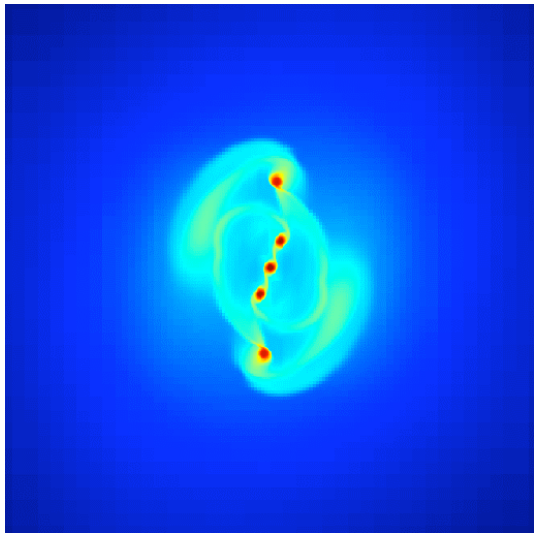
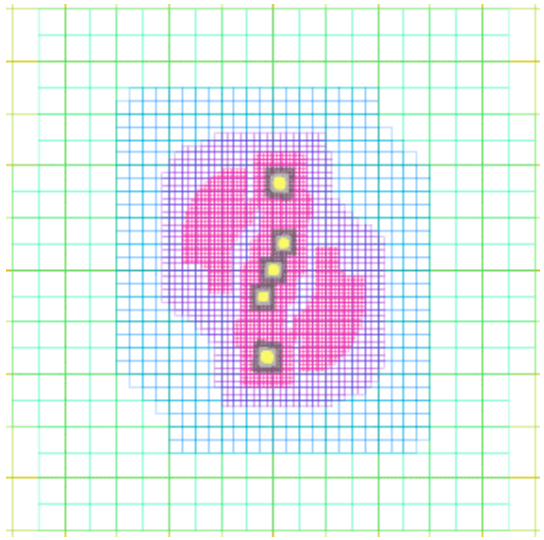
(Numerical) study of the collapse and of the fragmentation of prestellar dense core





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Outline

1. Context - Model

2. Radiative transfer for star formation calculation
   - → Flux Limited Diffusion Approximation
   - → Radiation Hydrodynamics equations solver

3. AMR vs SPH
   - → Fragmentation study

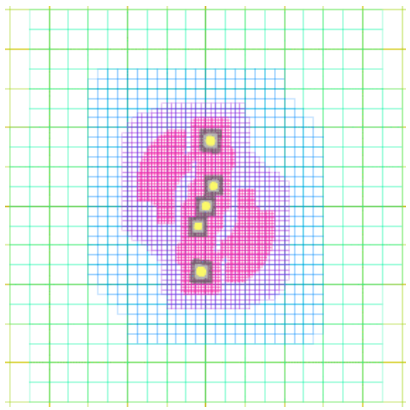
4. 3D AMR RMHD with FLD and ideal MHD
   - → 3D RHD collapse calculations
   - → 3D RMHD collapse calculations

2 main numerical methods:

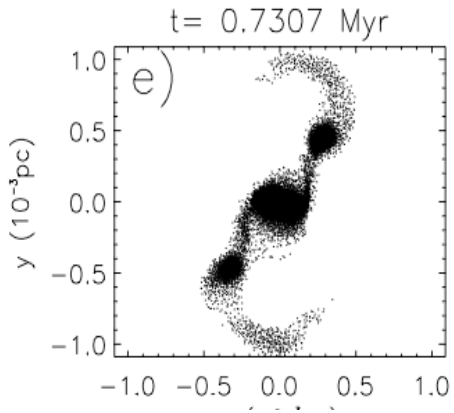
- Grid based : Hennebelle ,Fromang & Teyssier 08 (MHD), Krumholz et al 07 (RHD), Banerjee & Pudritz 06 (MHD), etc…
- SPH: Bate et al. (RMHD), Stamatellos et al 08 (RHD), etc…

Debate on the accuracy of both methods:

=> Are these methods appropriate to study structure formation? Are they converging?



- **DRAGON** code from Cardiff University Star Formation Group (eg. *Goodwin et al 2004*) Standard version, OpenMP & parallel Tree



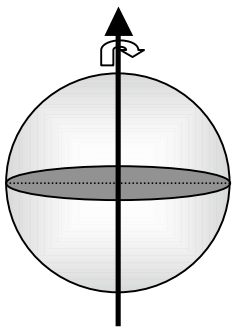
- **RAMSES** code (*Teyssier 2002)*

Finite volume, 2nd order Godunov scheme

Ideal MHD solver (*Fromang et al. 2006*)

MPI parallelized

Collapse with a *m*=2 perturbation - Set up



Model: isolated dense core of 1 $M_\odot$ in solid body rotation

1/ Small scales

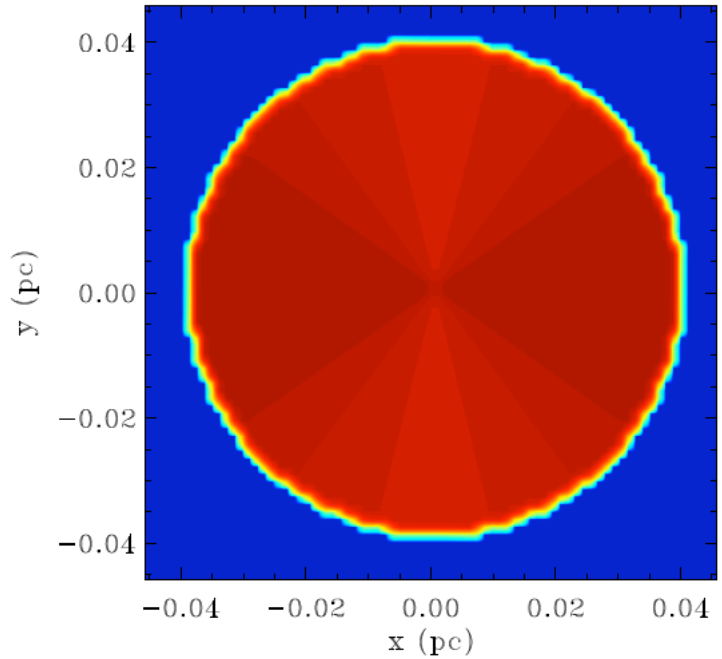
2/ Fragmentation

Fragmentation : IMF, disk stability

↪ Azimuthal density perturbation ==> spiral arms

– Perturbation *m*=2, amplitude *A*=0.1

$$\rho = \rho_0[1 + A cos(m\theta)]$$



↪ 2 parameters to set the system

– Thermal support: α = 0.5 (E_{th}/E_{grav})

