Large- and Small-Scale Dynamo Action

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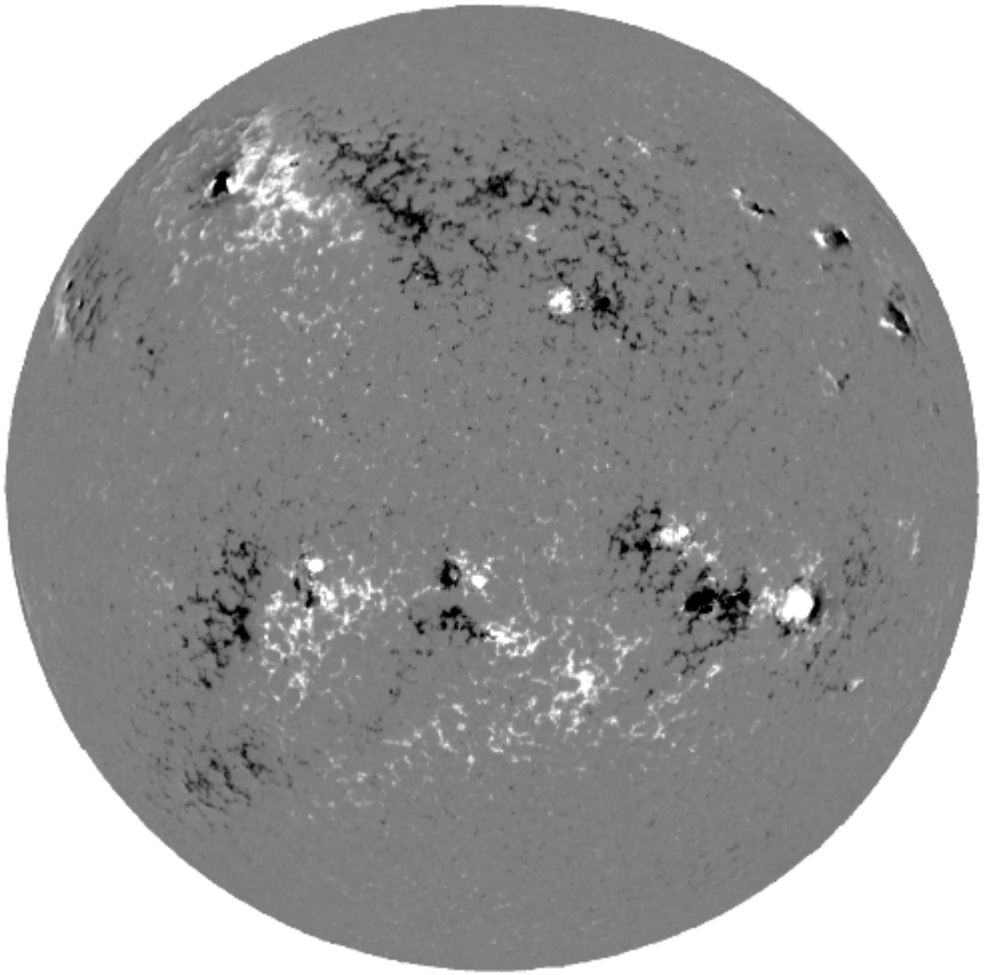
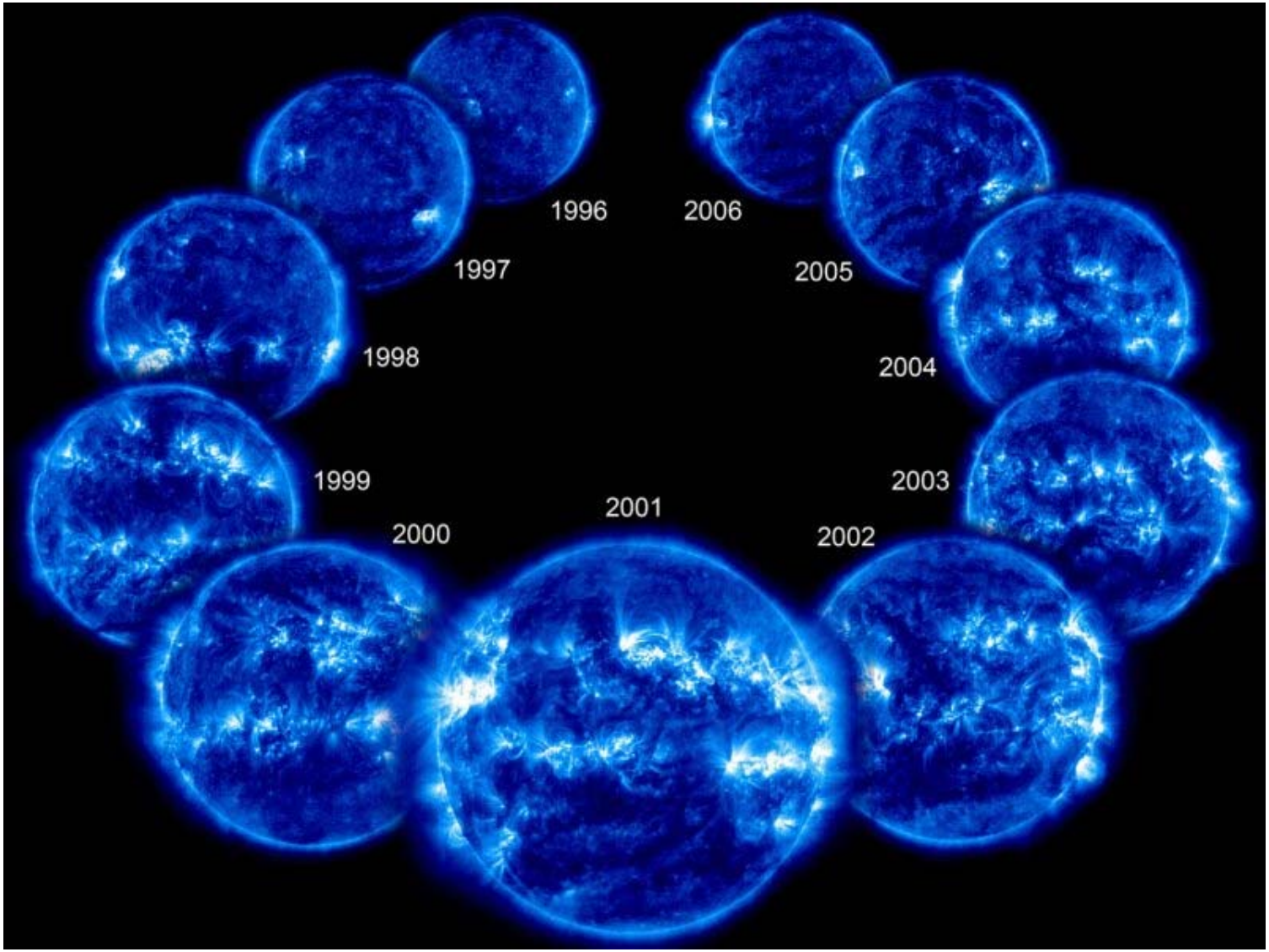




