Small-scale dynamo action in compressible convection

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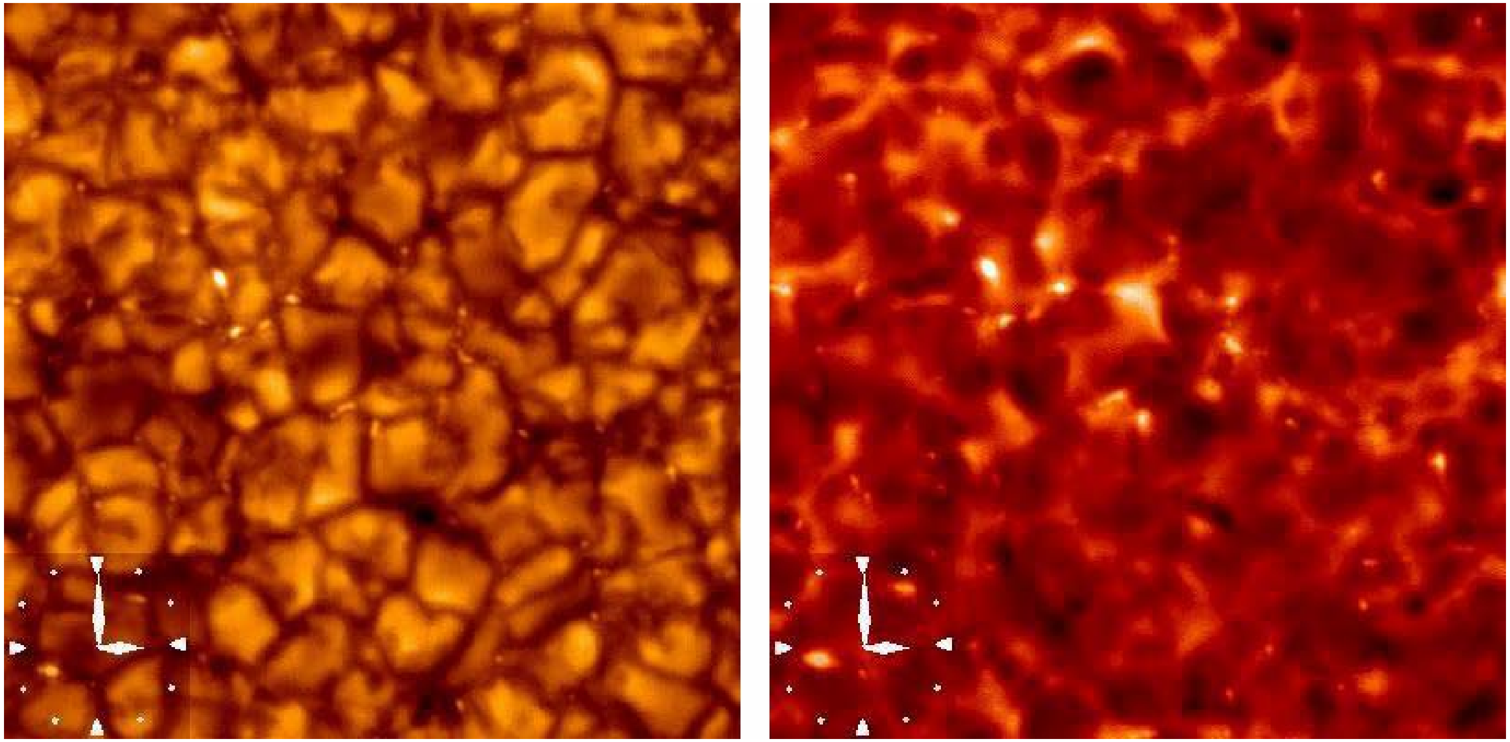
Observations

The quiet Sun:

Below: Convective motions and small-scale magnetic flux in the quiet Sun. Bright regions correspond to localised mixed parity magnetic flux concentrations.

Left: G-Band (430nm); **Right:** Ca II H (397nm) (Hinode SOT)

(http://solarb.msfc.nasa.gov/news/movies.html)

