Simulations of Solar Wind Turbulence

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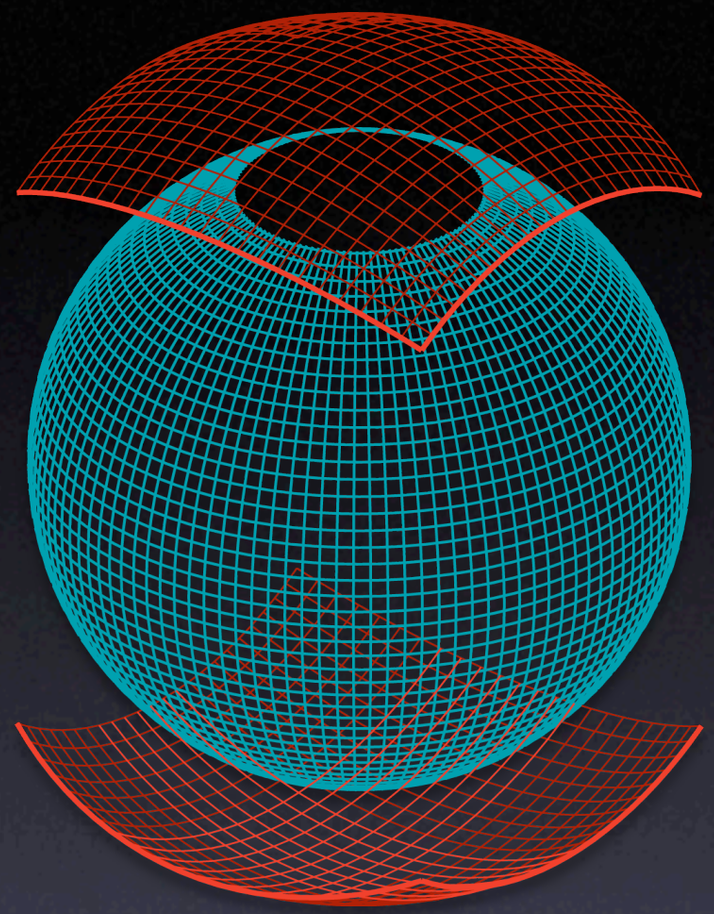
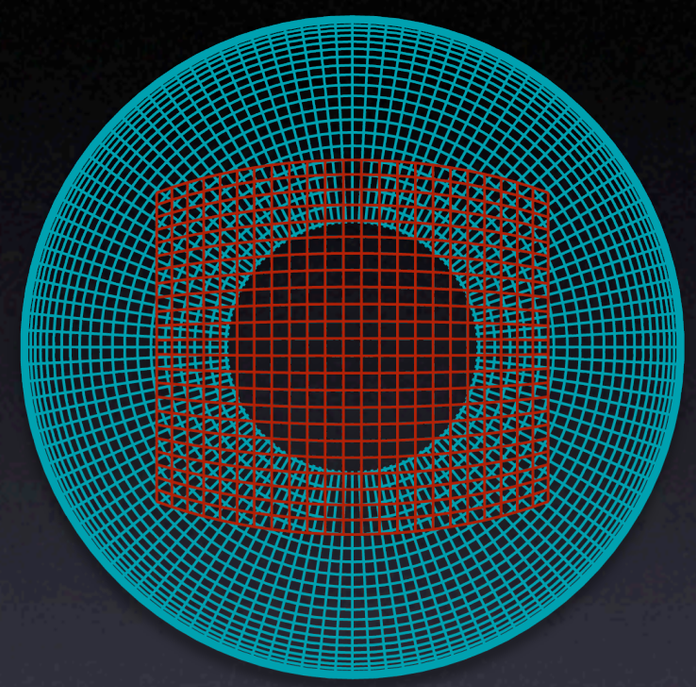
Outline — Part I

- General introduction
- A (time-relaxation) solution from 1 $R_\odot$ to 20 $R_\odot$ and a (stationary) solution from 20 $R_\odot$ to 1.5 AU
- A three-dimensional, time-dependent solution from 20 $R_\odot$ to 1.5 AU with the addition of an Alfvénic wave packet at the 20 $R_\odot$ boundary (with fairly general inner boundary conditions thanks to Gábór Toth)
- Evolution of waves and non-linear interactions that initiate a cascade

Composite Grid





- No singularity on the pole axis
- No condensation of grid points toward the poles
- Same equations for all 3 grids
- Interpolation between grids at boundaries

Numerics: CWENO (***Central Weighted Essentially Non-Oscillatory***)

Grid spacing: $\Delta\theta=\Delta\varphi=2^{\circ}$, linear spacing along radius

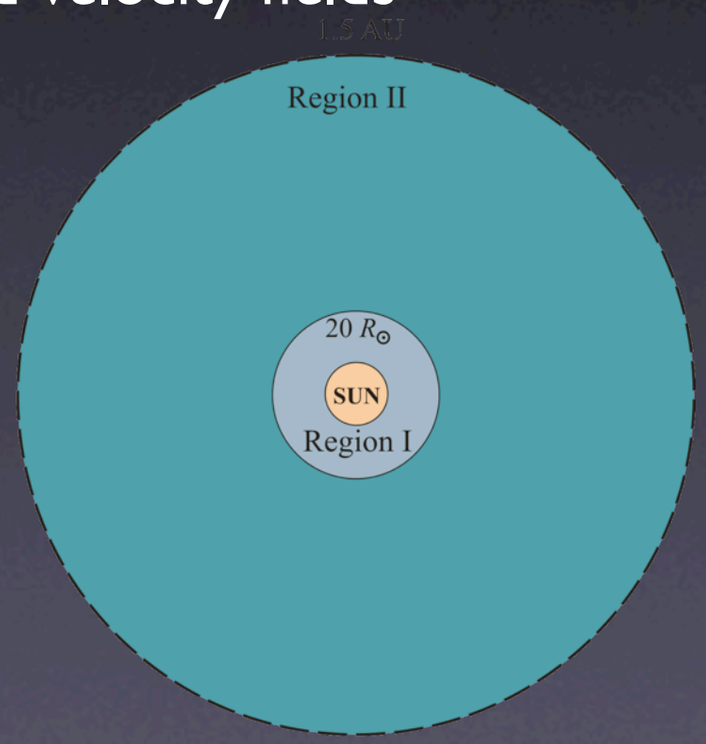
The Numerical Grid…

Step 1
Region I: 1–20 $R_\odot$

Type of flow: transonic, trans-Alfvénic, mixed initial-boundary value problem
Initial state: Parker-type flow with WKB Alfvén waves
Method: Time relaxation
Boundary conditions in the co-rotating frame: At one $R_\odot$ the magnetic and velocity fields are co-aligned.

Step 2
Region II: 20 $R_\odot$ – outer boundary

Type of flow: supersonic, super-Alfvénic
Problem: initial value (Cauchy) problem
Method: Outward integration along radius
Boundary conditions: from region I



Usmanov, A.V., and M.L. Goldstein, A tilted-dipole MHD model of the solar corona and solar wind, JGR., 108, doi: 10.1029/2002JA009777, 2003.

A Quiet, Time Stationary Initial State

Model Parameters

- Temperature at 1 $R_\odot$: 1.8×10^6 K
- Number density at 1 $R_\odot$: 5×10^7 cm^{-3}
- Driving amplitude of Alfvén waves at 1 $R_\odot$: 35 km/s
- Intensity of dipole-like magnetic field at 1 $R_\odot$: 11 Gauss
- Polytropic index:

  Region 1: $\gamma = 1.08$. Regions 2: $\gamma = 1.46$

Step 1: MHD Equations with WKB Alfvén Waves (Region I)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0$$

Corotating frame of reference

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot \left[\rho \mathbf{v}\mathbf{v} + \left(P + \frac{\mathcal{E}}{2} + \frac{B^2}{8\pi}\right)\mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{4\pi}\right] + \rho\left[\frac{GM_\odot}{r^2}\hat{r} + 2\mathbf{\Omega} \times \mathbf{v} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})\right]$$

Powell (1994)

$$= -\frac{\mathbf{B}(\nabla \cdot \mathbf{B})}{4\pi}$$

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = -(\nabla \cdot \mathbf{B})\mathbf{v}$$

$$\frac{\partial}{\partial t}\left[\frac{\rho}{2}(v^2 - |\mathbf{\Omega} \times \mathbf{r}|^2) + \frac{P}{\gamma - 1} + \frac{B^2}{8\pi} - \frac{\rho G M_\odot}{r} + \mathcal{E}\right]$$

$$+ \nabla \cdot \left\{\left[\frac{\rho}{2}(v^2 - |\mathbf{\Omega} \times \mathbf{r}|^2) + \frac{\gamma P}{\gamma - 1} - \frac{\rho G M_\odot}{r}\right]\mathbf{v} + \frac{\mathbf{B}}{4\pi} \times (\mathbf{v} \times \mathbf{B}) + \left(\frac{3}{2}\mathbf{v} + \mathbf{V}_A\right)\mathcal{E}\right\}$$

$$= -\frac{(\mathbf{v} \cdot \mathbf{B})}{4\pi}(\nabla \cdot \mathbf{B})$$

Effect of Alfvén waves

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot [(\mathbf{v} + \mathbf{V}_A)\mathcal{E}] + \frac{\mathcal{E}}{2}\nabla \cdot \mathbf{v} = -|\mathbf{v} + \mathbf{V}_A|\frac{\mathcal{E}}{L}$$