Astronum 2009

The large-scale dynamo problem

Sufficiently turbulent flows at high enough magnetic Reynolds numbers act as efficient dynamos. i.e. amplifiers of magnetic energy of fields with zero mean.

The key problem is to understand how such flows can act as generators of *large-scale* fields; i.e. fields with a significant component on scales much larger than a typical scale of the velocity (e.g. a typical turbulent eddy size).

Mean field dynamo theory

Most astrophysical dynamo models take, as their starting point, the mean field induction equation for the large-scale field $\boldsymbol{B}_0$:

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\mathbf{U}_0 \times \mathbf{B}_0) + \nabla \times (\alpha \mathbf{B}_0) + (\eta + \beta)\nabla^2 \mathbf{B}_0. \qquad (*)$$

α: regenerative term, responsible for large-scale dynamo action.
Relies on a lack of reflectional symmetry (handedness).

β: turbulent diffusivity.

Typically α and β are prescribed in a plausible (though essentially arbitrary) manner – and solar-like magnetic phenomena can be reproduced.

But is the large-scale field truly governed by equation (*)? (Cattaneo & Hughes 2009 *MNRAS)*

Mean field theory (kinematic)

Induction equation for mean field:

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times (\mathbf{U}_0 \times \mathbf{B}_0) + \nabla \times \langle \mathbf{u} \times \mathbf{b} \rangle + \eta \nabla^2 \mathbf{B}_0 .$$

Consider the induction equation for the fluctuating field:

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}_0) + \nabla \times (\mathbf{U}_0 \times \mathbf{b}) + \nabla \times \mathbf{G} + \eta \nabla^2 \mathbf{b},$$

where $\mathbf{G} = \mathbf{u} \times \mathbf{b} - \langle \mathbf{u} \times \mathbf{b} \rangle$.

Traditional approach is to assume that the fluctuating field is driven solely by the large-scale magnetic field.

i.e. in the absence of $\boldsymbol{B}_0$ the fluctuating field decays. ***No small-scale dynamo.***

Under this assumption, the relation between $\mathbf{b}$ and $\mathbf{B}_0$ (and hence between $\mathbf{E} = \langle \boldsymbol{u} \times \boldsymbol{b} \rangle$ and $\mathbf{B}_0$) is linear and homogeneous and we obtain the usual mean field induction equation.

Mean field theory: a small-scale dynamo

The equation for the fluctuating field can be written as:

$$\boxed{L(\mathbf{b}) = \nabla \times (\mathbf{u} \times \mathbf{B_0})}$$

Mean field formulation proceeds on the assumption that solutions to $L(\boldsymbol{b}) = 0$ decay.

This however is not in general true.
In turbulent flows, at high Rm, there will be exponentially growing solutions.
i.e. small-scale dynamo action.

Generation mechanisms

Large-scale (mean field) dynamos rely on a lack of reflectional symmetry, e.g. as provided by helical flows.

For small correlation times τ:

$$\alpha = -\frac{\tau}{3}\langle \mathbf{u} \cdot \nabla \times \mathbf{u} \rangle$$

Though the relationship between α and helicity is not, in general, straightforward.

Small-scale dynamos have mainly been studied in the context of fast dynamos (i.e. dynamos in the limit $Rm \rightarrow \infty$).
Fast dynamos require Lagrangian chaos in the flow.

In general, astrophysical flows will be both helical and chaotic.

The kinematic α-effect: interpretation

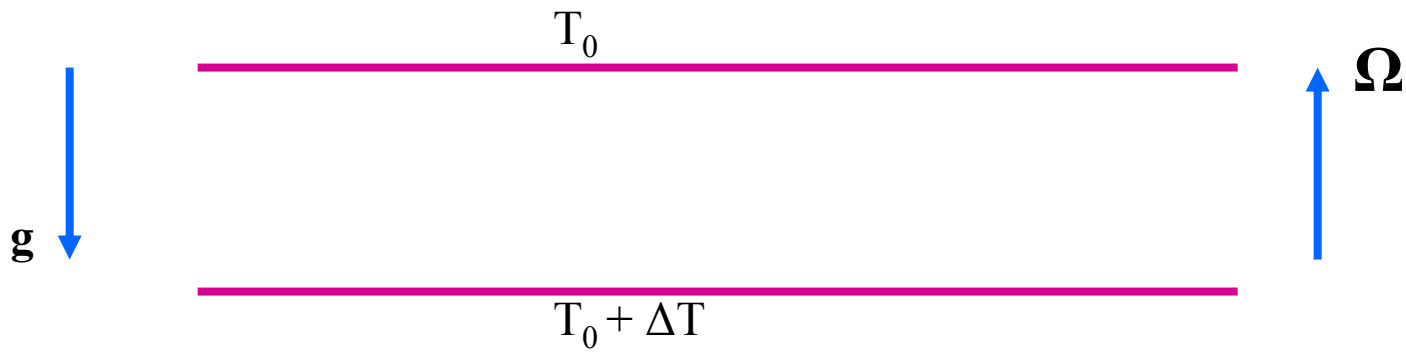
In the absence of small-scale dynamo action, knowledge of α (and β) provides information about the growth of a field on a large scale ~ $1/k$:

$$s \sim \alpha k - \beta k^2 \qquad (*)$$

Increasing the domain size from $L \sim \ell$ will allow dynamo action to set in when $L = \beta/\alpha$.

