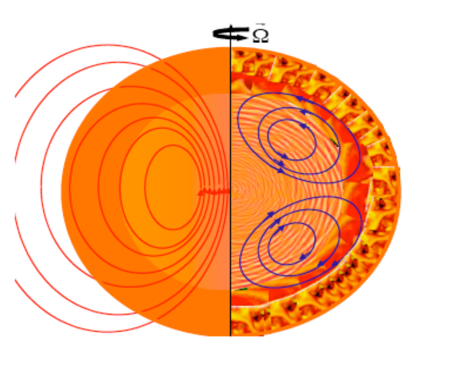
Secular transport in stellar interiors modelling



S. Mathis[1,8]

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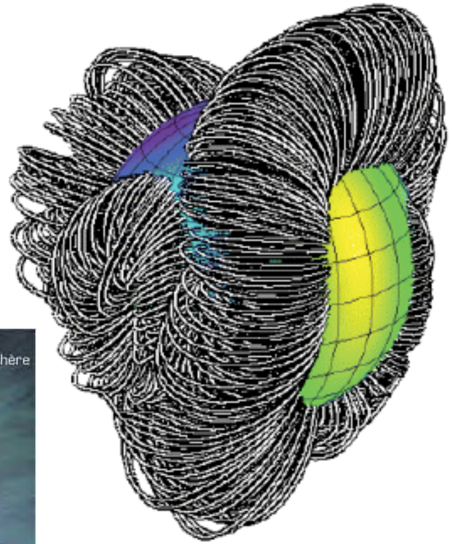
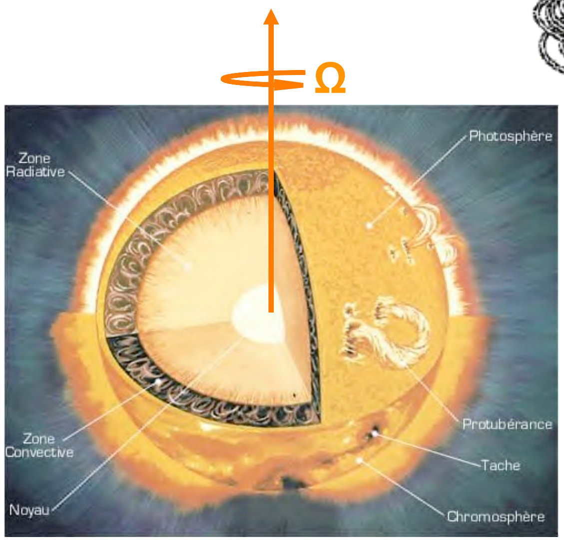
Astronum 2009;
29 June – 3 July 2009, Chamonix

Secular magnetohydrodynamics of stellar radiation zones

Complex magnetism

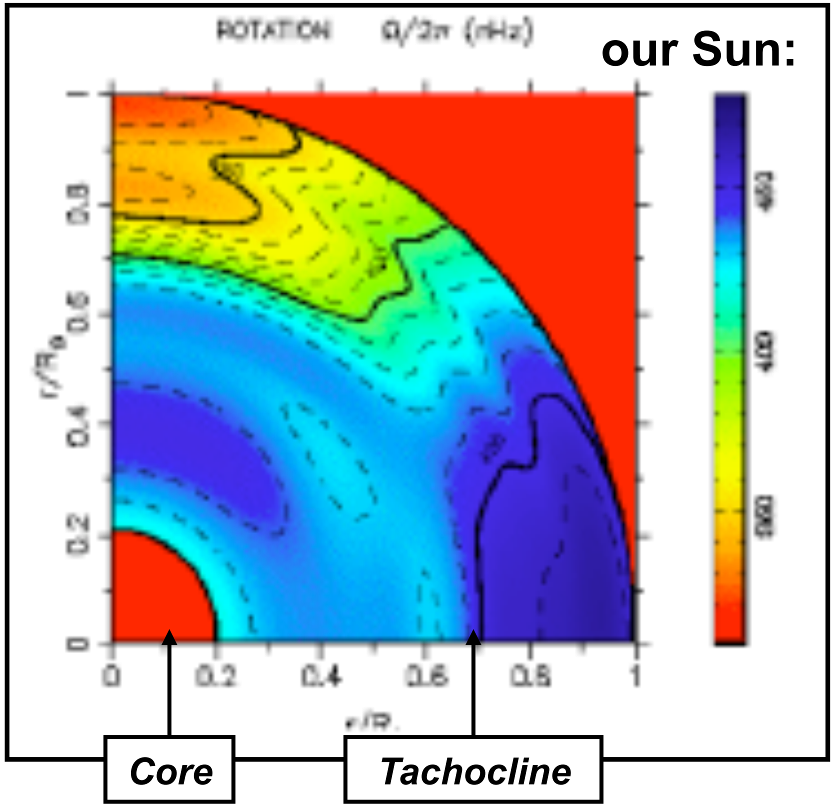
B0.2 main sequence star τ Sco (Donati et al. 2006)





our Sun

Differential rotation (Corbard 1998, Salabert et col., Garcia et al. 2007, Mathur et al. 2008, Eff-Darwich et al. 2008)



A coherent picture of the stars and their evolution
→ need to take into account the dynamical processes which operate in their radiation zones

Major impact of differential rotation

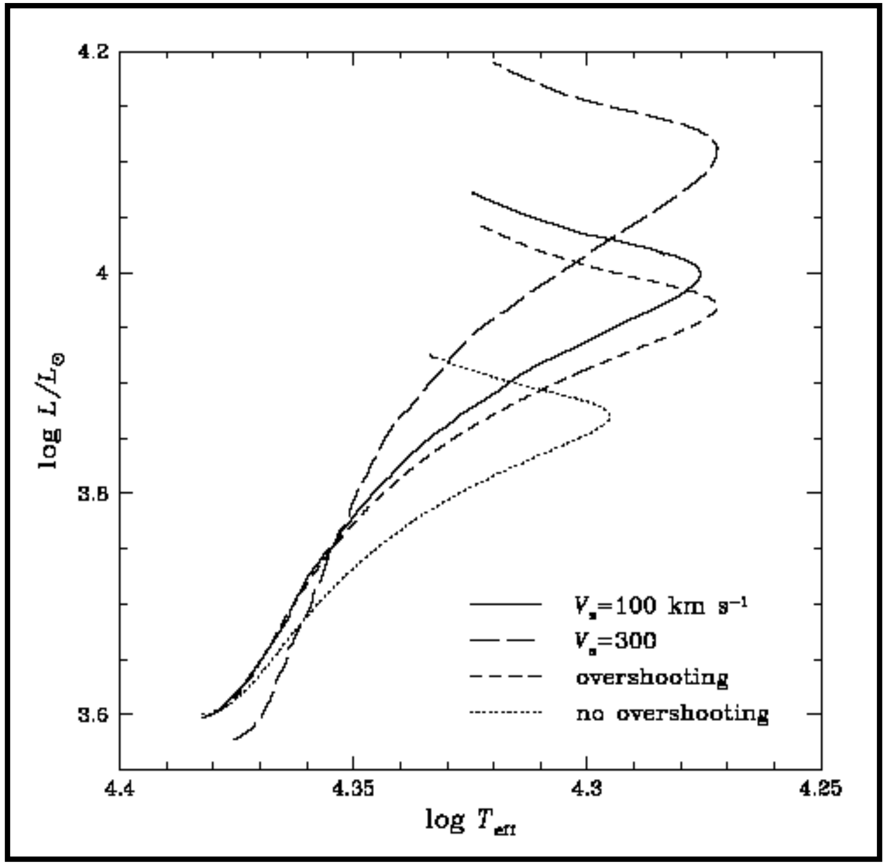
Rotation →
- large scale **circulations**
- **turbulence**
- **magnetic activity**
- **waves**
- **tidal potential** (binary systems)
- **stellar winds** (mass loss)

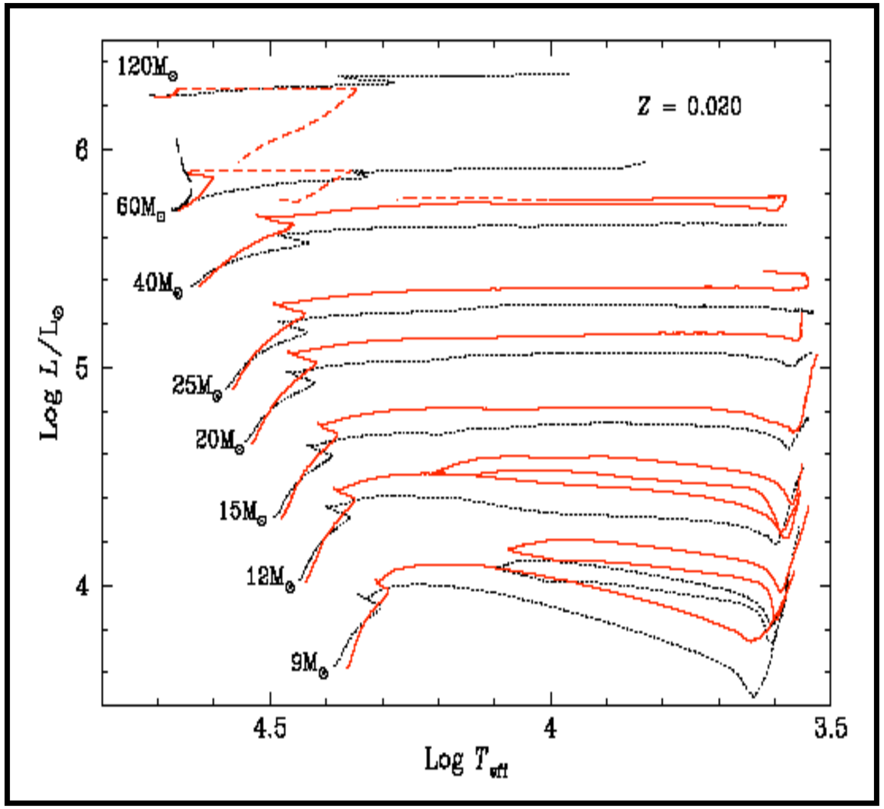
Those processes transport
- Angular Momentum → $\Omega(r,\theta)$
- Matter → modify chemical composition & nucleosynthesis