(Usmanov, 1996; Usmanov et al., 2000; Usmanov and Goldstein, 2003)

- ρ, $\mathbf{B}$, and P are the number density, magnetic field, and thermal pressure, respectively.
- $\mathbf{v}$ is the velocity in the frame of reference corotating with the Sun;
- $\mathbf{u}$ is the velocity in the inertial frame, $\mathbf{v} = \mathbf{u} - \mathbf{w}$ and $\mathbf{w} = \Omega \times \mathbf{r}$;
- Ω is the solar rotation rate;
- γ is the polytropic index ($\gamma = 1.47$);
- $M_\odot$ is the solar mass, G is the universal gravitation constant;
- $\hat{r}$ is a unit vector in the radial direction;

Step 2: Global State from 20 $R_\odot$->Outer Boundary (Region 2)

The solution at 20 $R_\odot$ is extended to the outer boundary by a radial integration using the following equations:

$$\nabla \cdot (\rho \mathbf{v}) = 0$$

$$\nabla \cdot \left[\rho \mathbf{u}\mathbf{v} - \frac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(P + \frac{B^2}{8\pi} \right) \mathbf{I} \right] = -\rho \left(\frac{GM_\odot}{r^2}\hat{\mathbf{r}} + \mathbf{\Omega} \times \mathbf{u} \right)$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \left[\mathbf{v} \left(\frac{\rho u^2}{2} + \frac{\gamma P}{\gamma - 1} + \frac{B^2}{4\pi} \right) - \frac{\mathbf{B}(\mathbf{u} \cdot \mathbf{B})}{4\pi} + \mathbf{w} \left(P + \frac{B^2}{8\pi} \right) \right] = -\rho u_r \frac{GM_\odot}{r^2}$$

$$\nabla \cdot (\mathbf{v} P^{1/\gamma}) = 0$$

Adding a Wave Packet: Time Integration Outward from 20 $R_\odot$

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Step 3: Equations with Alfvénic Fluctuations — Time Integration from 20 $R_\odot$ to 1.5 AU

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot \left[\rho \mathbf{u}\mathbf{v} - \frac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(P + \frac{B^2}{8\pi} \right) \mathbf{I} \right] = -\rho \left(\frac{GM_\odot}{r^2}\hat{\mathbf{r}} + \mathbf{\Omega} \times \mathbf{u} \right)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\frac{\partial}{\partial t}\left(\frac{\rho u^2}{2} + \frac{P}{\gamma - 1} + \frac{B^2}{8\pi} \right) + \nabla \cdot \left[\mathbf{v} \left(\frac{\rho u^2}{2} + \frac{\gamma P}{\gamma - 1} + \frac{B^2}{4\pi} \right) - \frac{\mathbf{B}(\mathbf{u} \cdot \mathbf{B})}{4\pi} + \mathbf{w} \left(P + \frac{B^2}{8\pi} \right) \right]$$
$$= -\rho u_r \frac{GM_\odot}{r^2}$$

$$\frac{\partial P^{1/\gamma}}{\partial t} + \nabla \cdot (\mathbf{v} P^{1/\gamma}) = 0$$

Defining the Initial Conditions for Including Waves

Yeh and Dryer [1985] pointed out that:

> For a solution of an initial-boundary value problem of magnetohydrodynamics, the initial data must be constrained by Gauss' law of magnetic solenoidality and the boundary data must be constrained by Faraday's law of magnetic induction. These are necessary and sufficient conditions. **...the spatial operation of divergence cannot be performed mathematically at boundary points.**

$$\nabla \cdot \mathbf{B} = 0 \quad \text{at} \quad t = t_0,$$

$$\left(\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} \right) \cdot \mathbf{l}_n = 0 \quad \text{for} \quad t > t_0.$$

where $\mathbf{l}_n$ is the unit normal at the boundary surface

Boundary Conditions ...

$$\frac{\partial B_r}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})|_r$$

This equation is satisfied if $B_r \neq B_r(t)$ and $\mathbf{v} \times \mathbf{B} = \nabla\psi$, where ψ is an arbitrary function.

$$\mathbf{v} = \mathbf{v}_0 + \delta\mathbf{v}, \quad \mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}$$

If $\mathbf{v}_0 \parallel \mathbf{B}_0$ and $\delta\mathbf{v} = \pm\delta\mathbf{B}/(4\pi\rho)^{1/2}$ (Alfvén waves) then

$$\pm\delta\mathbf{B} \times \mathbf{V}_A + \mathbf{v}_0 \times \delta\mathbf{B} = \nabla\psi \quad \text{or} \quad \bar{\mathbf{v}}_0 \times \delta\mathbf{B} = \nabla\psi$$

where

$$\mathbf{V}_A \equiv \mathbf{B}_0/(4\pi\rho)^{1/2}, \quad \bar{\mathbf{v}}_0 \equiv \mathbf{v}_0 \mp \mathbf{V}_A$$

$$\nabla_\theta\psi = \bar{v}_{0\phi}\delta B_r - \bar{v}_{0r}\delta B_\phi, \quad \nabla_\phi\psi = \bar{v}_{0r}\delta B_\theta - \bar{v}_{0\theta}\delta B_r$$

If $\delta B_r = 0$ then

$$\nabla_\theta\psi = \frac{\partial\psi}{\partial\theta} = -\,\bar{v}_{0r}\delta B_\phi, \quad \nabla_\phi\psi = \frac{1}{\sin\theta}\frac{\partial\psi}{\partial\phi} = \bar{v}_{0r}\delta B_\theta$$

$$\rightarrow \quad \frac{\partial B_r}{\partial t} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\nabla_\phi\psi) - \frac{\partial(\nabla_\theta\psi)}{\partial\phi}\right] \equiv 0$$

Our (present) choice: $\psi \equiv \eta u_0 B_0 \sin\theta \sin^2(m\theta)\sin(n\phi)\sin(t)$.

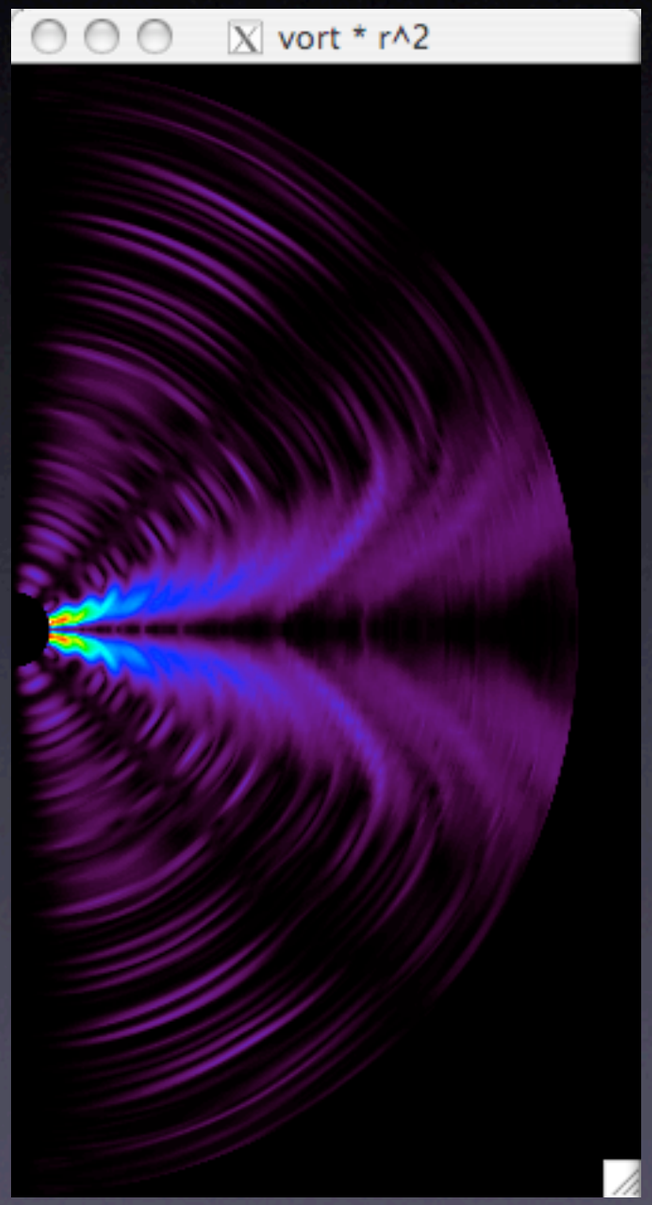
$$\delta B_\theta = \frac{1}{\bar{v}_{0r}\sin\theta}\frac{\partial\psi}{\partial\phi} = \eta B_0 \frac{u_0}{\bar{v}_{0r}} n \sin^2(m\theta)\cos(n\phi)\sin(t)$$

$$\delta B_\phi = -\frac{1}{\bar{v}_{0r}}\frac{\partial\psi}{\partial\theta} = -\,\eta B_0 \frac{u_0}{\bar{v}_{0r}}\sin(m\theta)[\cos\theta\sin(m\theta) + 2m\sin\theta\cos(m\theta)]\sin(n\phi)\sin(t)$$

