Self-Similar Evolution of Cosmic-Ray Modified Shocks

(& current status of DSA)

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Outline

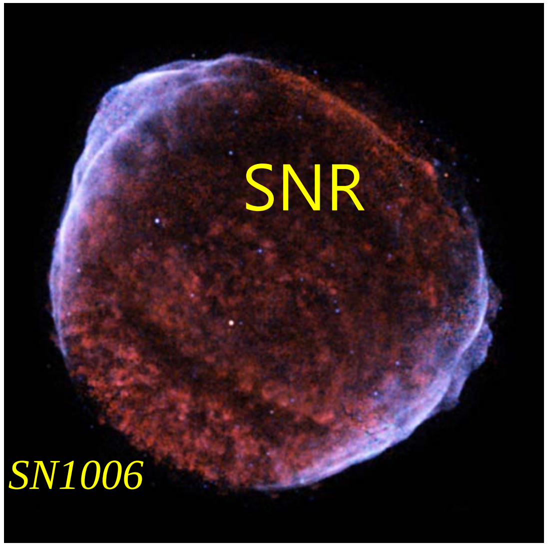
- Introduction: shocks & CRs in Astrophysics
- Key Physics of Diffusive Shock Acceleration
  * Fermi 1st order acceleration process
  * test-particle spectrum, f(p) → nonlinear feedback
  * wave-particle interactions: injection, wave generation
- Numerical Methods to study DSA

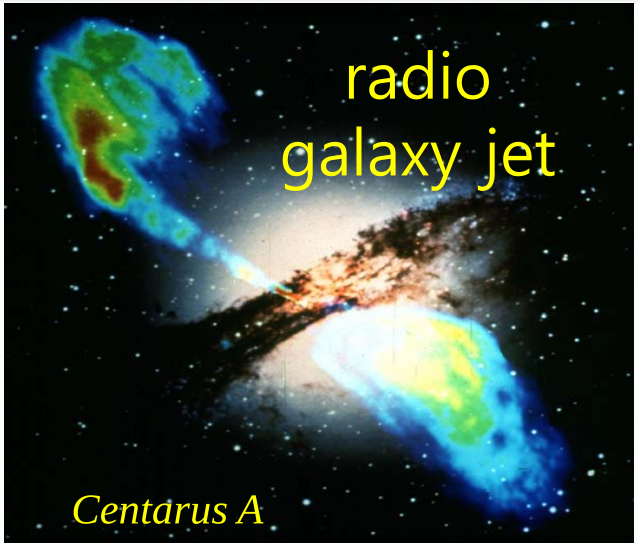
  Time-dependent Kinetic simulations using

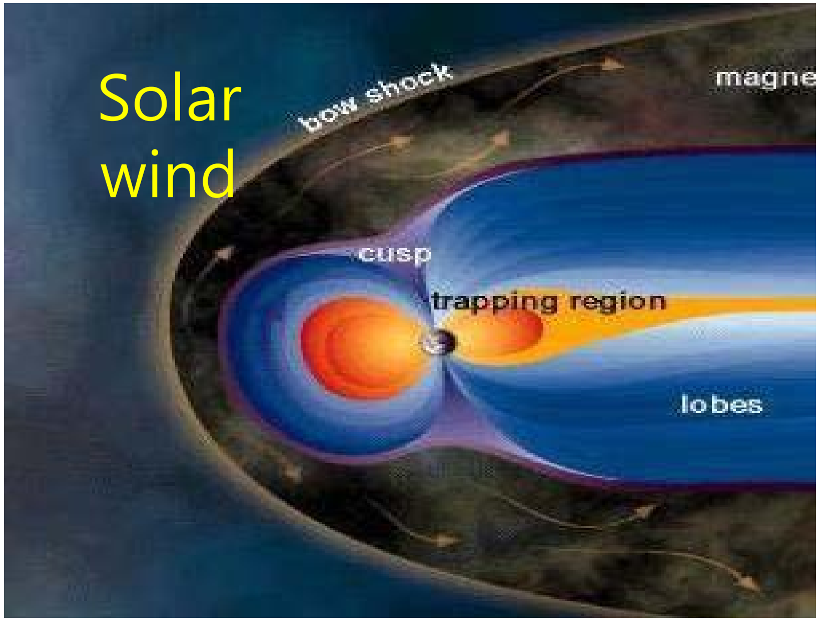
  CRASH (Cosmic Ray Acceleration Shock) code
- Self-Similar Evolution of CR modified shocks

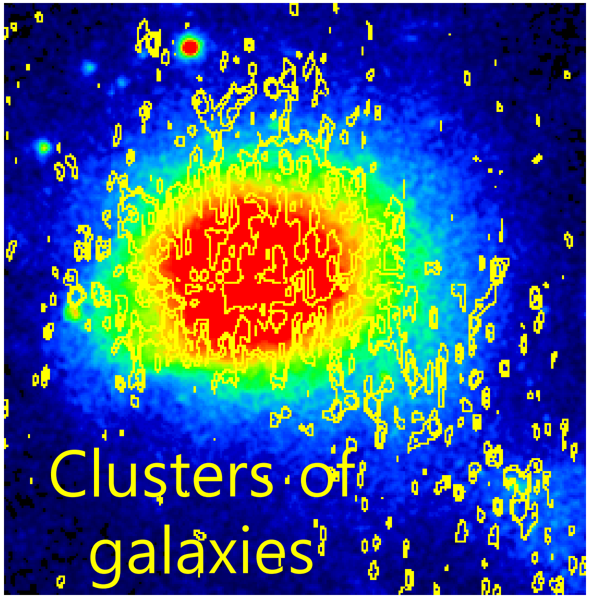
  Analytic form for time-dependent CR spectrum, $f(p,t)$

Shocks and Cosmic Rays in Astrophysical Environments







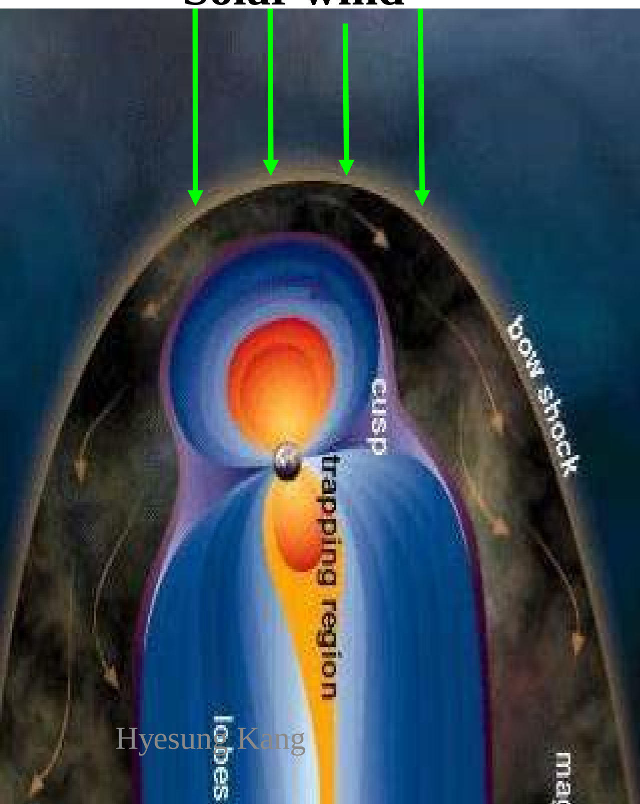


- Cosmic rays = relativistic charged particles
- in Astrophysical plasmas: CRs = nonthermal particles

  i.e. tail above Maxwellian distribution in *p* space
- suprathermal particles leak out of thermal pool into CRs

direct measurement at Earth's bow shock

Solar wind



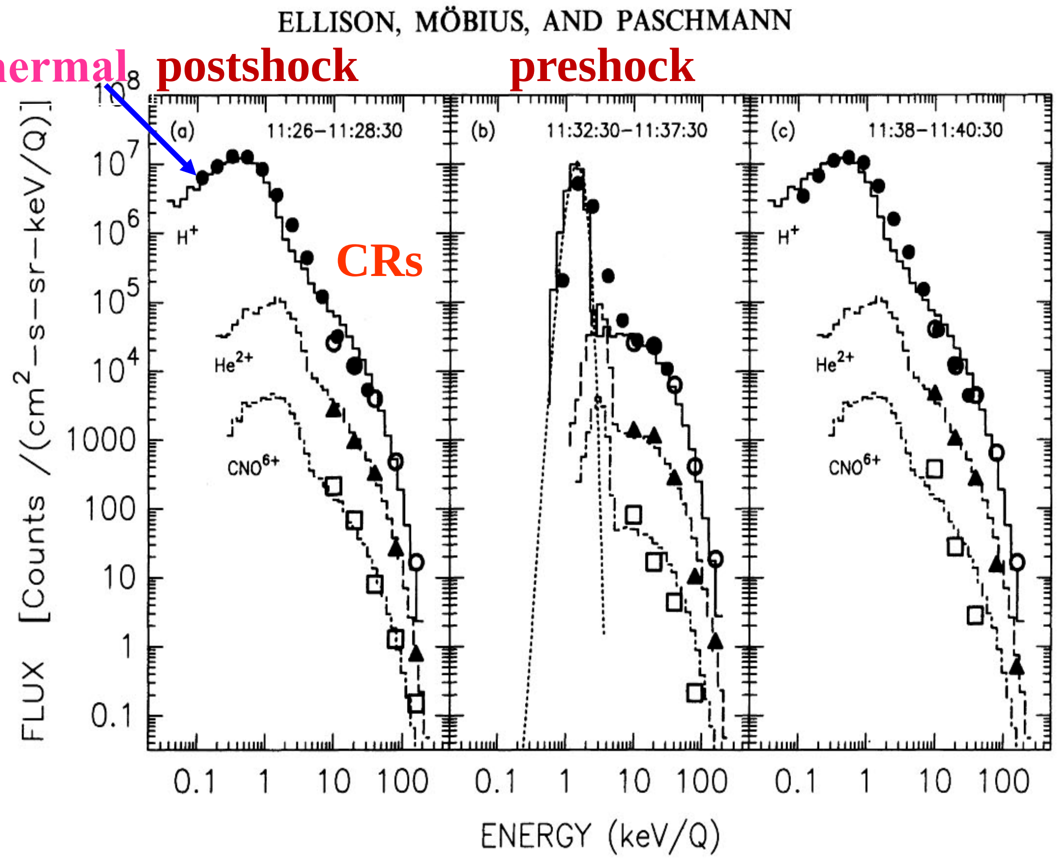


FIG. 6.—Monte Carlo simulation fits to observed downstream, upstream, downstream sequence of spectra shown in (*a*), (*b*), and (*c*) of Fig. 2. Fi...les are protons, triangles are He^{2+}, and squares are CNO^{6+}. The simulation uses $\alpha = 1$, and all parameters are the same in each of the three panels, ...ervation position and the normalization. For the downstream spectra ([*a*] and [*c*]), $D_{obs} > 1\lambda_0$ and $N_e V_p = 5.8 \times 10^7$ cm^{-2} s^{-1}. For the upstream ...$_s = -40\,\lambda_0$ and $N_e V_p = 1.0 \times 10^8$ cm^{-2} s^{-1}. The dotted line in (*b*) is a shifted Maxwellian with a bulk speed of 525 km s^{-1}, a normalization of N_e ...

