding dynamical equations, which correspond to those adopted by Efroimsky & Lainey (2007), who studied the impact of the rheology of solids on related tidal dissipation and evolution (Sect. 3). In Sect. 4, we study highly resonant tidal dissipation in fluid layers and discuss the strong differences with solids. Finally, we discuss astrophysical consequences for the evolution of planetary systems.

2. Set-up and dynamical equations

2.1. Model

To study the impact of rheology on tidal evolution and of the related variation of Q as a function of χ, we chose to follow

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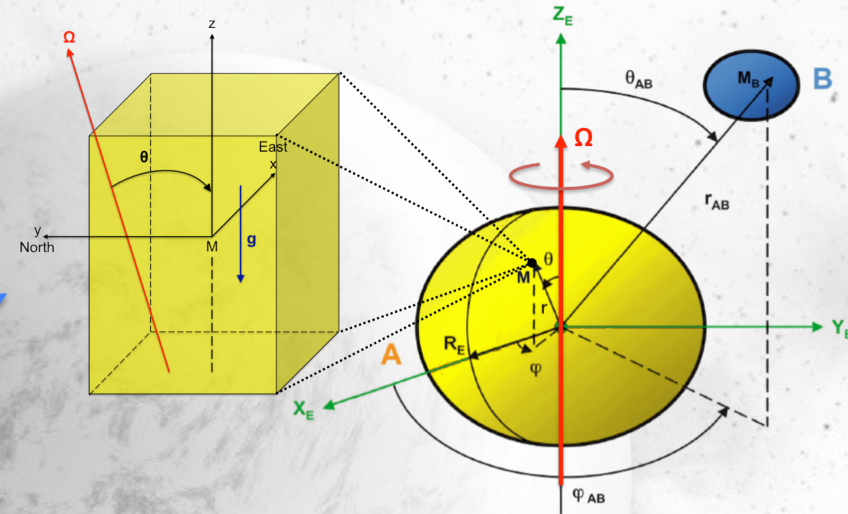
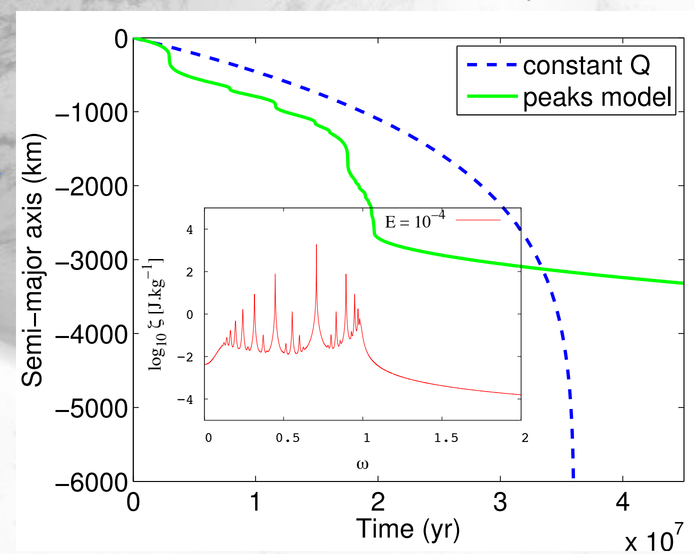




Letter A&A (2014)
Auclair-Desrotour, Le Poncin-Lafitte, Mathis

Publication 2 - *Understanding tidal dissipation in stars and fluid planetary regions I – Rotation, stratification & thermal diffusivity*

Astronomy & Astrophysics manuscript no. ADMLP2015-p-rev ©ESO 2015
June 19, 2015

Scaling laws to understand tidal dissipation in fluid planetary regions and stars I - Rotation, stratification and thermal diffusivity