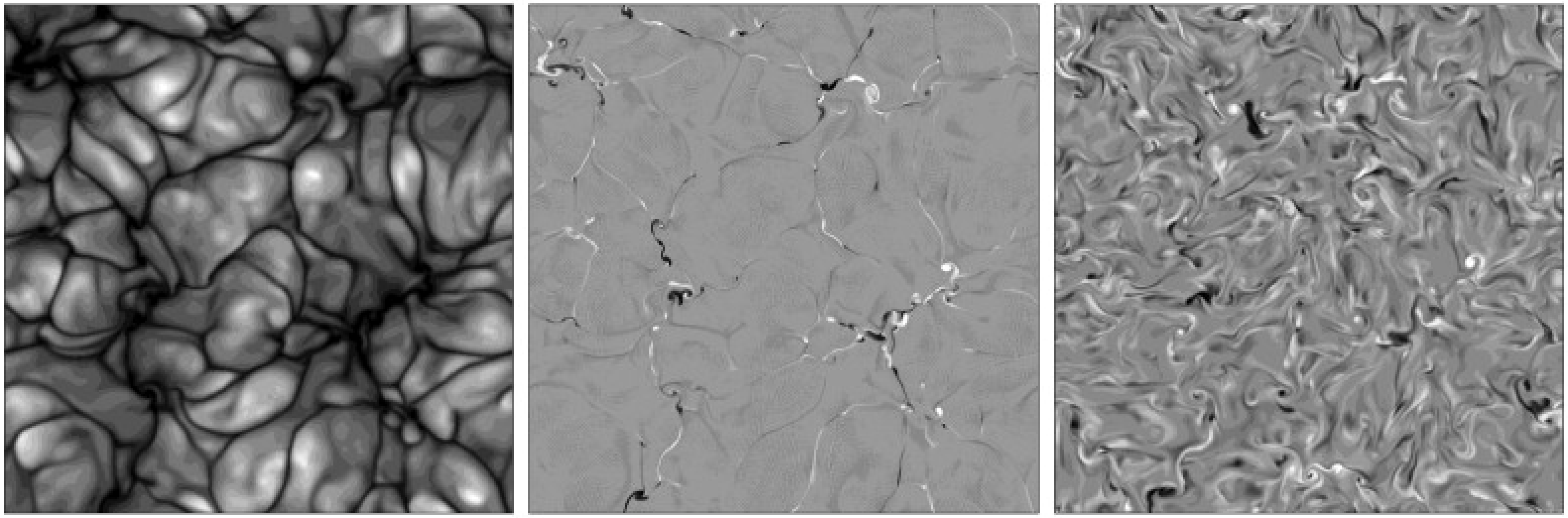


The origin of quiet Sun magnetic fields

Key question: Is a significant fraction of the quiet Sun magnetic flux generated near the surface by the small-scale convective motions?

Previous studies:

• Boussinesq (incompressible) convection: non-rotating (Cattaneo 1999); rotating (Cattaneo & Hughes 2006)

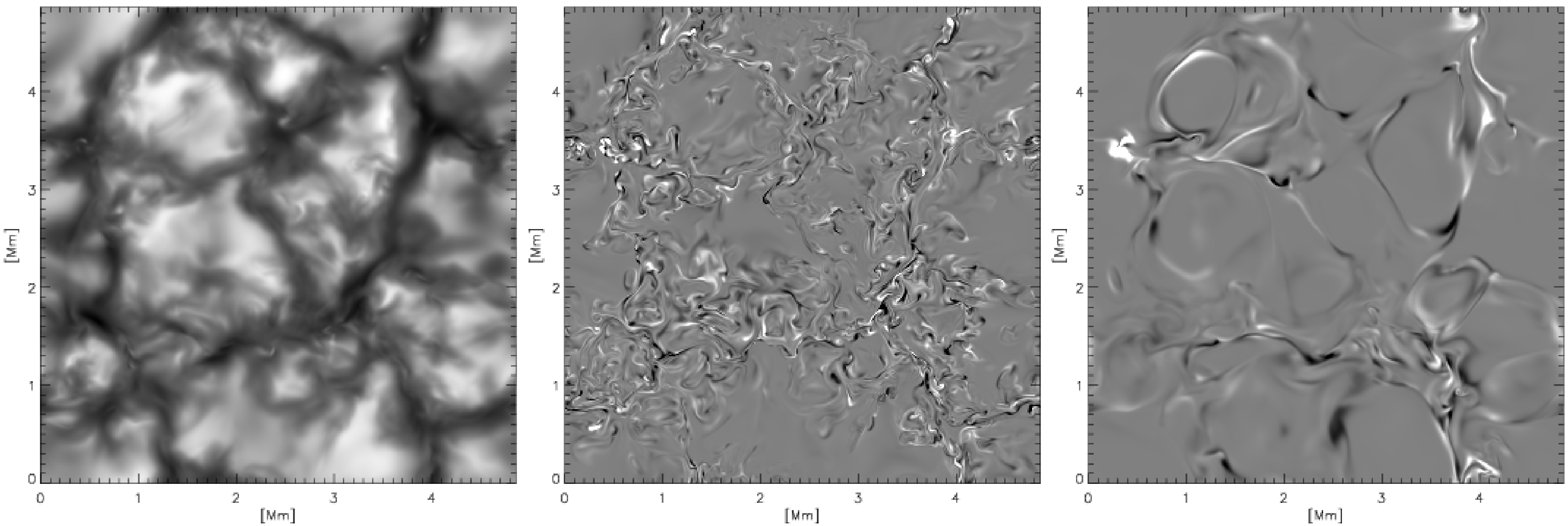


(Taken from Cattaneo 1999)

The origin of quiet Sun magnetic fields (cont.)