CR energy spectrum observed at Earth

power-law spectrum

-N(E) ~ $E^{-2.7}$ below the knee

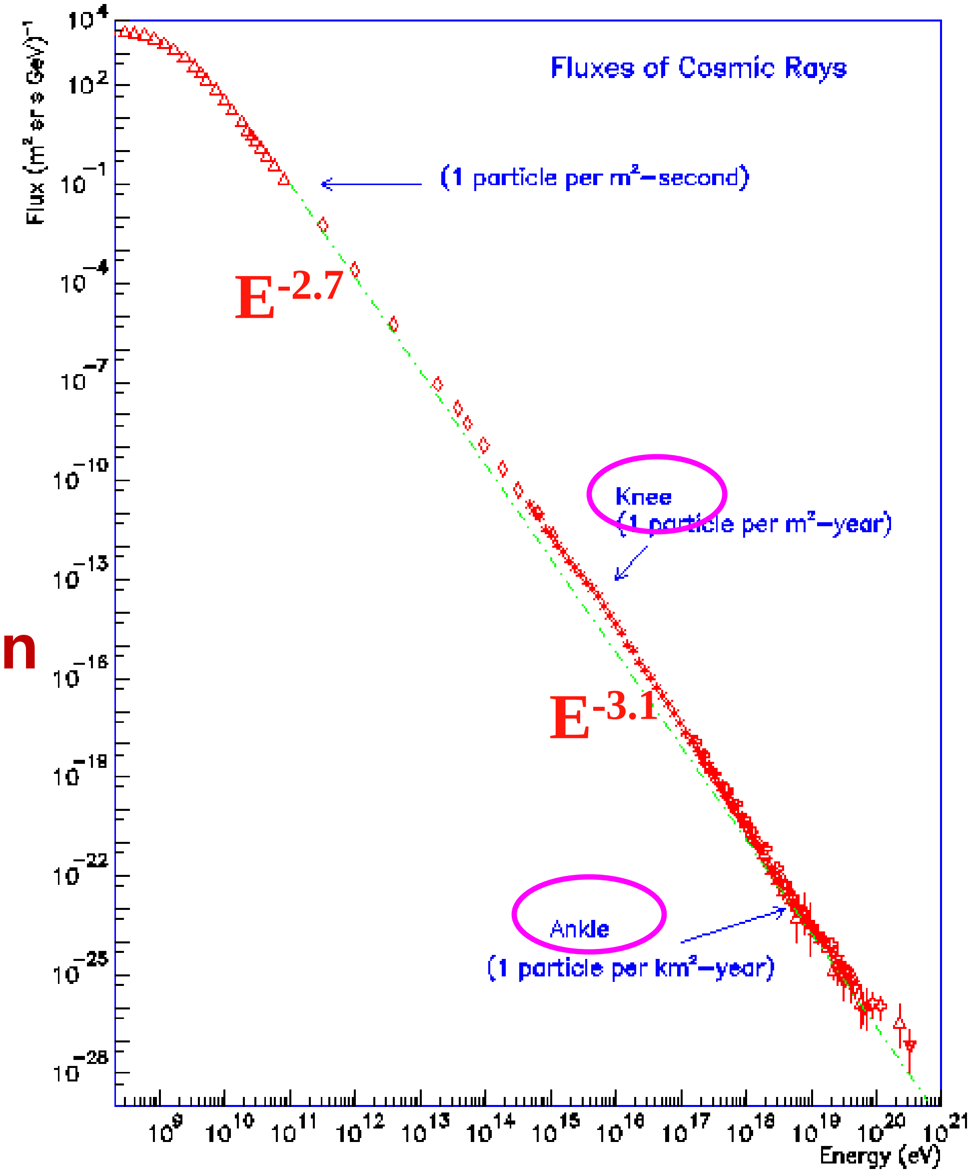
knee energy ~ 10^{15} eV

N(E) ~ $E^{-3.1}$ below the Ankle

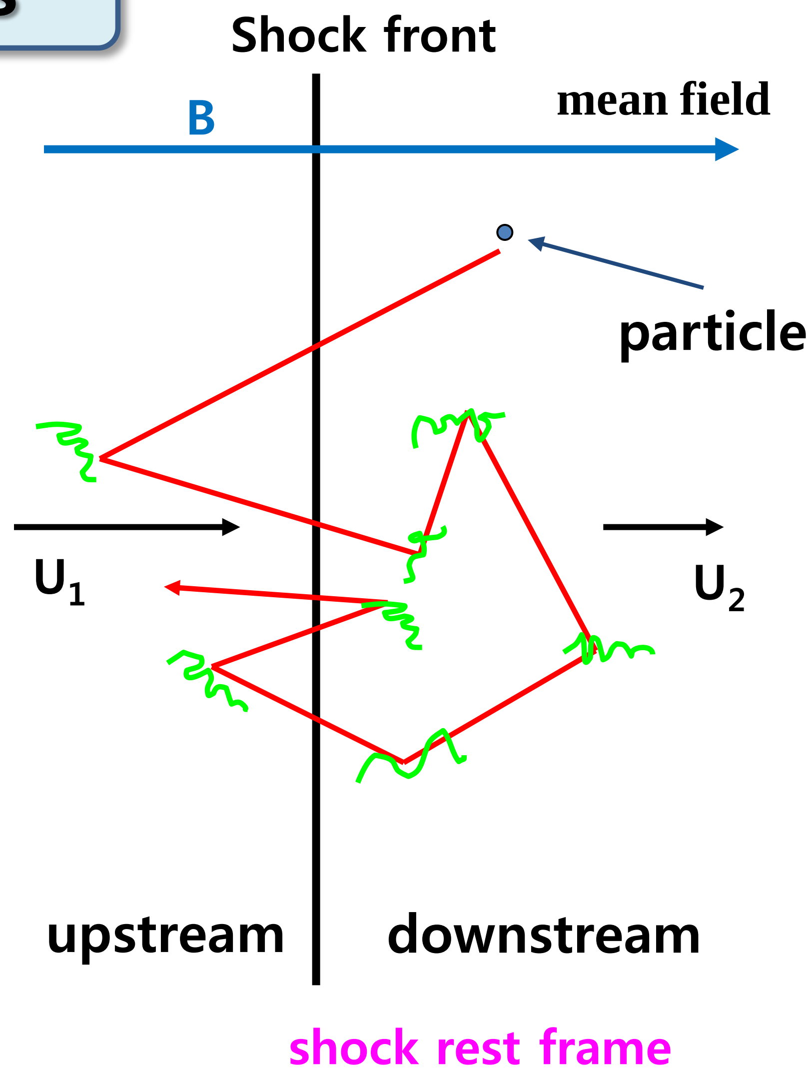
ankle energy ~ $10^{18.5}$ eV

- **"universal" power-law**

by a **"universal" acceleration mechanism** working on a wide range of scales and environments



"Fermi first order process"



In quasi-parallel shocks

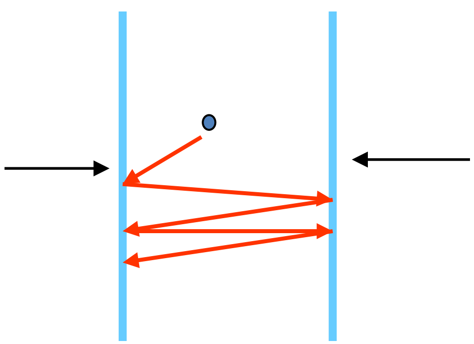
Alfven waves in a converging flow act as converging mirrors

→ particles are scattered by waves

→ cross the shock many times

Every collision is head-on.

$$\frac{\Delta p}{p} \sim \frac{u_1 - u_2}{v} \text{ at each shock crossing}$$



converging mirrors

Prediction of DSA theory in test particle limit

(when non-linear feedback due to CR pressure is insignificant)

$$\boxed{\frac{\Delta \mathrm{p}}{\mathrm{p}} \sim \frac{\mathrm{u}_1 - \mathrm{u}_2}{\mathrm{v}}, \quad p_{\mathrm{esc}} = \frac{\mathrm{u}_2}{\mathrm{v}} (\text{escape prob.}) \Rightarrow f(p) \propto p^{-q}}$$

where the slope, $q = 3\sigma/(\sigma-1) = 3u_1/(u_1-u_2)$

($\sigma = \rho_2/\rho_1 = u_1/u_2$ determined by the shock Mach No.)

for strong gas shock : $\sigma \rightarrow 4$, $q \rightarrow 4$,

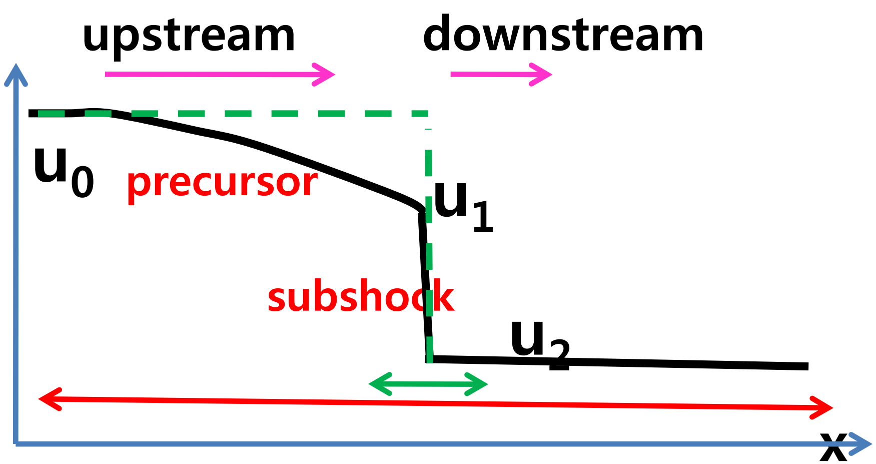
(for $\gamma = 5/3$ adiabatic index) **independent of *M***

So this could explain the universal power-law of f(p).