→ **Major impact on the internal dynamics,**

the evolution and the environment of the stars

Impact on stellar evolution





Influence of the rotation on the evolutionary track of a 9M$_\odot$ star in the HR diagram
Talon et al., 1997

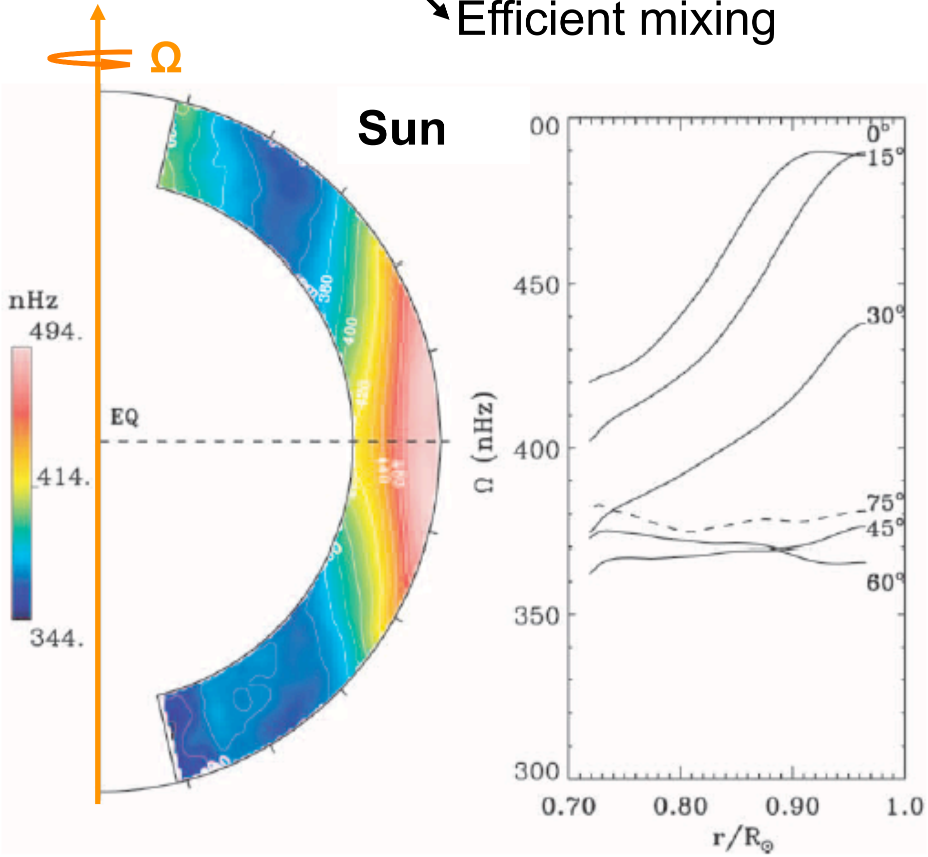
Evolutionary tracks for stars without (in black) and with (in red) rotation
Meynet & Maeder, 2000

Strongly modifies the last stages of stellar evolution

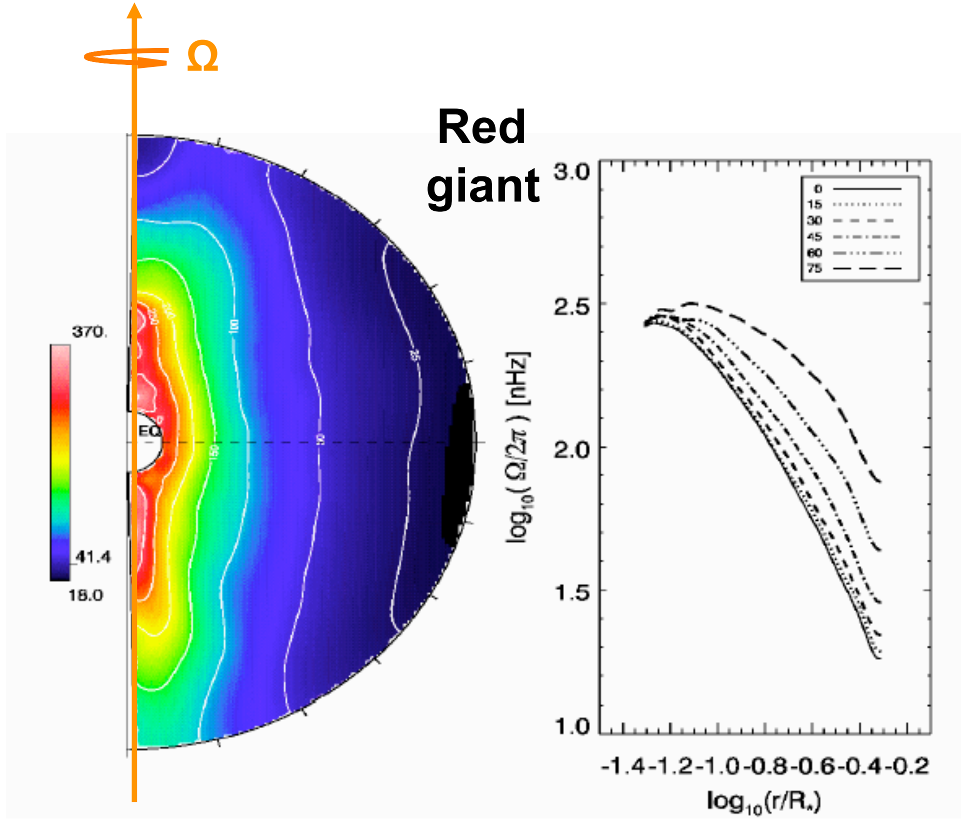
Transport in Convection Zones

Turbulence → Angular Momentum transport → differential rotation

Turbulence → Efficient mixing



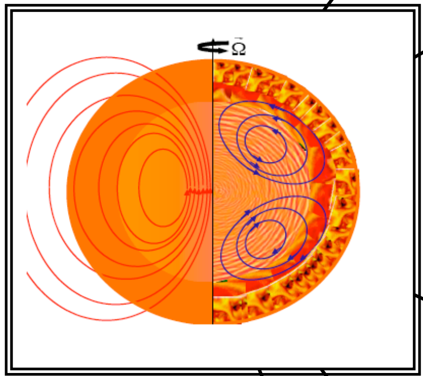
Brun & Toomre 2002



Brun & Palacios, subm.

Massive parallel simulations - no simple prescription)

Transport processes in radiation zones



Meridional circulation (diff. rot. and A. M. transport) (ADVECTION)

(Busse 1981; Zahn 1992; Maeder & Zahn 1998; Garaud 2002; Rieutord 2004; Mathis & Zahn 2004; Rieutord 2006; Espinosa Lara & Rieutord 2007; Decressin et al. 2009)

Turbulence (shear of the diff. rot.) (DIFFUSION)

(Talon & Zahn 1997; Lignères et al. 1999; Garaud 2001; Maeder 2003; Mathis et al. 2004-2007)

Magnetic field

Secular torque

(Charbonneau & Mac Gregor 1993; Gough & McIntyre 1998; Garaud 2002; Mathis & Zahn 2005; Brun & Zahn 2006; Garaud & Garaud 2008)

Instabilities

(Tayler; Spruit 1999-2002; Maeder & Meynet 2004; Braithwaite 2006-2007; Brun & Zahn 2006; Arlt et al. 2007; Kitchatinov & Rüdiger 2007; Gilman et al. 2007; Zahn, Brun, Mathis 2007)

Internal waves

(Press 1981; Schatzman 1993; Talon et al. 2002; Talon & Charbonnel 2005; Rogers et al. 2005-2006-2008; Pantillon et al. 2007; Mathis et al. 2008)

excited at the borders with C. Z.

propagating inside R. Z.

A. M. settled where they are damped

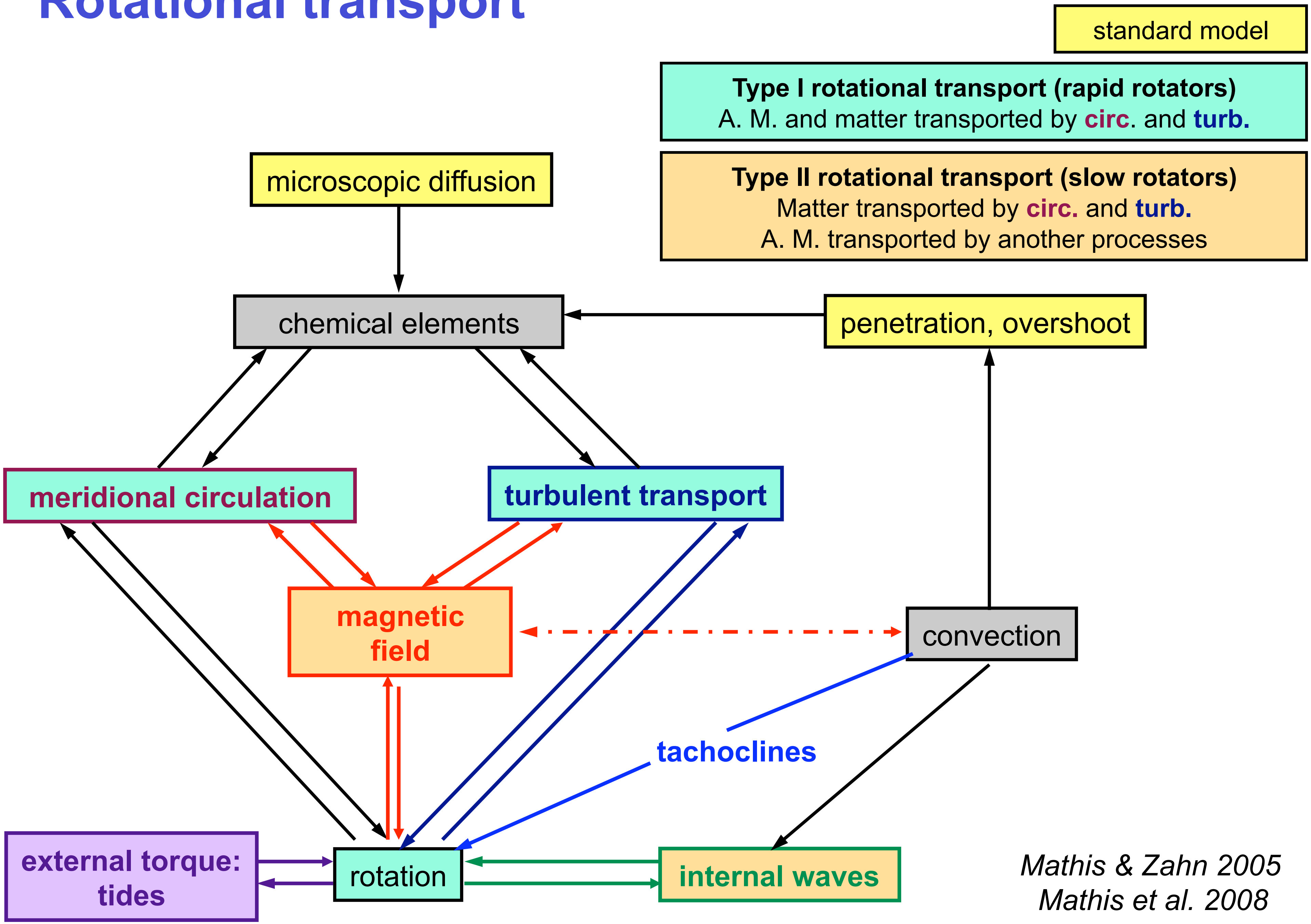
(Goldreich & Nicholson 1989)