• Dynamo action in compressible convection (less well understood):

LES simulations (Vögler & Schüssler 2007); Weakly-superadiabatically stratified convection (Brummell & collaborators, unpublished)



(Taken from Vögler & Schüssler 2007)

Our approach: DNS of dynamo action in three-dimensional compressible convection at comparatively modest Reynolds numbers (no radiative transfer etc…)

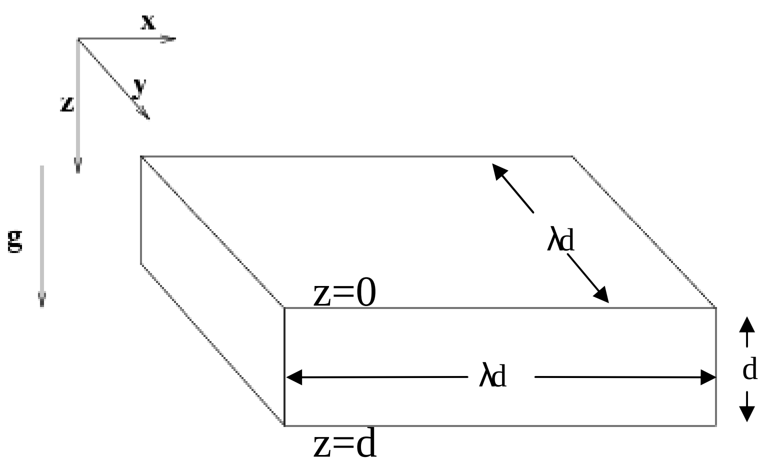
Model Setup

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \qquad P = \mathcal{R}\rho T$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla P + \rho g \hat{\mathbf{z}} + \frac{1}{\mu_0}(\nabla \times \mathbf{B}) \times \mathbf{B} + \mu \left[\nabla^2 \mathbf{u} + \frac{1}{3}\nabla(\nabla \cdot \mathbf{u})\right]$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \qquad \nabla \cdot \mathbf{B} = 0$$

$$\rho c_{\mathrm{v}} \left[\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T\right] = -P\nabla \cdot \mathbf{u} + K\nabla^2 T + Q_\nu + Q_\eta$$



Initially:

Hydrostatic equilibrium – a convectively unstable polytropic layer heated from below (polytropic index=1; ratio of specific heats, γ=5/3)

Density and temperature vary by an order of magnitude across the layer.

A horizontally-periodic Cartesian domain of depth, d (λ typically 4 or 8)

Upper and lower boundaries**:** Impermeable, stress-free, vertical field, fixed T

Numerical Methods

Although this model is an idealised representation of the solar photosphere, we still need to solve the equations of 3-dimensional compressible magnetoconvection → Large-scale numerical simulations

Numerical method

• Mixed finite-difference/pseudo-spectral scheme, explicit 3rd order Adams-Bashforth time-stepping

• Horizontal derivatives evaluated in Fourier space

• Fourth order finite differences (either upwinded or centred) are used to calculate vertical derivatives

• Depending upon the domain size, computational resolution is either 256x256x160 or 512x512x160

• Code parallelised using MPI

Many of these simulations were carried out using the UKMHD Cluster

Numerical results

Right: Statistically-steady hydrodynamic convection (Reynolds number, Re ~ 150)

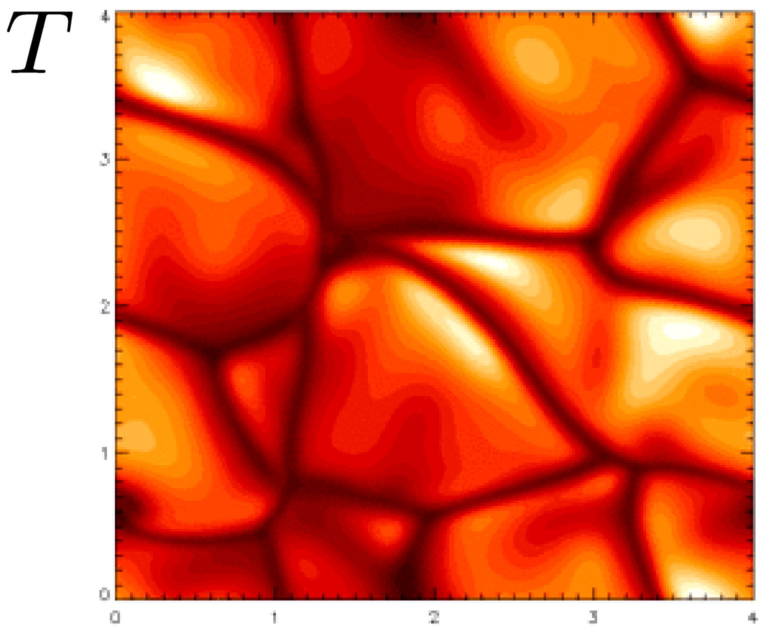
We insert a seed magnetic field:

