(Numerical) study of the collapse and of the fragmentation of prestellar dense core

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1. **Context - Model**

2. **Radiative transfer for star formation calculation**
   - Flux Limited Diffusion Approximation
   - Radiation Hydrodynamics equations solver

3. **AMR vs SPH**
   - Fragmentation study

4. **3D AMR RMHD with FLD and ideal MHD**
   - 3D RHD collapse calculations
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Numerical star formation - State of the art

2 main numerical methods:
- Grid based: Hennebelle, Fromang & Teyssier 08 (MHD), Krumholz et al 07 (RHD), Banerjee & Pudritz 06 (MHD), etc…
- SPH: Bate et al. (RMHD), Stamatellos et al 08 (RHD), etc…

Debate on the accuracy of both methods:
=> Are these methods appropriate to study structure formation? Are they converging?

RAMSES code (Teyssier 2002)
Finite volume, 2nd order Godunov scheme
Ideal MHD solver (Fromang et al. 2006)
MPI parallelized

Collapse with a $m=2$ perturbation - Set up

Model: isolated dense core of 1 $M_\odot$ in solid body rotation
- 1/ Small scales
- 2/ Fragmentation

Fragmentation: IMF, disk stability

⇒ Azimuthal density perturbation $\Rightarrow$ spiral arms
- Perturbation $m=2$, amplitude $A=0.1$
  $$\rho = \rho_0[1 + A\cos(m\theta)]$$

⇒ 2 parameters to set the system
- Thermal support: $\alpha = 0.5$  \(E_{\text{th}}/E_{\text{grav}}\)
- Rotational support: $\beta = 0.04$  \(E_{\text{rot}}/E_{\text{grav}}\)
- Tsuribe & Inutsuka (1999): $\alpha < 0.55 - 0.65\beta$
• **Euler equations + Gravity** for a perfect gas:
  - Continuity
  - Linear momentum conservation
  - Total energy conservation
  - Closure relation: \[ P = (\gamma - 1)\rho \left( e - \frac{1}{2}u^2 \right) \]

• **Jeans length**
  \[ \lambda_J = c_s \sqrt{\frac{\pi}{G \rho_0 \gamma}} \]

• **Barotropic EOS to mimic the thermal behaviour of the gas**

  \[ \frac{P}{\rho} = C_s^2 = C_0^2 \left[ 1 + \left( \frac{\rho}{\rho_c} \right)^{2/3} \right] \]

  \[
  \begin{cases}
    - \gamma = 1 & \text{si } \rho \ll \rho_c \rightarrow \text{ISOTHERMAL} \\
    - \gamma = 5/3 & \text{si } \rho \gg \rho_c \rightarrow \text{ADIABATIC}
  \end{cases}
  \]
- Why must we model radiative transfer?
  - Interaction gas-dust during the collapse => opacities
  - More realistic
  - Radiative pressure effect for high mass star formation (> 20 M\(_\odot\))
  - With FLD (Bate 09), number of object/5 in giant molecular clouds!
  - Comparison with observations (L\(_{\text{acc}}\) 1\(^{\text{st}}\) core...)
  - Holy grail: Have access and control the entropy level of the protostar...

⇒ The radiative transfer equation:

\[
\left( \frac{1}{c} \frac{\partial}{\partial t} + n \cdot \nabla \right) I(x, t; n, \nu) = \eta(x, t; n, \nu) - \chi(x, t; n, \nu) I(x, t; n, \nu)
\]

TOO HEAVY for multi-D dynamic calculations!...
Approximations with grey opacities

⇒ Grey Flux Limited Diffusion

Optically thick (mean free path $\ll L_{sys}$) $\implies$ diffusion approximation: $P_r=1/3$ $E_r$, $\partial_t F_r = 0$

$\implies$ Solve a diffusion equation on the radiative energy:

$$\frac{\partial E_r}{\partial t} - \nabla \cdot \left( \frac{c \lambda}{\rho \kappa_R} E_r \right) = \kappa_P \rho (4\pi B - c E_r)$$

Flux limiter (e.g. Minerbo 78)

✓ 2 sets of recent opacity tables

⇒ Semenov et al 03 at low temperature ($< \sim 600$ K).

⇒ Fergusson et al. 05 model at high temperature
Flux Limited Diffusion in RAMSES

⇒ RHD solver in the comoving frame:

\[
\begin{align*}
\partial_t \rho & + \nabla [\rho \mathbf{u}] = 0 \\
\partial_t \rho \mathbf{u} & + \nabla [\rho \mathbf{u} \otimes \mathbf{u} + P_T] = -(\lambda - \frac{1}{3}) \nabla E_T \\
\partial_t E_T & + \nabla [\mathbf{u} (E_T + P_T)] = -(\lambda - \frac{1}{3}) \nabla E_T \cdot \mathbf{u} + \nabla \frac{c\lambda}{\kappa R \rho} \nabla E_T \\
\partial_t E_r & + \nabla [\mathbf{u} E_r] + \frac{1}{3} E_r \nabla \cdot \mathbf{u} = -(\lambda - \frac{1}{3}) E_r \nabla \cdot \mathbf{u} + \kappa_P \rho (4\pi B - c E_r) + \nabla \frac{c\lambda}{\kappa R \rho} \nabla E_T
\end{align*}
\]