Tides

Equilibrium tide torque *(Zahn 1966; Mathis & Zahn in prep.)*

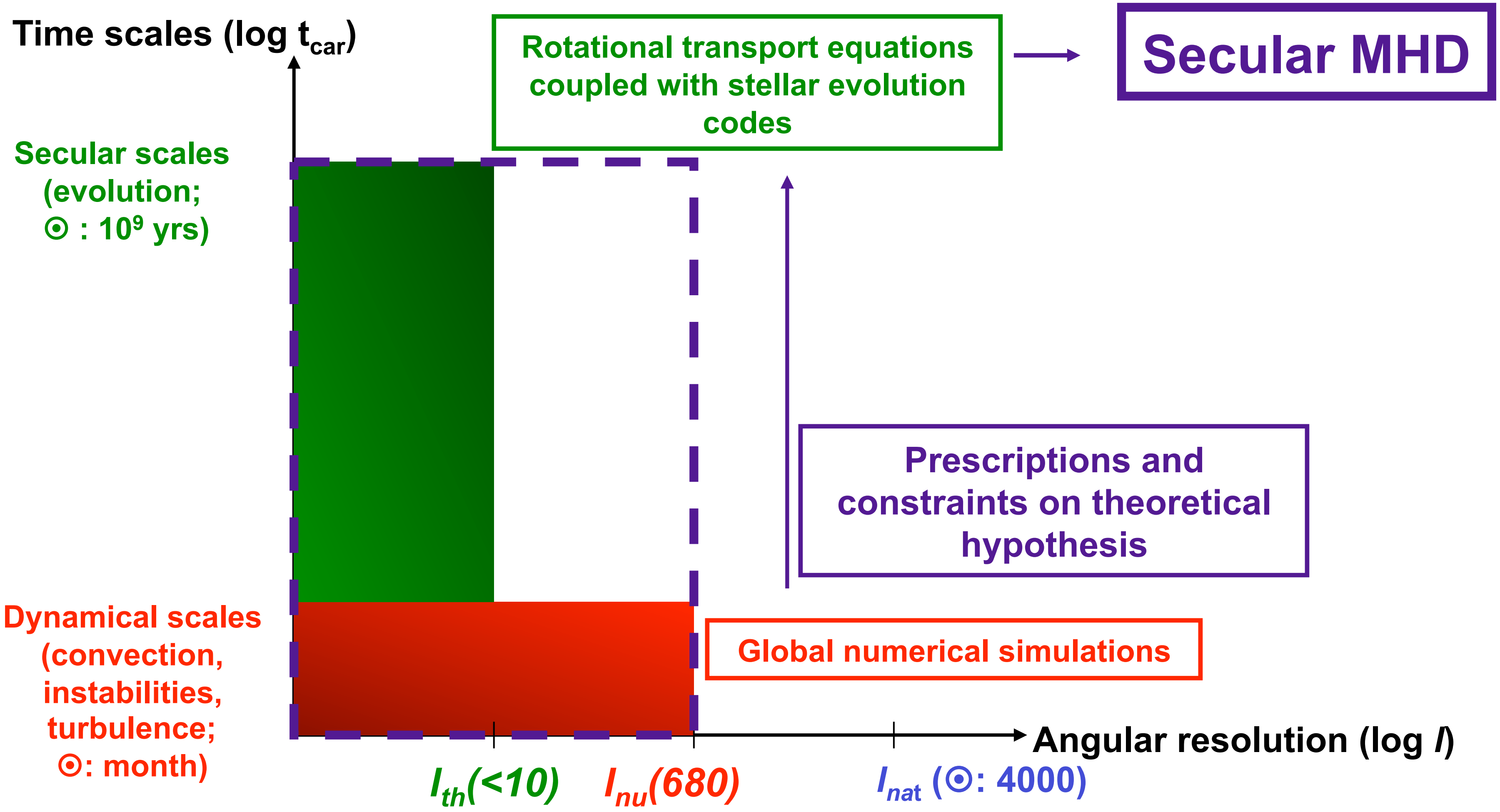
Dynamical tide *(Zahn 1975; Mathis & Zahn in prep.)*

Rotational transport



Mathis & Zahn 2005
Mathis et al. 2008

A multi-scales problem in time and space



Preliminary definitions

- Internal macroscopic velocity field:

$$\boldsymbol{V} = \underbrace{r\sin\theta\,\Omega(r,\theta)\,\widehat{\mathbf{e}}_\varphi}_{\text{Differential rotation}} + \underbrace{\dot{r}\,\widehat{\mathbf{e}}_r}_{\text{Contraction -dilatation}} + \underbrace{\boldsymbol{\mathcal{U}}_M(r,\theta)}_{\text{Meridional circulation}} + \underbrace{\boldsymbol{u}(r,\theta,\varphi,t)}_{\text{Waves velocity field}}$$

where $$\Omega(r,\theta) = \underbrace{\overline{\Omega}(r)}_{\text{Average}} + \underbrace{\widehat{\Omega}(r,\theta)}_{\text{Fluctuation}} = \overline{\Omega}(r) + \sum_{l>0} \Omega_l(r)\, Q_l(\theta)$$

Stable stratification of radiation zones
Anisotropic TURBULENT transport
($\nu_v \ll \nu_h$, $D_v \ll D_h$)

and $$\boldsymbol{\mathcal{U}}_M = \sum_{l>0}\left[U_l(r)P_l(\cos\theta)\,\widehat{e}_r + V_l(r)\frac{\mathrm{d}P_l(\cos\theta)}{\mathrm{d}\theta}\,\widehat{e}_\theta\right]$$ with $$\underbrace{V_l(r) = \frac{1}{l(l+1)\rho r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\rho r^2 U_l\right)}_{\text{Anelastic approximation}}$$

- Temperature and mean molecular weight:

$$T(r,\theta) = \underbrace{\overline{T}(r)}_{} + \underbrace{\delta T(r,\theta)}_{} \quad \text{where} \quad \delta T(r,\theta) = \sum_{l\geq 2}\left[\Psi_l(r)\,\overline{T}\right] P_l(\cos\theta)$$

$$\mu(r,\theta) = \underbrace{\overline{\mu}(r)}_{} + \underbrace{\delta\mu(r,\theta)}_{} \quad \text{where} \quad \delta\mu(r,\theta) = \sum_{l\geq 2}\left[\Lambda_l(r)\,\overline{\mu}\right] P_l(\cos\theta)$$

- Magnetic field:

$$\boldsymbol{B}(r,\theta) = \underbrace{\boldsymbol{\nabla}\wedge\boldsymbol{\nabla}\wedge\left(\xi_P(r,\theta)\,\widehat{e}_r\right)}_{\text{Poloidal part}} + \underbrace{\boldsymbol{\nabla}\wedge\left(\xi_T(r,\theta)\,\widehat{e}_r\right)}_{\text{Toroidal}}$$

$$\begin{cases} \xi_P(r,\theta) = \sum_{l=1}^{\infty} \xi_0^l(r)\, Y_l^0(\theta) \\ \xi_T(r,\theta) = \sum_{l=1}^{\infty} \chi_0^l(r)\, Y_l^0(\theta) \end{cases}$$

Transport equations system

A. M.: $$\underline{\rho \frac{\mathrm{d}}{\mathrm{d}t}}\left(r^2\overline{\Omega}\right) - \underbrace{\frac{1}{5r^2}\partial_r\left(\rho r^4 \overline{\Omega} U_2\right)}_{\text{Advection}} = \underbrace{\frac{1}{r^2}\partial_r\left(\rho \nu_v r^4 \partial_r \overline{\Omega}\right)}_{\text{Diffusion}} + \underbrace{\overline{\Gamma}_{\mathcal{F}_\mathcal{L}}(\boldsymbol{B})}_{\text{Lorentz torque}} - \underbrace{\frac{1}{r^2}\partial_r\left[r^2 \mathcal{F}_J(r)\right]}_{\text{Waves}}$$

$$\underbrace{\frac{\mathrm{d}}{\mathrm{d}t}}_{\text{Lagrangian derivative}} = \partial_t + \dot{r}\partial_r$$

idem $\Omega(\theta)$

Advection | Diffusion | Lorentz torque | Waves

Lagrangian derivative

Induction equation:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t}\xi_0^l - \underline{r\mathcal{P}_{\mathrm{Ad};l}(\Omega, U_l, \boldsymbol{B})} = \underline{\eta_h r \Delta_l\left(\frac{\xi_0^l}{r}\right)} \\ \frac{\mathrm{d}}{\mathrm{d}t}\chi_0^l + \partial_r(\dot{r})\chi_0^l - \underline{\mathcal{T}_{\mathrm{Ad};l}(\Omega, U_l, \boldsymbol{B})} = \underline{\left[\partial_r\left(\eta_h \partial_r \chi_0^l\right) - \eta_v l(l+1)\frac{\chi_0^l}{r^2}\right]} \end{cases}$$

Heat:

Non stationarity

$$\underline{\frac{M(r)}{L(r)}C_p\overline{T}\frac{\mathrm{d}\Psi_l}{\mathrm{d}t}} + \frac{M(r)}{L(r)}C_p\overline{T}\left[\Phi\frac{\mathrm{d}\ln\overline{\mu}}{\mathrm{d}t}\Lambda_l + \underline{\frac{N^2}{\overline{g}}U_l}\right] = \mathcal{T}_l(r) + \frac{M(r)}{L(r)}\frac{\mathcal{J}_l(\boldsymbol{B})}{\overline{\rho}}$$

$$\begin{aligned} \mathcal{T}_l = {} & \underbrace{2\left[1 - \frac{\overline{f}_{\mathcal{P}}(\Omega,\boldsymbol{B})}{4\pi G\overline{\rho}} - \frac{\left(\overline{\epsilon} + \overline{\epsilon}_{\mathrm{grav}}\right)}{\epsilon_m}\right]\frac{\widetilde{g}_l(\Omega,\boldsymbol{B})}{\overline{g}} + \frac{\widetilde{f}_{\mathcal{P},l}(\Omega,\boldsymbol{B})}{4\pi G\overline{\rho}} - \frac{\overline{f}_{\mathcal{P}}(\Omega,\boldsymbol{B})}{4\pi G\overline{\rho}}\left(-\delta\Psi_l + \varphi\Lambda_l\right)}_{\text{Thermal diffusion perturbing force (barotropic term)}} \\ & \underbrace{+ \frac{\rho_m}{\overline{\rho}}\left[\frac{r}{3}\partial_r\left(H_T\partial_r\Psi_l - (1-\delta+\chi_T)\Psi_l - (\varphi+\chi_\mu)\Lambda_l\right) - \frac{l(l+1)H_T}{3r}\left(1+\frac{D_h}{K}\right)\Psi_l\right]}_{\text{Thermal diffusion}} \\ & \underbrace{+ \frac{\left(\overline{\epsilon} + \overline{\epsilon}_{grav}\right)}{\epsilon_m}\left\{\left(H_T\partial_r\Psi_l - (1-\delta+\chi_T)\Psi_l - (\varphi+\chi_\mu)\Lambda_l\right) + (f_\epsilon\epsilon_T - f_\epsilon\delta + \delta)\Psi_l + (f_\epsilon\epsilon_\mu + f_\epsilon\varphi - \varphi)\Lambda_l\right\}}_{\text{Nuclear energy production and heating due to gravitational adjustements}} \end{aligned}$$

Thermal diffusion perturbing force (barotropic term)

Thermal diffusion

Nuclear energy production
and heating due to gravitational adjustements

Thermal wind equation (baroclinic equation):

$$\varphi\Lambda_l - \delta\Psi_l = \frac{r}{\overline{g}}\mathcal{D}_l(\Omega, \boldsymbol{B})$$

Shear-induced turbulence modelling

Vertical shear Ω(r)

- If there is a vorticity maximum: linear instability
- If not: non-linear finite-amplitude instability
- Stabilizing effect of stratification reduced by the thermal diffusion and the horizontal turbulence

$$D_v = \frac{R_{ic}}{N_T^2/(K+D_h) + N_\mu^2/D_h}\left[r\partial_r\overline{\Omega}\right]^2$$ *Talon & Zahn 1997*

Horizontal shear Ω(θ)

Assumptions:

- The instability tends to inhibit its origin: i. e. the horizontal differential rotation Ω(θ) (*cf. Richard 2001*)
- Anisotropic turbulent transport: $D_h >> D_v$

property: changes vertical advection into a vertical diffusion for the chemicals

$$\left(\frac{\mathrm{d}X_i}{\mathrm{d}t}\right)_{M_r} = \frac{\partial}{\partial M_r}\left[(4\pi r^2\rho)^2\,(\underline{D_v} + \underline{D_{\mathrm{eff}}})\,\frac{\partial X_i}{\partial M_r}\right] + \left(\frac{\mathrm{d}X_i}{\mathrm{d}t}\right)_{\mathrm{micro}} \qquad D_{\mathrm{eff}} = \sum_{l>0}\frac{(rU_l)^2}{l(l+1)(2l+1)D_h}$$ *Chaboyer & Zahn 1992*

Numerical modelling

→ **We have to treat the problem of secular (magneto)hydrodynamics of a stably stratified differentially rotating region coupled with its chemical evolution (nucleosynthesis)**

Method

- The fourth-order PDE for the angular momentum transport is broken into a system of four first-order PDE's which is solved using Newton-Raphson method (Henyey scheme). The radial discretization is achieved using second-order finite differences.

- The temporal problem is solved using totally implicit scheme.

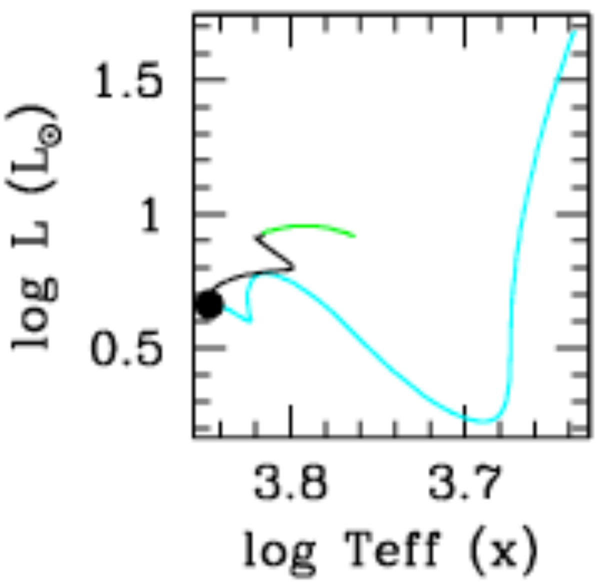
Numerical simulation of Type I Rotational Transport (I)

Hydrodynamical shellular case with Ω(r,θ)=Ω(r) (l=2); STAREVOL

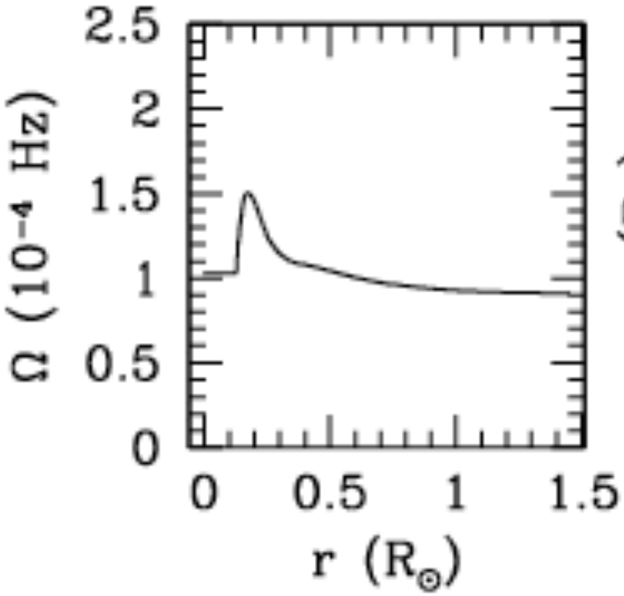
Siess et al. 2000
Decressin et al. 2009

1.5 $M_\odot$
$Z=Z_\odot$
V_i=100 $km.s^{-1}$
Magnetic braking

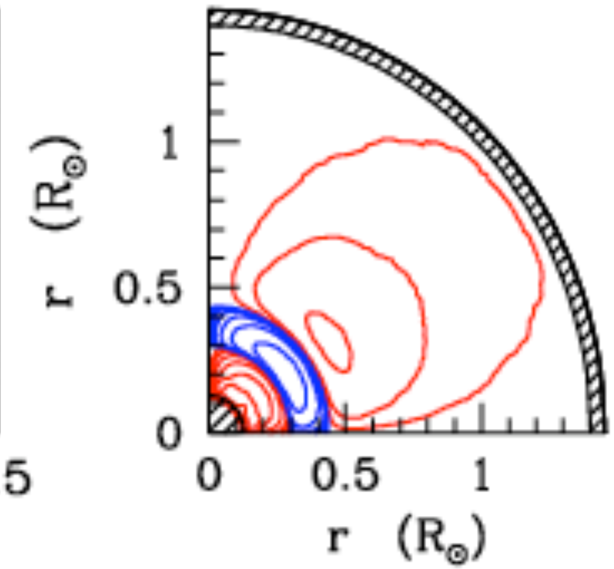
Hertzsprung-Russel diagram



Differential rotation



Stream lines M. C.



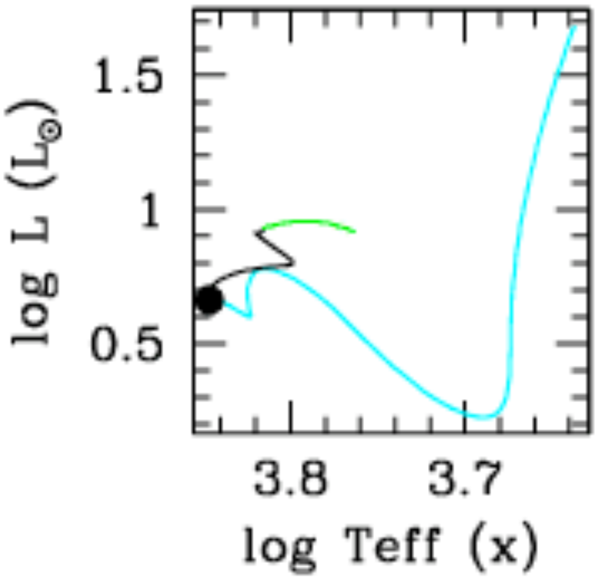
Numerical simulation of Type I Rotational Transport (II)