However, if there is small-scale dynamo action then, by definition, there is dynamo action when $L \sim \ell$.
This will be essentially independent of domain size L.
Any average of the magnetic field at intermediate scales will grow with the same growth rate.
The growth of the magnetic field on large scales has nothing to do with that predicted by equation (*).

Dynamo action driven by rotating convection



Rotating Boussinesq convection.
Boundary conditions: impermeable, stress-free, fixed temperature, perfect electrical conductor.

Childress & Soward 1972
Soward 1974
St Pierre 1993
Jones & Roberts 2000
Rotvig & Jones 2002
Stellmach & Hansen 2004

Equations, boundary conditions and parameters

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u} + \sigma T a^{1/2}\mathbf{e}_z \times \mathbf{u} = -\nabla p + \mathbf{j}\times\mathbf{B} + \sigma Ra\,\theta\,\mathbf{e}_z + \sigma\nabla^2\mathbf{u},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{u}\times\mathbf{B}) + (\sigma/\sigma_m)\nabla^2\mathbf{B},$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u}.\nabla\theta = w + \nabla^2\theta,$$

$$\nabla.\mathbf{u} = \nabla.\mathbf{B} = 0.$$

$$\partial_z u = \partial_z v = w = \partial_z B_x = \partial_z B_y = B_z = \theta = 0 \quad \text{on } z = 0, 1.$$

Taylor number, $Ta = 4\Omega^2 d^4/\nu^2 = 5 \times 10^5$.
Prandtl number $\sigma = \nu/\kappa = 1$.
Magnetic Prandtl number $\sigma_m = \nu/\eta = 5$.
Critical Rayleigh number for onset of convection = 59 008.
Critical Rayleigh number for onset of dynamo action ≈ 170 000.

Cattaneo & Hughes 2006 *JFM*
Hughes & Cattaneo 2008 *JFM*

Numerical method

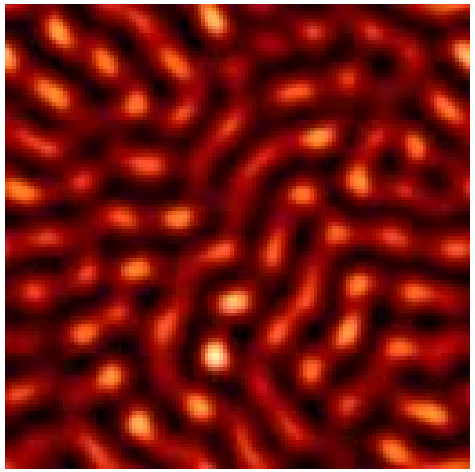
Pseudo-spectral code. Nonlinear terms calculated in configuration space.
All other terms evaluated in phase space, with FFTs to move between the two.

Diffusive terms calculated via an integrating factor.

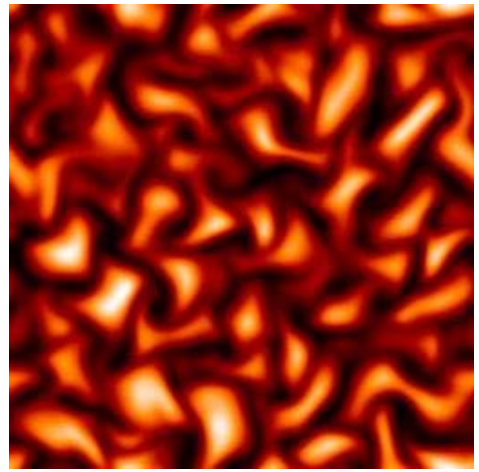
Projection method used to keep the velocity and magnetic fields divergence-free.

Time stepping using 3rd order Adams Bashforth.