But DSA is quite efficient → shock structure is modified by CR pressure.

$$q(p) = \frac{3U(p)}{U(p) - u_2} + \frac{d \ln(U(p) - u_2)}{d \ln p}$$

U(p) is the precursor velocity that particles with *p* sample.



$$\sigma_s = \frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \text{subshock comp.}$$

$$\sigma_t = \frac{u_0}{u_2} = \frac{\rho_2}{\rho_0} = \text{total shock comp.}$$

for $\kappa(p) = \kappa_n p^\alpha$

momentum dependent

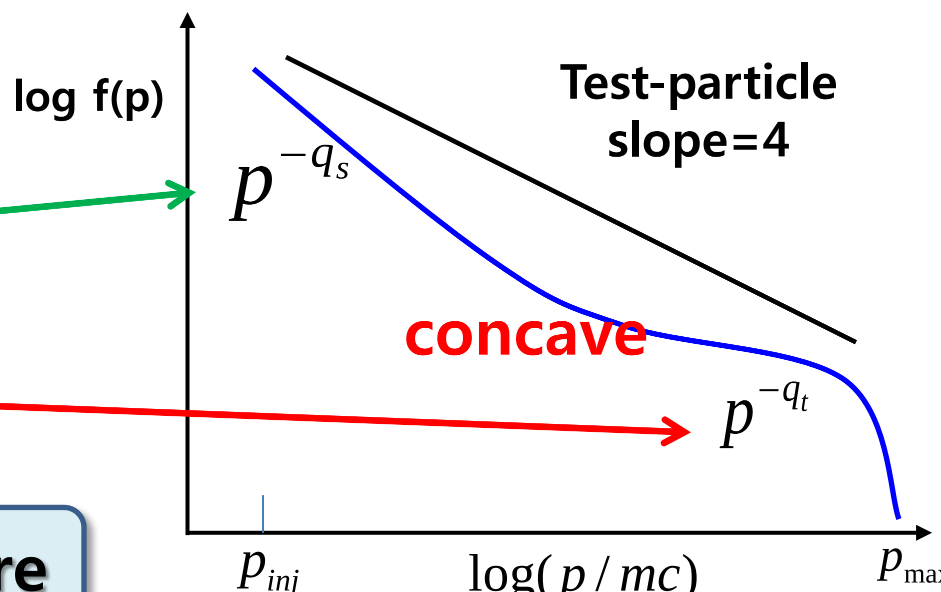
$$l_{diff}(p_{\min}) = \frac{\kappa(p_{\min})}{u_s}$$ → **feel only σ_s**

$$l_{diff}(p_{\max}) = \frac{\kappa(p_{\max})}{u_s}$$ → **feel σ_t**

Particles with different *p* experience different Δu.

$$q_s = \frac{3 \cdot \sigma_s}{\sigma_s - 1} > 4 : \text{subshock}$$

$$q_t = \frac{3 \cdot \sigma_t}{\sigma_t - 1} < 4 : \text{total shock}$$

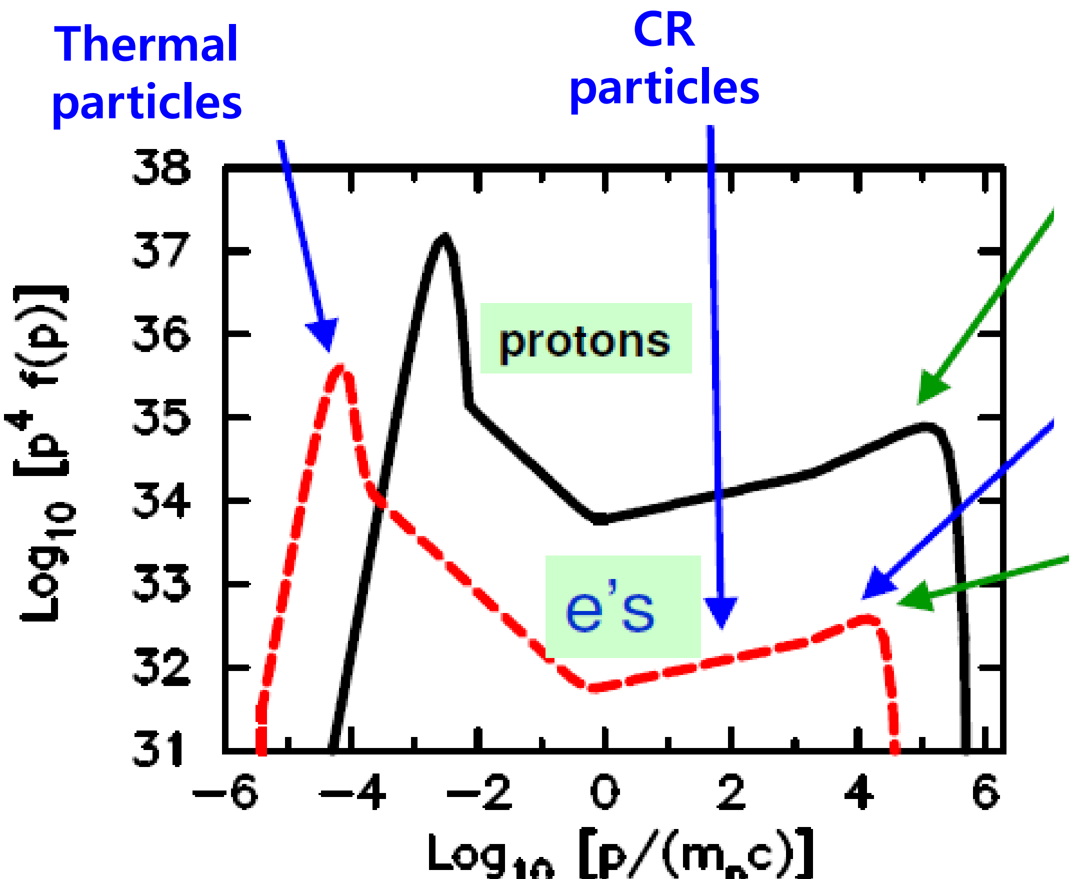


CR modified shock structure

e.g. semi-analytic model for f(p): Berezhko & Ellison

Electron and Proton distributions from efficient DSA

(from Berezhko & Ellison accel. model)



$$3\ \text{power-laws:}$$

$$f(p) \propto p^{-q_s}\ \text{subshock}$$

$$\propto p^{-q_i}\ \text{intermediate}$$

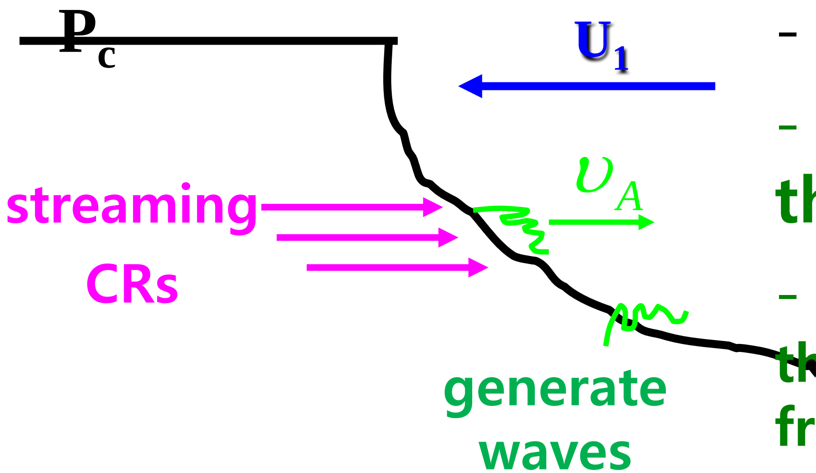
$$\propto p^{-q_t}\ \text{total shock}$$

$$q_s = \frac{3\cdot\sigma_s}{\sigma_s - 1}: \text{subshock comp.}$$

$$q_t = \frac{3\cdot\sigma_t}{\sigma_t - 1}: \text{total shock comp.}$$

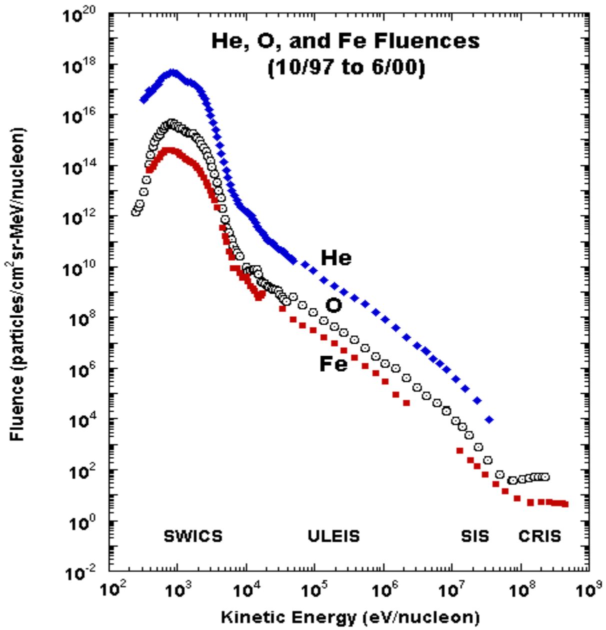
Spherical CR shock model for Supernova Remnants