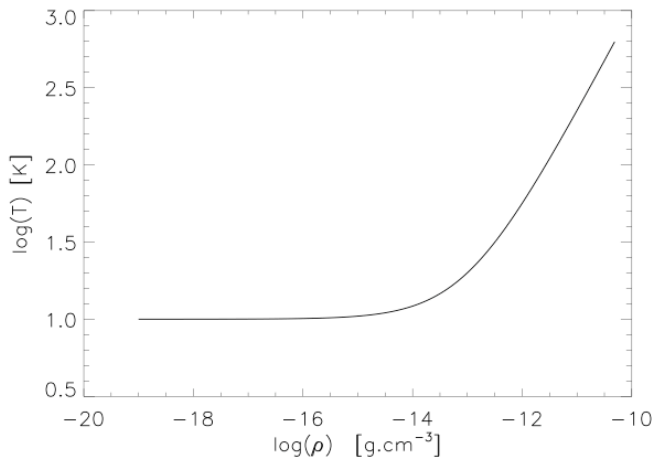
– Rotational support: β = 0.04 (E_{rot}/E_{grav})

– Tsuribe & Inutsuka (1999): $\alpha < 0.55 - 0.65\beta$

Usual gas equation - Jeans length

- **Euler equations + Gravity for a perfect gas:**
  - Continuity
  - Linear momentum conservation
  - Total energy conservation
  - Closure relation: $P = (\gamma - 1)\rho\left(e - \frac{1}{2}u^2\right)$

- **Jeans length** $\lambda_J = c_s\sqrt{\frac{\pi}{G\rho_0\gamma}}$



- **Barotropic EOS to mimic the thermal behaviour of the gas**

$$\frac{P}{\rho} = C_s^2 = C_0^2\left[1 + \left(\frac{\rho}{\rho_c}\right)^{2/3}\right] \quad \begin{cases} -\ \gamma = 1 & \text{si } \rho << \rho_c \rightarrow \textbf{ISOTHERMAL} \\ -\ \gamma = 5/3 & \text{si } \rho >> \rho_c \rightarrow \textbf{ADIABATIC} \end{cases}$$

- **Why must we model radiative transfer ?**
  - Interaction gas-dust during the collapse ==> opacities
  - More realistic
  - Radiative pressure effect for high mass star formation (> 20 $M_\odot$)
  - With FLD (*Bate 09)*, number of object/5 in giant molecular clouds!
  - Comparison with observations (L_{acc} 1st core…)
  - Holy grail : Have access and control the entropy level of the protostar…

⇒ **The radiative transfer equation:**

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \mathbf{n}\cdot\nabla\right)I(\mathbf{x},t;\mathbf{n},\nu) = \eta(\mathbf{x},t;\mathbf{n},\nu) - \chi(\mathbf{x},t;\mathbf{n},\nu)I(\mathbf{x},t;\mathbf{n},\nu)$$

TOO HEAVY for multi-D dynamic calculations!…

Approximations with grey opacities

⇒ Grey Flux Limited Diffusion

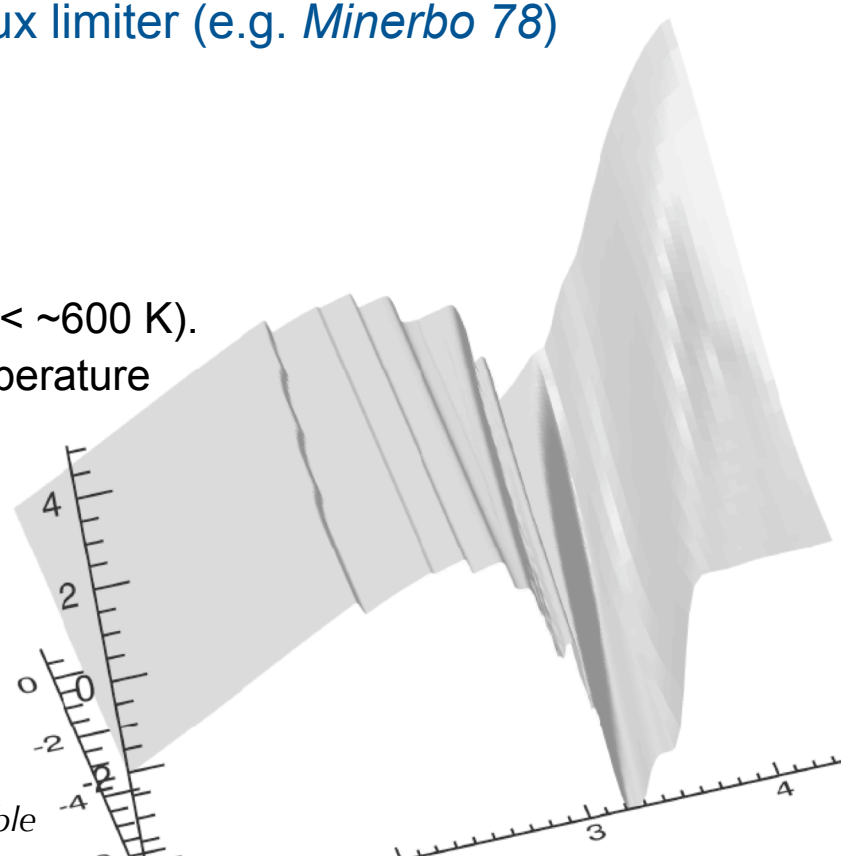
Optically thick (mean free path <<L_{sys}) ==> diffusion approximation: P_r=1/3 E_r, $\partial_t \mathbf{F}_\mathrm{r} = 0$
==> Solve a diffusion equation on the radiative energy:

$$\frac{\partial E_\mathrm{r}}{\partial t} - \nabla \cdot \left(\frac{c\lambda}{\rho \kappa_\mathrm{R}} \nabla E_\mathrm{r} \right) = \kappa_\mathrm{P} \rho (4\pi B - c E_\mathrm{r})$$

Flux limiter (e.g. *Minerbo 78*)

✓ 2 sets of recent opacity tables

- ⇒ *Semenov et al 03* at low temperature (< ~600 K).
- ⇒ *Fergusson et al. 05* model at high temperature