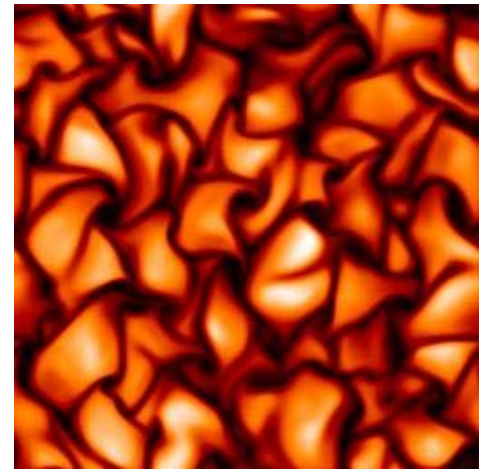
Rotating convection: large aspect ratio



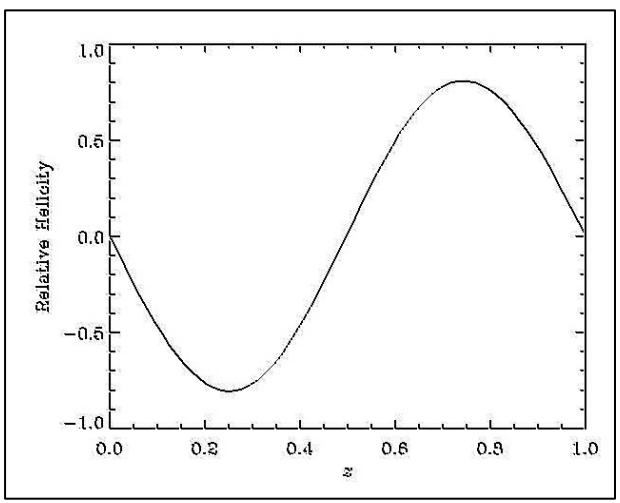
$Ra = 70{,}000$

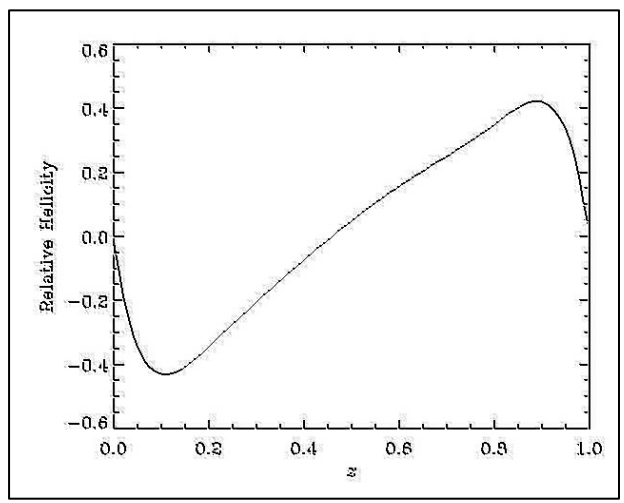


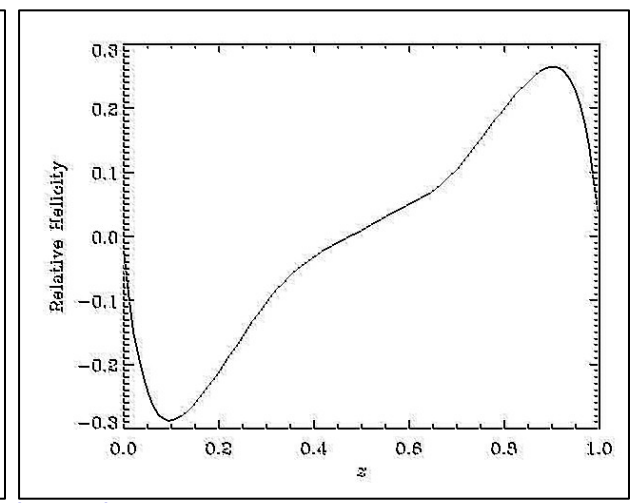
$Ra = 150{,}000$



$Ra = 500{,}000$

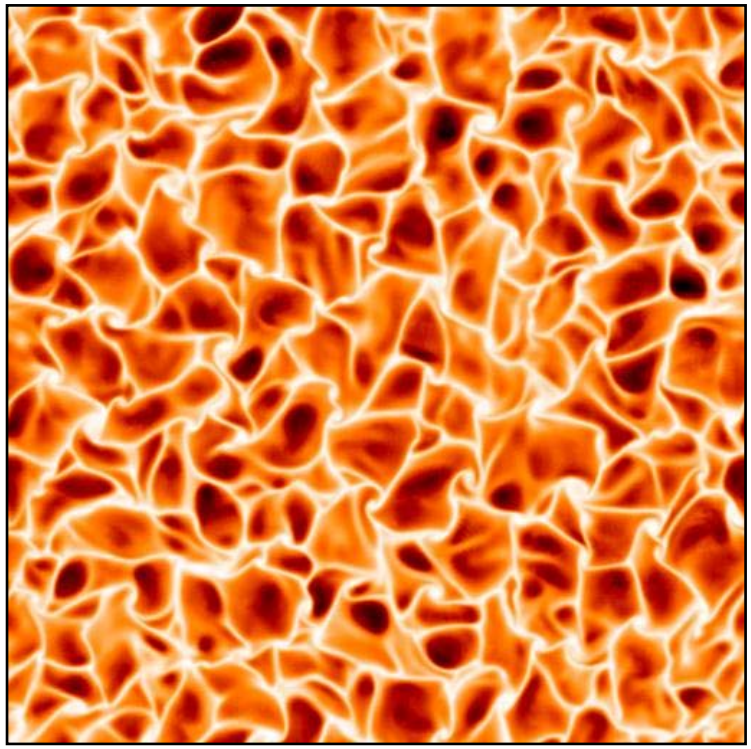