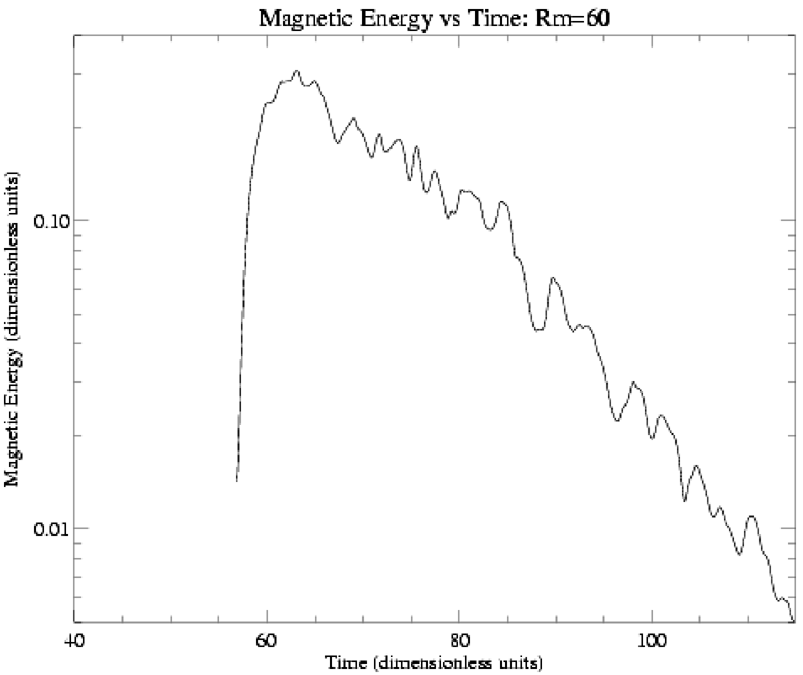
$$\mathrm{B} = \epsilon \cos(2\pi x/\lambda)\cos(2\pi y/\lambda)\hat{\mathbf{z}}$$

Crucial parameter: $$Rm = \frac{U_{rms} d}{\eta}$$

This magnetic Reynolds number must be large enough that inductive effects due to the flow outweigh magnetic diffusion. For a given flow, can vary Rm by varying η…..

Right: A decaying dynamo at Rm=60 (magnetic energy vs. time on a log-linear scale)





The kinematic regime

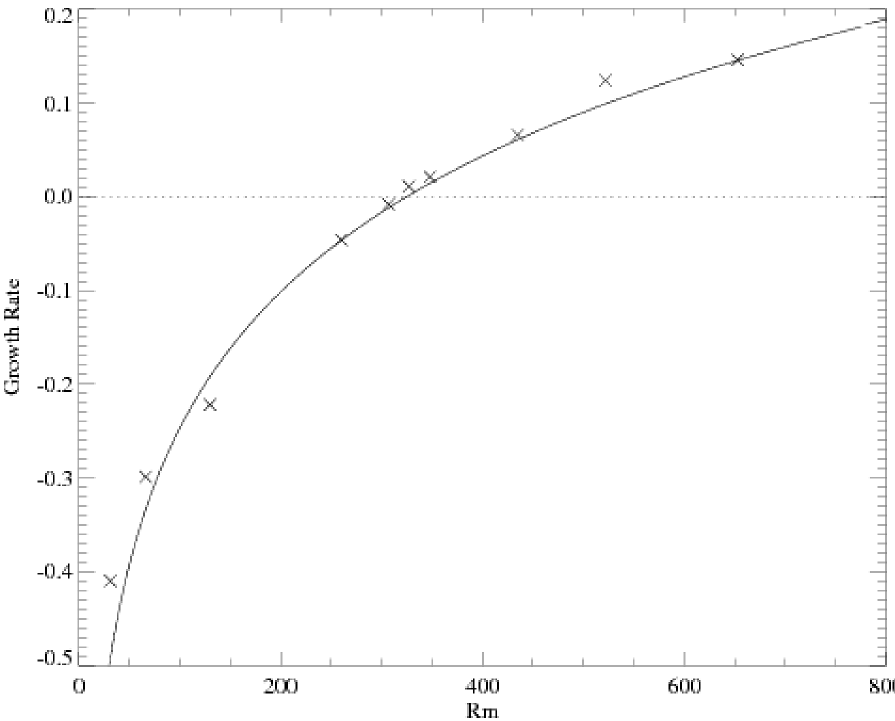
Focus initially upon the **kinematic** regime - no Lorentz feedback upon the flow.