Rosseland grey opacity table

Flux Limited Diffusion in RAMSES

⇒ RHD solver in the comoving frame:

$$\begin{cases} \partial_t \rho & + & \nabla[\rho \mathbf{u}] & = & 0 \\ \partial_t \rho \mathbf{u} & + & \nabla[\rho \mathbf{u} \otimes \mathbf{u} + P_\mathrm{T}] & = & -(\lambda - \frac{1}{3})\nabla E_\mathrm{r} \\ \partial_t \mathrm{E_T} & + & \nabla[\mathbf{u}(\mathrm{E_T} + P_\mathrm{T})] & = & -(\lambda - \frac{1}{3})\nabla E_\mathrm{r} \cdot \mathbf{u} \quad + \quad \nabla \frac{c\lambda}{\kappa_\mathrm{R}\rho} \nabla E_\mathrm{r} \\ \partial_t E_\mathrm{r} & + & \nabla[\mathbf{u} E_\mathrm{r}] + \frac{1}{3} E_\mathrm{r} \nabla \cdot \mathbf{u} & = & -(\lambda - \frac{1}{3}) E_\mathrm{r} \nabla \cdot \mathbf{u} \quad + \quad \kappa_\mathrm{P}\rho(4\pi B - cE_\mathrm{r}) + \nabla \frac{c\lambda}{\kappa_\mathrm{R}\rho} \nabla E_\mathrm{r} \end{cases}$$

Riemann solver - explicit Corrective terms - explicit Coupling + Diffusion - implicit

⇒ Largest fan of solution with speed:

$$\begin{cases} u - \sqrt{\frac{\gamma P}{\rho} + \frac{4E_\mathrm{r}}{9\rho}} \\ u \\ u + \sqrt{\frac{\gamma P}{\rho} + \frac{4E_\mathrm{r}}{9\rho}} \end{cases}$$

⇒ Implicit solved with an iterative conjugate gradient algorithm:

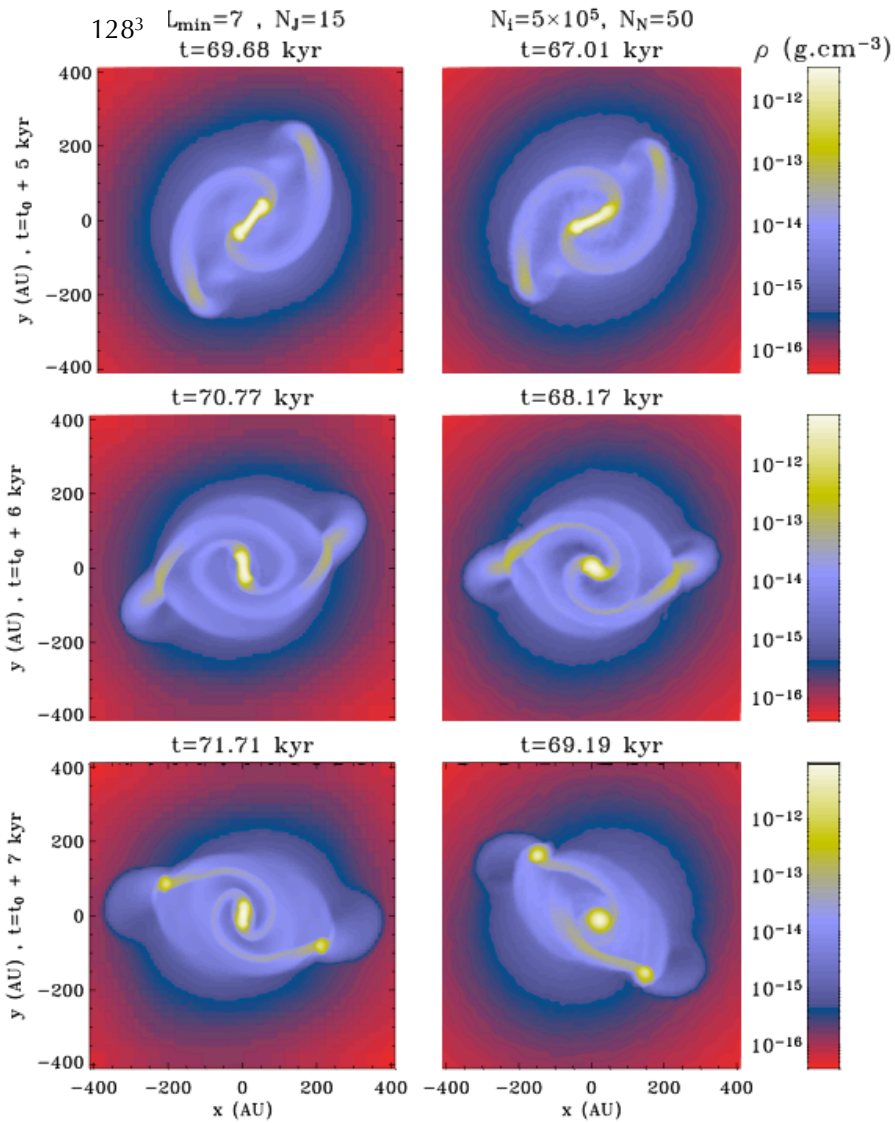
$$\frac{C_v T^{n+1} - C_v T^n}{\Delta t} = -\kappa_\mathrm{P}^n \rho^n \mathrm{c}(a_\mathrm{R}(T^{n+1})^4 - E_\mathrm{r}^{n+1})$$

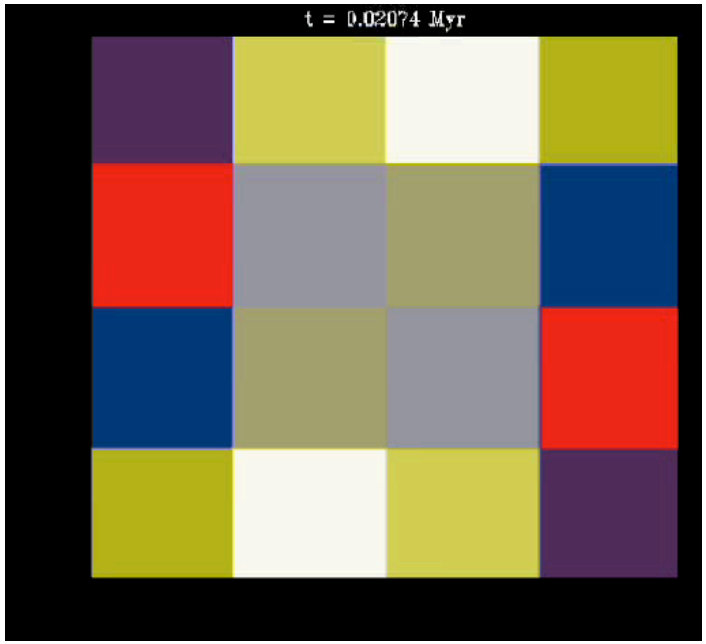
$$\frac{E_\mathrm{r}^{n+1} - E_\mathrm{r}^n}{\Delta t} - \nabla \frac{\mathrm{c}\lambda^n}{\kappa_R^n \rho^n} \nabla E_\mathrm{r}^{n+1} = +\kappa_\mathrm{P}^n \rho^n \mathrm{c}(a_\mathrm{R}(T^{n+1})^4 - E_\mathrm{r}^{n+1})$$