Riemann solver - explicit  Corrective terms - explicit  Coupling + Diffusion - implicit

⇒ Largest fan of solution with speed:

\[
\begin{align*}
\mathbf{u} & - \sqrt{\frac{\gamma P}{\rho} + \frac{4E_T}{9\rho}} \\
\mathbf{u} & \\
\mathbf{u} & + \sqrt{\frac{\gamma P}{\rho} + \frac{4E_T}{9\rho}}
\end{align*}
\]

⇒ Implicit solved with an iterative conjugate gradient algorithm:

\[
\frac{C_v T^{n+1} - C_v T^n}{\Delta t} = -\kappa_P^n \rho^n c(a_R (T^{n+1})^4 - E_r^{n+1})
\]

\[
\frac{E_r^{n+1} - E_r^n}{\Delta t} - \nabla \frac{c\lambda^n}{\kappa_R \rho^n} \nabla E_r^{n+1} = +\kappa_P^n \rho^n c(a_R (T^{n+1})^4 - E_r^{n+1})
\]

⇒ Linearize \((T^{n+1})^4 = 4(T^n)^3 T^{n+1} - 3(T^n)^4\)
AMR vs. SPH; Fragmentation using a barotropic EOS

**AIM:** study the dependency of the results on numerical parameters

AMR: $64^3 (L_{\text{min}}=6) ; N_j=15$

SPH: $N_p=5 \times 10^5 ; N_N=50$

i.e. ~ 5300 particles/Jeans mass!
3D Collapse with RHD

\[ \alpha = 0.50, \beta = 0.04, m=2, A=0.1 \]

Cool & dense regions ==> low Jeans mass
+ Fragments compared to the barotropic EOS case
3D Collapse with RHD: when does it fragment?

\[ \Gamma_g = P \nabla \cdot u \]
\[ \Lambda_{\text{diff}} = \frac{\Delta x^2}{c/3\kappa_R \rho} \]

Inside fragment: \( \Lambda_{\text{diff}} \sim \Gamma_g \)

\[ \tau_{\text{min}} = \min \left[ \int \kappa_R \rho \, dr \right]_{x^+, -, y^+, -, z^+, -} \]

-Cooling efficiency depends only on the density
Collapse 3D with RHD

FLD modified fragmentation (number of object, time, etc..) compared to the barotropic case...

But dense core magnetized => Need a magnetic field that will inhibit fragmentation
Collapse with magnetic field

Ideal MHD $\Longleftrightarrow$ flux freezing: $\phi \propto BR^2$
Magnetic field lines are compressed: $P_{mag} \uparrow$ + twisted

$\Rightarrow$ Outflow
(Machida et al. 05, Banerjee & Pudritz 06, Hennebelle & Fromang 08)

Ratio $E_{mag}/E_{grav} : B^2R^3/(M^2/R) \propto (\phi/M)^2$
$\Rightarrow$ independent of $R$, $B$ & dilute gravity
- $(\phi/M) > (\phi/M)_{crit}$ subcritical cloud
- $(\phi/M) < (\phi/M)_{crit}$ supercritical cloud, collapse

Parameter: $\mu = (\phi/M)_{crit} / (\phi/M)$
(observations $\mu \sim 2-5$)
Outflow, no fragmentation...

\[ \alpha = 0.37, \beta = 0.045, \mu = 5 \]

Impact on outflow structure & launching
In progress - First RMHD calculations at this scale!

\[ \alpha = 0.37, \beta = 0.045, \mu = 5 \]

Magnetic field lines
In progress - RMHD calculations!

\[ \alpha = 0.37, \beta = 0.045, \mu = 20 \]

Barotropic EOS: gas hot in optically thin region!!
Radiation escape in the vertical direction
\[ \alpha = 0.37, \beta = 0.045, m=2, A=0.1 \]

- Sink accretion according to a density threshold

\[ \Delta x_{\text{min}} < 1 \text{ AU} \]

- All accreted matter converted into accretion luminosity:

\[ L_{\text{acc}} = \frac{GM_{\text{sink}} \dot{M}}{R_*} \]

\[ R_* = 2R_\odot \]

\[ \rho_{\text{sink}} = 5 \times 10^{-11} \text{g.cm}^{-3} \]
Conclusions and Perspectives

✓ AMR vs SPH: Convergence!

✓ Radiative transfer:
  – Dramatic impact on fragmentation & outflow launching
  – Small scale physics very important

✓ 3D ==> several objectives (next or distant future…):
  – Outflow barotrop vs FLD
  – Prestellar core fragmentation with RMHD:
    • Radiative Feedback from protostars (sink particles)
    • Brown Dwarfs formation
    • 2nd collapse: no fragmentation of the 1st core ==> 2nd core fragmentation?
  – Massive star formation:
    • Radiative pressure effect (M > 20 M_☉) ==> stop collapse
    • Mass deficit in calculations: Yorke & Sonnhalter (2002) ==> 42 M_☉