(If $m = 2l$, $l = 1, 2, ...$ then $\delta B_\phi = 0$ on the equator)

The Evolution of an "Alfvénic" Wave Packet 0° Tilt

Vorticity × r^2

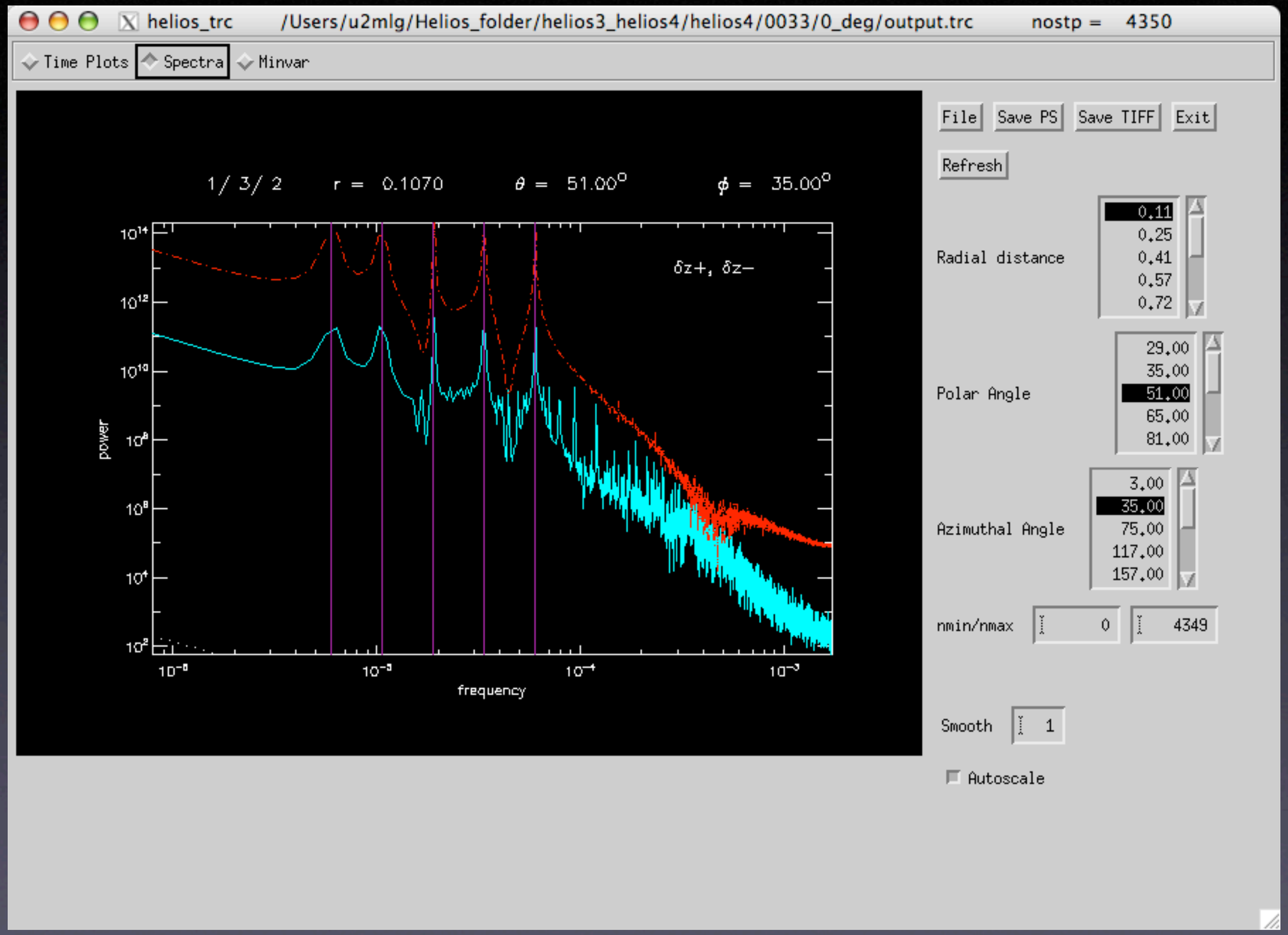


Results that follow:

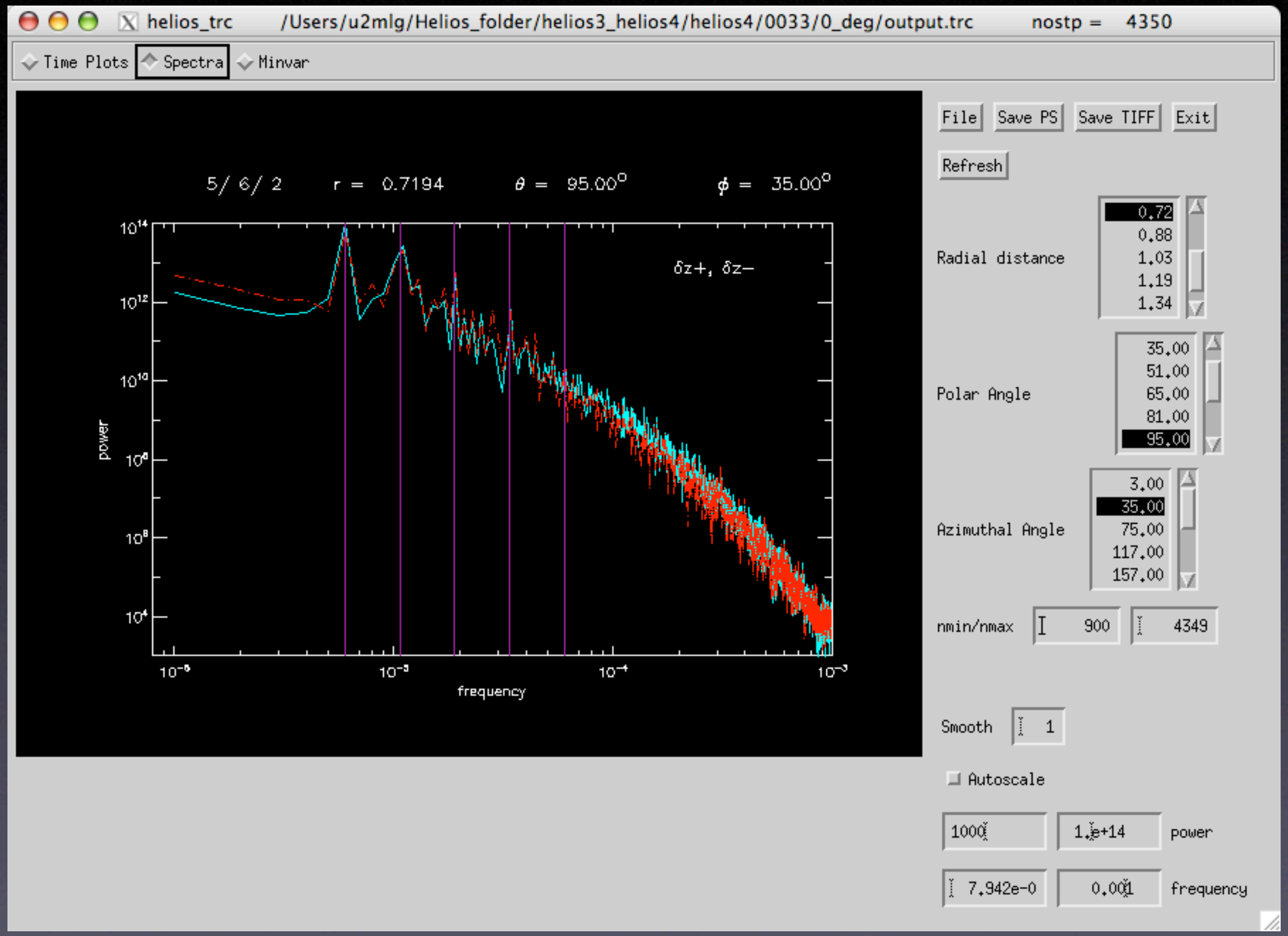
Power spectrum at the inner boundary (**20 $R_\odot$**), near **0.75 AU** in the current sheet and at high latitude, and near **1.5 AU** in the current sheet and high latitude

All results shown at only one azimuthal angle.