⇒ Linearize $(T^{n+1})^4 = 4(T^n)^3 T^{n+1} - 3(T^n)^4$

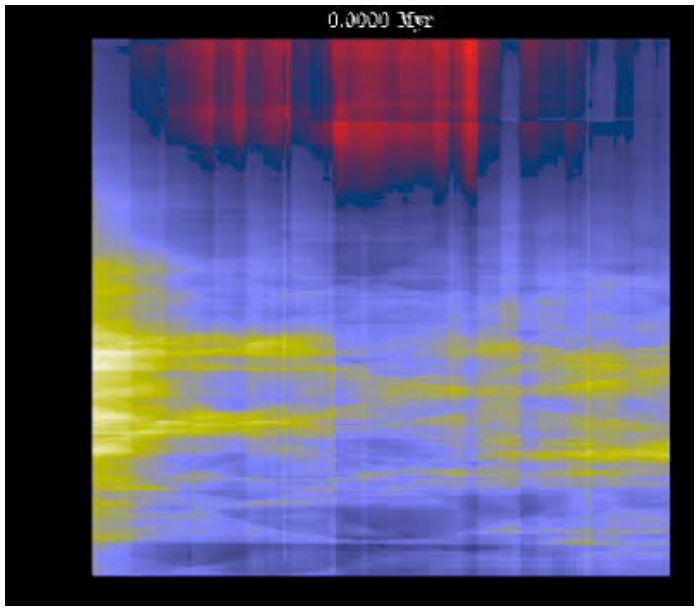
AMR vs. SPH; Fragmentation using a barotropic EOS

AIM: **study the dependency of the results on numerical parameters**





AMR



SPH

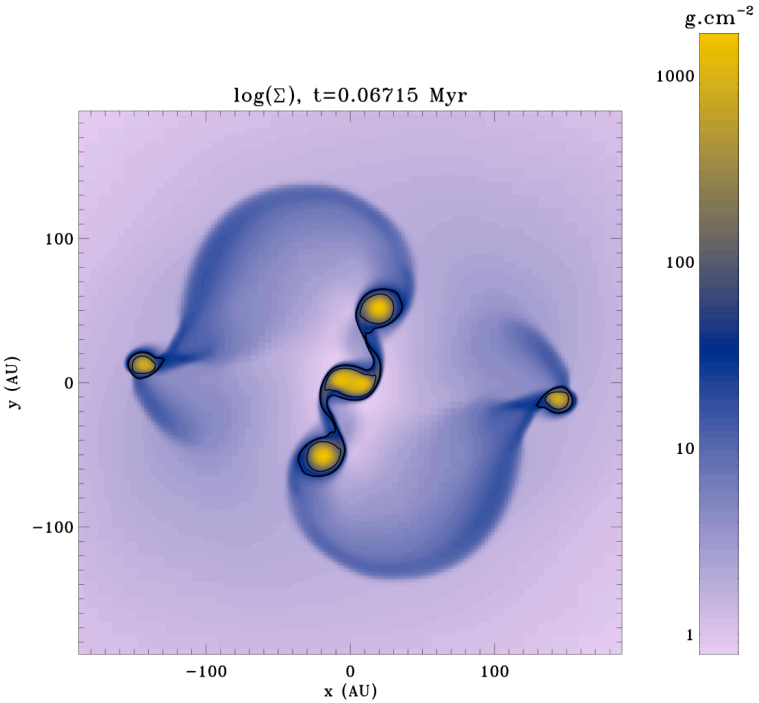
AMR: 64^3 (L_{min}=6) ; N_J=15 !

SPH: N_p=5x10^5 ; N_N=50

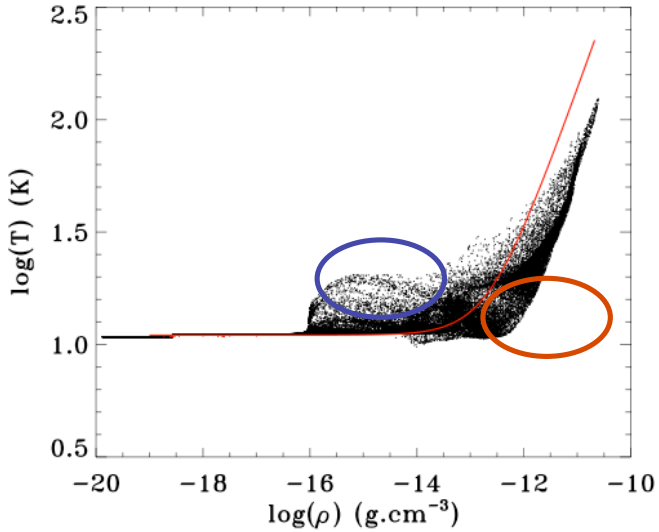
i.e. ~ 5300 particles/Jeans mass !