Relative Helicity $= \dfrac{\langle \mathbf{u} \cdot \boldsymbol{\omega} \rangle}{\langle u^2 \rangle^{1/2} \langle \omega^2 \rangle^{1/2}}$

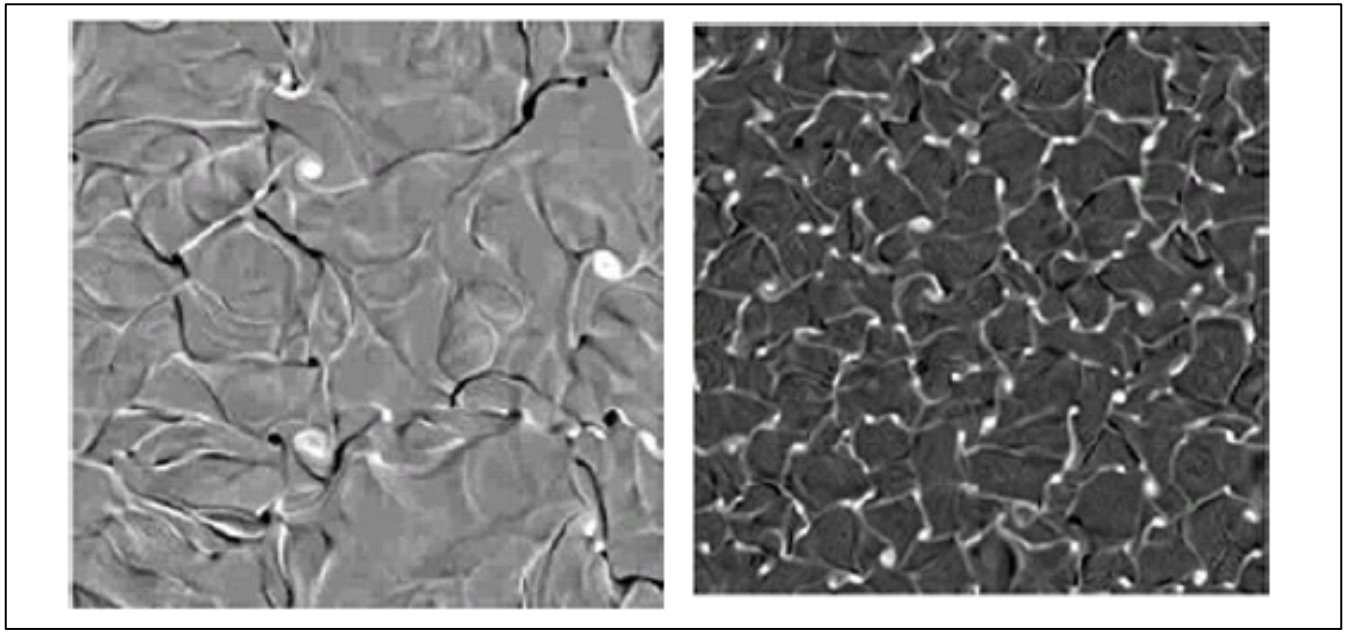
A potentially large-scale dynamo



$Ra = 10^6$, $Ta = 5 \times 10^5$
Box size: 10 x 10 x 1,
Resolution: 512 x 512 x 97
Snapshot of temperature.
No imposed mean magnetic field.