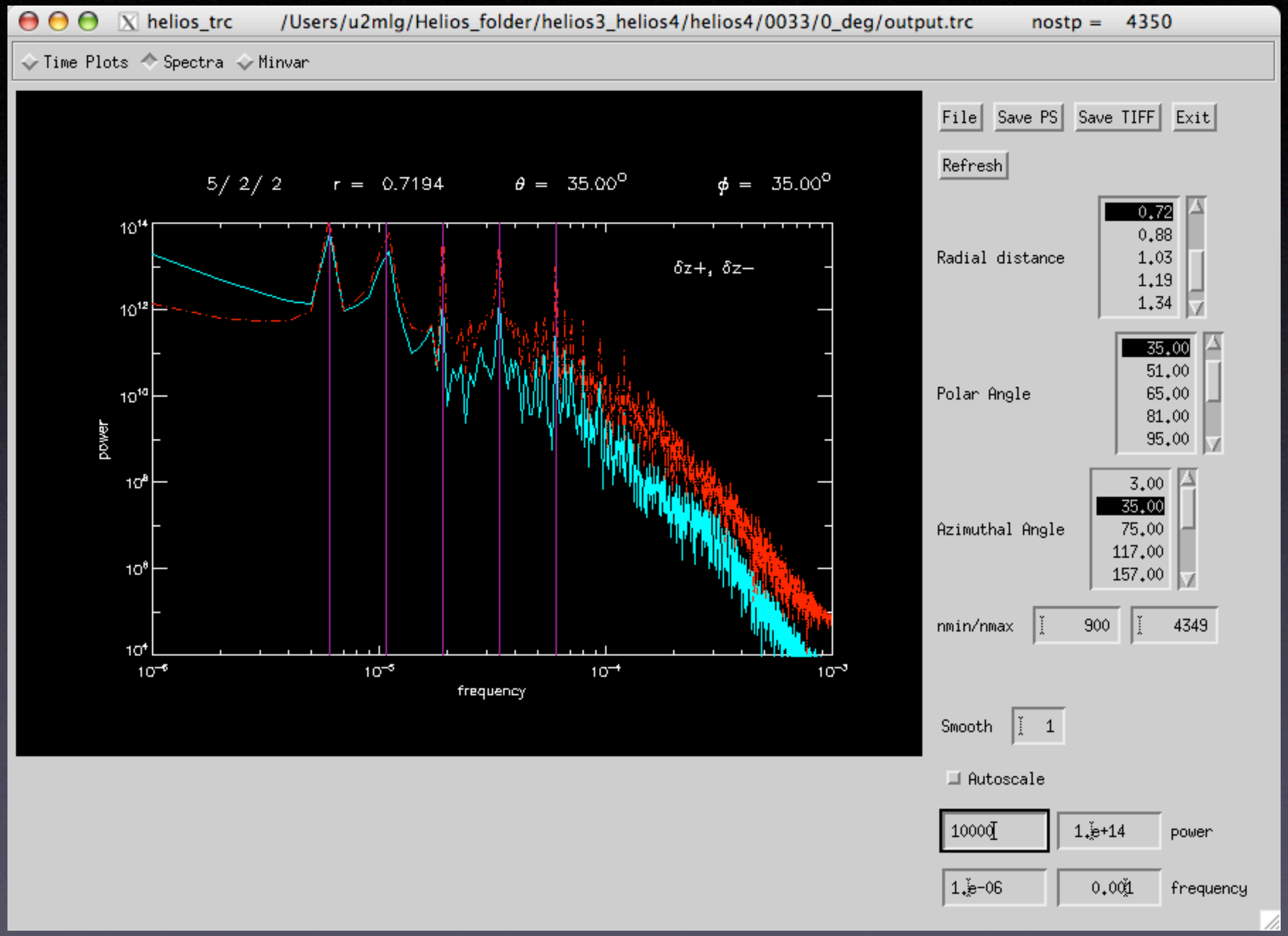
Spectrum at the 20 $R_\odot$ Inner Boundary



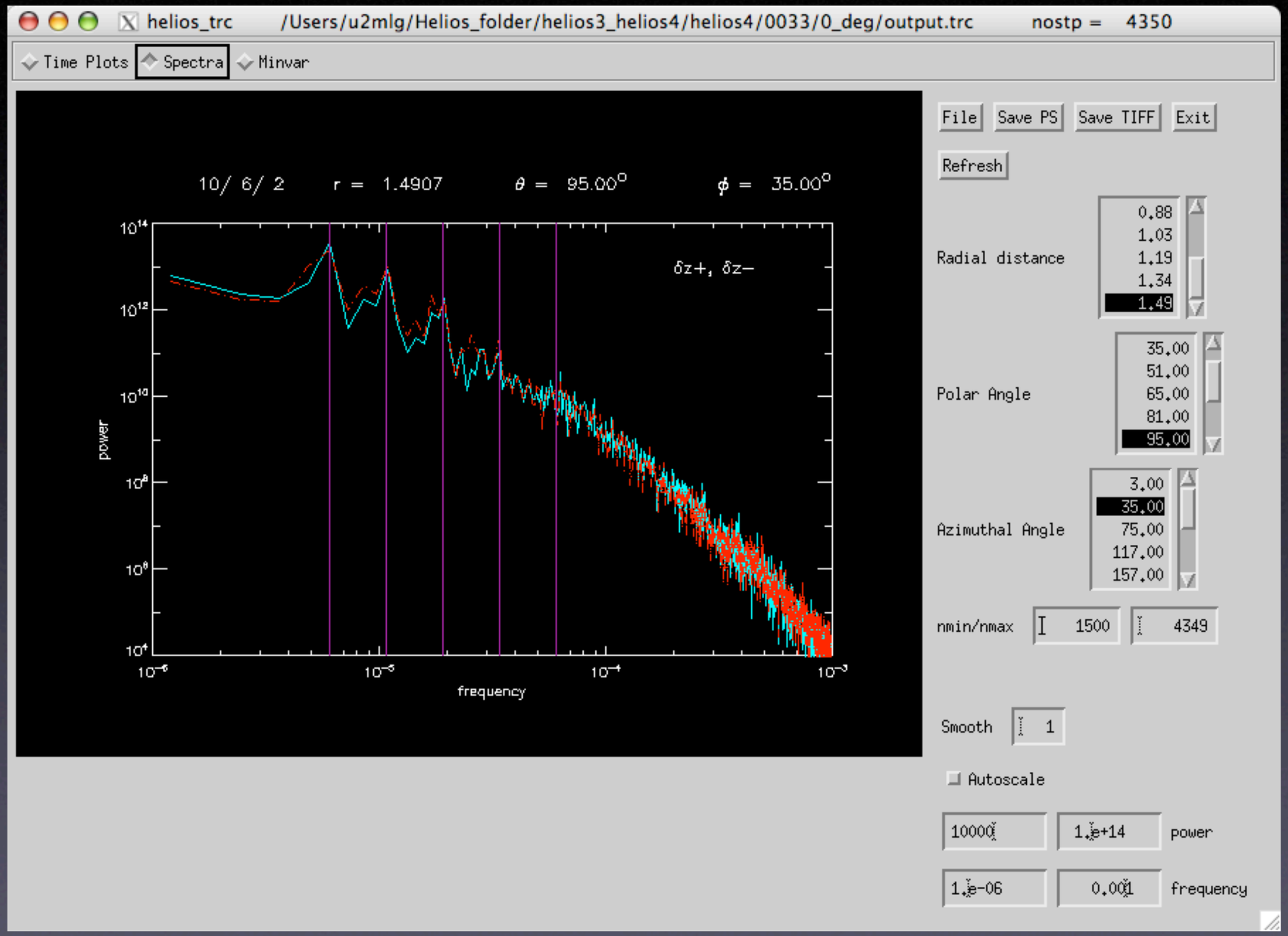
At 0.72 AU (in the current sheet)



