Unstaggered constrained-transport methods for the three-dimensional ideal magnetohydrodynamic equations

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Numerical methods for solving the ideal magnetohydrodynamic (MHD) equations in more than one space dimension must confront the challenge of controlling errors in the discrete divergence of the magnetic field. One approach that has been shown successful in stabilizing MHD calculations are constrained-transport (CT) schemes.

We consider unstaggered CT methods in which an evolution equation for the magnetic potential is solved during each time step and a divergence-free update of the magnetic field is computed by taking the curl of the magnetic potential. Rossmanith [1] developed such a CT method for the discretization of the two-dimensional MHD equations on Cartesian grids. In this case a scalar transport equation is solved for the out-of-plane component of the magnetic potential.

In [2, 3], we extended this CT method to the three-dimensional case. In this case we obtain an evolution equation for the magnetic vector potential $A$ of the form

$$\partial_t A + (\nabla \times A) \times u = -\nabla \psi,$$

where $\psi$ is an arbitrary scalar function. In order to get a closed set of evolution equations for the vector potential one needs an additional constraint (i.e., the gauge condition). Our approach is based on the Weyl gauge, which means we set $\nabla \psi = 0$. In this case the resulting evolution equation for the magnetic vector potential is \textit{weakly hyperbolic}, which required a special numerical treatment.

In [2], we used different operator splitting methods to discretize the weakly hyperbolic system for the magnetic potential on Cartesian grids. In [3], we developed an unsplit method for the weakly hyperbolic system. This unsplit method can be used on Cartesian grids as well as on logically rectangular mapped grids.