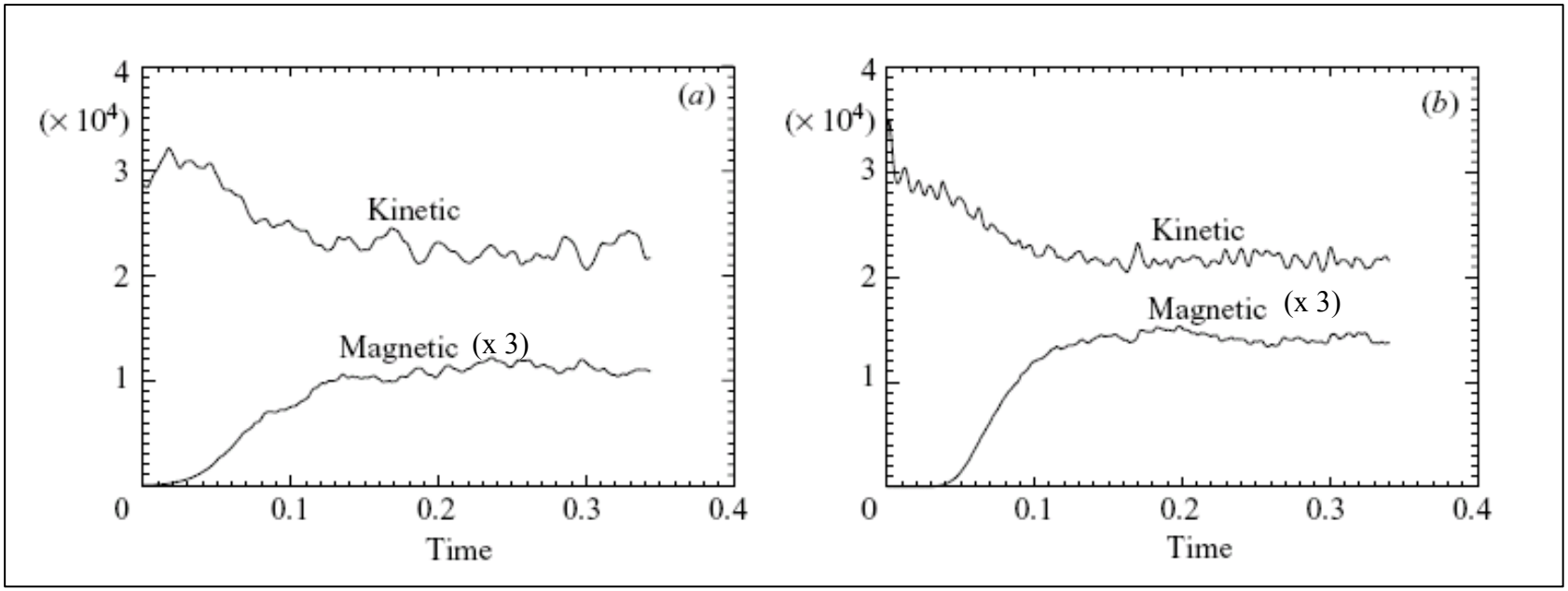
For comparison we consider a case with no rotation and with $Ra = 5 \times 10^5$.