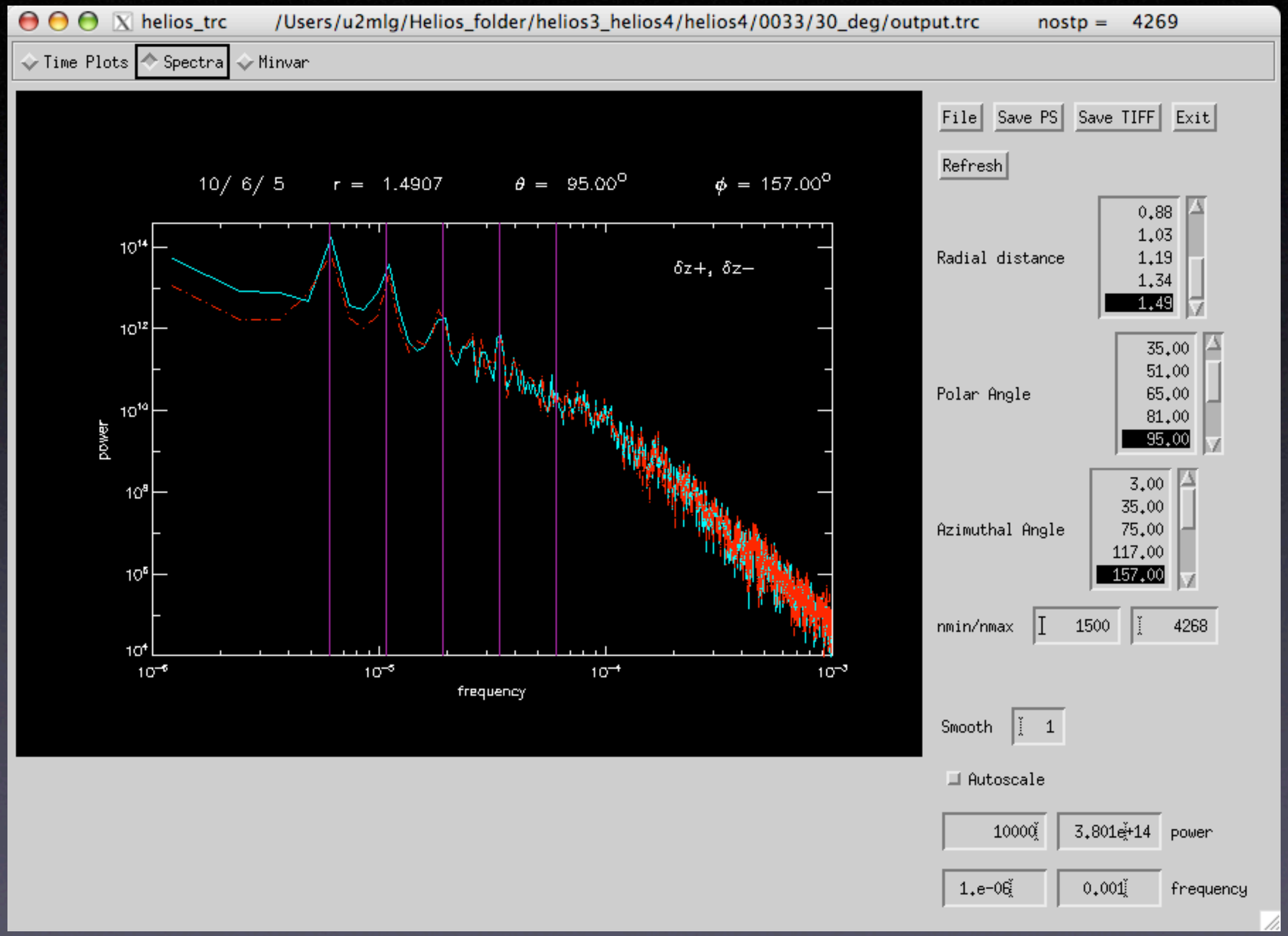
0.72 AU at High Latitudes



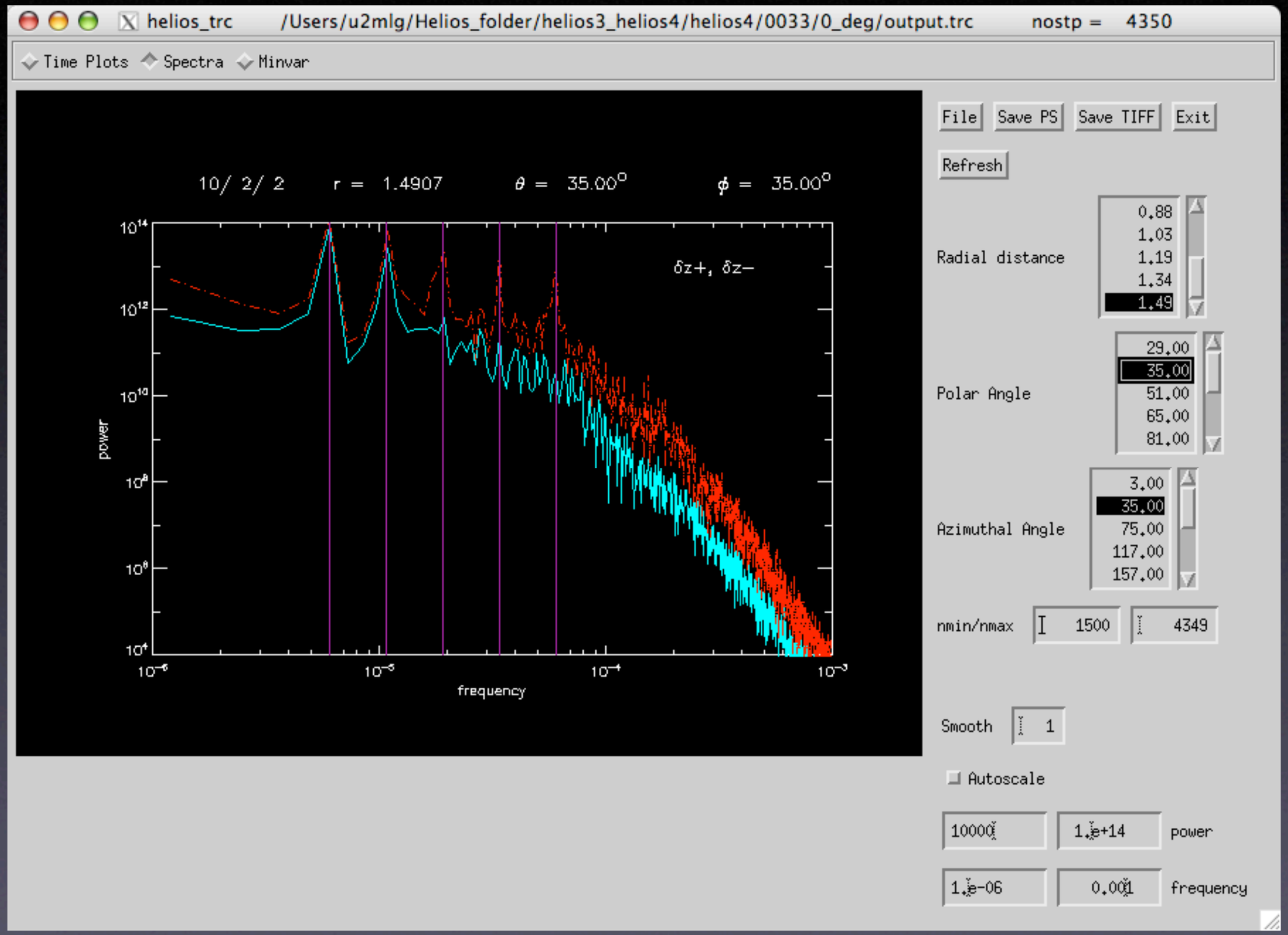
At 1.49 AU Near the Current Sheet



At 1.49 AU Near the Current Sheet

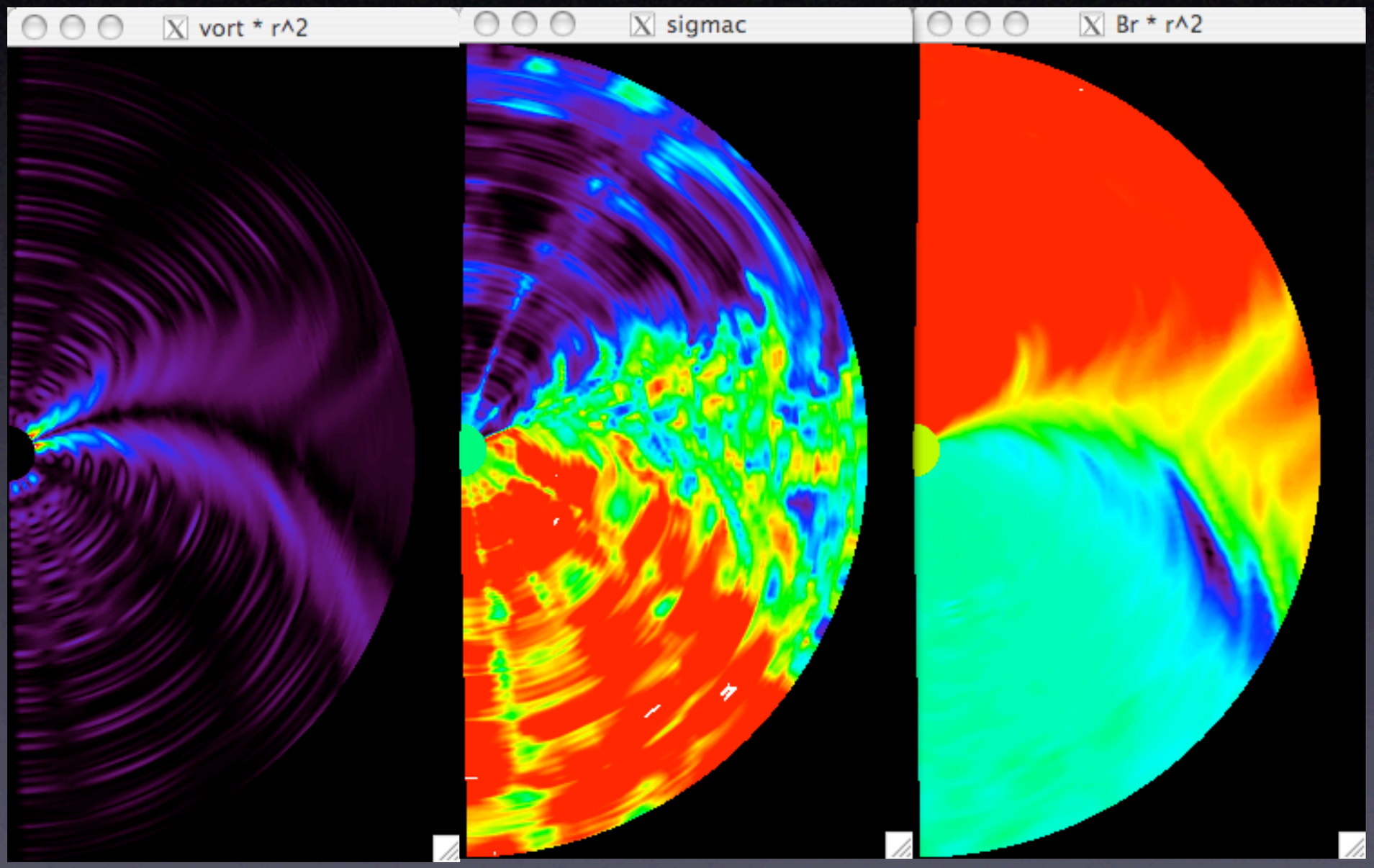


1.49 AU at High Latitudes



Vorticity, Cross Helicity, Radial Magnetic Field

Vorticity × r^2 | σ_c | $B_r \times r^2$



Conclusions — Part I

- We have performed time-dependent 3D MHD simulations to study the turbulent dynamics in the solar wind from 0.1 $R_\odot$ to 1.5 AU. The initial condition is a steady state, self-consistent solution of the MHD equations that fills the simulation domain.
- The simulation shows a significant latitude-dependent evolution of an initial "Alfvénic" wave packet that appears to be driven, in part, by proximity to the heliospheric current sheet together with the velocity shear between fast and slow wind.