Right: Magnetic energy growth rate as a function of Rm

$$Rm_{crit} \sim 325$$

Growth rate $\approx 0.21 \log\left(\frac{Rm}{Rm_{crit}}\right)$

Logarithmic fit for the growth rate similar a result of Rogachevski & Kleeorin (1997) for forced turbulence



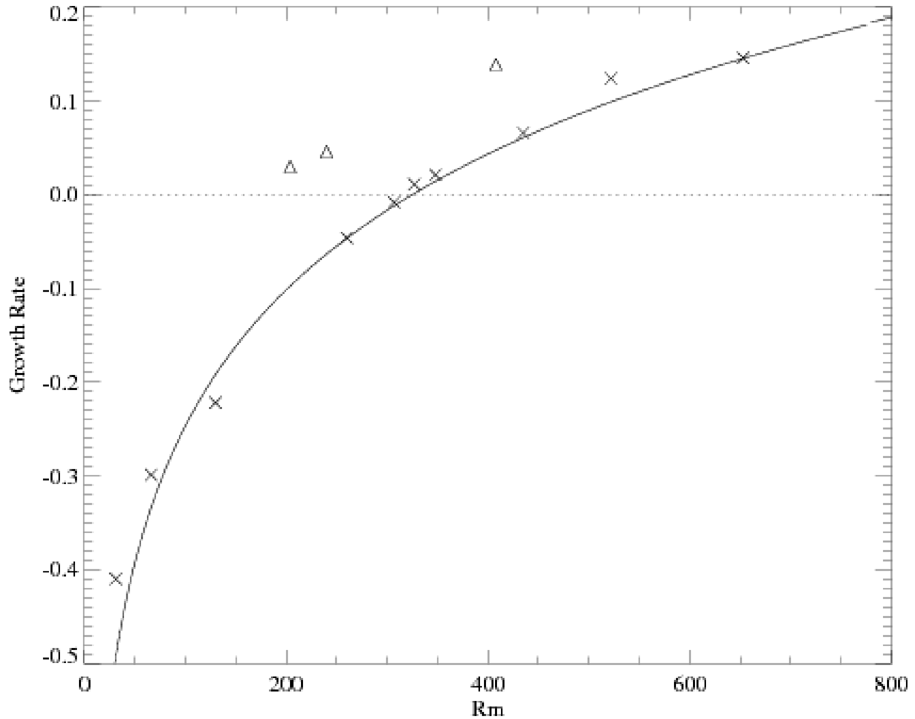
• We have not yet reached a “fast” regime in which the growth rate is independent of Rm.

• Like Boussinesq calculations, growth rate (at higher values of Rm) is comparable to the convective turnover time

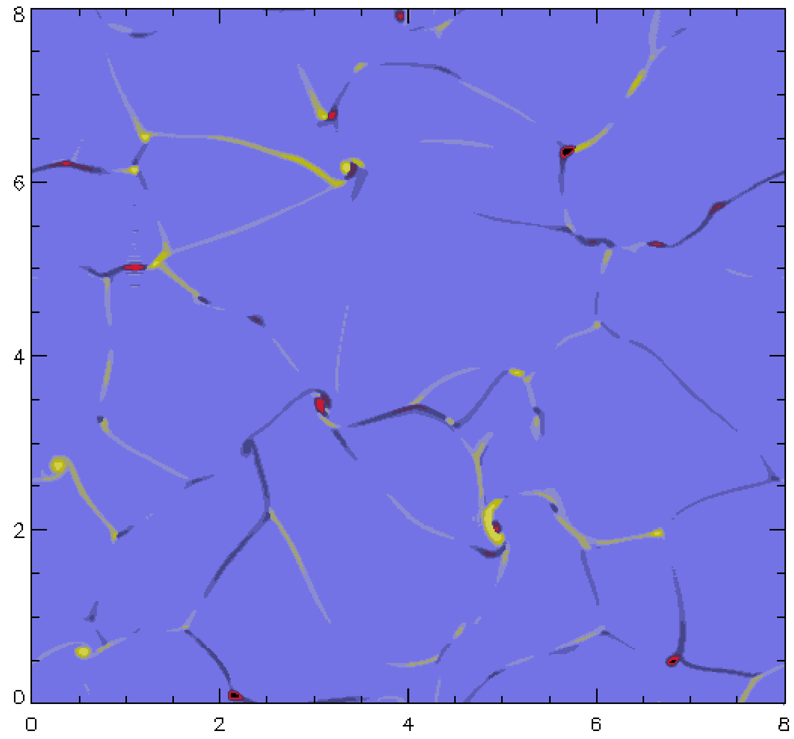
The kinematic regime (cont.)

Effect of increasing the box size: What happens to these kinematic dynamos if we increase the aspect ratio from λ=4 to λ=8?