Comparison of vorticity



$Ta = 0$, $Ra = 5 \times 10^5$ $\qquad$ $Ta = 5 \times 10^5$, $Ra = 10^6$

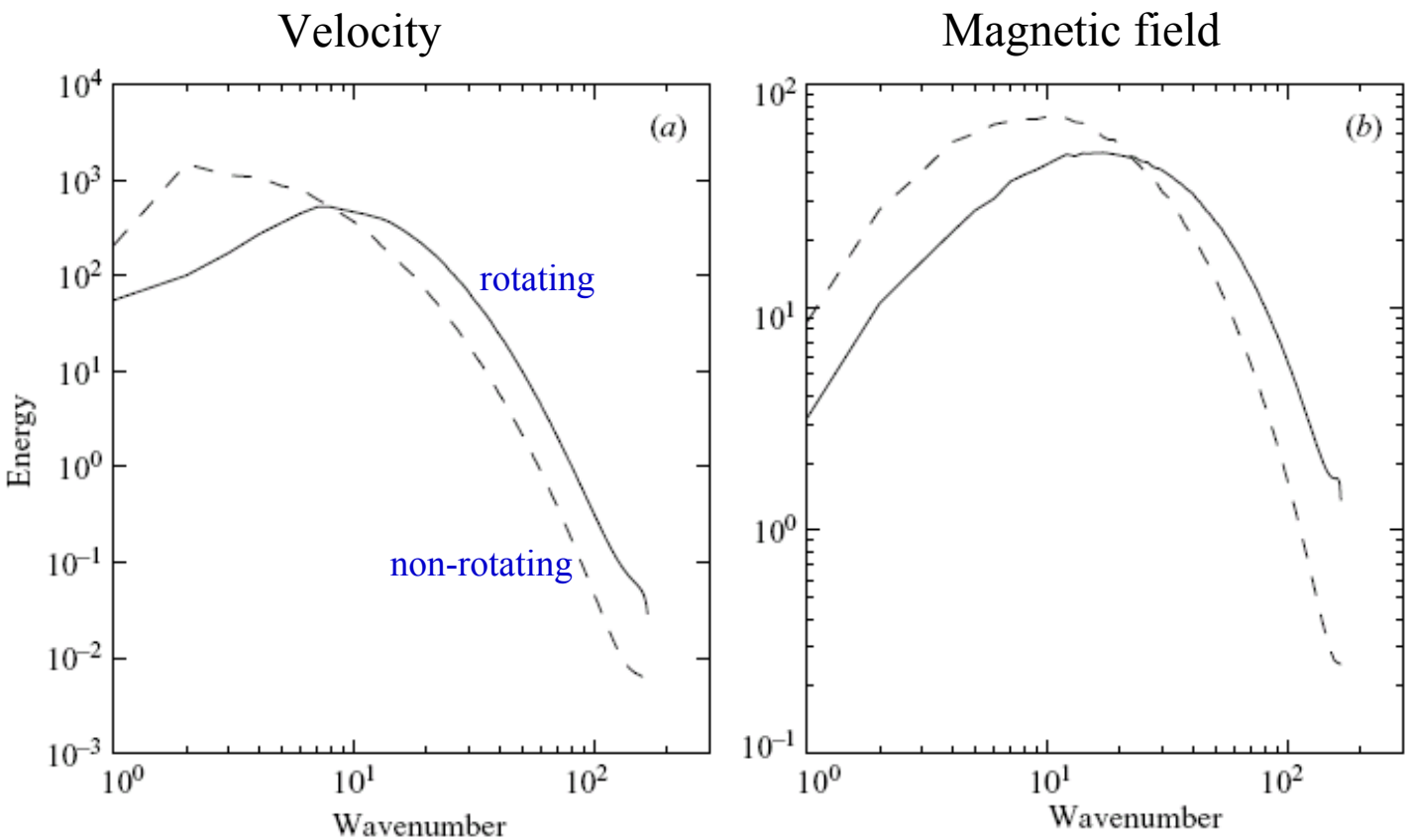
Dynamo action



$Ta = 0,\ Ra = 5 \times 10^5$

$Ta = 5 \times 10^5,\ Ra = 10^6$

Horizontal power spectra



Conclusions

- Turbulent flows at high *Rm* will generate, preferentially, a small-scale dynamo – irrespective of the helicity of the flow.

- So how does the Sun do it? What are the effects of a large-scale shear flow?

- A large-scale shear flow *is* beneficial in the absence of small-scale dynamo action (Hughes & Proctor 2009 *PRL*). However it is far from clear that it will transform a small-scale dynamo into a large-scale dynamo.

- Other possibilities:

- The dynamo may be intrinsically nonlinear; cf. the "strong field" branch in geodynamo models.

- The dynamo may be driven by a combination of shear and an "α-effect" driven by an instability.