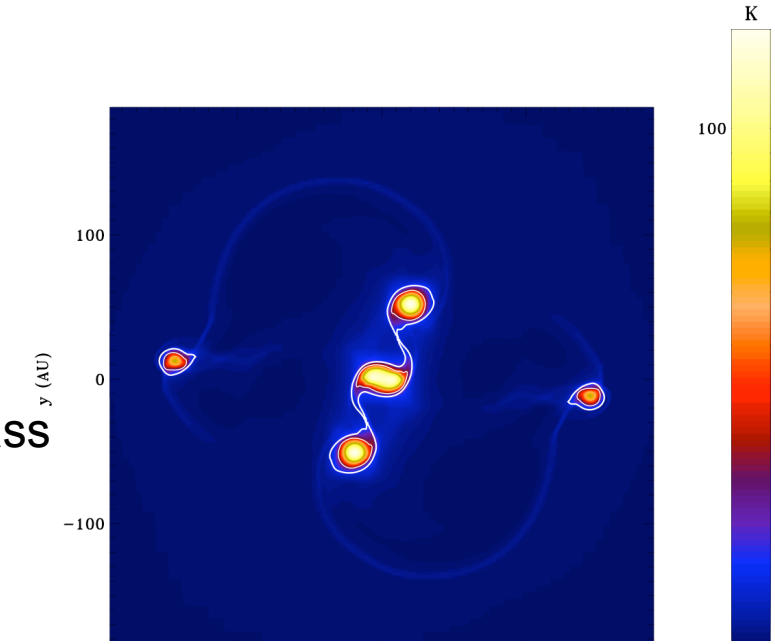
3D Collapse with RHD

α = 0.50 , β = 0.04 , m=2 , A=0.1



Minerbo flux limiter





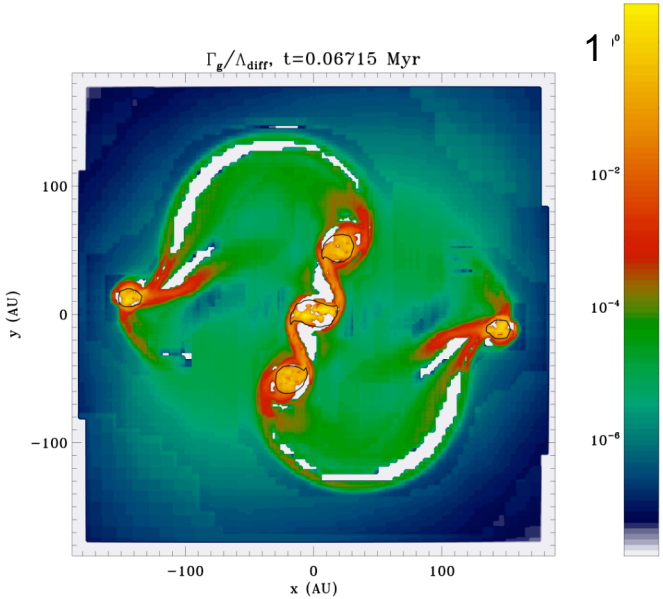
Cool & dense regions ==> low Jeans mass

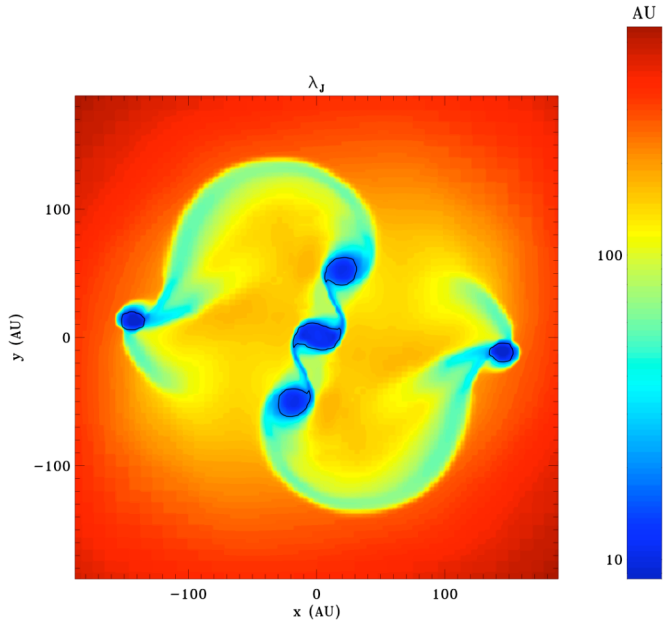
+ Fragments compared to the barotropic EOS case

3D Collapse with RHD: when does it fragment?

$$\Gamma_{\rm g} = P\nabla\cdot u$$

$$\Lambda_{\rm diff} = \frac{\Delta x^2}{c/3\kappa_{\rm R}\rho}$$

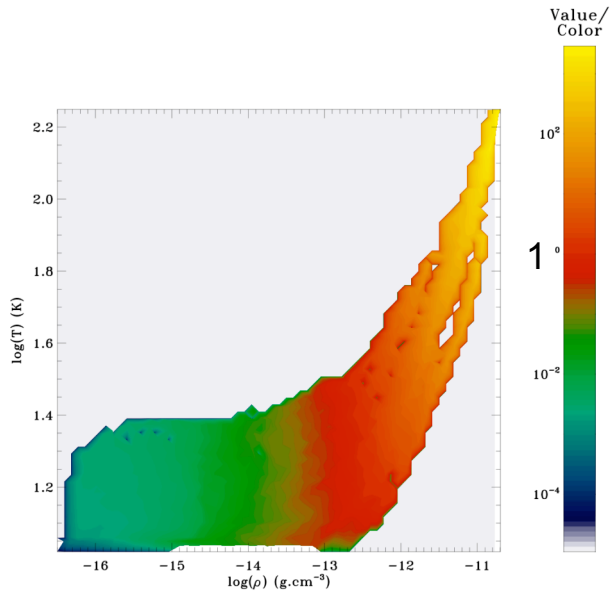




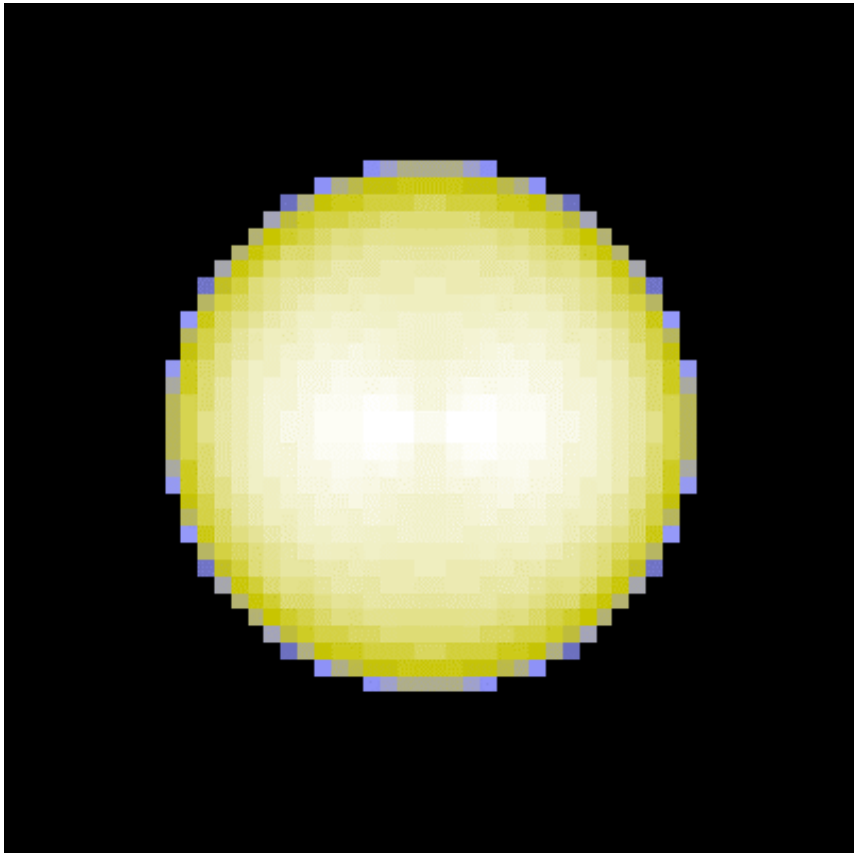
Inside fragment: $\Lambda_{\rm diff} \sim \Gamma_{\rm g}$

$$\tau_{\rm min} = \min\left[\int \kappa_{\rm R}\rho dr\right]_{x^{+,-},y^{+,-},z^{+,-}}$$