P. Auclair Desrotour[1,2], S. Mathis[2,3], and C. Le Poncin-Lafitte[4]

[1] IMCCE, Observatoire de Paris, CNRS UMR 8028, 77 Avenue Denfert-Rochereau, 75014 Paris, France
e-mail: pierre.auclair-desrotour@obspm.fr
[2] Laboratoire AIM Paris-Saclay, CEA/DSM - CNRS - Université Paris Diderot, IRFU/SAp Centre de Saclay, F-91191 Gif-sur-Yvette, France
[3] LESIA, Observatoire de Paris, CNRS UMR 8109, UPMC, Univ. Paris-Diderot, 5 place Jules Janssen, 92195 Meudon, France
e-mail: stephane.mathis@cea.fr
[4] SYRTE, Observatoire de Paris, CNRS UMR 8630, UPMC, LNE, 61 Avenue de l'Observatoire, 75014 Paris, France
e-mail: christophe.leponcin@obspm.fr

Received ... / accepted ...

ABSTRACT

Context. Tidal dissipation in planets and stars is one of the key physical mechanisms driving the evolution of star-planet and planet-moon systems. Several signatures of its action are observed in planetary systems thanks to their orbital architecture and the rotational state of their components.
Aims. Tidal dissipation inside the fluid layers of celestial bodies are intrinsically linked to the dynamics and the physical properties of the latter. This complex dependence must be characterized.
Methods. We compute the tidal kinetic energy dissipated by viscous friction and thermal diffusion in a rotating local fluid Cartesian section of a star/planet/moon submitted to a periodic tidal forcing. The properties of tidal gravito-inertial waves excited by the perturbation are derived analytically as explicit functions of the tidal frequency and local fluid parameters (i.e. the rotation, the buoyancy frequency characterizing the entropy stratification, viscous and thermal diffusivities) for periodic normal modes.
Results. The sensitivity of the resulting possibly highly resonant dissipation frequency-spectra to a control parameter of the system is either important or negligible depending on the position in the regime diagram relevant for planetary and stellar interiors. For corresponding asymptotic behaviors of tidal gravito-inertial waves dissipated by viscous friction and thermal diffusion, scaling laws for the frequencies, number, width, height and contrast with the non-resonant background of resonances are derived to quantify these variations.
Conclusions. We characterize the strong impact of the internal physics and dynamics of fluid planetary layers and stars on the dissipation of tidal kinetic energy in their bulk. We point out the key control parameters that really play a role and demonstrate how it is now necessary to develop ab-initio modeling for tidal dissipation in celestial bodies.

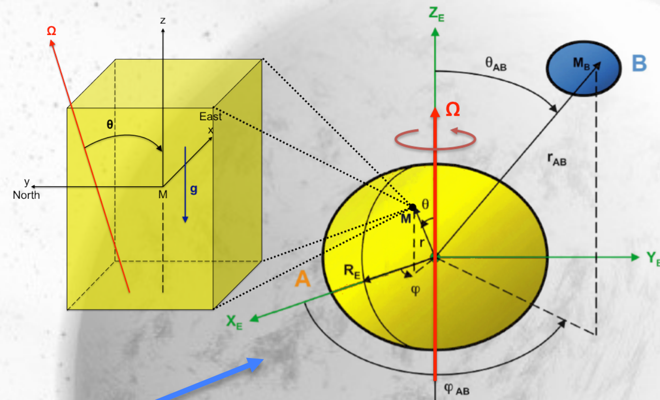
Key words. hydrodynamics – waves – turbulence – planet-star interactions – planets and satellites: dynamical evolution and stability