Particle injection, Wave generation, drift & dissipation



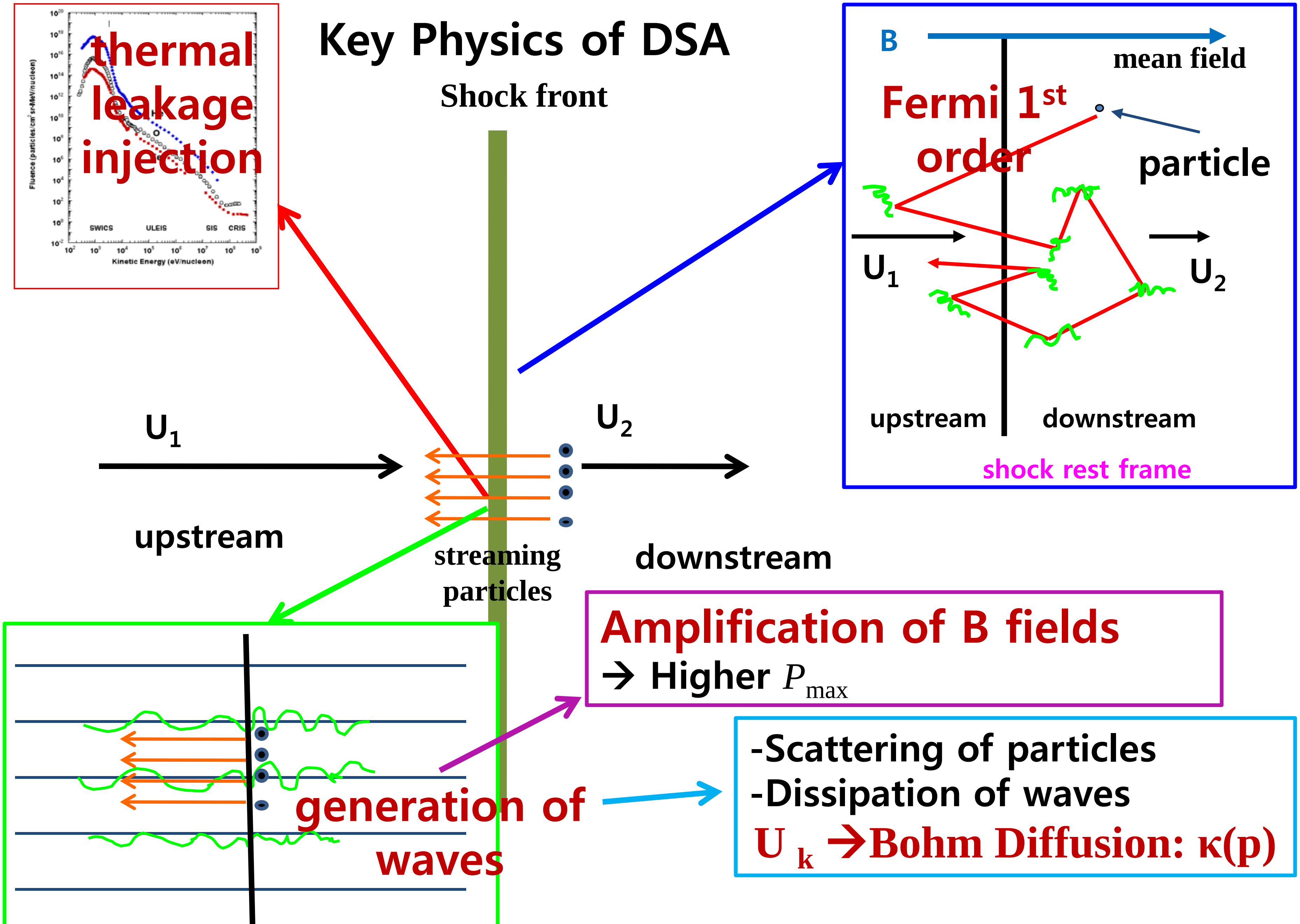
- waves drift upstream with $u_w = \upsilon_A$
- waves dissipate energy & heat the gas.
- CRs are scattered and isotropized in the wave frame rather than the fluid frame.

suprathermal particles
→leak upstream : becomes CRs
(thermal leakage injection)
→ self-generation of waves by wave-particle interactions & Amplification of B fields



particle spectra in Solar wind
(Mewaldt et al 2001)

Key Physics of DSA



Following individual particle trajectories and evolution of fields are impractical.

Complex wave-particle interactions are simplified.

→diffusion approximation (assuming isotropy of velocity distribution in local wave frame)

→Solve for Diffusion-Convection EQ for

f(p) = isotropic part of particle distribution function

$$\frac{\partial f}{\partial t} + (u+u_w)\frac{\partial f}{\partial x} = \frac{1}{3}\frac{\partial}{\partial x}[(u+u_w)]\, p\frac{\partial f}{\partial p} + \frac{\partial}{\partial x}[\kappa(x,p)\frac{\partial f}{\partial x}] + Q(x,p)$$

$u_w = \mathrm{B}/\sqrt{4\pi\rho}$ in upstream, $u_w = 0$ in downstream,

$\kappa(x,p)$ is spatial diffusion coefficient,

$Q(x,p)$ represents the injection term.

Numerical Methods to study DSA-1

- **Full & Hybrid plasma simulations: 3D is required**
  - **follow individual particles and magnetic fields**
  - **provide the most complete picture, but computationally very expensive**

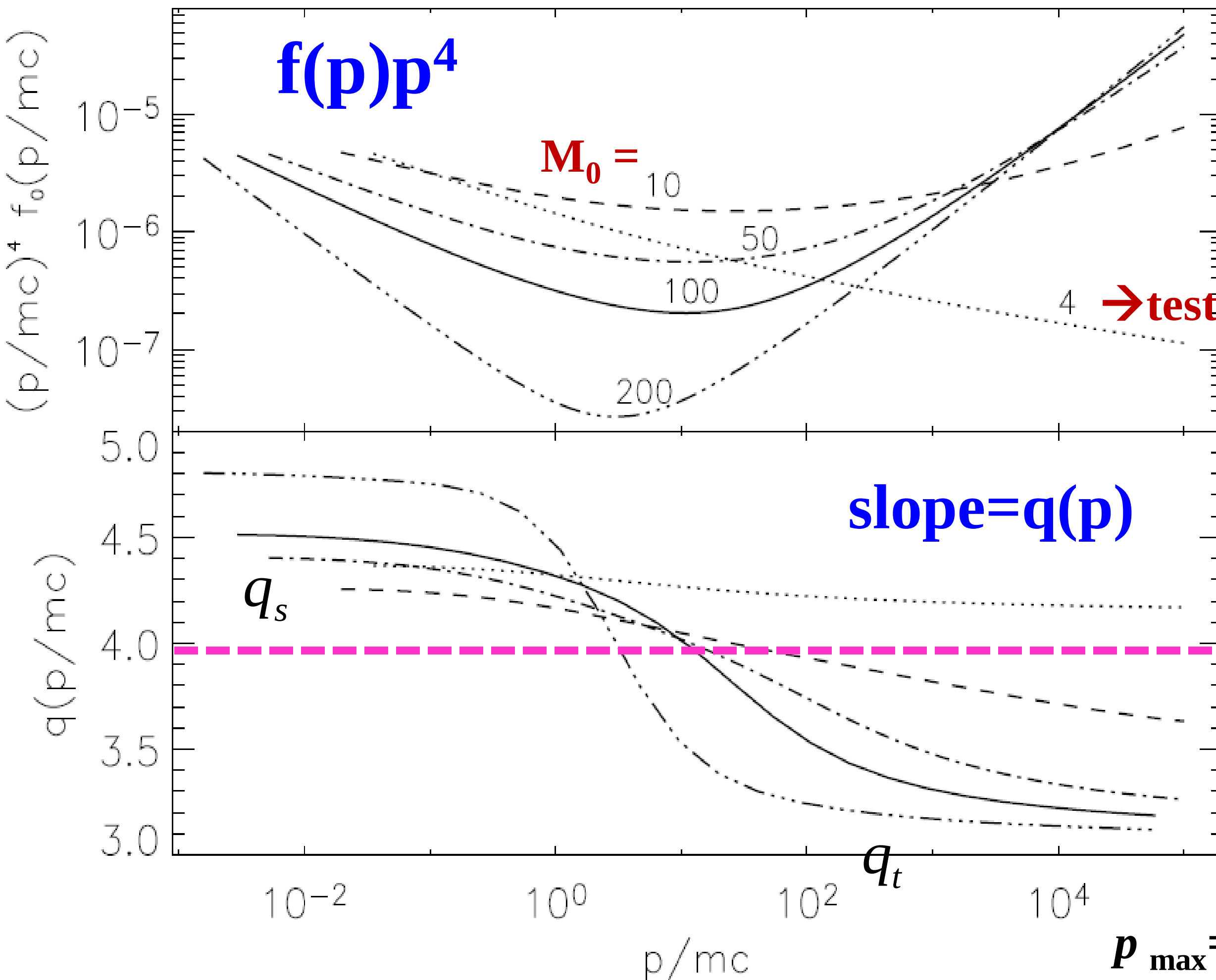
  (e.g. Bell & Lucek, Giacalone & Jokipii)
- **Monte Carlo Simulations with a scattering model:**
  - **scattered with a *prescribed scattering model*,**
  - **assume a steady-state shock structure with FEB** $\lambda \propto \frac{R^\alpha}{\rho} = \lambda_0 \left(\frac{A}{Q}\right)^\alpha \left(\frac{v}{u_2}\right)^\alpha \left[\frac{\rho_2}{\rho(x)}\right],$

  (e.g. Ellison & Jones, Baring)
- **Semi-analytic Method:**
  - **assume a steady-state shock structure with a fixed** p_{max}
  - **find the self-consistent shock structure**

  (e.g. Malkov & Voelk, Blasi & Amato)

e.g. Semi-analytic model: Amato & Blasi 2005, 2006

Solve DC equation for *f(x,p)* along with gasdynamic equations in the steady state limit



→**test-particle at low M**

$$q(p) = \frac{d \ln f}{d \ln p}$$

$$q_s = 3\sigma_s/(\sigma_s - 1)$$

$$q_t = 3\sigma_t/(\sigma_t - 1)$$

steeper at low *p*
flatter at high *p*

p $_{\mathbf{max}}$**=10**^{5} **mc**
(upper momentum boundary)

Numerical Methods to study DSA-2

- **Kinetic Simulations** : **for quasi-parallel shocks**

diffusion approximation based on isotropy of particle distribution

follow time dependent evolution of *f(x,p)* + gasdynamics EQs

*** Berezkho, Voelk, & collaborators:**

- 1D spherical geometry, piston driven shock , applied to SNRs,
- renormalization of space variables with diffusion length
- successful in predicting nonthermal radiation from SNRs

*** Kang & Jones : CRASH (Cosmic Ray Acceleration SHock code)**

- 1D plane-parallel and spherical grid comoving with a shock
- shock-tracking and AMR techniques are adopted
- numerical models for thermal leakage injection & wave drift are implemented.