-Cooling efficiency depends only on the density



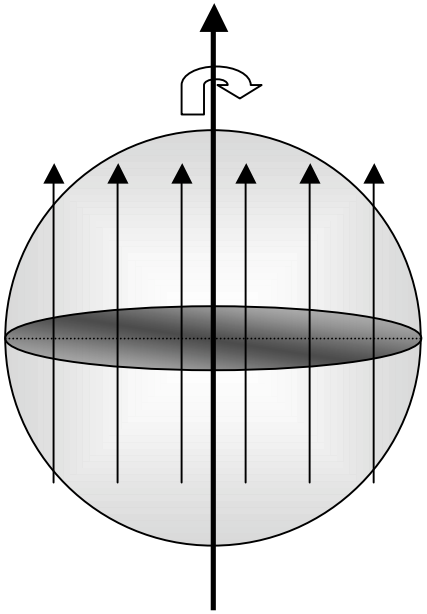
Collapse 3D with RHD



FLD modified fragmentation (number of object, time, etc..) compared to the barotropic case…

But dense core magnetized ==> Need a magnetic field that will inhibit fragmentation

Collapse with magnetic field



Ideal MHD <==> flux freezing: $\phi \propto BR^2$
Magnetic field lines are compressed : P_{mag} ↗
+twisted

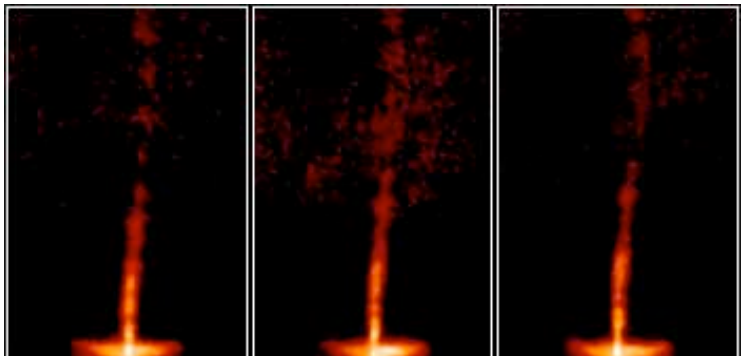
==> *Outflow*
(Machida et al. 05, Banerjee & Pudritz 06, Hennebelle & Fromang 08)

Ratio E_{mag}/E_{grav} : $B^2R^3/(M^2/R) \propto (\phi/M)^2$
==> independent of R, B & dilute gravity

- $(\phi/M) > (\phi/M)_{crit}$ subcritical cloud
- $(\phi/M) < (\phi/M)_{crit}$ supercritical cloud, collapse

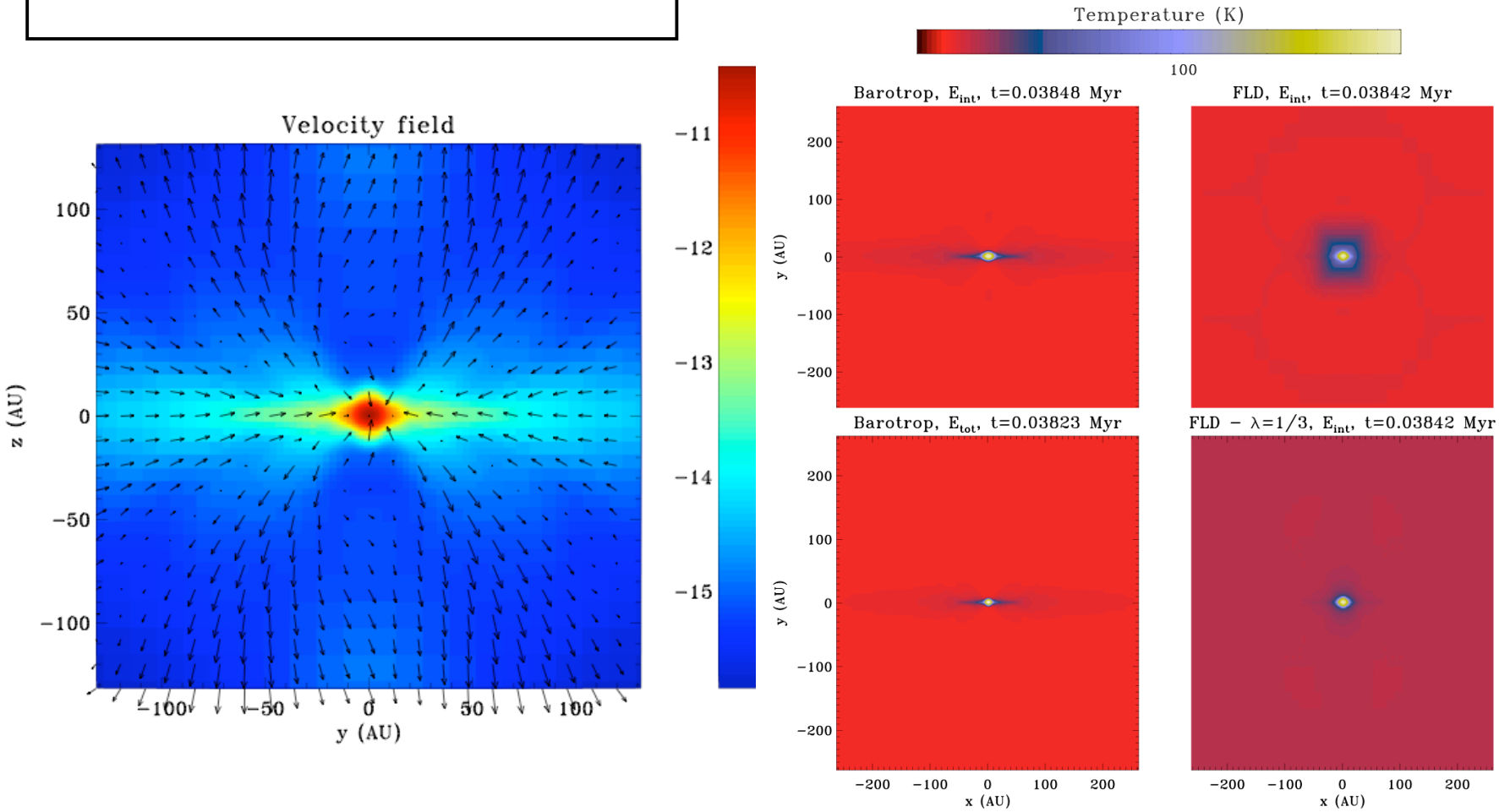
Parameter : $\mu = (\phi/M)_{crit} / (\phi/M)$
(observations $\mu \sim 2$-5)