- Higher growth rates
- Lower Rm_{crit}



B_z



This suggests that the eigenfunction with the highest growth rate cannot be accommodated in the narrower computational domain

The nonlinear regime

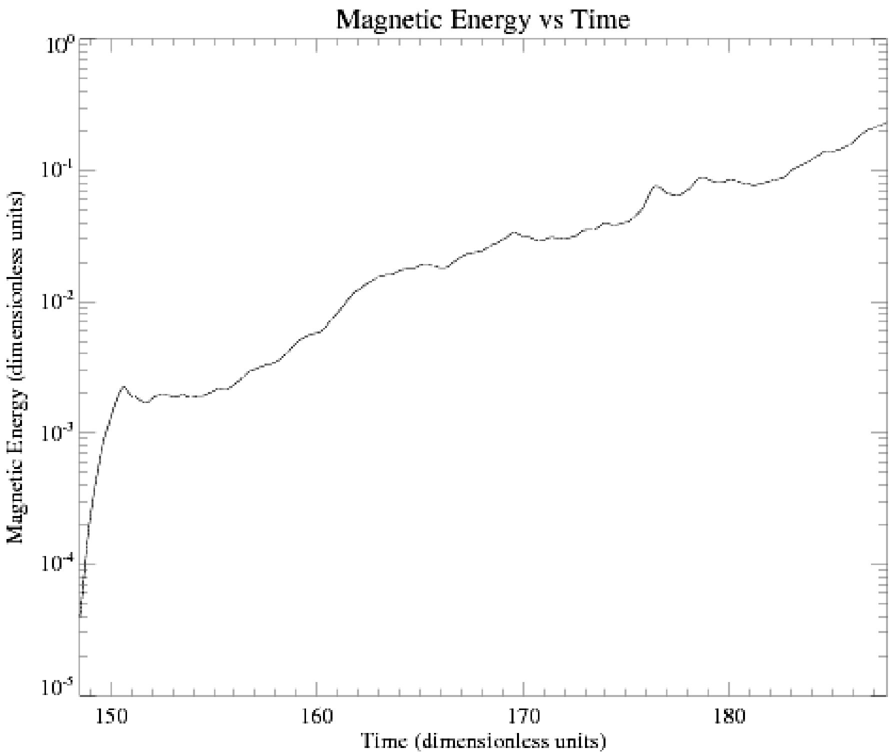
All the results so far are for **kinematic** dynamos - what about the nonlinear case?

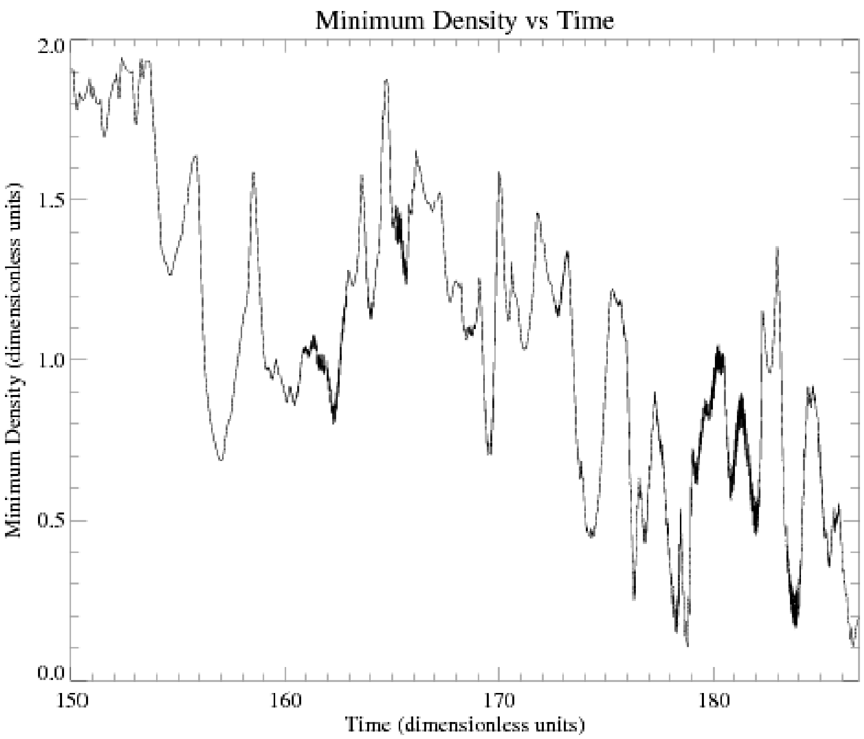
$$\lambda = 4 \quad Rm \sim 520 \quad Re \sim 150$$

Numerical resolution: $512 \times 512 \times 160$

Below left: Magnetic energy against time

Below right: Minimum density against time





The nonlinear regime (cont.)

Physical explanation for partial evacuation:

• Surface magnetic flux accumulates in the convective downflows, which drain fluid away from the surface

• Strong magnetic pressure gradient inhibits converging convective flows

→ Imbalance between incoming and outgoing mass fluxes

→ Partial evacuation of the surface regions

Numerical consequences:

Alfvén speed: $V_A \sim \frac{B_o}{\sqrt{\rho}}$ Coefficient of thermal diffusion: $\sim \frac{K}{\rho}$

Both of these become large in the partially-evacuated region

→Shorter timescales for Alfvenic disturbances and thermal diffusion

→Smaller critical time-step for stability of this explicit scheme

→Significantly increased runtime

The nonlinear regime (cont.)