Part II: MHD Modeling of the Solar Wind with Turbulence Transport and Heating

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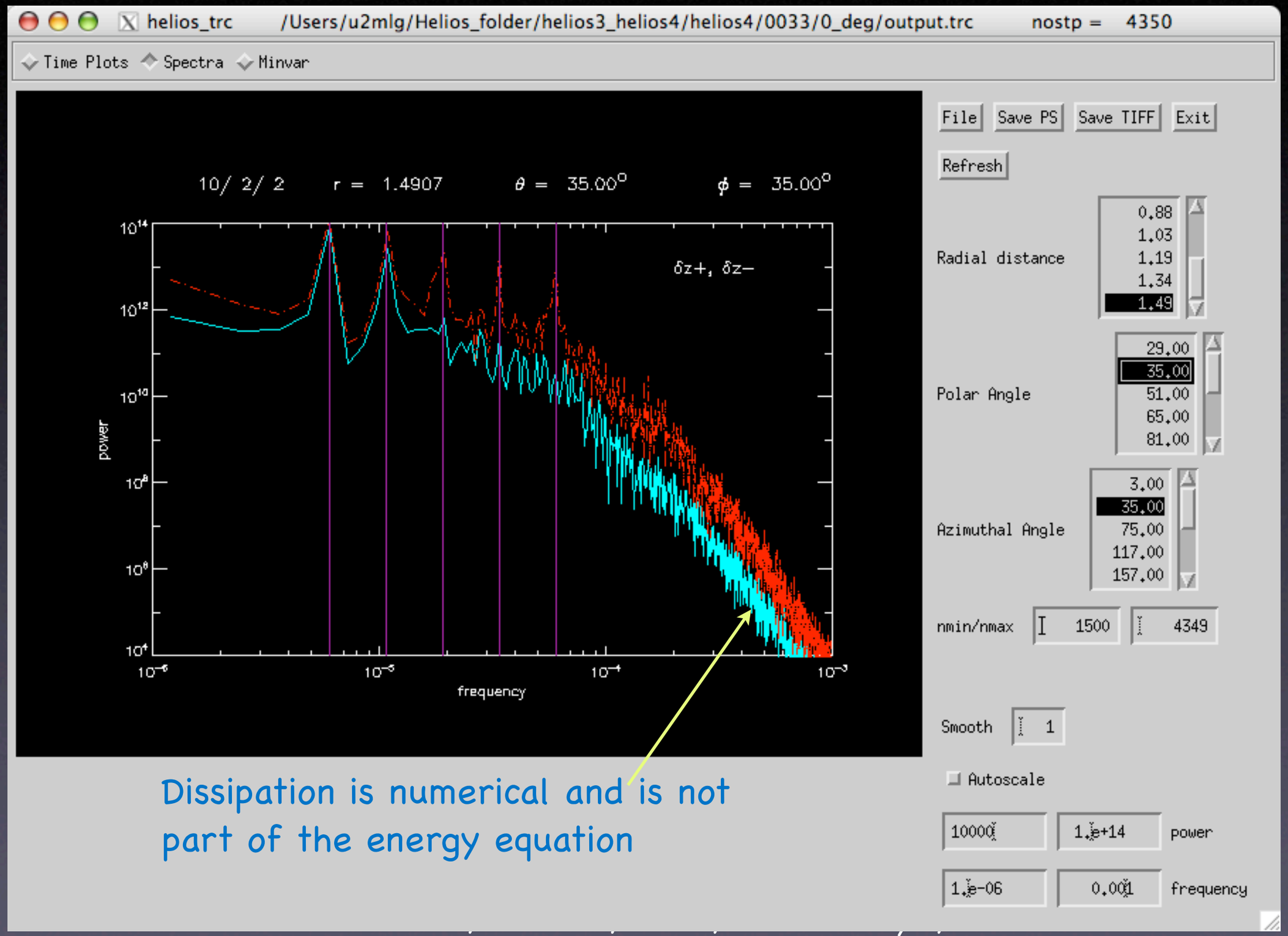
Outline – Part II

- Motivation
- Reynolds averaging equations
- Initial results: dipole tilted by 10°
- Initial results: dipole tilted by 30°
- Evolution of waves and non-linear interactions that initiate a cascade

Motivation — Numerical



Dissipation is numerical and is not part of the energy equation

Motivation — Observations

- An additional source of heat is needed to produce the observed speed, density and temperature of fast wind (Parker, 1958) to avoid using very small values of γ
- The temperature of the solar wind falls off less steeply than expected (Gazis 1984), even when shock heating is absent (Verma et al., 1995)
- Turbulence cascades appear adequate to provide the additional heat needed
- Present models use small polytropic indices (γ) to mimic an addition of heat from wave damping or the dissipation of turbulence
- Our goal is to solve equations that include the "sub-grid-scale" effects on the solar wind of turbulence transport and heating with more realistic values of γ.

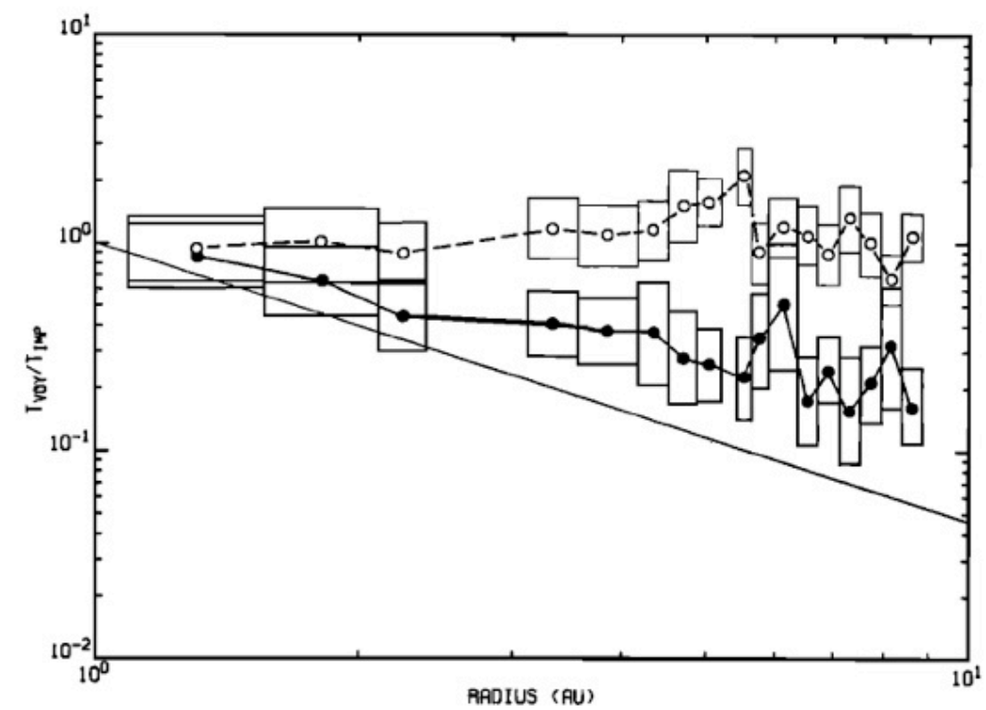


Fig. 4. Two-rotation averaged proton temperature observed at Voyager 1 plotted versus heliocentric distance. The data observed at Voyager have been divided by the data observed at IMP, as described for Figure 3*a*, in an effort to eliminate the effect of temporal variations. Each IMP data point has been plotted above the corresponding Voyager data point. The IMP 8 data points have been divided by 5.6×10^4 K, the value of the first IMP 8 data point, so that it will fit on the same dimensionless scale. The width of each rectangle represents the motion of the spacecraft during each averaging period. The height of each rectangle represent the width of the distributions of temperatures seen during each averaging period. The straight line varies as $R^{-4/3}$.

Reynolds Averaging

- The dependent variables in MHD equations are decomposed into large-scale and small-scale contributions.