Protostellar outflow



Outflow, no fragmentation…

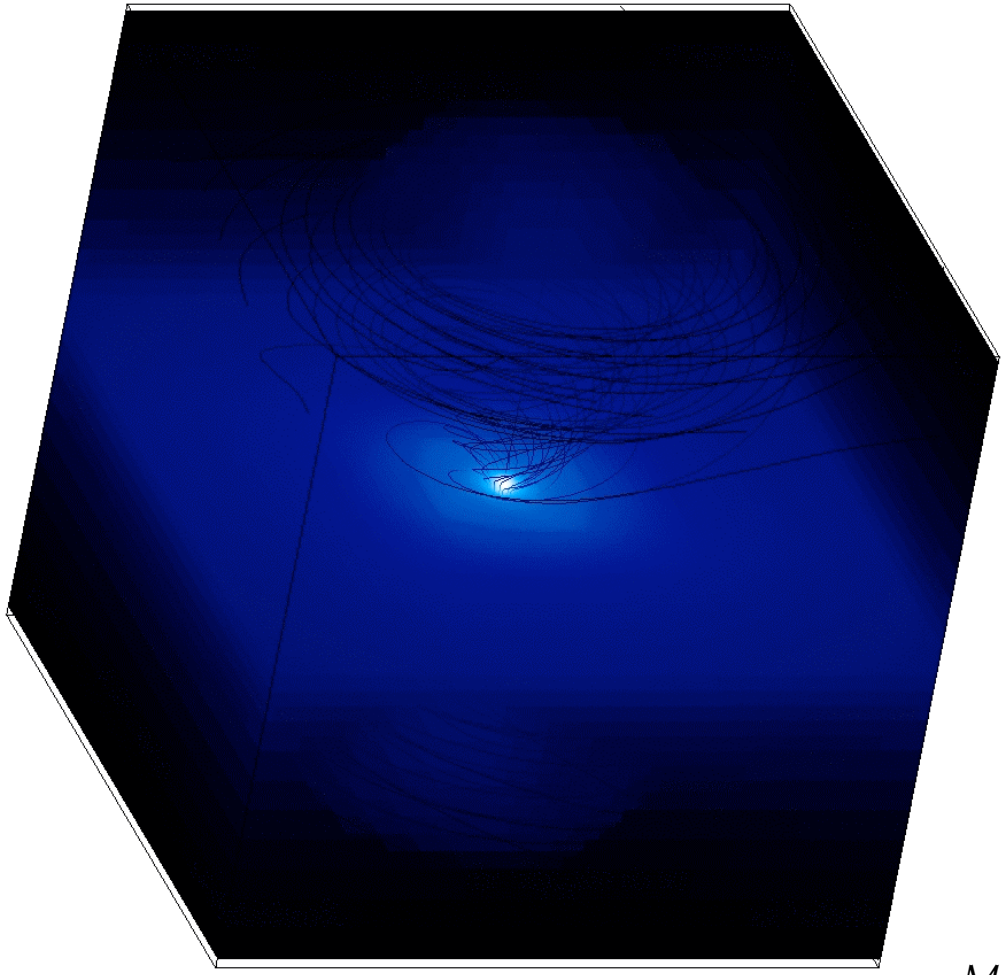
$\alpha = 0.37$, $\beta = 0.045$, $\mu = 5$



Impact on outflow structure & launching

In progress - First RMHD calculations at this scale!

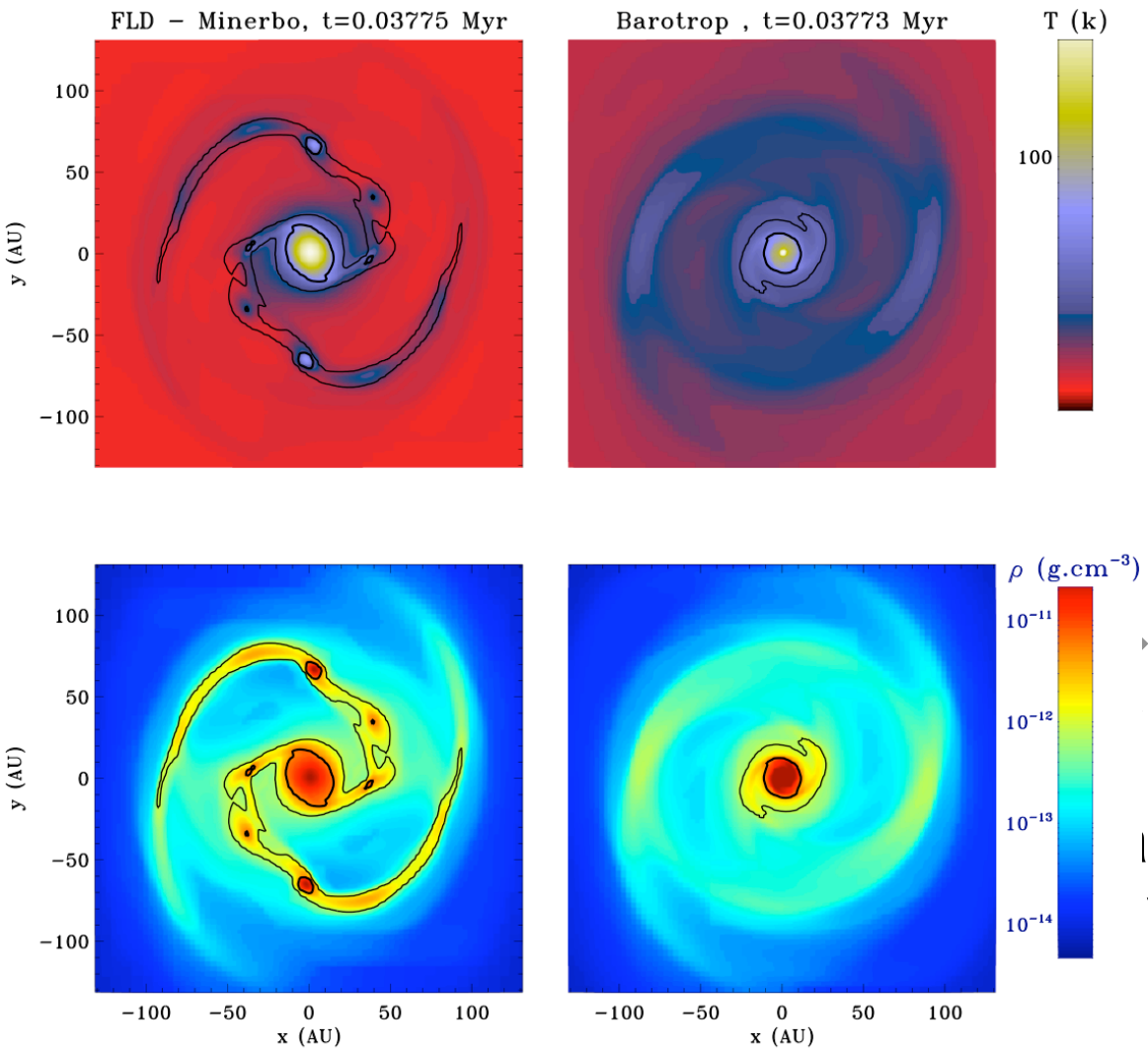
$\alpha = 0.37$, $\beta = 0.045$, $\mu = 5$