A “Solution”:

In order to run these simulations on a reasonable time-scale, we artificially impose a “floor” on the density

A marginal dynamo calculation should also limit the levels of partial evacuation

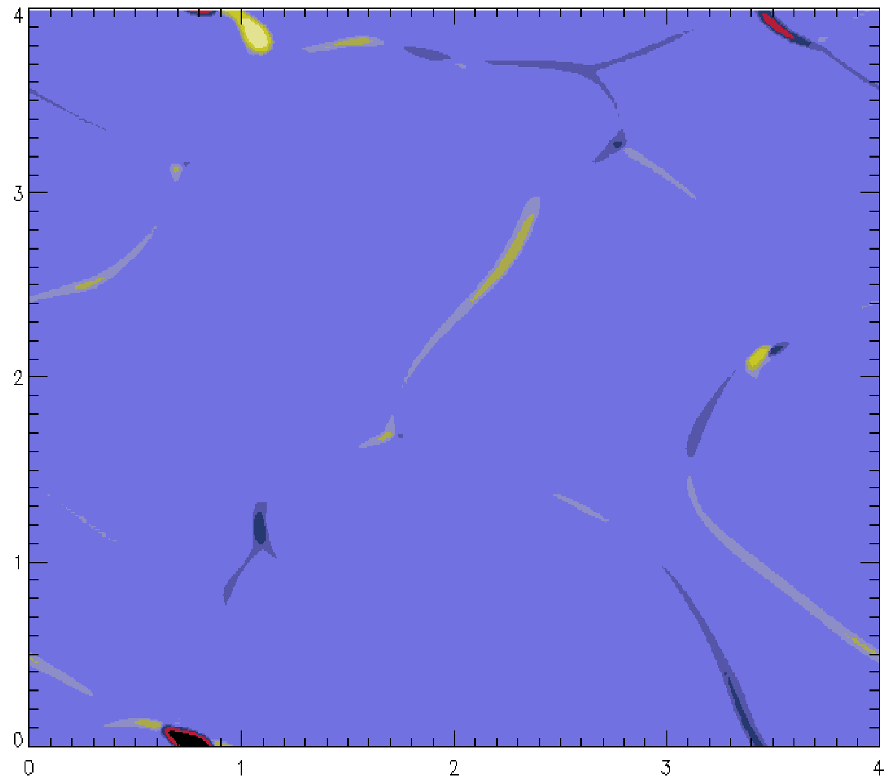
$$\lambda = 4 \qquad Re \sim 150$$

$$Rm \sim 350 \sim 1.1 Rm_{crit}$$

Numerical resolution:

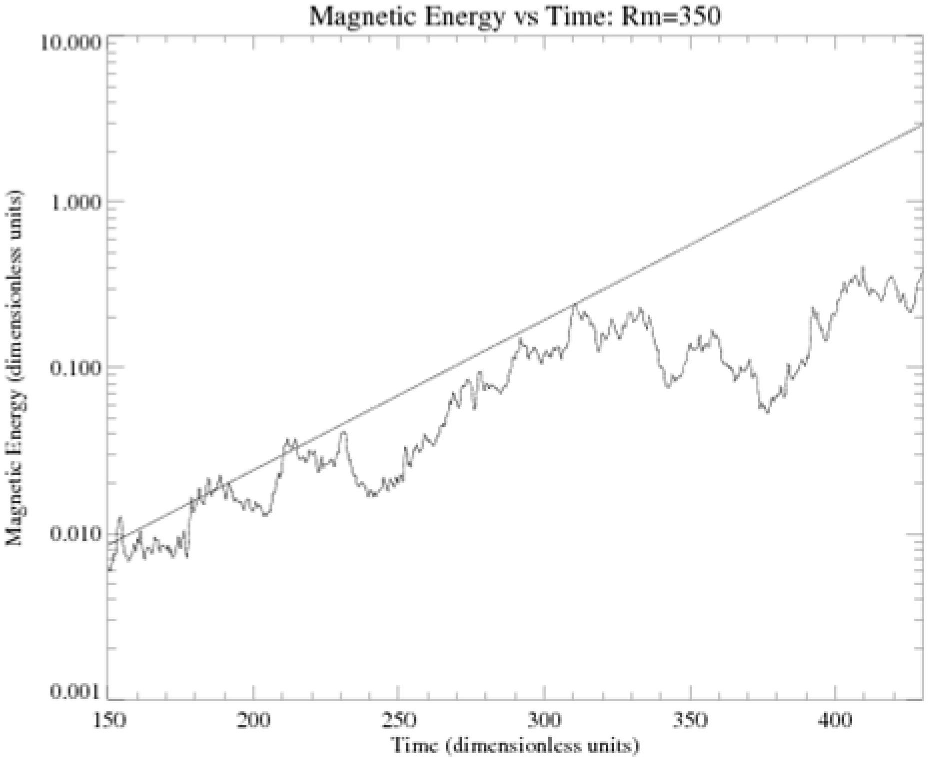
$$256 \times 256 \times 160$$

B_z



The nonlinear regime (cont.)