  The small-scale variables (primed) represent unresolved fluctuations (turbulence) that are modeled separately.

$$\tilde{\mathbf{v}} = \mathbf{v} + \mathbf{v}', \quad \tilde{\mathbf{B}} = \mathbf{B} + \mathbf{B}', \quad \tilde{\rho} = \rho + \rho', \quad \tilde{P} = P + P'$$

- < ... > is the ensemble average that accomplishes the separation according to the rules

$$\rho = \langle \tilde{\rho} \rangle, \quad \langle \rho' \rangle = 0, \quad \mathbf{v} = \langle \tilde{\mathbf{v}} \rangle, \quad \langle \mathbf{v}' \rangle = 0, \quad \mathbf{B} = \langle \tilde{\mathbf{B}} \rangle, \quad \langle \mathbf{B}' \rangle = 0, \ldots$$

- We further assume that the small-scale turbulence is locally incompressible ($\rho'=0$), while the large-scale flow is fully compressible.

Large-scale (Reynolds Averaged) MHD Equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot \left[\rho \mathbf{v}\mathbf{v} - \frac{1}{4\pi}\mathbf{B}\mathbf{B} + \left(P + \frac{\langle B'^2 \rangle}{8\pi} + \frac{B^2}{8\pi}\right)\mathbf{I} + \langle \rho \mathbf{v}'\mathbf{v}' - \frac{1}{4\pi}\mathbf{B}'\mathbf{B}' \rangle\right] + \rho\left[\frac{GM_\odot}{r^2}\hat{\mathbf{r}} + 2\mathbf{\Omega} \times \mathbf{v} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r})\right] = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} + \langle \mathbf{v}' \times \mathbf{B}' \rangle)$$

$$\frac{\partial P}{\partial t} + (\mathbf{v} \cdot \nabla)P + \gamma P(\nabla \cdot \mathbf{v}) = -\nabla \cdot \langle \mathbf{v}' P' \rangle + Q(\mathbf{r})$$

- ρ, $\mathbf{B}$, and P are the number density, magnetic field, and thermal pressure, respectively;
- $\mathbf{u}$ is the velocity in the inertial frame;
- $\mathbf{v}$ is the velocity in the frame of reference corotating with the Sun, $\mathbf{v} = \mathbf{u} - \mathbf{w}$ and $\mathbf{w} = \Omega \times \mathbf{r}$;
- γ is the polytropic index;
- Ω is the solar rotation rate;

Small-scale (Subgrid-Scale) Turbulence Transport Equations (V_A << V_{SW})

$$\frac{\partial Z^2}{\partial t} + (\mathbf{v}\cdot\nabla)Z^2 + \frac{Z^2}{2}\nabla\cdot\mathbf{u} + \sigma_D Z^2\left[\frac{\nabla\cdot\mathbf{u}}{2} - \hat{\mathbf{B}}\cdot(\hat{\mathbf{B}}\cdot\nabla)\mathbf{u}\right] = -\frac{\alpha f^+(\sigma_c)Z^3}{\lambda} + \dot{E}_{PI}$$

$$\frac{\partial(Z^2\sigma_c)}{\partial t} + (\mathbf{v}\cdot\nabla)(Z^2\sigma_c) + \frac{Z^2\sigma_c}{2}\nabla\cdot\mathbf{u} - \frac{2\boldsymbol{\epsilon}_m\cdot(\nabla\times\mathbf{u})}{\sqrt{4\pi\rho}} = -\frac{\alpha f^-(\sigma_c)Z^3}{\lambda}$$

$$\frac{\partial\lambda}{\partial t} + (\mathbf{v}\cdot\nabla)\lambda = \beta f^+(\sigma_c)Z - \frac{\beta\lambda\dot{E}_{PI}}{\alpha Z^2}$$

- $Z^2 = \langle v'^2\rangle + \langle B'^2\rangle/4\pi\rho$ is the incompressible turbulent energy (flow plus magnetic) per unit mass (Elsässer variance);
- σ_c is the normalized cross helicity (or Alfvénicity, or Alfvénic correlation);
- λ is the similarity (correlation) length scale;
- σ_D is the normalized energy difference, typically $\sigma_D \approx -1/3$ in the solar wind [e.g., Matthaeus and Goldstein, 1982a; Tu and Marsch, 1995];

[Zhou and Matthaeus, 1990; Matthaeus et al., 1994; Hossain et al., 1995; Zank et al., 1996; Matthaeus et al., 1996; Matthaeus et al., 1999; Smith et al., 2001, Matthaeus et al., 2004; Breech et al., 2005, 2008; ...]

Composite Set of Equations (V_A << V_{SW})

large-scale

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial(\rho \mathbf{u})}{\partial t} + \nabla \cdot \left\{ \rho \mathbf{u}\mathbf{v} - \frac{\eta}{4\pi}\mathbf{B}\mathbf{B} + \left[P + \frac{(1+\sigma_D)\rho Z^2}{4} + \frac{B^2}{8\pi} \right] \mathbf{I} \right\} = -\rho \left(\frac{GM_\odot}{r^2}\hat{\mathbf{r}} + \mathbf{\Omega} \times \mathbf{u} \right)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{v}\mathbf{B} - \mathbf{B}\mathbf{v}) = -\nabla \times \boldsymbol{\epsilon}_m$$

$$\frac{\partial E}{\partial t} = -\nabla \cdot \left[\mathbf{v}E + \mathbf{u}\bar{P} - \frac{\eta \mathbf{B}(\mathbf{u}\cdot\mathbf{B})}{4\pi} \right] - \frac{\mathbf{B}}{4\pi} \cdot (\nabla \times \boldsymbol{\epsilon}_m) + \frac{\rho \dot{E}_{PI}}{2}$$

small-scale

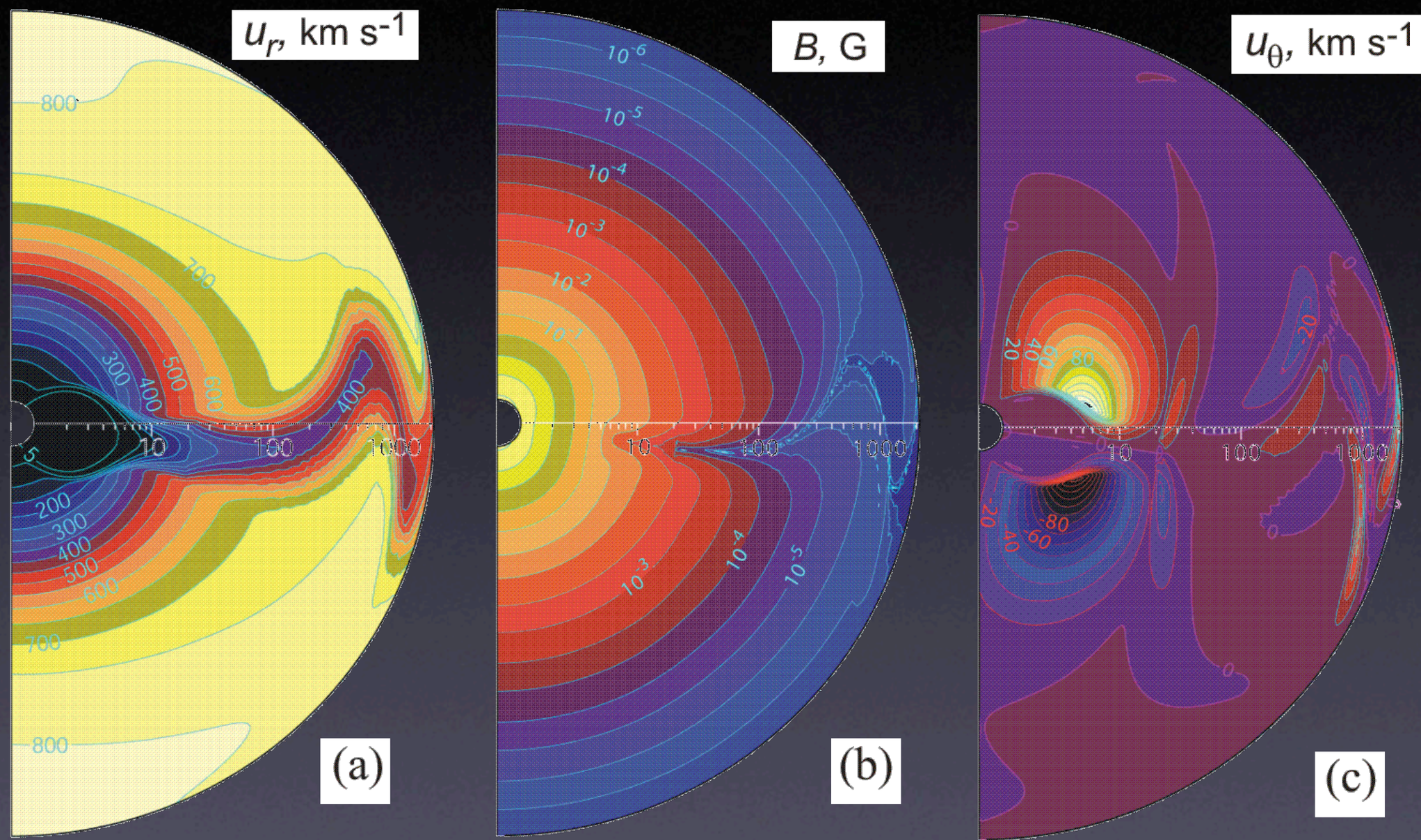
$$\frac{\partial Z^2}{\partial t} + (\mathbf{v}\cdot\nabla)Z^2 + \frac{Z^2}{2}\nabla\cdot\mathbf{u} + \sigma_D Z^2 \left[\frac{\nabla\cdot\mathbf{u}}{2} - \hat{\mathbf{B}}\cdot(\hat{\mathbf{B}}\cdot\nabla)\mathbf{u} \right] = -\frac{\alpha f^+(\sigma_c) Z^3}{\lambda} + \dot{E}_{PI}$$