Magnetic field lines

In progress - RMHD calculations!

$\alpha = 0.37$, $\beta = 0.045$, $\mu = 20$



rotropic EOS: gas hot in optically thin region!!

ıdiation escape in the vertical direction

α = 0.37 , β = 0.045 , m=2 , A=0.1

$$\rho_{\rm sink} = 5 \times 10^{-11} {\rm g.cm^{-3}}$$

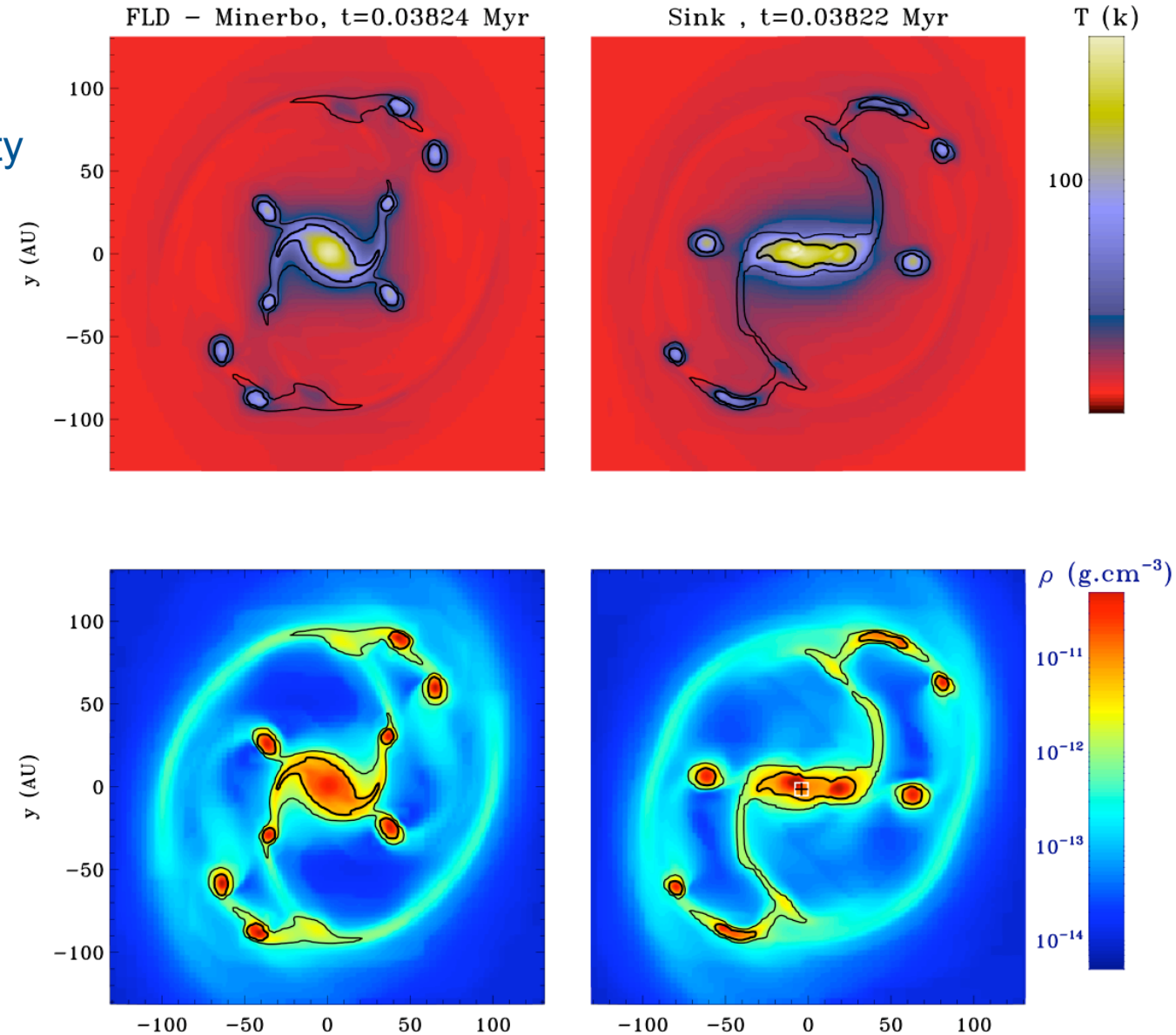
- Sink accretion according to a density threshold

$$\Delta x_{\rm min} < 1 \ {\rm AU}$$

- All accreted matter converted into accretion luminosity:

$$L_{\rm acc} = \frac{G M_{\rm sink} \dot{M}}{R_\star}$$

$$R_\star = 2 R_\odot$$



Conclusions and Perspectives

- ✓ **AMR vs SPH:** Convergence!

- ✓ **Radiative transfer:**
  - **Dramatic impact on fragmentation & outflow launching**
  - **Small scale physic very important**

- ✓ **3D ==> several objectives (next or distant future…) :**
  - **outflow barotrop vs FLD**
  - **Prestellar core fragmentation with RMHD:**
    - **Radiative Feedback** from protostars (sink particles)
    - **Brown Dwarfs formation**
    - **2nd collapse** : no fragmentation of the 1st core ==> 2nd core fragmentation?
  - **Massive star formation :**
    - Radiative pressure effect ($M > 20\ M_s$) ==> stop collapse
    - Mass deficit in calculations : Yorke & Sonnhalter (2002) ==> 42 M