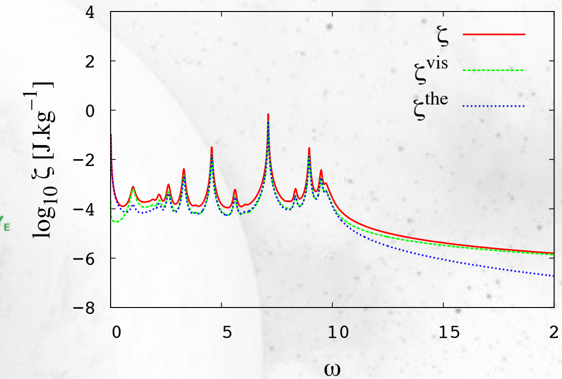
1. Introduction and context

Tides have a strong impact on the evolution of star-planet and planet-moon systems over secular time scales. Indeed, because of the dissipation of the kinetic energy of flows and displacements they induce in celestial bodies, they drive their rotational and orbital evolutions (see e.g. Ogilvie (2014) and references therein for the cases of stars and giant planets and Correia & Laskar (2003); Correia et al. (2008) for telluric planets). In the case of the Earth-Moon system, their causes and effects are now strongly constrained and disentangled thanks to satellite altimeter high-precision observations of ocean tides (Egbert & Ray 2000, 2001; Ray et al. 2001). In the Solar System, the actions of tides are detected and estimated from high-precision geodesic or/and astrometric observations (see e.g. Konopliv & Yoder (1996) for Venus, Williams et al. (2014) for the Moon, Lainey et al. (2007); Jacobson (2010); Konopliv et al. (2011) for Mars, Lainey et al. (2009, 2012) for Jupiter and Saturn and the attempt by Emelyanov & Nikonchuk (2013) for Uranus).

Finally, a new extremely important astronomical laboratory to explore and constrain the physics of tides is constituted by the numerous exoplanetary systems discovered over the last twenty years (Mayor & Queloz 1995; Perryman 2011). Indeed, they are composed of a large diversity of planets (from hot Jupiters to super-Earths) and host stars while their orbital architecture and the configuration of planetary and stellar spins strongly differ from the one observed in our Solar system (e.g. Albrecht et al. 2012; Fabrycky et al. 2012; Valsecchi & Rasio 2014). In this context, the understanding of the tidal formation and evolution of planetary systems is one of the most important problems of modern dynamical astronomy (e.g. Laskar et al. 2012; Bolmont et al. 2012; Ferraz-Mello 2013) while the needed understanding and quantitative prediction of tidal dissipation in celestial bodies is still a challenge (e.g. Mathis & Remus 2013; Ogilvie 2014, for complete reviews).

We owe the first theoretical work about a tidally deformed body to Lord Kelvin (Kelvin 1863). Then, a physical formalism has





Scaling laws

Domain		$A \ll A_{11}$			$A \gg A_{11}$	
$P_r \gg P_{r;11}^{reg}$		$l_{mn} \propto E$ $H_{mn} \propto E^{-1}$ $H_{bg} \propto E$	$\omega_{mn} \propto \frac{n}{\sqrt{m^2+n^2}} \cos\theta$ $N_{kc} \propto E^{-1/2}$ $\Xi \propto E^{-2}$		$l_{mn} \propto E$ $H_{mn} \propto E^{-1}$ $H_{bg} \propto A^{-1}E$	$\omega_{mn} \propto \frac{m}{\sqrt{m^2+n^2}} \sqrt{A}$ $N_{kc} \propto A^{1/4}E^{-1/2}$ $\Xi \propto AE^{-2}$
$P_r \ll P_{r;11}^{reg}$	$P_r \gg P_{r;11}$	$l_{mn} \propto E$ $H_{mn} \propto E^{-1}P_r^{-1}$ $H_{bg} \propto EP_r^{-1}$	$\omega_{mn} \propto \frac{n}{\sqrt{m^2+n^2}} \cos\theta$ $N_{kc} \propto E^{-1/2}$ $\Xi \propto E^{-2}$	$P_r \gg P_{r;11}^{diss}$	$l_{mn} \propto EP_r^{-1}$ $H_{mn} \propto E^{-1}P_r^2$ $H_{bg} \propto A^{-1}E$	$\omega_{mn} \propto \frac{m}{\sqrt{m^2+n^2}} \sqrt{A}$ $N_{kc} \propto A^{1/4}E^{-1/2}P_r^{1/2}$ $\Xi \propto AE^{-2}P_r^2$
	$P_r \ll P_{r;11}$	$l_{mn} \propto AEP_r^{-1}$ $H_{mn} \propto A^{-2}E^{-1}P_r$ $H_{bg} \propto EP_r^{-1}$	$\omega_{mn} \propto \frac{n}{\sqrt{m^2+n^2}} \cos\theta$ $N_{kc} \propto A^{-1/2}E^{-1/2}P_r^{1/2}$ $\Xi \propto A^{-2}E^{-2}P_r^2$	$P_r \ll P_{r;11}^{diss}$	$l_{mn} \propto EP_r^{-1}$ $H_{mn} \propto A^{-1}E^{-1}P_r$ $H_{bg} \propto A^{-2}EP_r^{-1}$	$\omega_{mn} \propto \frac{m}{\sqrt{m^2+n^2}} \sqrt{A}$ $N_{kc} \propto A^{1/4}E^{-1/2}P_r^{1/2}$ $\Xi \propto AE^{-2}P_r^2$