$$\frac{\partial (Z^2\sigma_c)}{\partial t} + (\mathbf{v}\cdot\nabla)(Z^2\sigma_c) + \frac{Z^2\sigma_c}{2}\nabla\cdot\mathbf{u} - \frac{2\boldsymbol{\epsilon}_m\cdot(\nabla\times\mathbf{u})}{\sqrt{4\pi\rho}} = -\frac{\alpha f^-(\sigma_c)Z^3}{\lambda}$$

$$\frac{\partial \lambda}{\partial t} + (\mathbf{v}\cdot\nabla)\lambda = \beta f^+(\sigma_c) Z - \frac{\beta\lambda\dot{E}_{PI}}{\alpha Z^2}$$

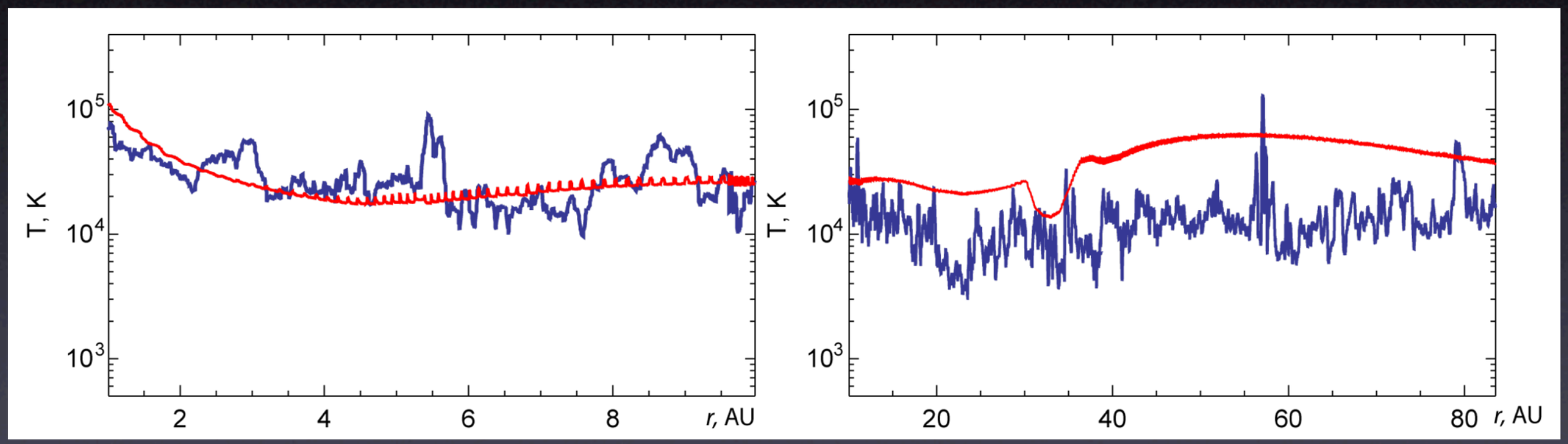
$$E = \frac{\rho u^2}{2} + \frac{P}{\gamma - 1} + \frac{B^2}{8\pi} - \frac{\rho G M_\odot}{r} + \frac{\rho Z^2}{2}, \quad \bar{P} = P + \frac{(1+\sigma_D)\rho Z^2}{4} + \frac{B^2}{8\pi}, \quad \eta = 1 + \frac{2\pi\sigma_D\rho Z^2}{B^2}$$

Dipole Tilted by 10°: Example of Solution for Region 1 & 2 (1–60 $R_\odot$) with WKB Alfvén Waves



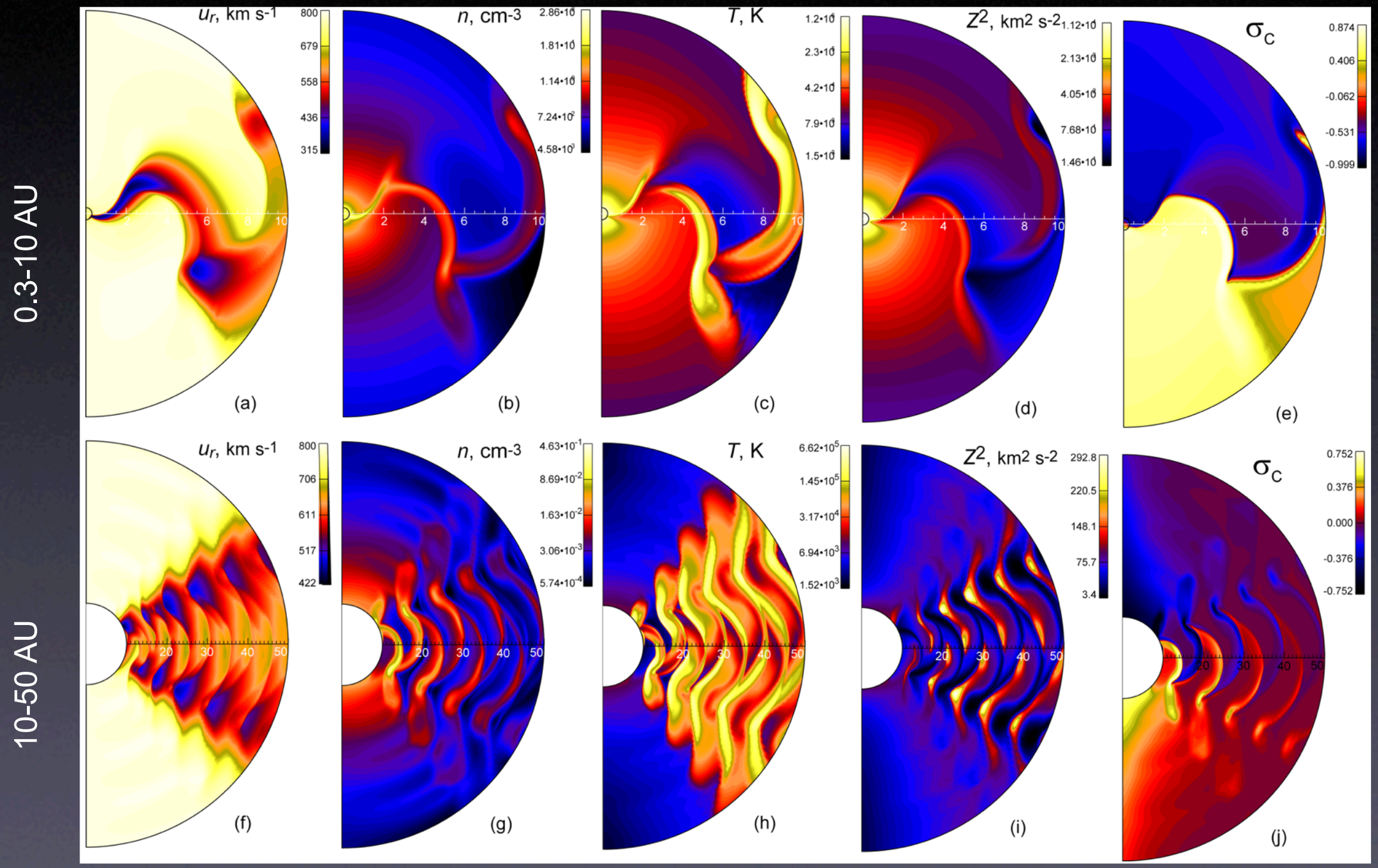
[*Usmanov and Goldstein*, JGR, 2003]

Dipole Tilted by 10°: Comparison with Voyager 2 Proton Temperature



Heating by corotating shocks may be important

Dipole Tilted by 30°: Meridional Plane, 0.3–100 AU



Summary — Part II

- We have developed a three-dimensional global steady-state MHD solar wind model with turbulence transport and heating.

- The model is a generalization of the approach developed by Matthaeus et al. for modeling of the solar wind turbulence and plasma heating in case of a constant-speed solar wind in a predetermined magnetic field.

- In addition to plasma and magnetic field parameters, the model describes the three-dimensional distribution of turbulence throughout the heliosphere: the turbulence energy Z^2 per unit mass, the cross helicity σ_C, and the correlation length λ.

- The model results appear to be in reasonable agreement with Ulysses observations and in some agreement with Voyager 2 observations (especially inside 10 AU).