Hydrodynamical shellular case with $\Omega(r,\theta)=\Omega(r)$ (l=2); STAREVOL

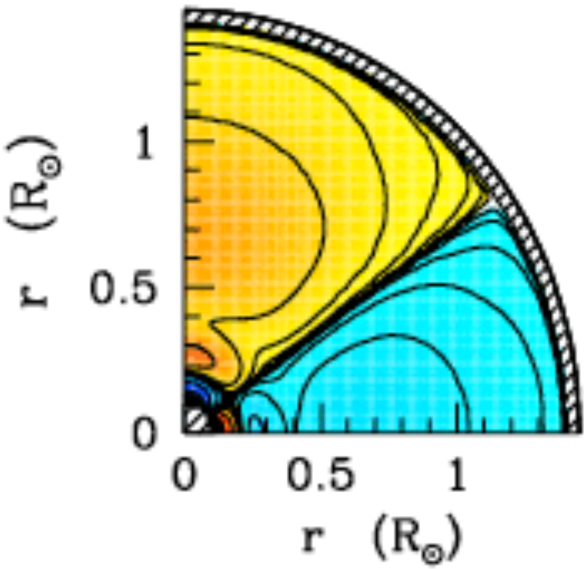
Decressin et al. 2009

$1.5\ M_{\odot}$
$Z=Z_{\odot}$
$V_i=100\ km.s^{-1}$
Magnetic braking

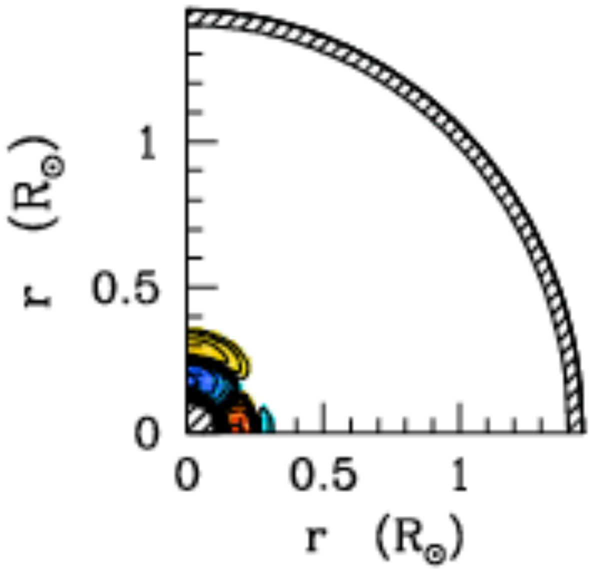
Hertzsprung-Russel diagram



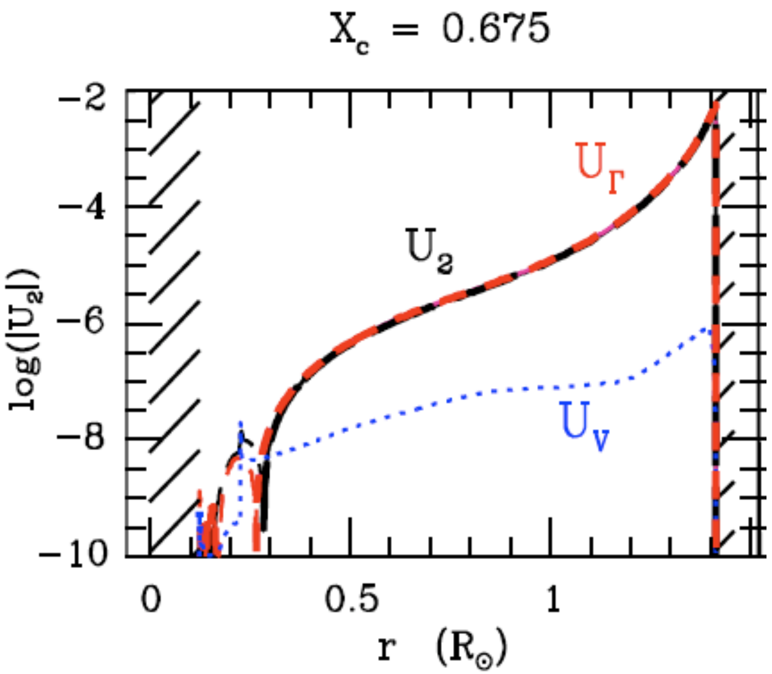
Temperature fluctuation



Chemical fluctuation



Diagnosis and identification (I)



Terms meridional circulation

$$U_2 = \frac{5}{\rho r^4 \overline{\Omega}} \left[\underbrace{\Gamma(m)}_{\text{Extraction}} - \underbrace{\rho \nu_v r^4 \partial_r \overline{\Omega}}_{\text{Viscous}} \right]$$

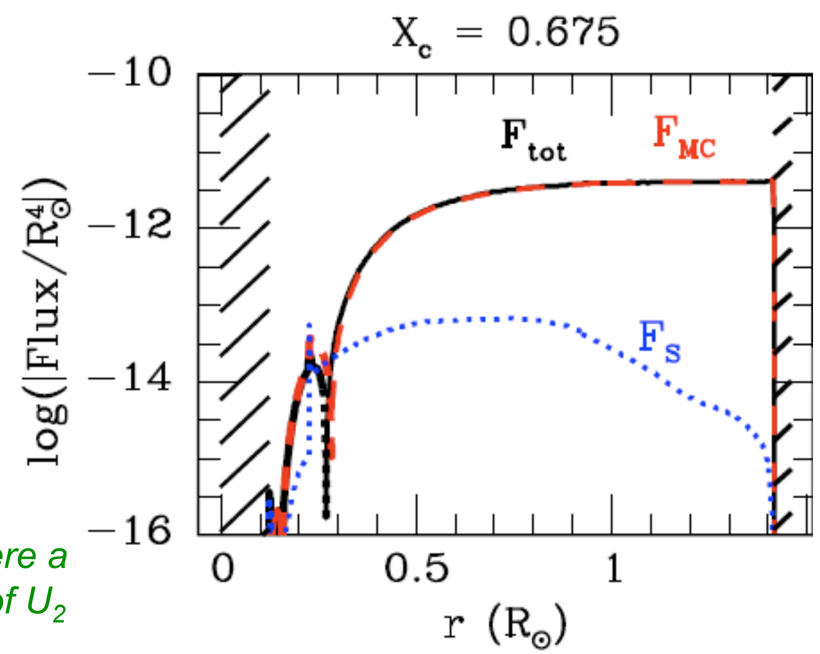
$$\Gamma(m) = \frac{1}{4\pi} \frac{\mathrm{d}}{\mathrm{d}t} \left[\int_0^{m(r)} r'^2 \overline{\Omega} \mathrm{d}m' \right]$$

→ **Wind-driven circulation** $t_{ES} = \frac{R}{U} \approx t_{KH} \left(\frac{1}{R\Omega^2} \frac{GM}{R^2} \right)$

Flux of Angular Momentum:

- **Meridional circulation** $R_\odot^4 F_{MC}(r) = \frac{1}{5} \rho r^4 \overline{\Omega} U_2$

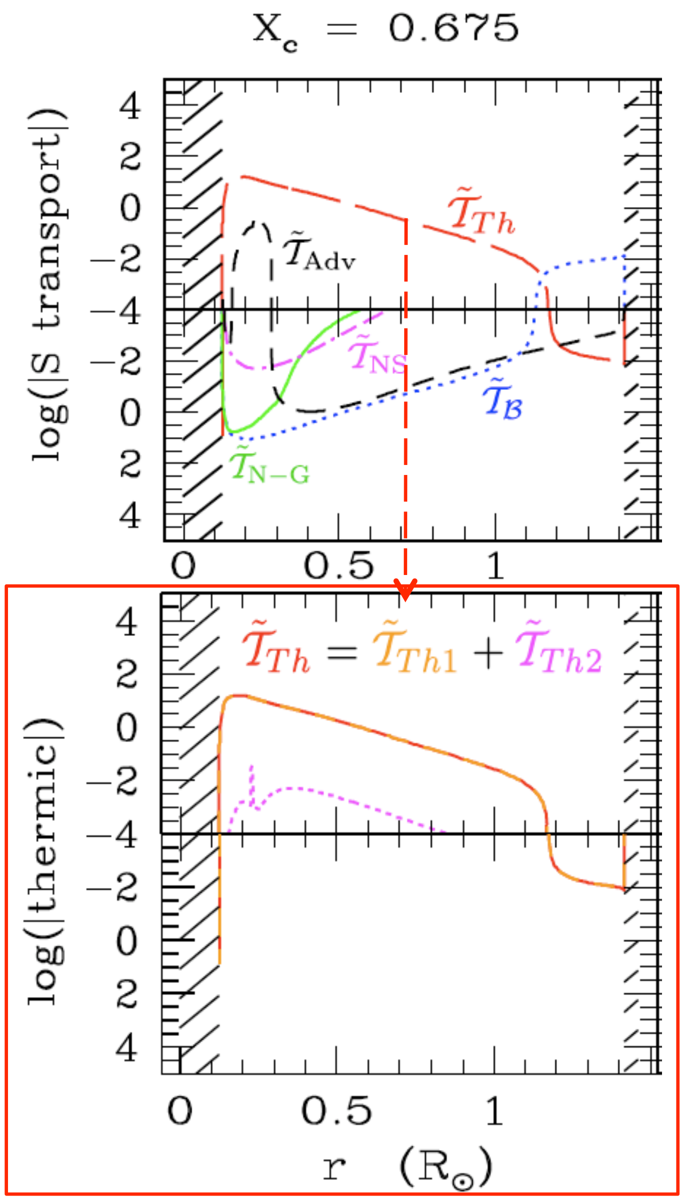
- **Shear induced turbulence** $R_\odot^4 F_S(r) = \rho \nu_v r^4 \partial_r \overline{\Omega}$



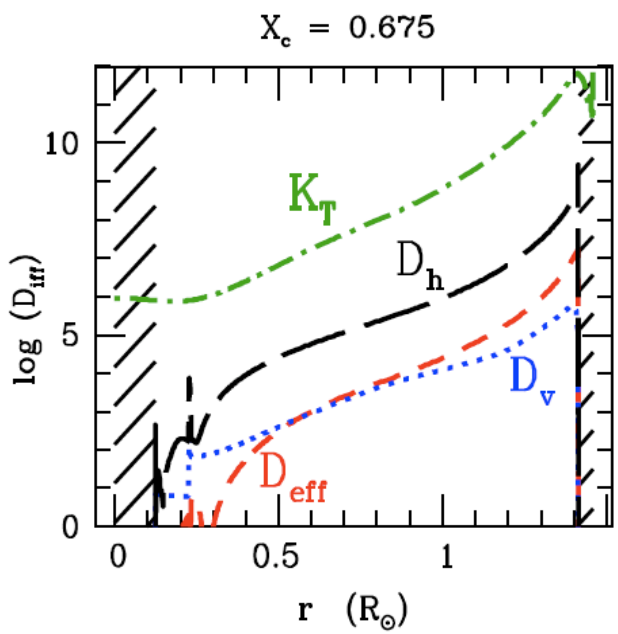
Same balances in massive stars except under the surface where a balance is established between extraction and viscous terms of U_2 and adv. and viscous transports of A. M.