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Atmospheric tides in Earth-like exoplanets

P. Auclair Desrotour[1,2], J. Laskar[1], and S. Mathis[2,3]

[1] IMCCE, Observatoire de Paris, CNRS UMR 8028, 77 Avenue Denfert-Rochereau, 75014 Paris, France
e-mail: pierre.auclair-desrotour@obspm.fr, jacques.laskar@obspm.fr
[2] Laboratoire AIM Paris-Saclay, CEA/DSM - CNRS - Université Paris Diderot, IRFU/SAp Centre de Saclay, F-91191 Gif-sur-Yvette, France
[3] LESIA, Observatoire de Paris, CNRS UMR 8109, UPMC, Univ. Paris-Diderot, 5 place Jules Janssen, 92195 Meudon, France
e-mail: stephane.mathis@cea.fr

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ABSTRACT

Context. Context.
Aims. Aim.
Methods. Method.
Results. Results.
Conclusions. Conclusions.

Key words. hydrodynamics – waves – turbulence – planet-star interactions – planets and satellites: dynamical evolution and stability

1. Introduction

2. Dynamics of a thick atmosphere forced thermally and gravitationally

The reference book "Atmospheric tides" (Chapman & Lindzen 1970) has set the bases of analytical approaches for atmospheric tides. This pioneering work focused on fast rotating telluric planets covered by a thin atmosphere, such as the Earth. We generalize it in this section, by establishing the equations that govern the dynamics of tides in a thick fluid shell in the whole range of possible tidal frequencies, from synchronization to fast rotation. We thus consider a spherical telluric planet of radius R covered by an atmospheric layer of typical thickness H_{atm}. The atmosphere rotates uniformly with the rocky part at the spin frequency $\mathbf{\Omega}$. Therefore, the dynamics will be written in the natural spherical co-rotating frame, $\mathscr{R}_{\mathrm{O}} : \left\{ O, \mathbf{e}_r, \mathbf{e}_\theta, \mathbf{e}_\varphi \right\}$, using the spherical coordinates (r, θ, φ). The atmosphere is assumed to be a perfect gas homogeneous in composition, of molar mass M, and stratified radially. Its pressure, density and temperature are denoted p, ρ and T respectively. For the sake of simplicity, all dissipative mechanisms, such as viscous friction and heat diffusion, are ignored except the radiative loss of the gas, which plays an important role near synchronization. Tides can be considered as a linear first order perturbation near a global equilibrium state. The corresponding quantities, for which we use the subscript $_0$ (p_0, ρ_0, T_0), are supposed to vary with the radial coordinate only. The Brunt-Vaisala frequency can be expressed as a function of these quantities :

$$N^2 = g\left[\frac{1}{\Gamma_1 p_0}\frac{dp_0}{dr} - \frac{1}{\rho_0}\frac{d\rho_0}{dr}\right], \quad (1)$$

where $\Gamma_1 = (\partial \ln p_0 / \partial \ln \rho_0)_S$ is the adiabatic exponent (S being the specific macroscopic entropy). Moreover, we assume that the fluid follows the perfect gas law,

$$p_0 = \mathscr{R}_s \rho_0 T_0, \quad (2)$$

where $\mathscr{R}_s = \mathscr{R}_{\mathrm{GP}}/M$ is the so-called specific gas constant of the atmosphere, $\mathscr{R}_{\mathrm{GP}} = 8.3144621$ J.mol^{-1}.K^{-1} (data from NIST) being the molar gas constant.