CR spectrum at RXJ1713
→ nonthermal radiation

$t \approx 1600\, yrs$

$B_d \approx 130 \mu G, B_u \approx 65 \mu G$

$n_{ISM} \approx 300 cm^{-3}$

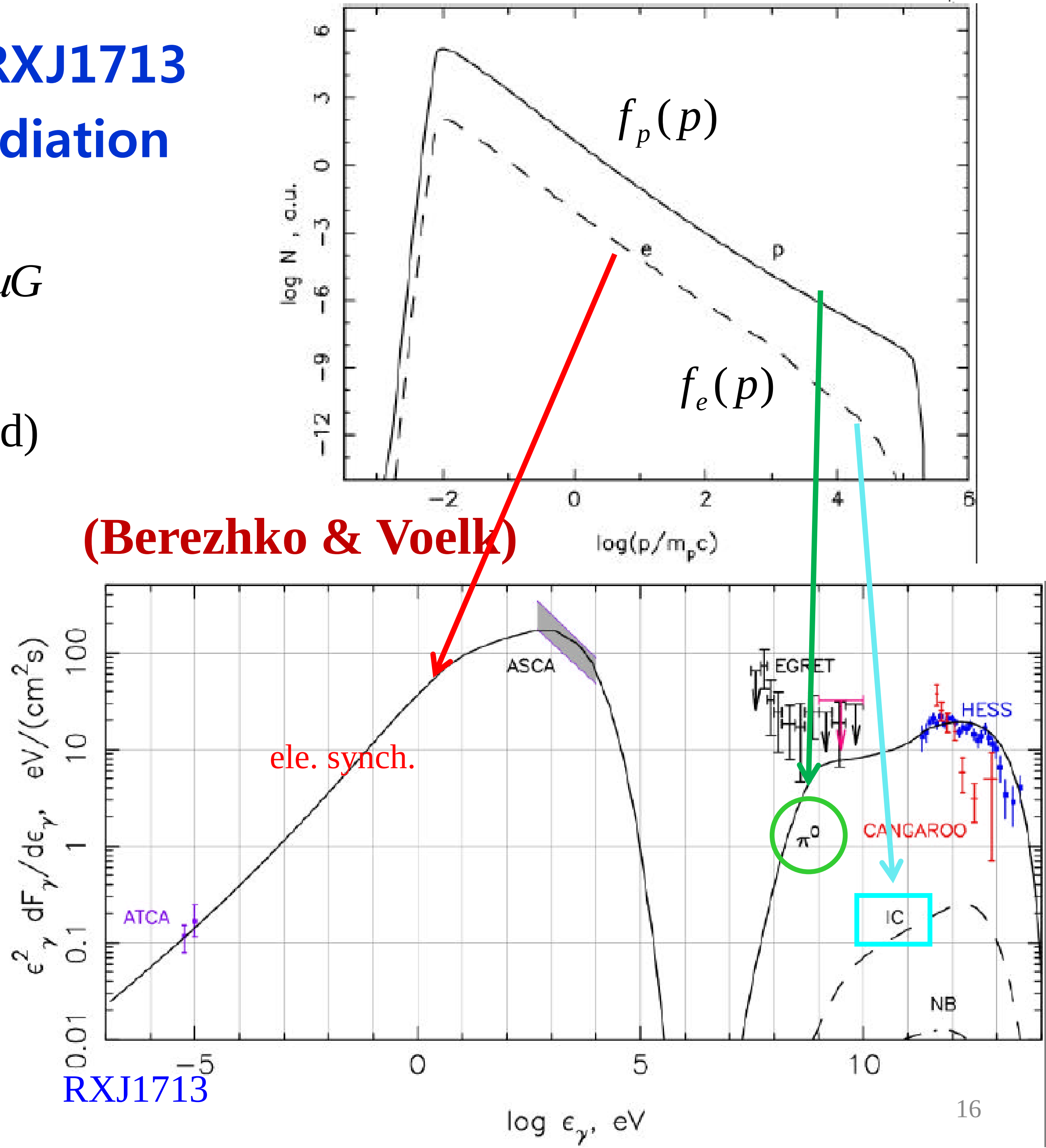
$\sigma_t \approx 6.3$ (CR modified)

$E_{\max} \approx 100$ TeV

$E_{SN} \approx 1.8 \times 10^{51}$ ergs

CR e + CMBR→
IC scattering
→TeV γ-ray

CR p + p→ π^0 decay
→ GeV-TeV γ-ray



RXJ1713

Basic Equations for Kinetic DSA Simulations

(1D plane-parallel)

$$\frac{\partial \rho}{\partial t} + \frac{\partial (u\rho)}{\partial x} = 0$$

$$\frac{\partial (\rho u)}{\partial t} + \frac{\partial}{\partial x}(\rho u^2 + P_g + \underline{P_c}) = 0$$

$$\frac{\partial (\rho e_g)}{\partial t} + \frac{\partial}{\partial x}(\rho e_g u + P_g u) = \underline{-u\frac{\partial P_c}{\partial x} + W - L}$$

ordinary gasdynamics EQs

+ P_c terms

Diffusion Convection Eq. for isotropic part of f(p)

$$\frac{\partial f}{\partial t} + (u + u_w)\frac{\partial f}{\partial r} = \frac{1}{3}\frac{\partial}{\partial x}(u + u_w)\cdot p\frac{\partial f}{\partial p} + \frac{\partial}{\partial x}[\boxed{\kappa(x, p)}\frac{\partial f}{\partial x}] + \boxed{Q(x, p)}$$

$$P_c = \frac{4}{3}\pi m_p c^2 \int_0^\infty f(p)\frac{p^4 dp}{\sqrt{p^2+1}}$$

***W*= wave dissipation heating, u_w = drift speed of waves**

***L*= thermal energy loss due to injection, Q= CR injection**

Numerical Tool: CRASH Code (Kang et al. 2001)

Bohm type diffusion: $\kappa(p) \propto p^2 / \sqrt{p^2+1}$

- **wide range of diffusion length scales to be resolved:** $l_{diff} = \kappa(p)/u_s$

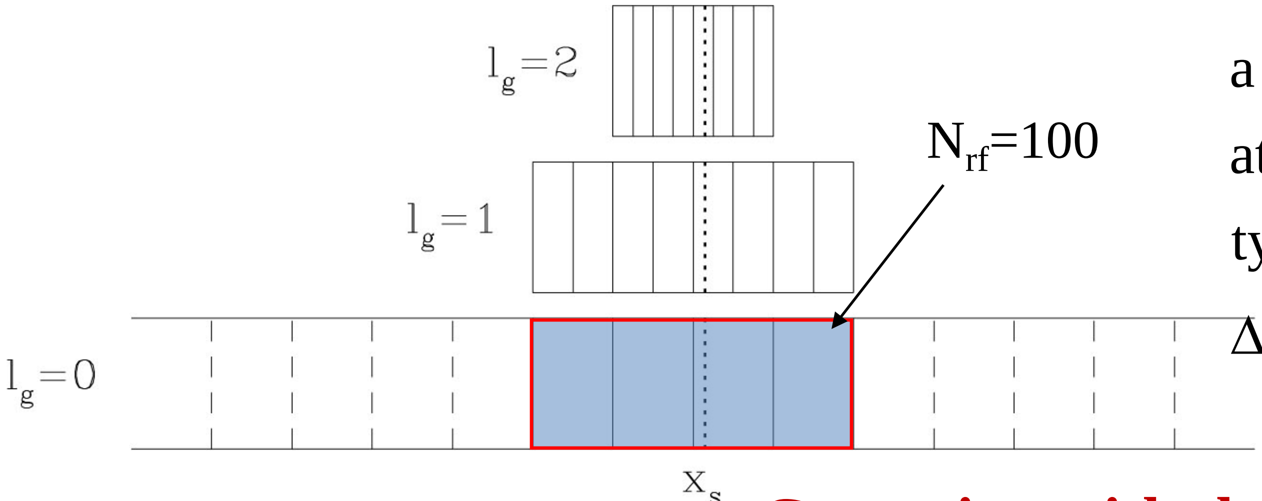
  from p_{inj}/mc(~10^{-2}) to outer scales for the highest p_{max}/mc (~10^6)

1) Shock Tracking Method (Le Veque & Shyue 1995)

- **tracks the subshock as an exact discontinuity**

2) Adaptive Mesh Refinement (Berger & Le Veque 1997)