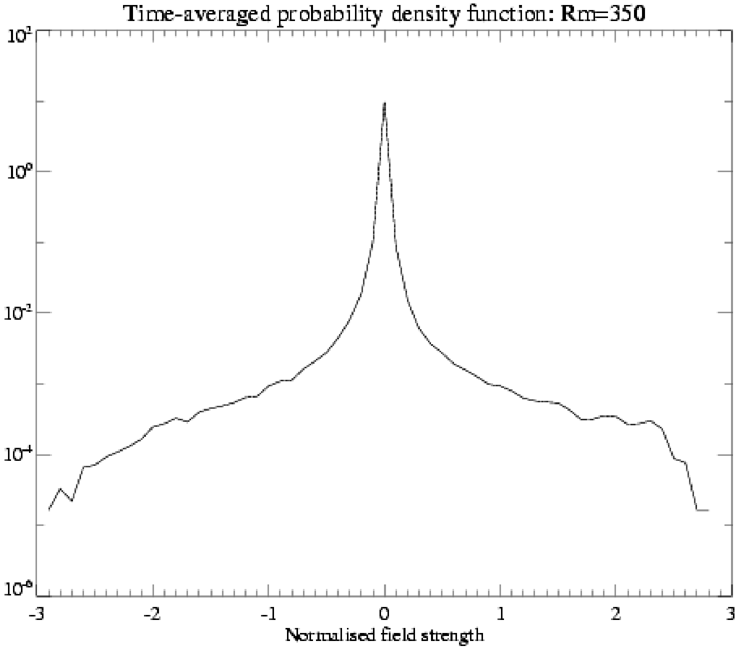
Below: A plot of magnetic energy against time. The straight line shows the estimated rate of growth from the kinematic calculations

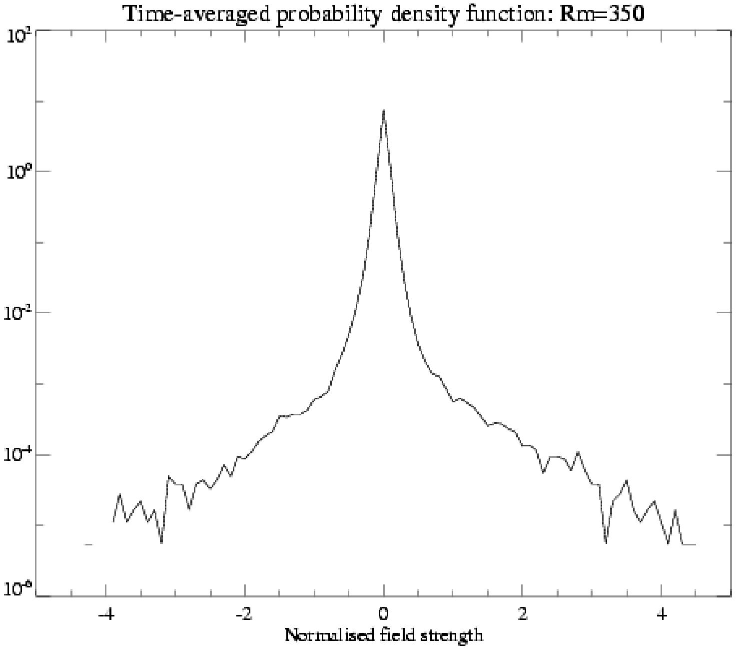


- The dynamo is probably still growing, but well into the nonlinear phase
- Global magnetic energy approximately 1% of the total kinetic energy
- Dynamo efficiency already comparable to that of Vögler & Schüssler (2007), despite the modest magnetic Reynolds number

The nonlinear regime (cont.)

Below: Probability density functions for the vertical component of the magnetic field at the upper surface (Left) and at the mid-plane (Right)





- These pdfs are qualitatively similar to those obtained by Vögler and Schüssler (2007), although more stretched at the extreme edges of the distribution

Summary

• Kinematic simulations indicate that convection can act as a small-scale dynamo, at least at moderate values of the Reynolds number.

• Apparent logarithmic dependence of growth rate upon the magnetic Reynolds number (at least in this range of values for Rm…)

• Marginal nonlinear dynamo appears to be nearing saturation. The magnetic energy is about 1% of the global kinetic energy. Comparable efficiency to LES simulations of Vögler & Schüssler (2007), despite modest Rm

Open questions:

• How well does this dynamo process actually work in the solar photosphere?

• How do the idealised boundary conditions influence the results?

• Is it possible to write a code that can handle these calculations in a more efficient manner?