2.1. Forced dynamics equations

The atmosphere is affected by the tidal gravitational potential U and the heat power per mass unit J. Considering that this forcing is of first order, we linearize the equations of dynamics by introducing the small variations induced by the tidal perturbation, the velocity field $\mathbf{V} = V_\theta \mathbf{e}_\theta + V_\varphi \mathbf{e}_\varphi + V_r \mathbf{e}_r$, and δp, $\delta\rho$, δT such as:

$$p = p_0 + \delta p, \quad \rho = \rho_0 + \delta\rho, \quad T = T_0 + \delta T. \quad (3)$$

It represents six unknown quantities to compute, and we will therefore solve a six-dimension system. First, we need the Navier-Stokes equation. We assume the *Cowling approximation* and ignore the feedback effect caused by the tidal potential due to the elongation. Thus, in the co-rotating frame, the linearized equation can be written:

$$\frac{\partial \mathbf{V}}{\partial t} + 2\mathbf{\Omega} \wedge \mathbf{V} = -\frac{1}{\rho_0}\mathbf{\nabla} p - \frac{g}{\rho_0}\delta\rho\,\mathbf{e}_r - \mathbf{\nabla} U, \quad (4)$$

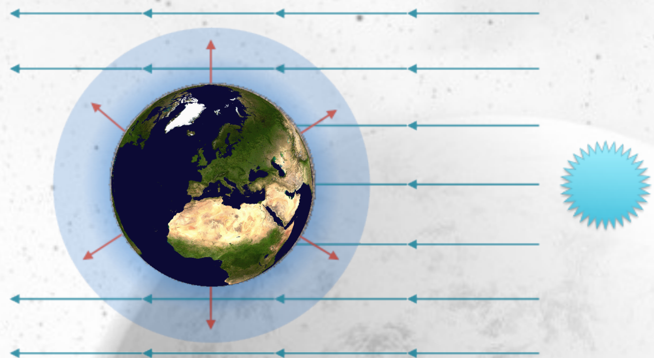
which is expanded:

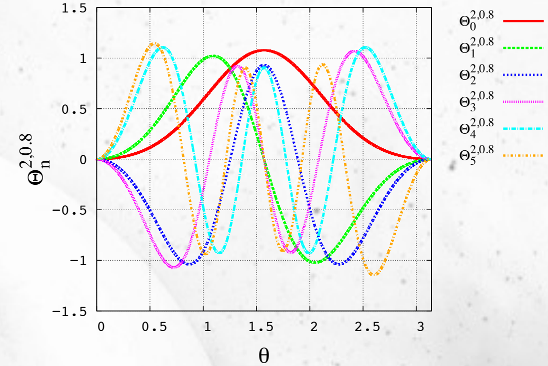
$$\frac{\partial V_\theta}{\partial t} - 2\Omega V_\varphi \cos\theta = -\frac{1}{r}\frac{\partial}{\partial\theta}\left(\frac{\delta p}{\rho_0} + U\right), \quad (5)$$

$$\frac{\partial V_\varphi}{\partial t} + 2\Omega\cos\theta V_\theta + 2\Omega\sin\theta V_r = -\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}\left(\frac{\delta p}{\rho_0} + U\right), \quad (6)$$

Article number, page 1 of 20

Article A&A (in prep.)
Auclair-Desrotour, Laskar, Mathis





$$\boxed{\delta p_n = \frac{\rho_0}{1-\varepsilon_{s;n}}\left\{\frac{\sigma^2}{H\Lambda_n}e^{\frac{x}{2}}\left[\frac{d\Psi_n}{dx} + \mathcal{A}\Psi_n\right] - \frac{\Gamma_1 - 1}{i\sigma + \sigma_0}\frac{\sigma^2}{\sigma_{s;n}^2}J_n - U_n\right\}}$$