- **refines region around the subshock with multi-level grids**

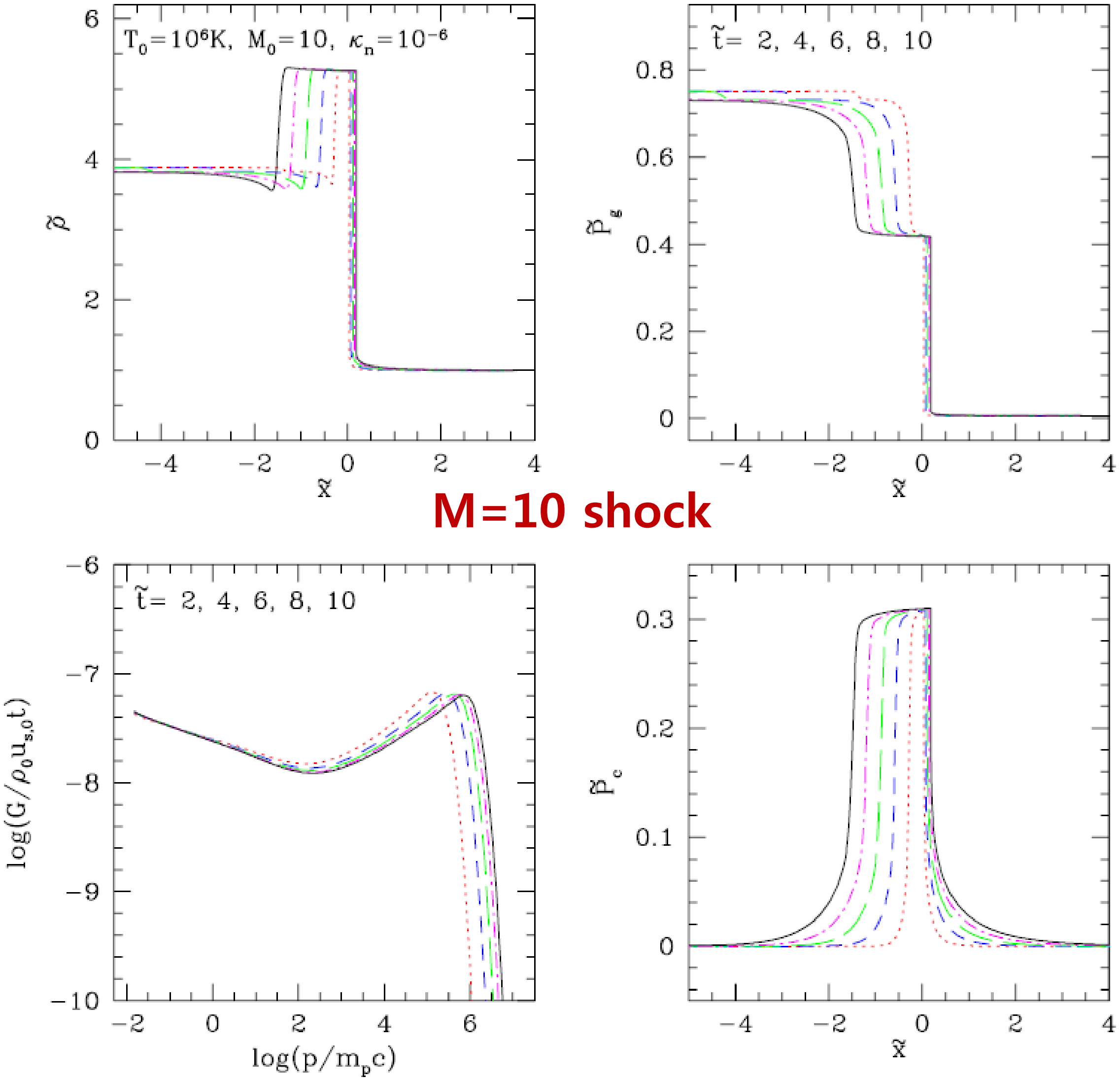


a factor of two refinement at each grid level, typically $l_{g,\max} = 8-10$

$\Delta x_{10} = \Delta x_0 / 1024$

Comoving with the shock

Kinetic Simulations of CR modified Shocks



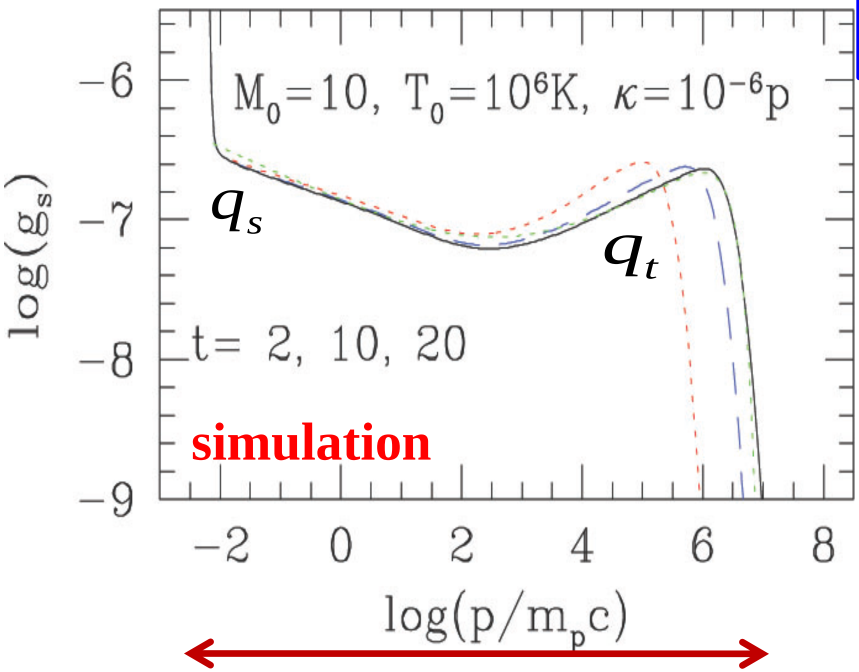
After $P_{c,2}$ reaches to an asymptotic value,

→ the shock flow becomes self-similar.

The shock structure stretches linearly with *t*, independent of κ(p).

$$l_{\max} \approx \frac{\kappa(p_{\max})}{u_s} \approx \frac{1}{8} u_s t$$

$$t_{acc} \approx 8 \frac{\kappa(p_{\max})}{u_s^2}$$



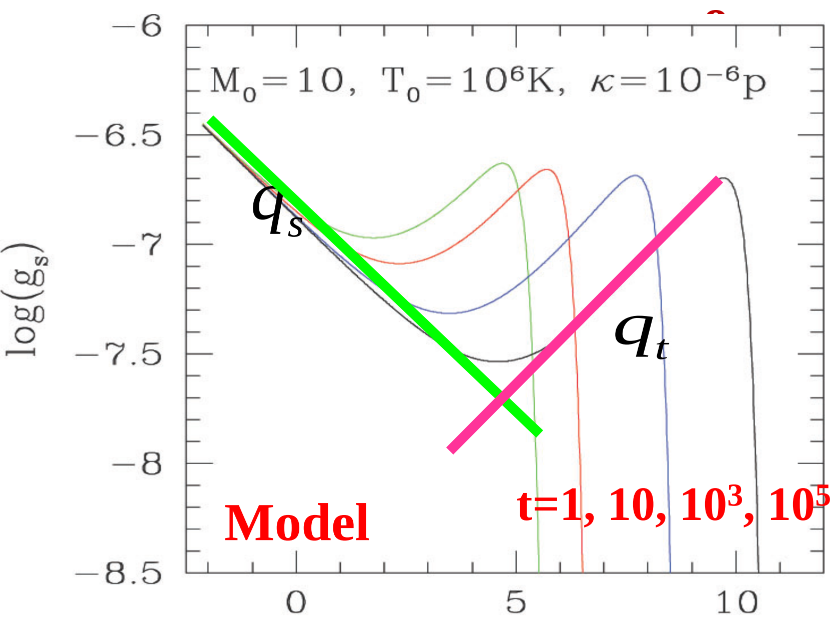
CR distribution function at shocks

$$q_s = \frac{3\sigma_s}{\sigma_s - 1} \quad \text{subshock jump at low } p$$

$$q_t = \frac{3\sigma_t}{\sigma_t - 1} \quad \text{total shock jump at high } p$$

$$p_{\min} = p_{\text{inj}} \approx 2 \cdot \left(\frac{u_s}{c}\right)$$

$$p_{\max} \approx \frac{u_s^2}{8\kappa_n} \cdot t\,, \quad where\ \kappa(\mathrm{p}) = \kappa_{\mathrm{n}}\, p\left(\frac{\rho}{\rho_o}\right)^{-1}$$



possible analytic form : **two power-laws**

$$f(x_s, p) = [\, \underline{f_1 \cdot (p / p_{\min})^{-q_s}}$$
$$+ \underline{f_2 \cdot (p / p_{\max})^{-q_t}}\,] \cdot \exp[-(p / p_{\max})^2\,]$$

where $f_1 = f_{th}(p_{\text{inj}})$ at thermal tail

f_2 : to be determined by $P_{c,2}$

Why CR modified shocks becomes self-similar ?

in the limit of $t \to \infty$, $p_{\max} \to \infty$

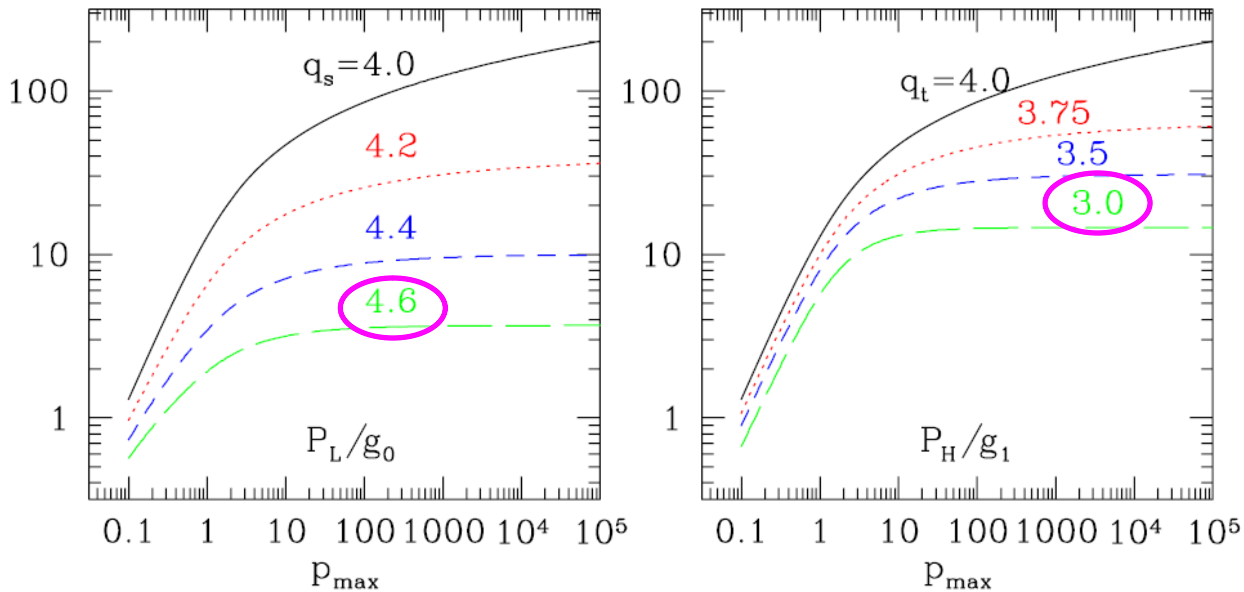
postshock $P_{c,2} \to$ constant

then shock structure $\to$ steady,

i.e. $P_{g,2}, \rho_1, \rho_2 \to$ constant

$$P_c = \frac{4\pi}{3} m_p c^2 \int_{p_{\rm inj}}^{\infty} g(p) \frac{p}{\sqrt{p^2+1}} \frac{dp}{p}. = P_L \ + \ P_H$$

$$\boxed{g_s(p) = \left[g_0 \cdot \left(\frac{p}{p_{\rm inj}}\right)^{-q_s+4} + g_1 \cdot \left(\frac{p}{p_{\max}}\right)^{-q_t+4} \right] \exp\left[- \left(\frac{p}{1.5 p_{\max}}\right)^{2\alpha} \right]}$$