Diagnosis and identification (II)

Entropy transport



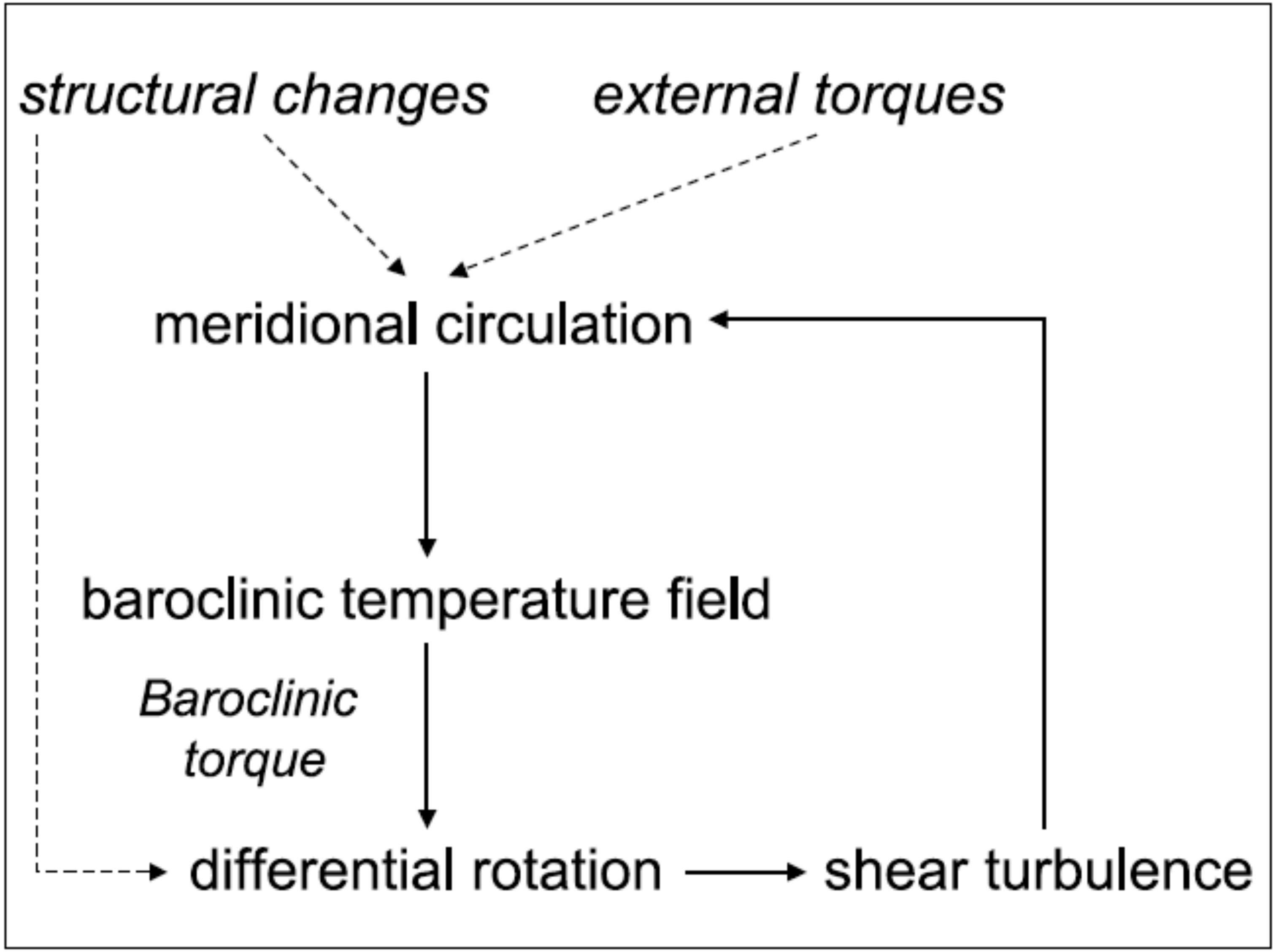
Transport coefficients



$$\nabla \cdot (\chi \nabla T) - \nabla \cdot \boldsymbol{F}_{\rm H} \approx -\overline{\rho}\,\overline{T}\, U_r \partial_r \overline{S}$$

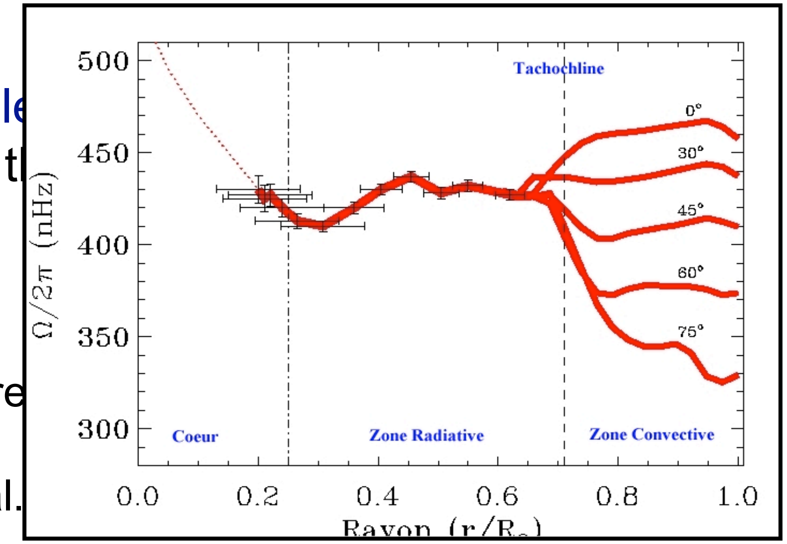
Thermal relaxation driving term: mean entropy advection

The transport loop
in differentially rotating stellar radiation zones



Type I Rotational Transport

The same processes (circulation and turbule
the transport of angular momentum and t



Successes:

- properties of massive stars (Meynet 2004 and re
- Chemical properties of subgiants (Palacios et al.
- Mixing of light elements for high temperature (Talon & Charbonnel 1998)

Weaknesses: for late type stars, predicts

- fast rotating core with a smooth gradient
  ≠ helioseismology (Pinsonneault et al. 1989; Chaboyer et al. 1995; Matias & Zahn 1997)
- Bad description of the mixing of light elements (Balachandran & Bell 1998, Balachandran 2002)

⟶ **Another process is responsible for the transport of angular momentum**

Type I Rotational Transport

The same processes (circulation and turbulence) are responsible for the transport of angular momentum and the mixing of chemicals

Successes:

- properties of massive stars (Meynet 2004 and references therein)

- Chemical properties of subgiants (Palacios et al. 2003, Palacios et al. 2004)

- Mixing of light elements for high temperature (Talon & Charbonnel 1998)

Weaknesses: for late type stars, predicts

- fast rotating core with a smooth gradient
$\neq$ helioseismology (Pinsonneault et al. 1989; Chaboyer et al. 1995; Matias & Zahn 1997)

- Bad description of the mixing of light elements (Balachandran & Bell 1998, Balachandran 2002)

$\longrightarrow$ **Another process is responsible for the transport of angular momentum**

Type II Rotational Transport

Circulation and turbulence are responsible for
the mixing of chemicals;

Another process operates for the transport of angular momentum;
has indirect impact on mixing, by shaping the rotation profile

Magnetic field ?

Internal Gravity Waves ?

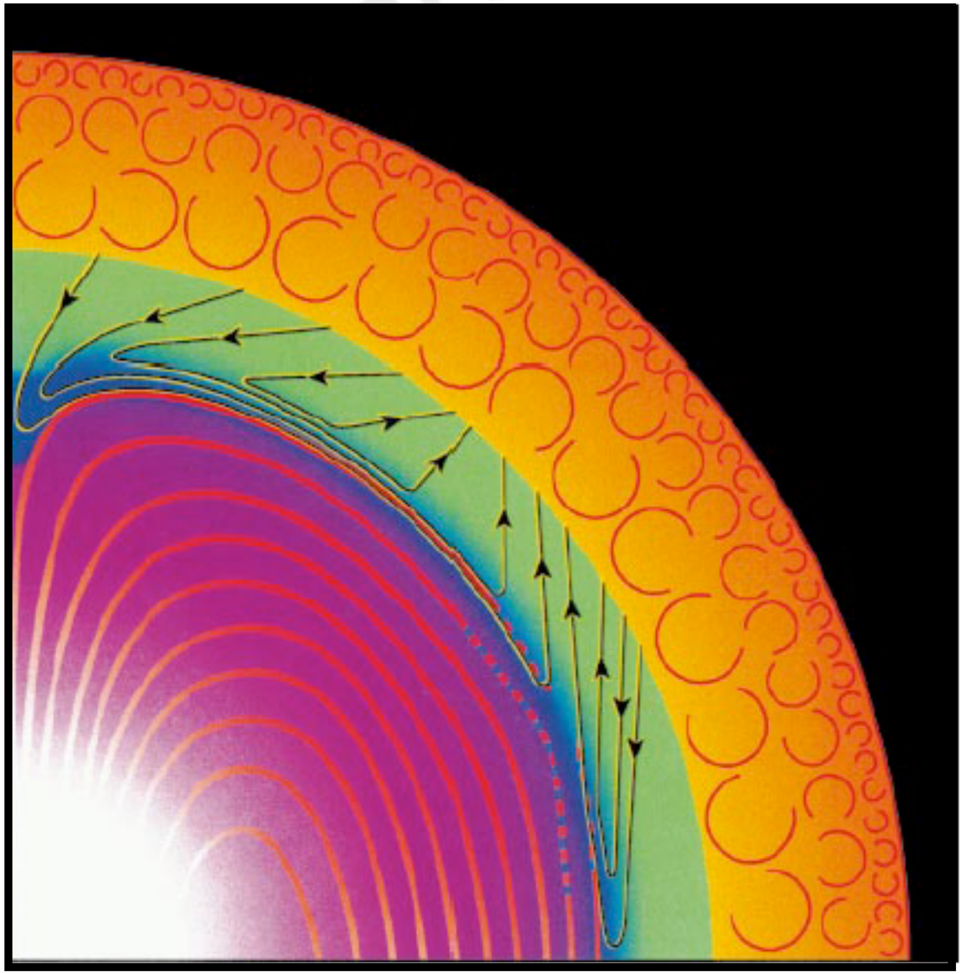
Magnetic field in radiative zones

-Does it prevent the spread of tachocline?
-Does it enforce uniform rotation?
-Is it a dynamo in radiation zones associated to N.-A. instabilities?