if $q_s > 4.0$ (i.e. $\sigma_s < 4.0$),

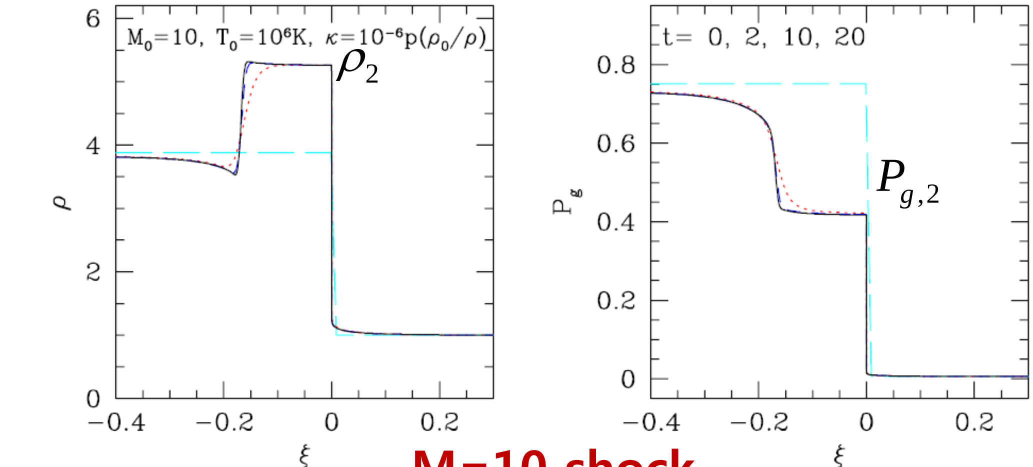
$P_L \to$ constant at large $p_{\max}$

if $q_t < 4.0$ (*i.e.* $\sigma_t > 4.0$),

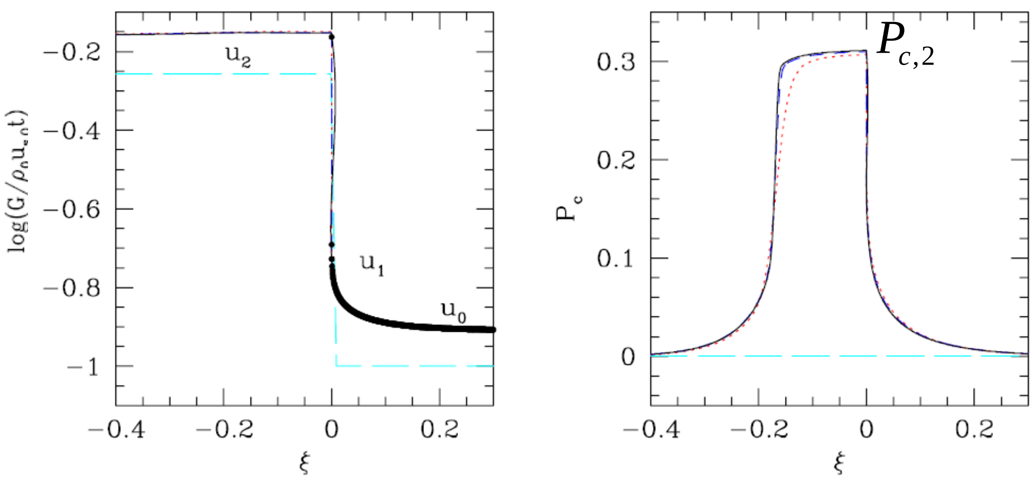
$P_H \to$ constantat large $p_{\max}$

true for CR modified shocks.

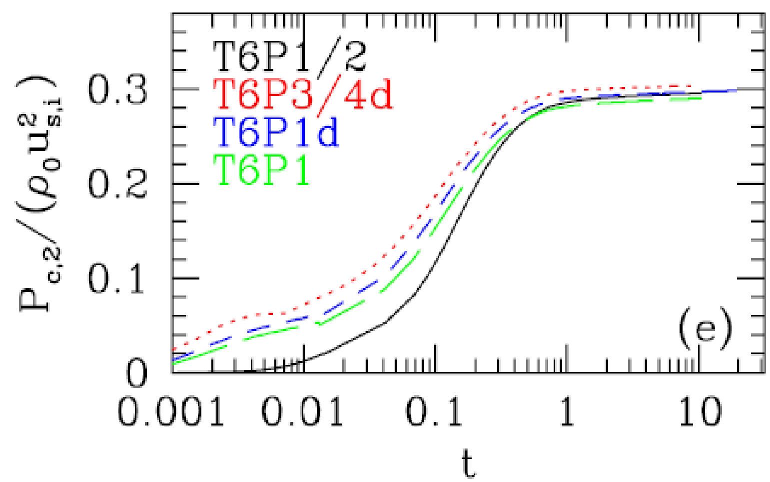
Self-similar Evolution of CR modified Shocks



M=10 shock

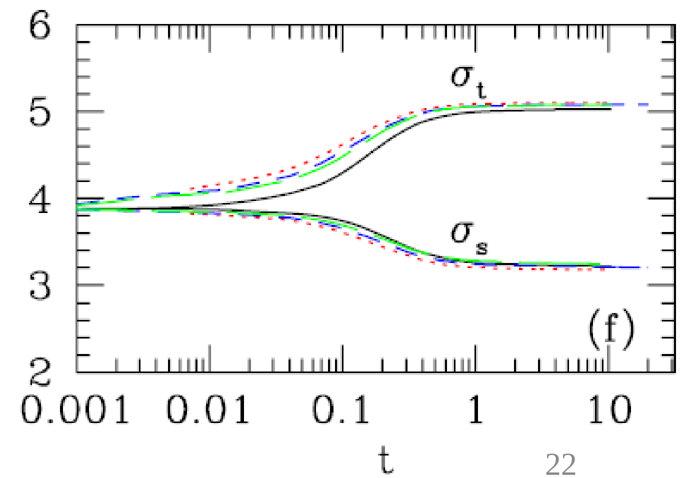


$P_{g,2}$, $P_{c,2}$ $\rightarrow$ constant in time



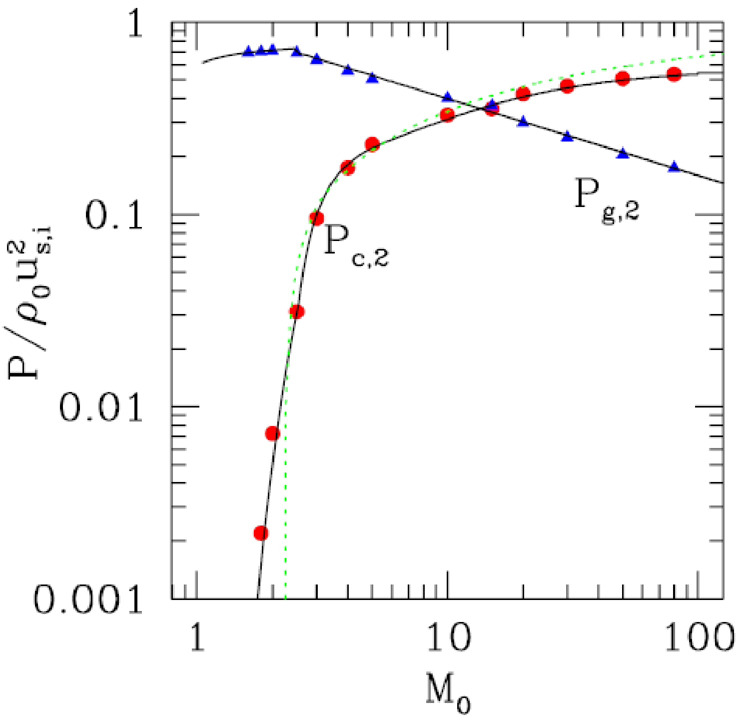
$\sigma_t = \rho_2 / \rho_0 \rightarrow$ constant in time

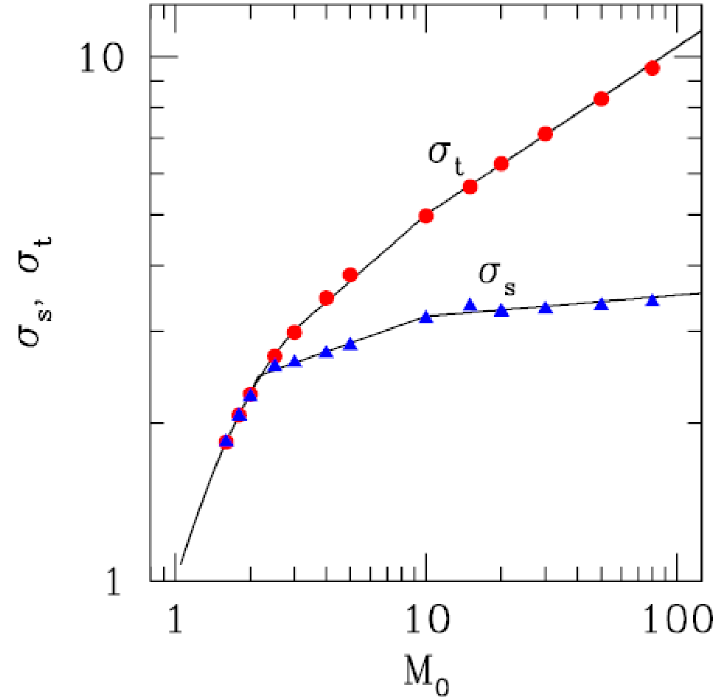
$\sigma_s = \rho_2 / \rho_1$



$$\xi = \frac{x}{l_{\max}(t)} \propto \frac{x}{u_s \cdot t}$$ = **similarity variable**

Time asymptotic solutions from DSA simulations





total compression

$$\sigma_t = \rho_2 / \rho_0$$

subshock compression

$$\sigma_s = \rho_2 / \rho_1 \sim 3$$

$$\boxed{\frac{P_{c,2}}{\rho_0 u_s^2} \approx 0.5 \ \text{for}\ M_0 \geq 20}$$

Note that these solutions cannot be obtained analytically from first principles (i.e. conservation laws), so they have to be found through DSA numerical solutions.