Convection Zone
Dynamo field

Tachocline

Radiation Zone
Fossil field



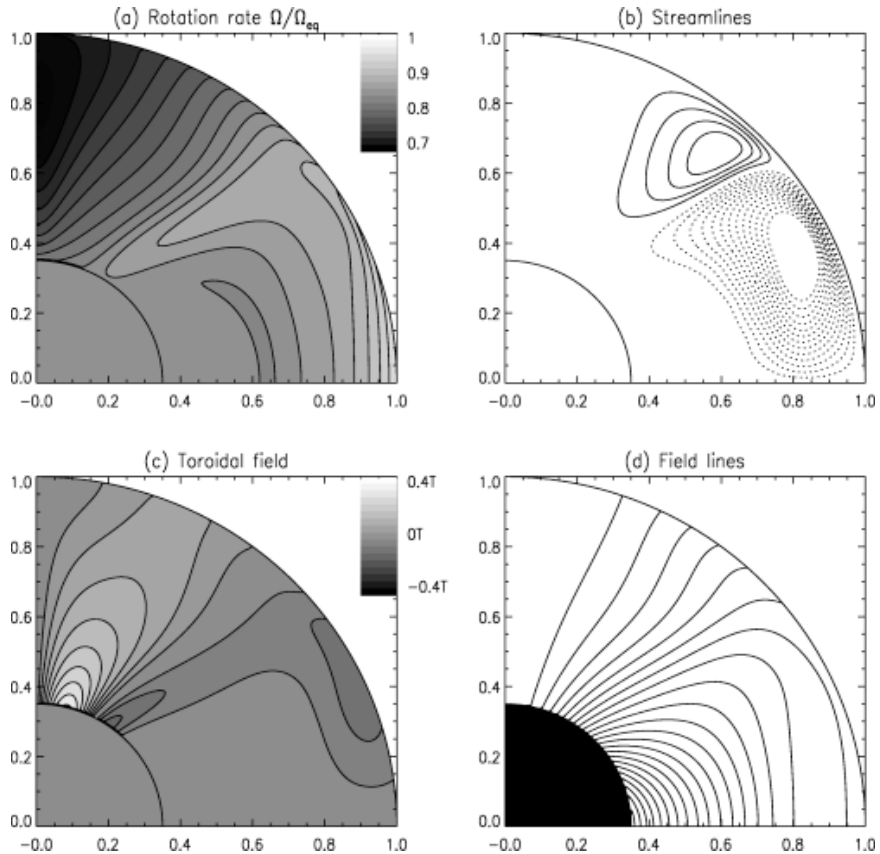
Gough & McIntyre 1998

Magnetic transport in radiation zones

Fossil field: 2D stationary solutions

Garaud 2002, Garaud & Garaud 2008

intermediate field case (13000 G)

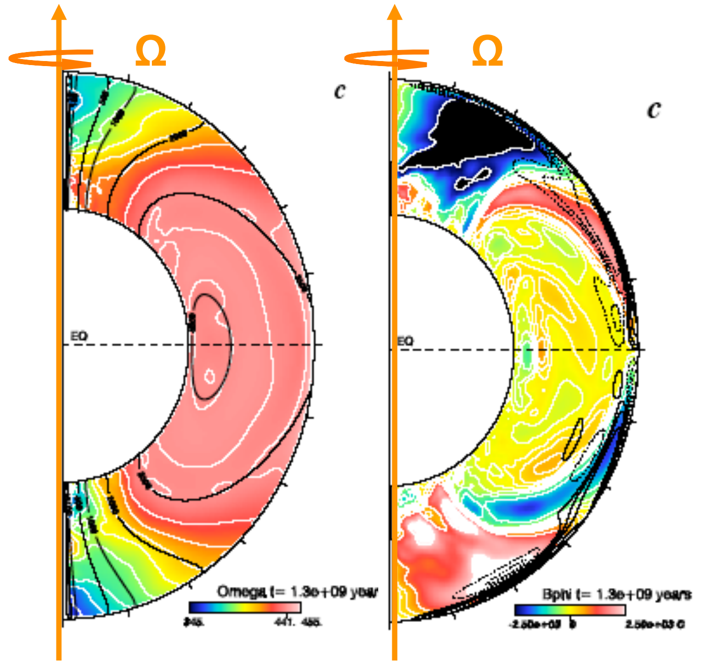


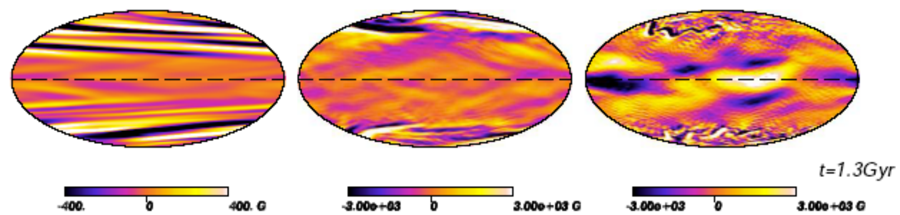
At high latitudes poloidal field threads through CZ enforces diff. rotation

Ferraro's law

3D solutions **ASH code**

Brun & Zahn 2006; Zahn, Brun, Mathis 2007





Ferraro's law and
3D non-axisymmetric MHD instabilities;
on the track of a potential dynamo

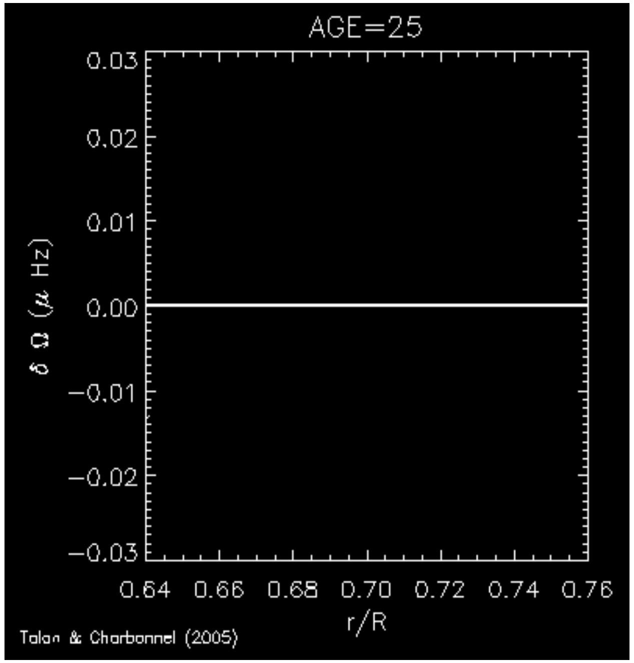
Transport of Angular Momentum by internal waves

If prograde and retrograde waves are equally excited:

- No differential rotation → no net deposition of A. M.
- Differential rotation → Doppler shift → net deposition of A. M.

High degree waves below the convective zone:

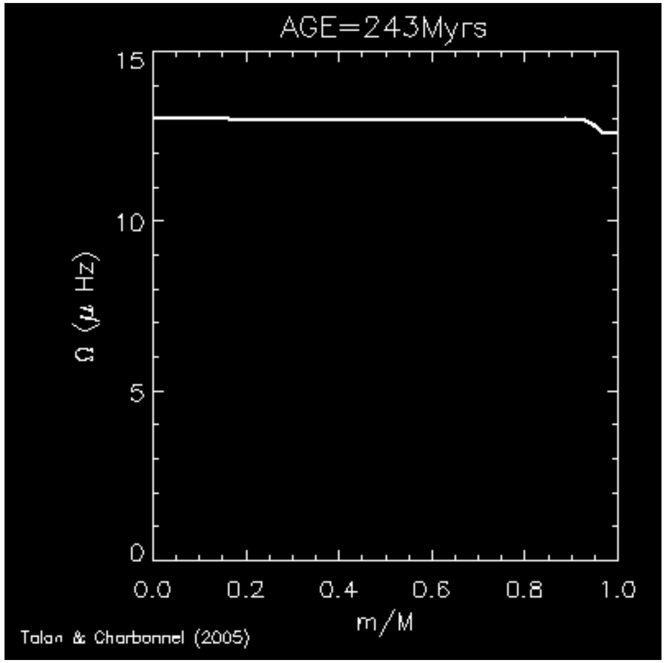
Shear Layer Oscillation



Transport by low degree (l≈10), low frequency waves (ν<5μHz)

Secular A. M. extraction driven by the wind (S.L.O. filtered out)

→ **nearly uniform rotation profile (cf. solar R. Z.)**



Talon & Charbonnel 2005

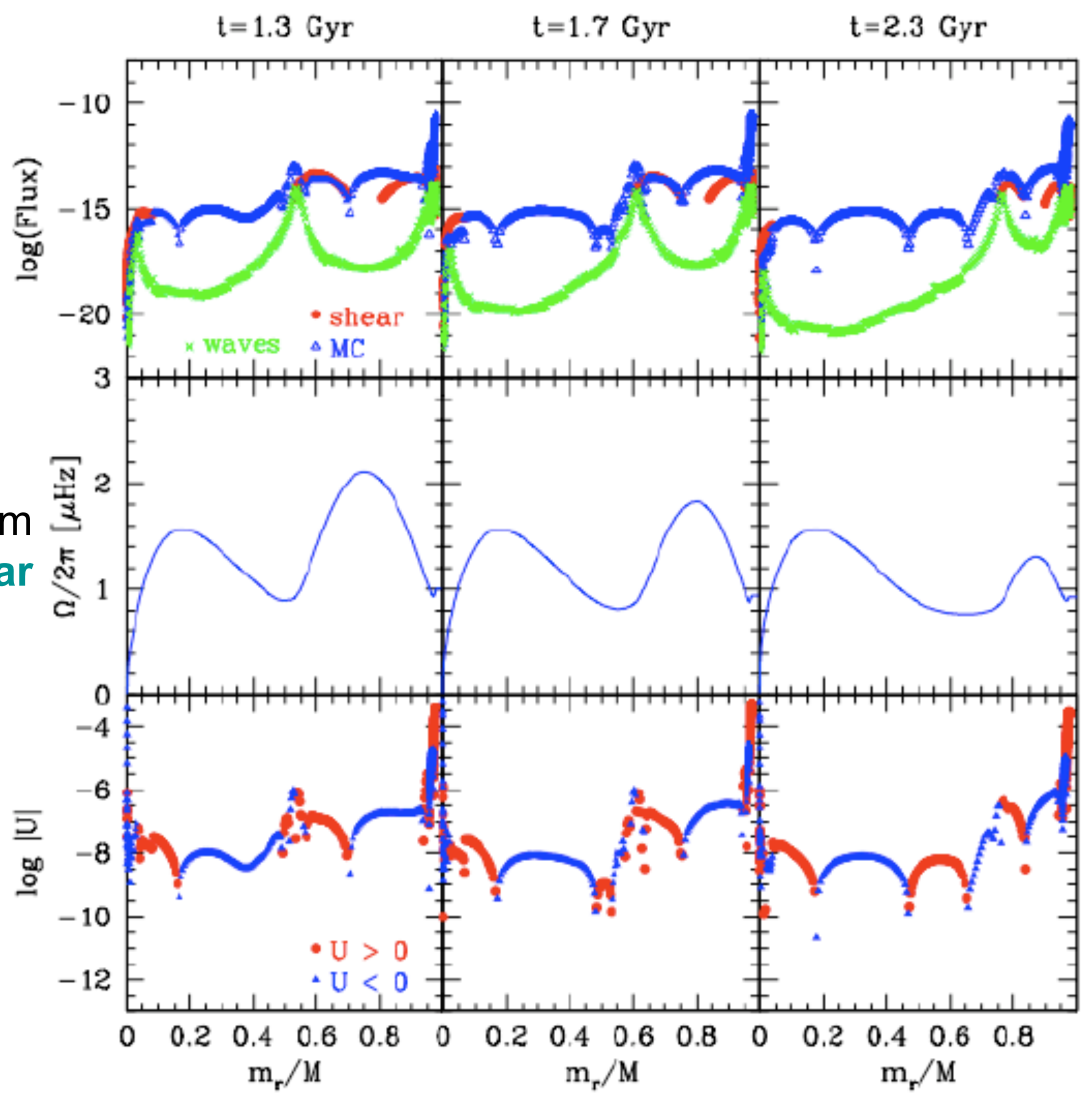
Dynamical vision of a 1 $M_\odot$ star with a magnetic braking (V_i=50 Km.s^{-1});
1D simulation: Ω averaged over latitude

Diagnosis and identification

Dynamical vision of the evolution of a 1$M_\odot$ star with magnetic braking (Vi=50 km.s-1)

→ Secular extraction of angular momentum with an associated **highly multi-cellular** meridional circulation

→ **Transport driven by the braking** at the surface and the **associated extraction fronts**



Mathis, Eggenberger, Talon, Charbonnel; in prep.

What should be done

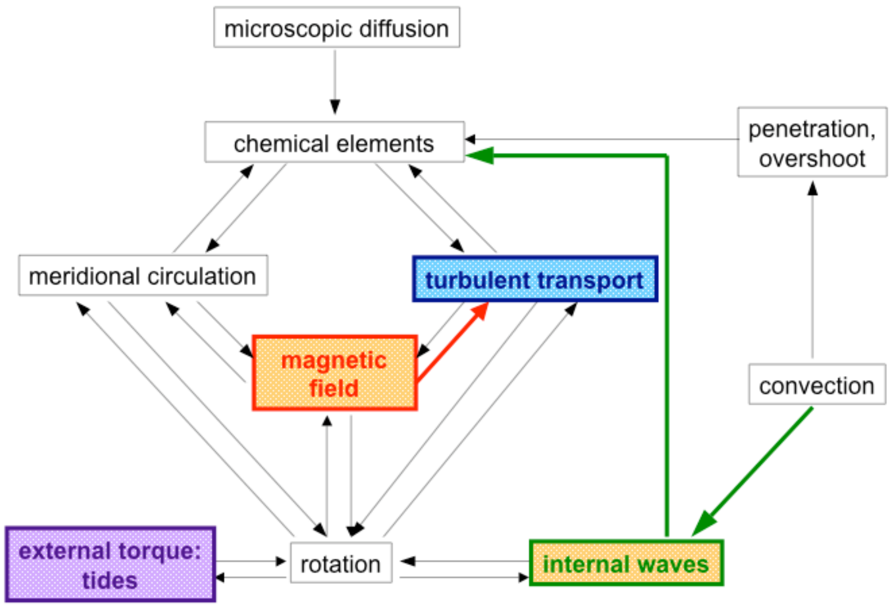
Work is in progress to implement differential rotation in latitude and transport by magnetic field and waves influenced by Ω & B and the associated diagnosis

→ Hydrodynamical (& MHD) vision of stellar evolution

MHD instabilities

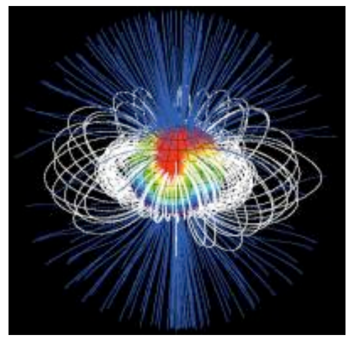
Waves excitation

Tidal Dissipation

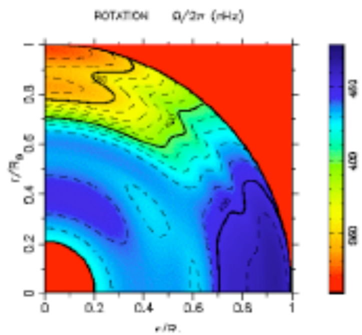


Major impact on:

-Stellar magnetism



-Stellar rotation

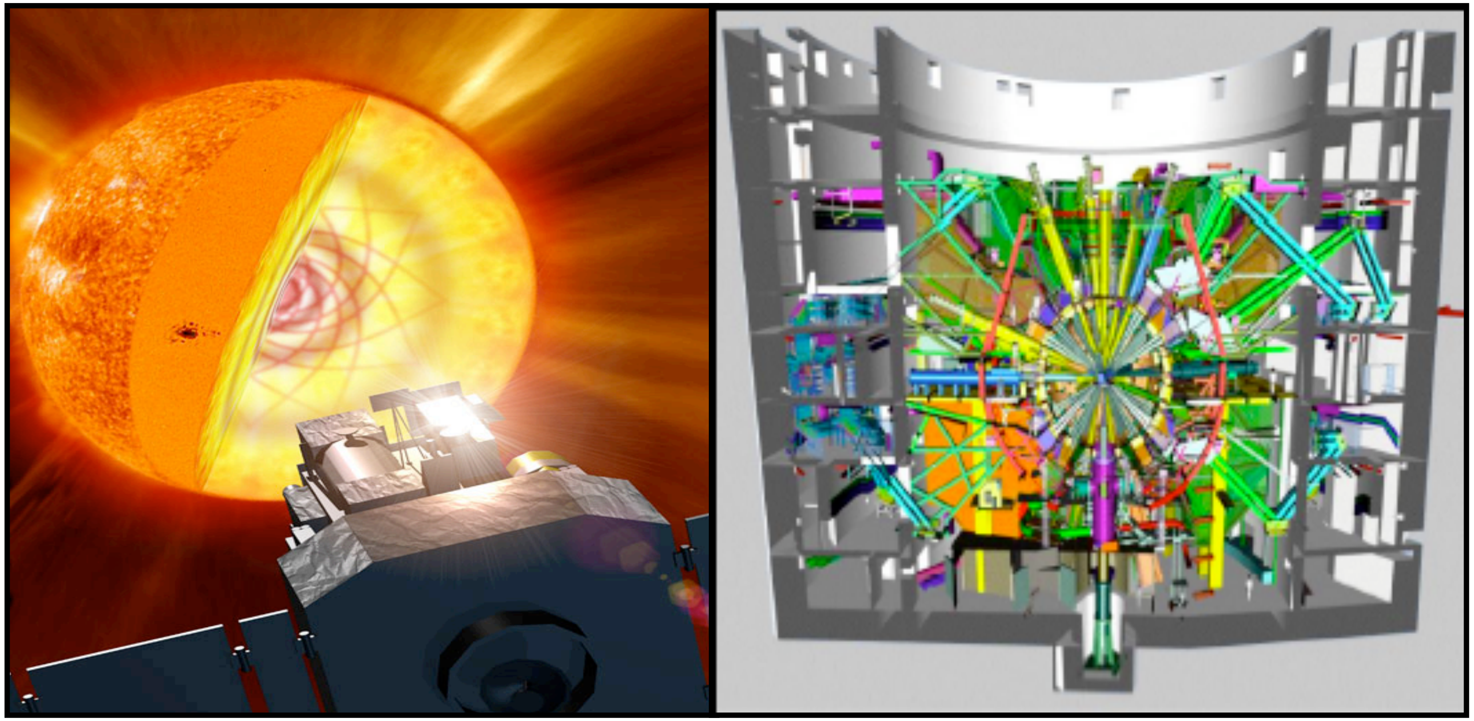


-Stellar evolution

Context

-Helio and asteroseismology spatial missions
(SOHO, SDO, PICARD, Golf-NG, Solar Orbiter, ASTROD; MOST, COROT, KEPLER, PLATO)

-Powerful ground-based instruments
(VLT, ESPADONS; BiSON, GONG; HARPS, SONG)



-Numerical simulation of stellar (magneto-)hydrodynamics (ASH, ESTER)

-Laboratory experiments relevant for astrophysical plasmas (LIL, LMJ, ITER)

Dynamical vision of the Hertzsprung-Russel diagram