SUMMARY

In CR modified shocks, the precursor & subshock transition approach the time-asymptotic state.

$$\boxed{P_{g,2}\,,\, P_{c,2},\, \sigma_t = \rho_2/\rho_0,\;\; \sigma_s = \rho_2/\rho_1 \rightarrow \text{constant}}$$

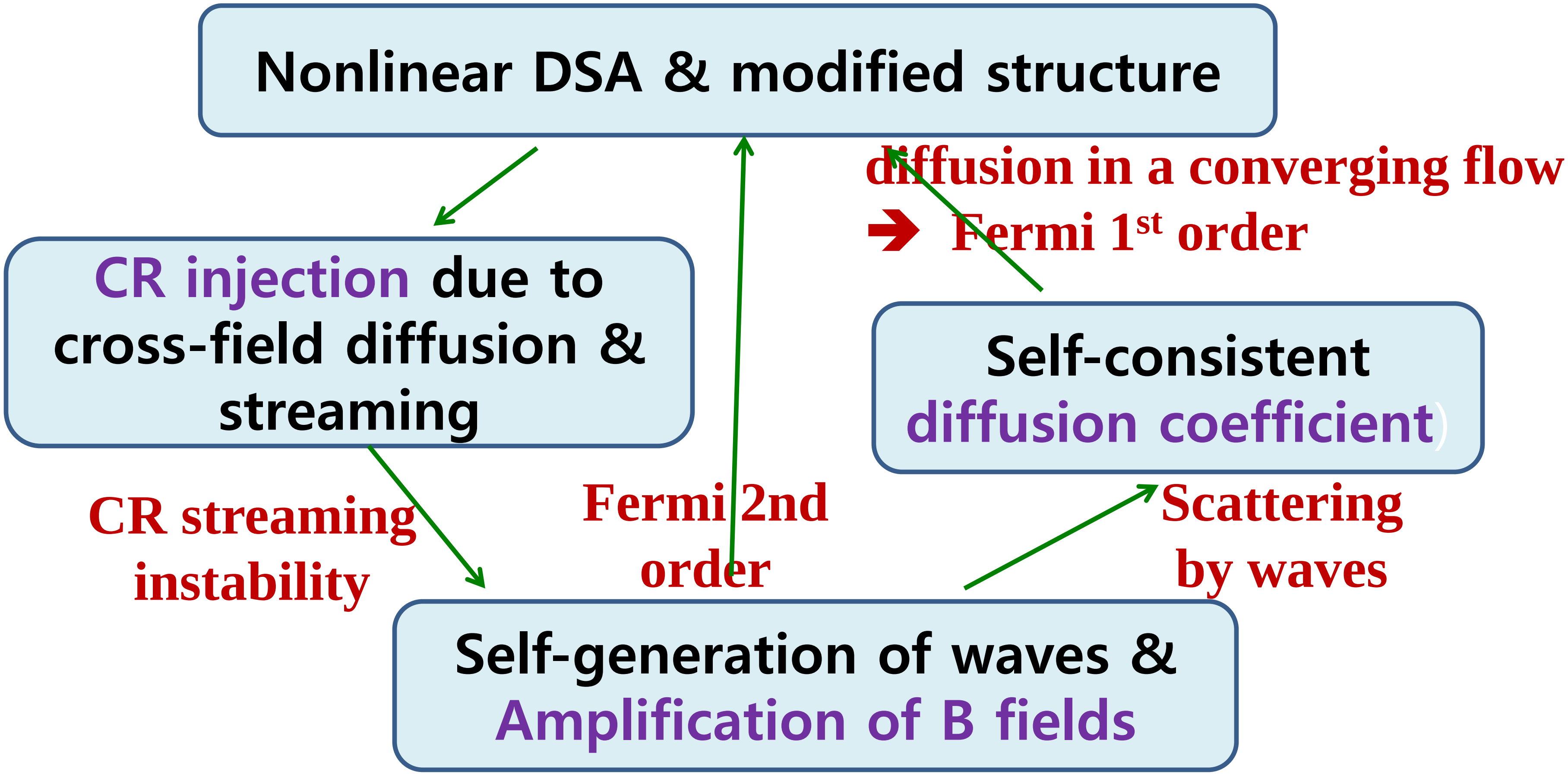
(need numerical simulations)

Then shock precursor structure evolves in a self-similar fashion, depending only on similarity variable, $\xi = x/(u_s\, t)$. During this self-similar stage, the CR distribution at the subshock maintains a characteristic form: two power-laws

$$f_s(p,t) = [\, f_1 \cdot (p/p_{\min})^{-q_s} + f_2 \cdot (p/p_{\max}(t))^{-q_t}\,] \cdot \exp[-(\frac{p}{1.5\,p_{\max}(t)})^{2\alpha}]$$

where $f_1 = f_{th}(p_{\rm inj})$ at thermal tail, f_2 : to be determined by $P_{c,2}$

$$q_s(M) = \frac{3\sigma_s}{\sigma_s - 1}\;,\quad q_t(M) = \frac{3\sigma_t}{\sigma_t - 1}\;,\; p_{\rm inj} \approx 2 \cdot (\frac{u_s}{c}),\; p_{\max}(t) \approx \frac{u_s^2}{8\kappa_n} \cdot t$$



wave-particle interactions at collisonless shocks

Growth & Damping of waves ➔ **Scattering, κ(x,p)**

➔

Predict $f(p)$, p_{max} non-thermal radiation (observation)

Numerical Model for Thermal Leakage Injection in CRASH

$\tau_{esc}(\varepsilon_B, \upsilon / u_d)$ **: filter function**

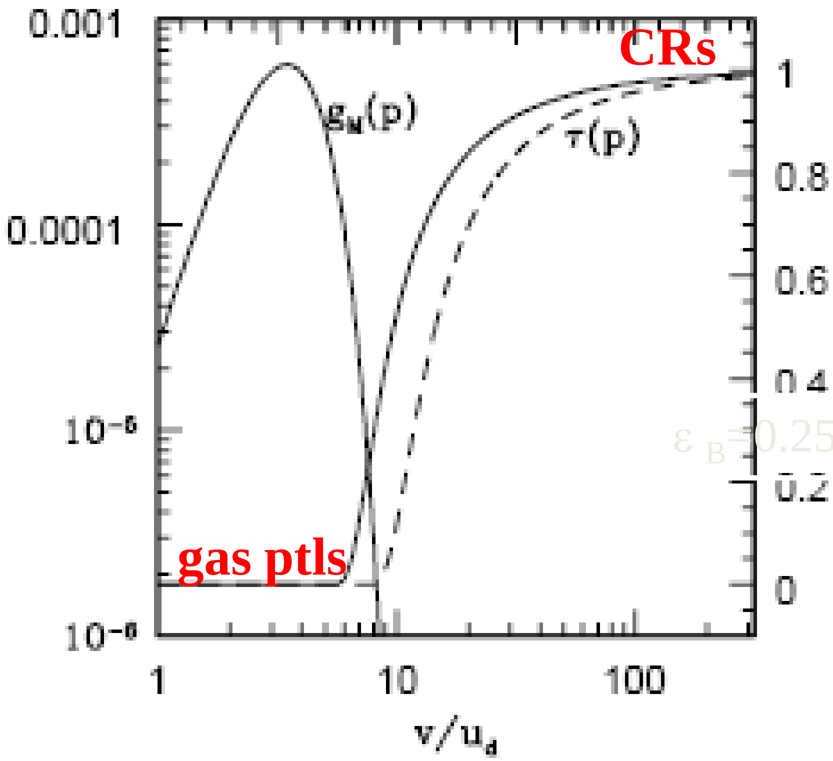
"Transparency function": probability that particles at a given velocity can leak upstream. e.g. $\tau_{esc} = 1$ for CRs

$\tau_{esc} = 0$ for thermal ptls

u_d = downstream flow speed

$$\frac{\upsilon}{u_d} \approx \frac{\upsilon_{th}}{u_d} = fcn(M_s)$$

$$\varepsilon_B = \frac{B_0}{B_\perp} = \frac{\text{mean field}}{\text{turbulent field}}$$



Smaller ε_B : stronger turbulence,
→ difficult to cross the shock,
→ less efficient injection

For a given values of ε_B , injection rate is controlled by the shock Mach number.

Monte Carlo simulation Vladimirov, Ellison, & Bykov (2006)
B field Amplification in CR dominated shocks

CR streaming instability (eg. McKenzie & Volk 1982)

$$\left[\frac{d}{dt}U(x,k)\right]_{\text{stream}} = V_G\left[\frac{\partial P_p(x,p)}{\partial x}\frac{dp}{dk}\right]_{p=\bar{p}(k)};$$

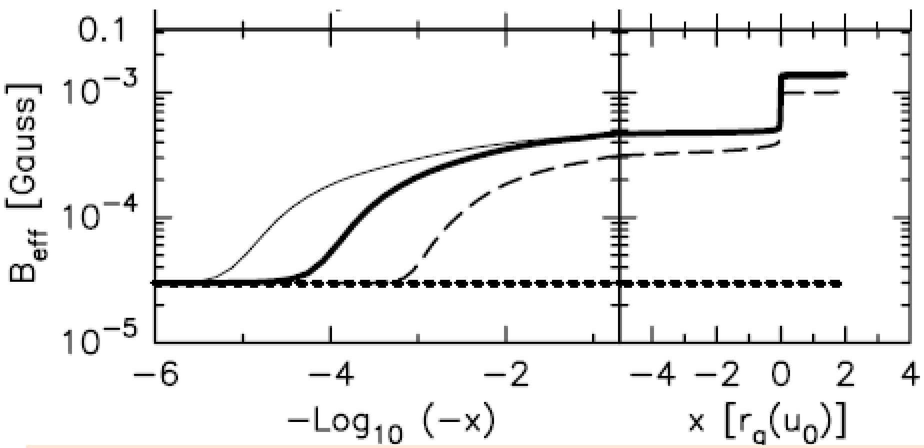
growth of B turbulent energy

With a prescribed scattering model

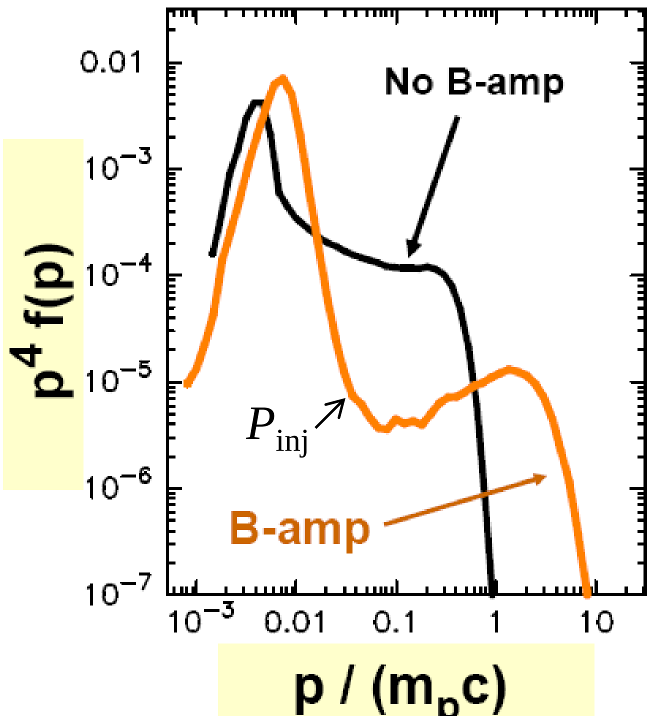
$$\lambda \propto \frac{R^\alpha}{\rho} = \lambda_0\left(\frac{A}{Q}\right)^\alpha\left(\frac{v}{u_2}\right)^\alpha\left[\frac{\rho_2}{\rho(x)}\right],$$

→ *f(x,p)*

+ Gasdynamic equations for a steady-state shock



B_{eff}/B_0 ~ 30-400 : amplification factor
→ can achieve higher p_{max}



All parameters are the same in these cases except one has B-amplification

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